ECONOMICS OF FORESTRY IN AN EVOLVING SOCIETY*

PAUL A. SAMUELSON
Massachusetts Institute of Technology

A debate that has raged for centuries is unlikely to be resolved by me in one lecture. However, I shall do my best to set forth the issues and indicate what ought to be the crucial factors that a jury should consider in rendering its verdict on the matter. The issue is one between forestry experts and the general public on the one side and professional economists and profit-conscious businessmen on the other. At first blush this would seem to suggest that economists are on the side of the interests and are not themselves members of the human race. But, as I hope to show, sound economic analysis is needed to do justice to the cases put forward by either of the adversary parties.

SUSTAIN OR NOT SUSTAIN?

To vulgarize and oversimplify, *there has been a tradition in forestry management which claims that the goal of good policy is to have sustained forest yield, or even "maximum sustained yield" somehow defined.* And, typically, economists have questioned this dogma.

If laissez-faire enterprisers tended to be led by that invisible hand Adam Smith talked about to achieve in fact sustained forest yields, and even maximum sustained forest yields, no doubt there would be a school of economists called into existence to give their blessings to the doctrine of maximum sustained yield. In that case there would be no great debate. The economists in the liberal arts division of the university, on those rare occasions when they deign to think about the practical problems of forest management, would come out with the same conclusions and dicta as would the professional foresters in the school of forestry. Moreover, the professors in the biological departments, and the lay public generally, would heartily approve of the actual solution in this best of all possible worlds.

Life is not like that and it hasn't been for a long time. The medieval forests of Britain, and of Europe, tended to be chopped down as society moved into the Industrial Revolution. The virgin forests that graced the New World when Columbus arrived here have increasingly been cut

*This paper was presented initially as a lecture at a symposium on "The Economics of Sustained Yield Forestry," at the University of Washington, Seattle, Washington, November 23, 1974. I owe thanks to the National Science Foundation for financial aid, and to Kate Crowley for editorial assistance. Also to Vicki Elms for help with the bibliography, and to Barbara Feldstein for composition. It was Professor Barney Dowdle of the University of Washington who, knowing of my innocence of forestry economics, inveigled me into making these preliminary researches.

down once the calculus of dollar advantage began to apply. When I informed a graduate student that I was preparing this lecture, he mentioned to me the rumor that a nearby consulting firm had applied dynamical programming analysis to the problem of how old — or rather how young — a tree should be when it is to be optimally cut in the steady state. Allegedly, its computer spun out of control and generated a negative, or for all I know, imaginary, root for the equation: apparently at realistic profit rates, it doesn't pay to keep a forest in existence at all. This is probably only a tall story, but it does well illustrate the fact that standard managerial economics, and actual commercial practice, both tend to lead to an optimal cutting age of a forest that is much shorter than the 80 or even 100 years one often encounters in the forestry literature.

EXTERNALITIES AND INTERVENTION

This apparent clash between economists and foresters is not an isolated one. Biological experts in the field of fisheries are sometimes stunned when they meet economists who question their tacit axiom that the stock of fish in each bank of the ocean ought to be kept as a goal at some maximum sustained level. Similarly, hard-boiled economists are greeted with incredulity if and when they opine that it may be optimal to grow crops in the arid plain states only until the time when the top soil there has blown away to its final resting place in the ears and teeth of Chicago pedestrians.

Everybody loves a tree and hates a businessman. Perhaps this is as it should be, and perhaps after the profession of economics is 1,000 rather than 200 years old, the human race will be as conditioned to abhor economists as it has become to abhor snakes. But really, these matters need arguing in court so the informed jury, and I do mean the informed jury of human beings, can make up its mind.

Let me say in advance of the argument, there is no ironclad presumption that profit seeking laissez-faire will lead to the social optimum. Thus, suppose that a living redwood tree helps purify the air of smoggy Los Angeles. Suppose sowing the land to short-lived pine trees prevents floods 500 miles downstream. Then we may well have here a case of what modern economists recognize as "externalities." We economists these days spend much of our time analyzing the *defects* of competitive free entry and push-shove equilibrium when important externalities are involved. If therefore in the great historic debate on sustained yields, foresters and conservationists had brought into court an elaboration of the respects in which forestry is an activity beset with important externalities, carefully and objectively described, Ph.D.'s in economics would be found on both sides of the case under trial. Indeed, if the externalities involved could be shown to be sufficiently important, I am naive enough to believe that all economists would be found on the side of the angels, sitting thigh next to thigh with the foresters. (*All* economists agree? Well, almost all.)

