

Problem Set - Experiments

- Assume that a linear regression is appropriate for estimating the effect of a treatment T on outcome Y conditional on a confounding covariate x . Sketch hypothetical data (plot Y vs x , with treated units as X's and controls as O's) for the following situations
 - no treatment effect
 - constant treatment effect
 - treatment effect increasing with x
 - a difference in means would not capture the treatment effect, but the linear regression above would
 - a linear regression would not capture the treatment effect, but a nonlinear regression would
- The table below describes a hypothetical experiment with 2,400 participants.

| Category | # participants | x | T | Y^0 | Y^1 | Y |
|----------|----------------|-----|-----|-------|-------|-----|
| 1 | 300 | 0 | 0 | 4 | 6 | |
| 2 | 300 | 1 | 0 | 4 | 6 | |
| 3 | 500 | 0 | 1 | 4 | 6 | |
| 4 | 500 | 1 | 1 | 4 | 6 | |
| 5 | 200 | 0 | 0 | 10 | 12 | |
| 6 | 200 | 1 | 0 | 10 | 12 | |
| 7 | 200 | 0 | 1 | 10 | 12 | |
| 8 | 200 | 1 | 1 | 10 | 12 | |

where x is a pre-treatment predictor, T the treatment status, and Y^0, Y^1 the potential outcomes.

- Complete the last column in the table
 - What is the ATE?
 - Is it plausible that these data come from a RCT?
 - What is the relation between i) the difference in the average outcome between the treated and controls and ii) the ATE?
 - What is the estimate and standard error of the coefficient on T in a regression of Y on T and x ?
- We have a binary treatment T_i and observed outcome Y_i , where $Var(Y_i|T=0) = \sigma_0^2$ and $Var(Y_i|T=1) = \sigma_1^2$. We are interested in the average treatment effect δ , and $\hat{\delta} = \bar{Y}_1 - \bar{Y}_0$.
 - Show that $\sigma_0^2 = \sigma_1^2 = \sigma^2$ implies that there is no treatment effect heterogeneity (and the other way around).

In the lecture it was shown that balancing Type I error at a significance level α and Type II error with probability β , when $H_0 : \delta = 0$ and $H_1 : \delta = \delta_0$, implies the

following equality

$$\delta_0 = (t_{1-\alpha/2} + t_\beta) \sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_0^2}{N_0}} \quad (1)$$

where t_a is the a -th percentile of the student distribution.

(b) Derive the smallest sample sizes N_0 and N_1 that satisfy this equality if $\sigma_0^2 = \sigma_1^2 = \sigma^2$.

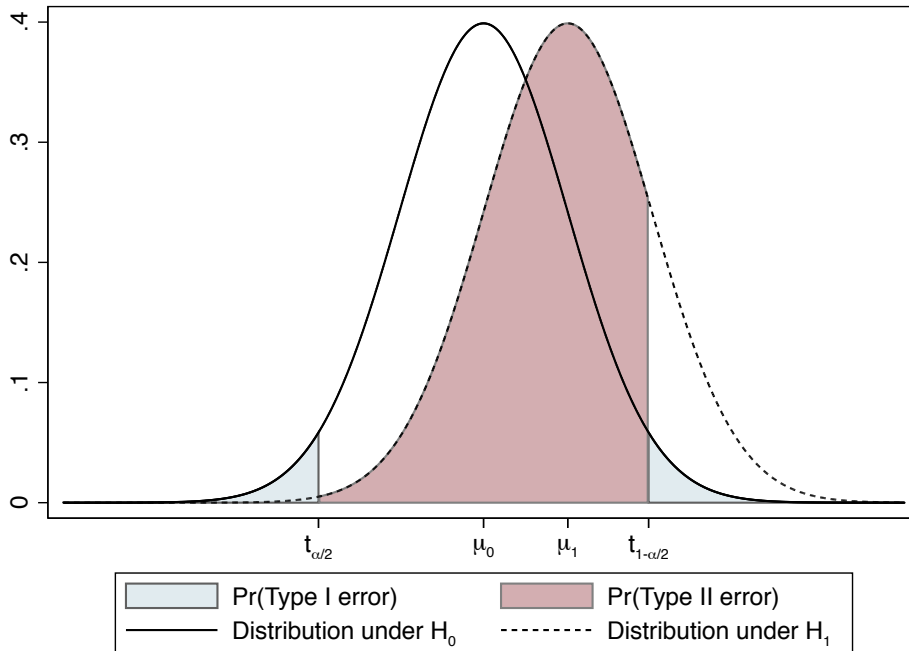
(c) (*) Derive the smallest sample sizes when the variances of the outcomes are not equal.

4. You want to set up an experiment where you (randomly) offer a school subsidy to children between 12 and 16. You are interested in how this affects school attendance of children. You know that the proportion attending pre-intervention is 0.4.

Policy makers have decided that an effect of 0.1 would make the program worthwhile. You decide that any hypothesis will be tested at a 5% significance level. Finally, you wish to reject the null with probability at least $p = 0.8$ if the true effect is larger than 0.1.

In what follows it is assumed that you can use the Normal distribution as an approximation for the Student distribution.

Remember Type I and Type II errors:



and power = 1 - Pr(Type II error).

For a given significance level α and $H_1 : \delta = \delta_0$, we can write the probability of a Type II error as

$$\begin{aligned} \Pr(t_{\alpha/2} < \frac{\hat{\delta}}{se(\hat{\delta})} < t_{1-\alpha/2}) &= \Pr(t_{\alpha/2} - \frac{\delta_0}{se(\hat{\delta})} < \frac{\hat{\delta} - \delta_0}{se(\hat{\delta})} < t_{1-\alpha/2} - \frac{\delta_0}{se(\hat{\delta})}) \\ &= \Phi(t_{1-\alpha/2} - \frac{\delta_0}{se(\hat{\delta})}) - \Phi(t_{\alpha/2} - \frac{\delta_0}{se(\hat{\delta})}) \\ &\approx \Phi(t_{1-\alpha/2} - \frac{\delta_0}{se(\hat{\delta})}) \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal CDF and the second equality follows because under H_1 we know that $\hat{\delta} - \delta_0 / V(\hat{\delta}) \sim N(0, 1)$, and the second equality because $\Phi(t_{\alpha/2} - \frac{\delta_0}{V(\hat{\delta})})$ will typically be close to zero (check the graph to see why).

- (a) For $N_0 = N_1 = 100$, suppose you intend to reject $H_0 : \text{ATE} = 0$ at the 5% level: What is the probability of a type II error (rejecting the null hypothesis when it is true) if the true effect is 0.5? What is the power? What is the power of your test if you test H_0 at the 10% level? And at the 1% level? What happens to the power of your test when $N_0 = N_1 = 200$?
 - (b) Use your answer in (a) to derive the optimal sample sizes when you restrict $N_0 = N_1$ in this hypothetical experiment.
 - (c) Use $\Pr(t_{\alpha/2} < \frac{\hat{\delta}}{se(\hat{\delta})} < t_{1-\alpha/2}) \approx \Phi(t_{1-\alpha/2} - \frac{\delta_0}{se(\hat{\delta})})$ to derive the optimal sample sizes when you restrict $N_0 = N_1$ in this hypothetical experiment
5. A researcher is interested in the effect of teacher quality on student test scores. Define the question more clearly / indicate what requires clarification by sketching an experimental design.

Standard normal table - $P(Z < z)$

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |