

## Dynamic oligopoly theory

### Collusion – price coordination

Illegal in most countries

- exemptions wide-spread
- explicit collusion not feasible

### Tacit collusion

Repeated interaction

Theory of repeated games

Deviation from an agreement to set high prices has

- a short-term gain: increased profit today
- a long-term loss: deviation by the others later on

Tacit collusion occurs when

long-term loss > short-term gain

Model: two firms, homogeneous good,  $C(q) = cq$

Prices in period  $t$ :  $(p_{1t}, p_{2t})$

Profits in period  $t$ :  $\pi^1(p_{1t}, p_{2t}), \pi^2(p_{1t}, p_{2t})$

History at time  $t$ :  $H_t = (p_{10}, p_{20}, \dots, p_{1, t-1}, p_{2, t-1})$

A firm's strategy is a rule that assigns a price to every possible history.

A subgame-perfect equilibrium is a pair of strategies that are in equilibrium after every possible history: Given one firm's strategy, for each possible history, the other firm's strategy maximizes the net present value of profits from then on.

$T$  – number of periods

$T$  finite: a unique equilibrium

period  $T$ :  $p_{1T} = p_{2T} = c$ , irrespective of  $H_T$ .

period  $T - 1$ : the same

and so on

$T$  infinite (or indefinite)

At period  $\tau$ , firm  $i$  maximizes

$$\sum_{t=\tau}^{\infty} \delta^{t-\tau} \pi^i(p_{1t}, p_{2t}), \quad \delta = \frac{1}{1+r}$$

The best response to  $(c, \dots)$  is  $(c, \dots)$

But do we have other equilibria? Can  $p > c$  be sustained in equilibrium?

Trigger strategies: If a firm deviates in period  $t$ , then both firms set  $p = c$  from period  $t + 1$  until infinity.

[Optimal punishment schemes? Abreu *J Econ Th* 1986]

Monopoly price:  $p^m = \arg \max (p - c)D(p)$

Monopoly profit:  $\pi^m = (p^m - c)D(p^m)$

A trigger strategy for firm 1:

- Set  $p_{10} = p^m$  in period 0
- In the periods thereafter,
  - $p_{1t}(H_t) = p^m$ , if  $H_t = (p^m, p^m, \dots, p^m, p^m)$
  - $p_{1t}(H_t) = c$ , otherwise

If a firm *collaborates*, it sets  $p = p^m$  and earns  $\pi^m/2$  in every period.

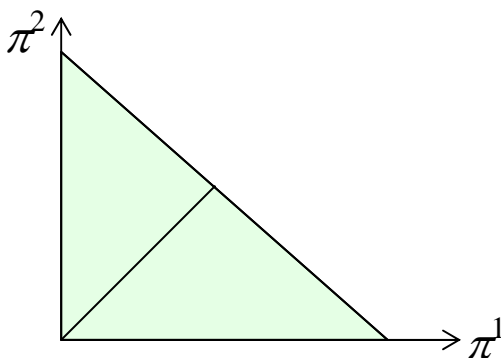
The *optimum deviation*:  $p^m - \varepsilon$ , yielding  $\approx \pi^m$  for one period.

An equilibrium in trigger strategies exists if:

$$\frac{\pi^m}{2}(1 + \delta + \delta^2 + \dots) \geq \pi^m + 0 + 0 + \dots$$

$$\Leftrightarrow \frac{1}{2} \frac{1}{1 - \delta} \geq 1 \Leftrightarrow \delta \geq \frac{1}{2}$$

The same argument applies to collusion on any price  $p \in (c, p^m]$ .  $\Rightarrow$  Infinitely many equilibria.



The Folk Theorem.

## Collusion when demand varies

Demand stochastic:

Periodic demand is

low:  $D_1(p)$  with probability  $\frac{1}{2}$

high:  $D_2(p)$  with probability  $\frac{1}{2}$

$D_1(p) < D_2(p), \forall p.$

The demand shocks are *i.i.d.*

Each firm sets its price after having observed demand.

What are the best collusive strategies for the two firms?

Trigger strategies: A deviation is followed by  $p = c$  forever.

What are the best collusive prices? One price in low-demand periods and one in high-demand periods:  $p_1$  and  $p_2$ .

$\pi_s(p)$  – total industry profit in state  $s$  when both firms set  $p$ .