"PRIVATE" VERSUS "COMMON" PROPERTY

Earlier I mentioned fisheries. Even those economists who ostrich-like tend always to play down externalities if they can, have long recognized that there is a "common property" element in hunting and fishing: even though I were to have to pay rent to someone who owns a particular acre of the ocean in order to put down my net there, my act of fishing there can hope to draw on fish with might migrate from nearby acres. So we have in the case of fisheries a special kind of externality that makes it nonoptimal to have *decentralized rent-charging owners* of subdivisions of a common fishing bank. Government regulation and centralized decision making for the whole fishing pool, if it can be arranged in this age of nationalism, is obviously preferable to free competition as Gordon (1954), Scott (1955), and Crutchfield (1962) have analyzed.

From a cursory glance at the literature of forestry, both technical and economic, I do not perceive foresters to be making as a case for timber what is true for fisheries, or for oil drilling. It is true that forest fires are a hazard that adjacent timber lands may face in common. And if the units of land owned by each forester-owner were very, very small, the externalities between adjacent plots would render decentralized competitive decision-making nonoptimal. However, for the most part, timber ownership will not under laissez-faire tend to stay so pulverized, since it is quite feasible to have the span of ownership widened to the optimal degree without creating monopoly or vitiating the assumptions of workable competition.

At the beginning, therefore, even before entering into the serious argument, let us make a deposition that the following would be a false issue in the debate:

Abolishing private ownership in land or abandoning public regulation of forest land owned by the government is not an alternative to maximum sustained yield that is advocated by anybody. This would certainly result in unnecessary decimation of the forest. Indeed, as Vernon Smith (1968) has shown in one of his models, it could result in extinction of *all* forests; but even if a realistic model of complete push-shove free-free entry led to a maintainable sustained-yield steady state, the average age of the forest stands in such a Hobbesian jungle might well involve rotation periods so short as to be absurd, which is why in medieval Germany severe limits were properly placed on the use by the public of crown and public forest lands.

COMPETITIVE LAND RENTS

The economists who oppose maximum sustained-yield do not advocate any such absurd push-shove procedure. They assume that the cultivator who plants a tree on one acre of land owns or rents the right to exclusive garnering of the fruits of that which he has planted. Similarly, if I own yonder acre or have leased it from a public or private owner, and if I desist in chopping down a tree that is not yet ripe, I expect to find it still there when I do come to chop it down. In return for this exclusive use of my own area of cultivation, I expect to pay a land rent. If I own the land outright, I pay it *to myself* at an opportunity cost rate that is perfectly well determined in a freely competitive market. Or I pay the rent to a private owner, who knows he can rent that land to somebody else like me if not to me. Or I pay a rent to a government that owns the land.

This rate of land rental can be high or low. If the total amount of land available for growing the timber that society needs, and which is close enough to the market to be able to avoid heavy transportation charges, is severely limited in amount, then the appropriate competitive land rent will be high. If on the other hand land is extremely plentiful, its scarcity rent will be very low. It will not even matter for the purpose of our analysis if well-located land is so plentiful as to be redundant. In that case, its competitive land rent will be zero, but even though land rent is zero I shall still need to have *exclusive* rights to the fruits of my earlier labor and other investment inputs, independently of whether in other acres of the redundant territory push-shove free-free entry is permitted.

ASSUMPTIONS FOR THE ANALYSIS

Let me first review the correct economic principles that would be applicable if forestry can be regarded simply as sources of wood saleable in competitive markets. This initially assumes away externalities such as flood control, pollution abatement, species preservation, vacationers' enjoyments, etc.

Although I am not a specialist in the field of forest economics, I have been reading a couple of dozen different analyses ranging over the last two centuries that grapple with optimal steady-state rotation periods. The economic analysis in most of them is wrong. In some it is very wrong. In others it is not quite right. In at least one case, the remarkable 1849 German article by Martin Faustmann, the analysis does come close to an essentially correct solution.

These remarks are not intended to give a harsh indictment of foresters or of economists who have worked in the field of forestry. The mistakes made in the forestry literature can be duplicated aplenty in the intermediate textbooks of pure economics.

