5.01.2016 | Ch 4: 10, 11, 12, 13, 14, 16.

4.10 ①

$$\begin{array}{c|ccc} & C & S & G \\ \hline C & 0.5 & 0.4 & 0.1 \\ S & 0.3 & 0.4 & 0.3 \\ G & 0.2 & 0.3 & 0.5 \end{array}$$ – transition matrix for $P(X_t \mid X_{t-1})$

$$P(X_{n+3} \neq G,\ X_{n+2} \neq G,\ X_{n+1} \neq G \mid X_n = C)$$

$$= 1 - P(X_{n+3} = G \,\|\, X_{n+2} = G \,\|\, X_{n+1} = G \mid X_n = C) =$$

$$= \left[y_n = \begin{cases} X_n, & n < N \\ A = G, & n \geq N \end{cases},\quad N := \min\{X_n = G\} \qquad \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.2 \\ 0 & 0 & 1 \end{pmatrix} = Q \right] =$$

$$= 1 - P(y_{n+3} = G \mid y_n = C) =$$

$$= 1 - Q^3_{CG} = 1 - 0.415 = 0.585$$

$$Q^3 = \begin{pmatrix} 0.293 & 0.292 & 0.415 \\ 0.219 & 0.220 & 0.561 \\ 0 & 0 & 1 \end{pmatrix}$$

ex 4.11

$$P = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$$

X_t - markov

2

$$P(X_{n+4} = 4 \mid X_n = 4,\ X_{n+4} \neq C,\ X_{n+3} \neq C \dots X_{n-3} \neq C)$$

$$= (\text{define } X_n = 4 \text{ as } X_0) =$$

$$P(\underbrace{X_4 = 4}_{A} \mid \underbrace{X_0 = 4}_{C}, \cancel{X_0} \neq C, \underbrace{X_1 \neq C, X_2 \neq C, \dots, X_4 \neq C}_{B})$$

$$= \left[\text{Bayes theorem } P(A \mid BC) = \frac{P(A, B \mid C)}{P(B \mid C)} \right] = \frac{P(X_4 = 4, X_1 \neq C, \dots, X_4 \neq C \mid X_0 = 4)}{P(X_4 \neq C, X_3 \neq C, \dots, X_1 \neq C \mid X_0 = 4)} =$$

the same as previously

$$= \left[y_n = \begin{cases} X_n, & n < N \\ A = C, & n \geq N \end{cases} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} = Q \right] = \frac{P(y_4 = 4, y_1, \dots, y_4 \neq C \mid y_0 = 4)}{P(y_4 \neq C, \dots, y_1 \neq C \mid y_0 = 4)}$$

$$= \frac{P(y_4 = 4 \mid y_0 = 4)}{1 - P(y_4 = C \mid y_0 = 4)} = \boxed{\frac{Q^4_{44}}{1 - Q^4_{C4}} = 0.542}$$

if today is not included for not being cheerful:

$$\frac{Q^4_{44}}{1 - Q^3_{C4}} = 0.40803$$

ex 4.12

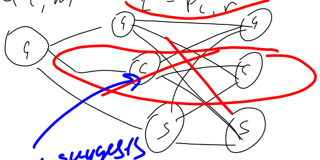
3

$X_n,\ n \geq 0$ with $Pr(P_{ij})$, $i, j \in ||S||$

$P(X_n = m \mid X_0 = i,\ X_k \neq r,\ k = 1, \dots, n) =$

$= [$ Book claims $] = Q^{*\,n}_{i,m}$, consider Gary from ex 4.11, then

$Q^*_{i,m} = \dfrac{P_{i,m}}{1 - P_{i,r}}$ $\forall i, m \neq r$



Book suggests

$$Q^* = \begin{pmatrix} \frac{0.4}{1-0.3} & \frac{0.3}{1-0.3} \\ \frac{0.3}{1-0.2} & \frac{0.5}{1-0.2} \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & \frac{3}{7} \\ \frac{3}{8} & \frac{5}{8} \end{pmatrix}$$

$P(X_4 = G \mid X_0 = G,\ X_1, \dots, X_4 \neq C) =$

$= Q^{*4}_{GG} = \left[Q^{*4} = \begin{pmatrix} 0.4674 & 0.5325 \\ 0.4660 & 0.5340 \end{pmatrix} \right]$ $\neq 0.532$!?

look at example with two steps:

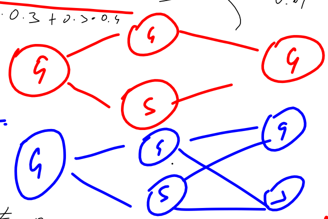
$P(X_2 = G \mid X_0 = G,\ X_1, \dots, X_2 \neq C) =$

$$= \left[G \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} \right] = \frac{P(X_2 = G,\ X_1 \neq C,\ X_2 \neq C \mid X_0 = G)}{P(X_1 \neq C,\ X_2 \neq C \mid X_0 = G)} =$$

$$= \frac{0.5 \cdot 0.5 + 0.3 \cdot 0.3}{0.5 \cdot 0.5 + 0.3 \cdot 0.3 + 0.5 \cdot 0.3 + 0.3 \cdot 0.4} = \frac{0.34}{0.61} = 0.5573$$

for numerator:

for denumerator:



$Q^{*2}_{GG} = 0.5513 \neq 0.5573$ — !? The formula is not true!

$\dfrac{Q^2_{GG}}{1 - Q^2_{GC}} = 0.5573$ — True

$$Q^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0.48 & 0.25 & 0.27 \\ 0.39 & 0.27 & 0.34 \end{pmatrix}$$

m

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

class 1 — recurrent

class 2 — recurrent

$= r$

— the formula would work — formula would be true!

class 3 — recurrent

$n > 1$

$P(X_t = m \mid X_t \neq r,\ X_{t-1} = i)$ — one step transitions, true

$$\frac{P(X_t = m,\ X_t \neq r \mid X_{t-1} = i)}{P(X_t \neq r \mid X_{t-1} = i)} =$$

$$\frac{P(X_t = m \mid X_{t-1} = i)}{1 - P(X_t = r \mid X_{t-1} = i)} = \frac{P_{i,m}}{1 - P_{i,r}}$$