With prices  $p_1$  and  $p_2$  in the two states, each firm's expected net present value is:

$$\begin{aligned} V &= \sum_{t=0}^{\infty} \delta^t \left[ \frac{1}{2} \frac{D_1(p_1)}{2} (p_1 - c) + \frac{1}{2} \frac{D_2(p_2)}{2} (p_2 - c) \right] \\ &= \frac{1}{4(1-\delta)} [D_1(p_1)(p_1 - c) + D_2(p_2)(p_2 - c)] \\ &= \frac{\pi_1(p_1) + \pi_2(p_2)}{4(1-\delta)} \end{aligned}$$

The best possible collusive price in state  $s$  is:

$$p_s^m = \arg \max (p - c)D_s(p), s = 1, 2.$$

$$\pi_s^m = (p_s^m - c)D_s(p_s^m), s = 1, 2.$$

If the firms can collude on these prices, then:

$$V = \frac{\pi_1^m + \pi_2^m}{4(1 - \delta)}$$

A deviation in state  $s$  receives a gain equal to:  $\pi_s^m$

For  $(p_1^m, p_2^m)$  to be equilibrium prices, we must have:

$$\pi_s^m \leq \frac{1}{2}\pi_s^m + \delta V \Leftrightarrow \pi_s^m \leq 2\delta V$$

The difficulty is state 2 (high-demand), since  $\pi_1^m < \pi_2^m$ .

The equilibrium condition becomes:

$$\pi_2^m \leq 2\delta \frac{\pi_1^m + \pi_2^m}{4(1 - \delta)}$$

$$\Leftrightarrow \delta \geq \frac{2}{3 + \frac{\pi_1^m}{\pi_2^m}} \equiv \delta_0$$

$$0 < \frac{\pi_1^m}{\pi_2^m} < 1 \Rightarrow \frac{1}{2} < \delta_0 < \frac{2}{3}$$

But what if  $\delta \in [\frac{1}{2}, \delta_0)$ ? Can we still find prices at which the firms can collude?

The problem is again state 2. We need to set  $p_2$  so that

$$\pi_2(p_2) \leq 2\delta \frac{\pi_1^m + \pi_2(p_2)}{4(1-\delta)}$$

$$\Rightarrow \pi_2(p_2) = \frac{\delta}{2-3\delta} \pi_1^m$$

$$\frac{1}{2} \leq \delta < \frac{2}{3} \Rightarrow \frac{\delta}{2-3\delta} \geq 1 \Rightarrow \pi_2 \geq \pi_1$$

So: prices below monopoly price in high-demand state – during boom. Could even be that  $p_2 < p_1$ .

But is this a price war?

More realistic demand conditions:

Autocorrelation – business cycle.

Collusion most difficult to sustain just as the downturn starts.

Haltiwanger & Harrington, *RAND J Econ* 1991

Kandori, *Rev Econ Stud* 1991

Bagwell & Staiger, *RAND J Econ* 1997

[Exercise 6.4]

## Empirical studies of collusion

- the railroad cartel
  - Porter *Bell J Econ* 1983
  - Ellison *RAND J Econ* 1994
- collusion among petrol stations
  - Slade *Rev Econ Stud* 1992
- collusion in the soft-drink market: prices and advertising
  - Gasmi, *et al.*, *J Econ & Manag Strat* 1992
- collusion in procurement auctions
  - Porter & Zona *J Pol Econ* 1993 (road construction)
  - Porter & Zona *RAND J Econ* 1999 (school milk)

## Infrequent interaction

Suppose the period length doubles.

$$\delta \rightarrow \delta^2$$

Collusion feasible if:

$$\delta^2 \geq \frac{1}{2} \Leftrightarrow \delta \geq \frac{1}{\sqrt{2}} \approx 0.71$$

## Multimarket contact

Market A: Frequent interaction, period length 1.  
Collusion if  $\delta \geq 1/2$ .

Market B: Infrequent interaction, period length 2.  
Collusion if  $\delta^2 \geq 1/2$ .

What if both firms operate in both markets?

Can the firms obtain collusion in both markets even in cases where  $\delta^2 < 1/2 < \delta$ ?

A deviation is most profitable when both markets are open.

Deviation yields:  $2\pi^m$

Collusion yields:

$[\pi^m/2]$  every period, plus

$[\pi^m/2]$  every second period (starting today)

Collusion can be sustained if:

$$\frac{\pi^m}{2} [1 + \delta + \delta^2 + \dots] + \frac{\pi^m}{2} [1 + \delta^2 + \delta^4 + \dots] \geq 2\pi^m$$

$$\Leftrightarrow \frac{1}{2} \frac{1}{1-\delta} + \frac{1}{2} \frac{1}{1-\delta^2} \geq 2$$

$$\Leftrightarrow 4\delta^2 + \delta - 2 \geq 0 \Leftrightarrow \delta \geq \frac{\sqrt{33}-1}{8} \approx 0.593$$