Thus, Irving Fisher was the greatest single economic writer on interest and capital, and his 1930 *Theory of Interest* summarized his life work in that field. Yet at MIT we ask graduate students on quizzes to identify and correct Fisher's false solution as to when a tree should be cut (a false solution that he seems to share with the great von Thunen (1826) and the brilliant Hotelling (1925) as well as with some excellent economists who have written on forestry in recent decades). Again, Kenneth Boulding is one of our leading economists; but his rule of maximizing the so-called "internal rate of return" has led many a forestry economist down the garden path (Boulding 1935). A 1960 review of the literature by G. K. Goundrey comes out with the wrong Boulding solution, and yet his analysis purports to lean on such excellent authorities as Wicksell, Scitovsky, Kaldor, Metzler, and Scott; alas, it did not lean more heavily on Faustmann (1849), Preinrich (1938), Alchian (1952), Bellman (1957), Gaffney (1957), Hirshleifer (1958), and perhaps Samuelson (1937).

If an unambiguous solution to the problem is to be definable, of course certain definite assumptions must be made. If the solution is to be simple, the assumptions must be heroic. These include: (1) knowledge of future lumber prices at which all outputs can be freely sold, and future wages of all inputs; (2) knowledge of future interest rates at which the enterprise can both borrow and lend in indefinite amounts; and (3) knowledge of technical lumber yields that emerge at future dates once certain expenditure inputs are made (plantings, sprayings, thinnings, fellings, etc.). Finally, it is assumed (4) that each kind of land suitable for forests can be bought and sold and rented in arm's length transactions between numerous competitiors; or, if the government owns public lands, it rents them out at auction to the highest of numerous alternative bidders and conducts any of its own forestry operations so as to *earn the same maximum rent* obtainable at the postulated market rate of interest. For the special steady-state model, the future prices and interest rates must be assumed to be known constants. Moreover, our problem is not one merely of managerial economics; rather we must deduce the competitive prices that clear the industry's market.

Assumptions would not be heroic if they could be easily taken for granted as being exactly applicable. Stochastic factors of climate, lightning, forest fires, and disease must in real life qualify the technical assumptions made in (3) above. At the least, therefore, as a second approximation, one must introduce probabilities and expected values into the decision calculus.

Similarly, tomorrow's lumber price is not knowable exactly, much less the price of lumber a score of years from now when today's seedling will mature. So, in other than a first approximation, the assumptions under (1) need to be complicated.

Finally, future interest rates are not knowable today. Moreover, the inherent uncertainties involved in interest and profit yields also serve to falsify the assumption in (2) that the enterprise is able at each date both to borrow and to lend in indefinite amounts at one interest rate (even one knowable at *that* date if not now). Once we recognize that the enterprise is in an imperfect capital market, we will not be able to deduce its optimal forestry decisions independently of knowledge about its owners' personal preferences concerning consumption outlays of different dates (and concerning their "liquidities" at different dates).

CORRECT CAPITAL ANALYSIS

Our problem is now well posed. What principles provide its solution? What is the exact nature of the solution?

(i) Does it yield a steady-state rotation period as long as that which achieves the foresters' traditional "maximum sustained yield"?

(ii) Is the optimal rotation that *shorter* period which maximizes the present discounted value over the first planting cycle of the cash receipts that come from the sale of cut lumber minus the cash expenses of planting and cutting inputs (excluding from the net cash receipts stream any adjustments for implicit and explicit land rent)?

(iii) Is the optimal rotation period that still shorter period which maximizes Boulding's "Internal Rate of Return," computed as that largest rate of interest which when applied to the net dollar cash receipts over one complete cycle reduces the resulting present discounted value to zero (and, be it noted, ignores land rent in setting up the net algebraic cash receipts!)?

(iv) Alternatively, is the optimum the rotation period that results from maximizing (a) the present discounted value of all net cash receipts excluding explicit or implicit land rents, but calculated over the *infinite chain* of cycles of planting on the given acre of land from now until Kingdom Come; or (b) what may sound like a different criterion, the rotation period that results from maximizing the present discounted value of net algebraic receipts over the first cycle, but with the market land rental included in those receipts, it being understood that the land rental that each small enterprise will be confronted with will be the *maximum* rental that ruthless Darwinian competition can contrive?

