Autumn 2011 #4

$$2x\dot{x} = (x^2+k)\, t e^{2t}$$

$k>0$ so $x^2+k \neq 0$. No constant solution

$$\underbrace{\frac{2x}{x^2+k}}_{\substack{u = x^2+k \\ du = 2x\,dx}} dx = t e^{2t}\, dt$$

$$\ln|x^2+k| = \int t e^{2t}\, dt$$

$$= \tfrac{1}{2} t e^{2t} - \tfrac{1}{2}\int e^{2t}\, dt$$

(*) $$= \tfrac{1}{2} t e^{2t} - \tfrac{1}{4} e^{2t} + C$$

$$x^2+k = e^C \cdot \exp\left[\tfrac{1}{4} e^{2t} \cdot (2t-1)\right]$$

$$x(t) = \pm\sqrt{Q \exp\left[\tfrac{1}{4} e^{2t}(2t-1)\right] - k}$$ , $Q > 0$

Part. solution with $x(1) = 2$:

(*) $\Rightarrow$ $$\ln(4+k) = \tfrac{1}{2}e^2 - \tfrac{1}{4}e^2 + C$$

$$4+k = e^{\frac{1}{4}e^2} \cdot \underbrace{e^C}_{Q}$$

$$Q = (4+k)\exp\left(-\tfrac{1}{4}e^2\right)$$

$$x(t) = \sqrt{(4+k)\exp\left[\tfrac{1}{4}\left\{e^{2t}(2t-1) - e^2\right\}\right] - k}$$

"+" since $x(1) > 0$