Dynamics of Small Open Economies

Econ 4330 Open Economy Macroeconomics Spring 2010

Lecture 4 Part A

Asbjørn Rødseth February 2010

$$CA_t = Y_t - \tilde{Y}_t - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{v}{1+r} W_t$$

Lessons about CA from the infinite horizon models

- Temporarily high output causes surpluses
- Temporarily high investment or gov. consumption cause deficits
- Temporarily high or low taxes do not matter.
- The more persistent shocks are, the lower is the initial CA effect
- Expectations of fast-output growth produce deficits
- A high degree of impatience among consumers produce deficits
- Countries with a high marginal productivity of capital tend to get an initial deficit if they open up to international capital markets
- Deficits or surpluses can continue indefinitely (do not self-correct)

Do the conclusions fit with experience?

Do the conclusions survive if we enrich the theory?

The effect of output growth on CA

Consumption growth determined by Euler equation: $u'(C_t) = \beta(1+r)u'(C_{t+1})$

Output growth determined by exogenous productivity growth (and investment)

Local consumption growth is independent of local output growth

Euler equation with CES-utility: $C_{t+1} = [\beta(1+r)]^{\sigma} C_t = (1+v)C_t$

v = rate of growth of consumption

If $\beta < 1$ and $\sigma < 1$, $v < r$. Assume this.

Compare two countries with different rates of output growth

Solution of the small open economy model

From last weeks lecture with $I_t = 0$ and $G_t = 0$

$$W_t = (1+r)B_t + \frac{1+r}{r}\tilde{Y}_t$$

$$C_t = \frac{r-v}{1+r}W_t = (r-v)B_t + \frac{r-v}{r}\tilde{Y}_t$$

$$CA_t = rB_t + Y_t - C_t = Y_t - \frac{r-v}{r}\tilde{Y}_t + vB_t$$

Consequences of different rates of trend output growth

$Y_{t+1} = (1+g)Y_t$

g = outupt growth rate, $r > g$ assumed

$$\tilde{Y}_t = \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(1+g)^{s-t}Y_t = \left(\frac{r}{1+r}\right)\left(\frac{1}{1-\frac{1+g}{1+r}}\right)Y_t = \frac{r}{r-g}Y_t$$

Insert this in the current account equation:

$$CA_t = Y_t - \frac{r-v}{r}\left(\frac{r}{r-g}\right)Y_t + vB_t = \frac{v-g}{r-g}Y_t + vB_t$$

$(v-g)/(r-g)$ is the savings rate out of current income from production

From interest income vB_t is saved, while $(r-v)B_t$ is spent

Stricter rule than for the Petroleum Fund!

$v > 0 \Rightarrow$, $B_t > 0$ contributes to $CA_t > 0$, and increased B_{t+1}. No self-correction.

$$CA_t = \frac{v-g}{r-g} Y_t + vB_t$$

The country with low output growth

$g < v$ Output growth lower than consumption growth

- The share of output saved $(v-g)/(r-g)$, is positive.
- The share can be huge even if *v-g* is small
- The savings are needed to raise future consumption possibilities faster than income.
- The staring level of consumption is low

The country with high output growth

$g > v$: Output growth higher than consumption growth

- The share of output saved $(v-g)/(r-g)$, is negative.
- The starting level of consumption is high.

What happens to the debt in the long run?

$$B_{t+1} = B_t + CA_t = (1+v)B_t + \frac{v-g}{r-g}Y_t$$

Asset ratio: $b_t = B_t/Y_t$ (negative of the debt ratio)

$$\frac{B_{t+1}(1+g)}{Y_t(1+g)} = (1+v)\frac{B_t}{Y_t} + \frac{v-g}{r-g}$$

$$b_{t+1}(1+g) = (1+v)b_t + \frac{v-g}{r-g}$$

$$b_{t+1} = \frac{1+v}{1+g}b_t + \frac{v-g}{(1+g)(r-g)}$$

First order difference equation, solution see Berck and Sydsæter

$$b_s = \left(\frac{1+v}{1+g}\right)^{s-t}\left(b_t + \frac{1}{r-g}\right) - \frac{1}{r-g}$$

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High output growth country $g > v$

Then $s \to \infty \Rightarrow b_s \to -\frac{1}{r-g}$

- Debt ratio goes to a (high) constant
- High output growth → consumption exceeds output to begin with.
- The share of consumption in output will tend to zero.
- The whole GDP is used to pay interest on the foreign debt

Low output growth country $g < v$

- $b_t > -1/(r-g)$, has to be positive initially (or country is bankrupt)
- b_s will become positive and grow without limit.
- Some of the interest is still consumed.