If you have been testing yourself by trying to answer the objective-type quiz that I have just propounded, you will receive a perfect A+ if you gave the following answers:

(i) No, the rotation period that maximizes sustained yield is so long that, at the postulated positive interest rate and inevitable market rent for land, it will bankrupt any enterprise that endeavors to realize it.

(ii) No, maximizing the present discounted value, over *one* planting cycle, of cash receipts from cut timber sold minus cash receipts for inputs that do planting, thinning, and cutting will give you a somewhat too long rotation period and will not enable you to cover the land rent that will be set by your more perspicacious competitors. However, your error will not be so very great in the case the length of each cycle is very great and/or the rate of interest per annum is very large, so that the discounted value today of a dollar payable at the end of the cycle is negligible. Still, employing this method that is so frequently advocated by sophisticated economists will lead you to the following absurdity: an increase in initial planting cost will have *no effect at all* on your optimal rotation period, up to the point that it makes it unprofitable to put the land you own into lumber, even when you are philanthropic enough to forego obtainable positive land rent. It is a solution that pretends that the Archimedean forest lever never needs the land fulcrum to work with.

(iii) No, ignoring land rent and maximizing the internal rate of return will give you so short a rotation period that, at the postulated interest rate, you will not be able to pay yourself the positive land rental set by competition. Moreover, maximizing the interal rate of return will give you the nonsensical result that you should select the same rotation period when the interest rate, the price of time, is high or low; and, when initial planting costs are zero, it will give a meaningless infinite return.

(iv) Finally, yes, (a) and (b), which really are exactly the same method, constitute the only correct method. The first formulation, in terms of an infinite chain of repeated cycles, was already proposed in the brilliant 1849 German article by Martin Faustmann. A glance at its recent English translation convinces me of his remarkable merit, even though at first glance one does not find in it the exact explicit conditions for optimal cutting age of the forest stand. I do not know that the economics literature caught up with this degree of sophistication prior to the 1938 *Econometrica* survey article on depreciation by Gabriel Preinrich, which was itself a notable anticipation of the dynamic programming that Richard Bellman made routine in the postwar period. The second approach, which I cannot recall seeing explicitly in print, will perhaps be more intuitively understandable at a first approach to the subject; and, in any case, land rent has tended not to be given the proper analysis it needs.

In a moment I shall illustrate all this by means of a specific model, which though not very realistic will be familiar to economists since the time of Stanley Jevons. From it, you will infer the presumption that commercial exploitation of forestry will lead to a departure from the goal of maximum sustained yield even greater than may have been realized by adherents and critics of forestry dogmas. The higher the effective rate of interest, the greater will be the shortfall of the optimal rotation age compared to the age that maximizes steady-state yield. As the interest rate goes to zero, the economists' correct optimum will reach the limit of the foresters' target of maximum sustained yield. Only if an explicit land rent charge is introduced into the cash stream will Boulding's maximized internal rate of return avoid incorrect results; but in that case, Ockham's razor can cut it down as redundant (worse than that, as involving incomplete, implicit theorizing.) Actually, as we have seen in (b) above, including in competitive land-rent can save from error the popular method of maximizing present discounted value calculated over only the first cycle; however, to know *how much* rent so to include, one must impose the condition that it be just large enough to reduce to zero that maximized discounted value over one cycle, and this rent so calculated will turn out to be after capitalization exactly what the Faustmann-Preinrich-Bellman-Hirshleifer solution deduced. It should be noted that, in the special case where the land for timber growth is redundant and therefore free, maximizing over a single cycle will singularly give the correct answer, and maximizing the internal rate of return will with equal singularity also give the same answer. Since at least one writer, Goundrey (1960), has alleged that timber land in Canada is so plentiful as to be free, it is worth emphasizing that even in this case the three methods nominated by economists will deduce a rotation period significantly shorter than the foresters' maximum because of the positive interest rate. The foresters, without realizing it, are correct only when the true interest rate is literally zero.

THE BOGEY OF COMPOUND INTEREST

I cannot conclude this general survey of wrong and right ways of analyzing the actual equilibrium that will emerge in the competitive steady-state without expressing my amazement at the low interest rates which abound in the forestry literature. Faustmann, writing in the middle of the nineteenth century, uses a four percent rate. Thunen, writing at the same time, uses a five percent interest rate. The 1960 Goundrey survey also uses a five percent rate. These will seem to an ordinary economist and businessman as remarkably low. The notion that for such gilt-edge rates I would tie up my own capital in a 50-year (much less a 100-year) timber investment, with all the uncertainties and risks that the lumber industry is subject to, at first strikes one as slightly daft. I can only guess that such low numbers have been used either as a form of wishful thinking so foresters or forest economists can avoid rotation ages so short they show up the forester's "maximum sustained yield"; or because the writers have not had the heart to face up to the discounting almost out of existence of receipts payable half a century from now.

Let us make no mistake about it. The positive interest rate is the enemy of long-lived investment projects. At six percent interest, money doubles in 12 years, quadruples in 24, grows 16-fold in 48 years, and 256-fold in 96 years. Hence, the present discounted value today of $1 of timber harvest 96 years from now is, at six percent, only 0.4 of one cent!

Foresters know this and fight against compound interest. Thus, an economist cannot help but be amused at the 1925 gem by the Assistant Chief, Board of Research, U.S. Forest Service, Ward Shepard. Entitled "The Bogey of Compound Interest," this argues that if you have a forest stand in the steady state, no interest need be involved: your cutting receipts exceed your planting expenses! This is so absurd as to be almost believable to the layman — up to the moment when the economist breaks the news to the farmer, lumber-company president, or government official that he can mine the forest by cutting it down without replanting and sell the land, thereafter putting the proceeds into the bank or into retiring the public debt and subsequently earn interest forever.

"Bogey" has two meanings. The first, which is Shepard's naive meaning, is that compound interest is a fictitious entity which, like the Bogey Man, is wrongfully used to frighten little children. The second and here more legitimate meaning of bogey is that defined in Webster's Dictionary as "a numerical standard of performance set up as a mark to be aimed at in competition." Compound interest is indeed the legitimate bogey that competitors must earn in forestry if they are not to employ their land, labor, and disposable funds in other more lucrative uses.

Competitive theory can be reassuring as well as frustrating to the forester. There is a popular notion that interest calculations may be applied to decisions for next year as against the immediately following years. "But," it is not infrequently argued, "when what is at issue is a tree or dam whose full fruits may not accrue until a century from now, the brute fact that our years are numbered as three score and ten prevents people from planting the trees that will not bear shade until after they are dead — altruism, of course, aside."

To argue in this way is to fail to understand the logic of competitive pricing. Even if my doctor assures me that I will die the year after next, I can confidently plant a long-lived olive tree, knowing that I can sell at a competitive profit the one-year-old sapling. Each person's longevity and degree of impatience to spend becomes immaterial in a competitive market place with a borrowing, lending, and capitalizing interest rate that encapsulates all which is relevant about society's effective time preferences.

INFLATION AND INCOME TAXES

What interest rate is appropriate for forestry? I hesitate to pronounce on such a complex matter. A dozen years ago I might incautiously have said 12 percent or more. And, just recently you could have got 12 percent per annum on $100,000 left with safety in the bank for three months. But this of course represented in part the 1974 10+ percent annual inflation rate, a rate which the price rise in lumber could also presumably share. Indeed timber lands are often recommended as an inflation hedge: if the interest rate is 12 percent and the price of lumber rises at 12 percent per annum, it is a standoff and in effect there is a zero real interest rate.

Fortunately, I was able to show back in 1937, correcting a misleading interpretation in Keynes' 1936 *General Theory*, that so long as price changes are anticipatable, it does not matter in what "own-rates-of-interest" you calculate to make decisions (such as at what optimal age to cut a tree), the optimal physical decision will always be invariant. This means that essentially all we need in order to discuss forest economics correctly is to concentrate on (1) the *real* rate of interest (i.e., the actual interest rate on money minus the presumed known rate of overall price inflation), and (2) the real price of lumber outputs and inputs (i.e., the percentage real rate of rise for $P_{lumber}/P_{general}$).

There is another complication. If marginal tax rates are (say) 50 percent, a 12 percent yield before tax is a 6 percent yield after tax. It would seem to make quite a difference for optimal rotation decisions whether we must use a 12 or a 6 percent discount rate. Actually, and this may seem discouraging to the foresters' dream of maximum sustained yield, one can correctly use the higher pre-tax rate in making optimal decisions provided the income tax authorities really do properly tax true money income at uniform prices. More specifically, I showed in Samuelson (1964) that, if foreseeable depreciation and appreciation are taxed when they occur, a person always in the 50 percent (or 99 percent) bracket will make the same optimal decisions as a person always in the zero percent bracket.