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The long run solution is not meaningful

If the output growth exceeds consumptio growth forever, the share of consumption in output will tend to zero.

Sooner or later the small country ceases to be small

Default risks

Is the present value budget constraint too permissive?

Constraints on the debt to GDP-ratio will force fast-growing countries to borrow less

Constant growth rates forever? No!

Are consumers really behaving as if they have an infinite horizon?

Bequests are common

Children do support their parents

Maximizing a dynastic welfare function?

Following social norms or biological urges?

Why do poor parents leave bequests to rich descendants?

Bequests determined by the parents' resources?

Or by the difference between the parent and the children?

Alternative theories of saving

Life-cycle saving:

- Saving for old age, college for the children, dowry, (bequests)
- Saving and accumulated wealth related to life-time income of present generations

Precautionary savings

- Risks that cannot be insured
- Risks not covered by social security
- Desired wealth stands in relation to income
- Credit rationing

Power and status

- Business, saving profits to expand the business
- Politics

Overlapping generations and life-cycle saving

Open Economy Macro: Lecture 4

Asbjørn Rødseth

University of Oslo

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Motivation

- Individuals have finite lives
- The economy persists
- Individual decision making, not dynastic

Different from infinite horizon model

- Positive relation between savings and growth
- Wealth to GDP ratios are bounded
- Timing of taxes matter for current accounts

Main assumptions

- Small open economy
- Output exogenous (endowment economy)
- Given world interest rate
- Consumers live for two periods
- Generations overlap
- No bequests and no gifts from children
- One representative consumer for each generation

Behavior of individual consumer

Utility function, generation borne at t:

$$U = u(c_t^Y) + \beta u(c_{t+1}^O) \tag{1}$$

Budget constraint:

$$c_t^Y + (1+r)^{-1} c_{t+1}^O = y_t^Y + (1+r)^{-1} y_{t+1}^O = w_t \tag{2}$$

w_t total life-time wealth of individual of generation t
First order condition:

$$u'(c_t^Y) = \beta(1+r)u'(c_{t+1}^O) \tag{3}$$

Example: log utility

$$u(c) = \ln c \tag{4}$$

Special case of CES-utility with $\sigma = 1$
First order condition:

$$1/c_t^Y = \beta(1+r)/c_{t+1}^O$$

or

$$c_{t+1}^O = \beta(1+r)c_t^Y$$

Insert in budget equation, solve and get:

$$c_t^Y = \frac{w_t}{1+\beta}, \qquad c_{t+1}^O = \frac{\beta(1+r)w_t}{1+\beta} \tag{5}$$

Saving when young

Individual saving (use (5) and (2)):

$$s_t^Y = y_t^Y - c_t^Y = \frac{1}{(1+\beta)(1+r)}\left[\beta(1+r)y_t^Y - y_{t+1}^O\right] \qquad (6)$$

$e =$ the growth rate of income from young to old,
$y_{t+1}^O = (1+e)y_t^Y$.

$$s_t^Y = \frac{1}{(1+\beta)(1+r)}\left[\beta(1+r) - (1+e)\right] y_t^Y$$

Savings rate of the young is then:

$$\mu = s_t^Y / y_t^Y = \frac{[\beta(1+r) - 1] - e)}{(1+\beta)(1+r)} \qquad (7)$$

Saving when young

$$\mu = \frac{[\beta(1+r) - 1] - e)}{(1+\beta)(1+r)}$$

Two reasons for saving:

- the return is high enough to overcome impatience $\beta(1+r) > 1$.
- income is lower when old $e < 1$

Retirement creates need for saving.

Saving when old

Saving when old is the negative of saving when young:

$$s_{t+1}^O = -s_t^Y$$

The sum of saving over the individual life-cycle is zero
+ Standard assumption $y^O << y^Y$

$$\Rightarrow$$

The young are saving, the old are dissaving.

Aggregate saving

The young save, the old dissave
+ Sum of savings over individual life-cycle is zero

$$\Rightarrow$$

Aggregate savings positive only if young are richer or more numerous than old.

Aggregation

Total savings

$$S_t = N_t s_t^Y + N_{t-1} s_t^O \tag{8}$$

N_t Size of young generation at t

Total financial assets of households at end of period t:

$$B_{t+1}^P = N_t s_t^Y \tag{9}$$

Total household income:

$$Y_t = N_t y_t^Y + N_{t-1} y_t^O \tag{10}$$

Growth and savings

- n growth rate of population $N_{t+1} = (1+n)N_t$
- g growth rate of income between generations $y^Y_{t+1} = (1+g)y^Y_t$
- e growth rate of income over life-cycle, $y^O_{t+1} = (1+e)y^Y_t$

$$\begin{aligned} S_t &= N_t \mu y^Y_t - N_{t-1}\mu y^Y_{(t-1)} \\ &= N_t \mu y^Y_t - N_t(1+n)^{-1}\mu y^Y_t (1+g)^{-1} \\ &= \mu N_t y_t[(1+n)(1+g)-1]/[(1+n)(1+g)] \qquad (11) \end{aligned}$$

No growth, no net saving

Growth and savings cont.

Aggreate output:

$$Y_t = N_t y_t^Y + N_{t-1} y_t^O = N_t y_t \frac{(1+n)(1+g)+(1+e)}{(1+n)(1+g)} \tag{12}$$

Aggregate savings rate:

$$\begin{aligned} \frac{S_t}{Y_t} &= \mu \frac{(1+n)(1+g)-1}{(1+n)(1+g)+(1+e)} \\ &= \left(\frac{[\beta(1+r)-1]-e}{(1+\beta)(1+r)} \right) \left(\frac{n+g+ng}{2+n+g+ng+e} \right) \end{aligned} \tag{13}$$

(Compare p. 150 in OR, where $n = 0$ and $\beta(1+r) = 1$).

Growth and savings cont.

Focus on case where $e < 0$ and $\beta(1+r) = 1$

$$\mu = \frac{-e}{(1+\beta)(1+r)} = \frac{-\beta e}{1+\beta}$$

$$\frac{S_t}{Y_t} = -e\left(\frac{n+g+ng}{2+n+g+ng+e}\right)\frac{\beta}{1+\beta}$$

Savings rate is

- decreasing in e
- Increasing in n
- Increasing in g

Life-cycle model and timing of taxes

- Infinte horizon consumers: Compensates for tax reduction by saving more because they know they have to pay-back through higher taxes later
- Life-cycle consumer: Spends the part of the tax reduction that is going to be paid by future generations

Life-cycle model - evaluation

- Can explain that fast growing countries save more
- Net foreign assets stays within limits
- Retirement saving
- Precautionary saving
- Borrowing constraints
- Other life-cycle related motives
- Bequests without dynastic optimization

Investment and growth

Production function (constant returns:

$$Y = F(K, AN) = ANF(K/AN, 1) = ANf(k) \tag{14}$$

$k = K/AN$

First order condition:

$$f'(k) = r \tag{15}$$

k constant when r constant

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{K_t}{A_t N_t} = k \tag{16}$$

Investment and growth

$$K_{t+1} = K_t \frac{A_{t+1} N_{t+1}}{A_t N_t} = K_t(1+g)(1+n) \tag{17}$$

$$I_t = K_{t+1} - K_t = [(1+g)(1+n) - 1]K_t = (g + n + gn)K_t \tag{18}$$

- Investement rate high in fast-growing economies
- Feldstein-Horioka puzzle

Saving in corporations

- Norway2008: Saving in the corporate sector six times saving in the houshold sector
- Housholds own corporations
- Governments own corporations
- Importance of income distribution for savings