But are actual U.S. or Canadian tax systems "fair" in their income taxation? Of course not. As a forest grows in size and value, instead of taxing this certain accretion of true income in the Henry Simons fashion, the tax is deferrable until the wood is cut. So forestry may provide a "tax loophole," which can distort decisions toward the longer rotation period of the foresters' maximum sustained yield, particularly if capital-gains tax-treatment is available at lower rates.

To sum up, I might mention that William Nordhaus (1974) recently showed at Brookings that *real* profit yields have been falling in recent years. Thus, his Table 5 suggests that real before-tax yields on corporate capital have tended to average only about 10 percent in the early 1970's as against over 15 percent 20 years earlier. This seems better for forest economics than in earlier decades, but still bad enough. Tax loopholes may further improve the viability of longer rotation periods. Also, I remember Frank Knight's being quoted as saying that, in effect, *real* lumber prices have risen historically about enough to motivate holding on to forest inventories — a dubious generalization, but one that reminds us that lumber price cannot be taken as a hard constant in realistic analysis. Before a nation, or regions it trades with, completely depletes a needed item, the price of that item can be expected to rise.

DEFINING MAXIMUM SUSTAINED YIELD IN A CLASSIC CASE

You might think that the practical man's notion of sustained yield, or of maximum sustained yield, would be clear-cut. But if you do, you haven't had much experience with analyzing so-called common sense notions. Certainly maximum sustained yield in forestry does not suggest all land wasted on soybeans and other goodies should be plowed and planted with trees. Nor does it mean that land devoted to forests should be manicured and fertilized by all the labor in society, labor not needed for subsistence calories and vitamins, in order to produce the most lumber that land is capable of in the steady state.

The amount of lumber a virgin forest is capable of producing in a wild state approaches closer to the notion's content. But biologists have long realized that Darwinian evolution leads to an ecological equilibrium in which many trees grow to be too old in terms of their wood-product efficiency; and, in any case, a virgin forest left unmolested by man is like a librarian's perverted dream of a library where no books are ever permitted to be taken out so that the inventory on the shelves can be as complete as possible.

One presumes that "maximum sustained yield" is shorthand for a reasonable notion like the following:

Cut trees down to make way for new trees when they are past their best growth rates. Follow a planting, thinning, and cutting cycle so the resulting (net?) lumber output, averaged over repeated cycles or, what is the same thing, averaged over a forest in a synchronized age class distribution, will be as large as possible.

Jevons, Wicksell, and other economists have for a century analyzed a simple "point-input point-output, time-phased" model that can serve as the paradigm of an idealized forest. Labor input of L_t does planting on an acre of forest land at time t. Then at time $t + T$, I can cut lumber of Q_{t+T}, freeing the land for another input of L_{t+T} and output $Q_{t+2T}, \ldots,$ and so forth in an infinite number of cycles. Biology and technology give me the production function relating inputs and output, namely

$$(1) \qquad \begin{aligned} Q_{t+T} &= f(T) \\ f'(T) > 0, \quad a < T < b; &\quad f'(b) = 0 \end{aligned}$$

Actually, as we'll see, $f(T)$ is short for $f(s, L; T)$, where L is labor input at the beginning of one planting cycle and s is the land used throughout that complete cycle (which can be set at $s = 1$).

In the steady state, a new part of each forest is being planted at every instant of time, an old part is being cut down at age T, and forest stands of all ages below T are represented in equal degree. If we wish to calculate the average product per unit of land of the synchronized forest, we can follow one cycle on one part of the forest, and divide the Q it produces in T periods by T to get average product per year. So one measure of gross sustained yield would be $f(T)/T$. However, this neglects the fact that workers must be paid wages. These are payable in dollars at rate W; and the lumber is sold in dollars at the competitive price P. But we could think of the workers who do the initial planting as being paid off in kind, in lumber they can sell at price P. So their wage in lumber, $(W/P)L$, must be subtracted from gross output $f(T)$ in order to form "average sustainable net lumber yield" of $[f(T) - (W/P)L]/T$.

Figure 1 shows the story. The point of maximum $f(T)$ is shown at B, where $f'(b) = 0 > f''(b)$. The point of maximum sustained gross yield T_q, is shown where a ray through the origin, OG, is tangent to the $f(T)$ curve. The point of maximum sustained net yield, is given by tangency at T_r of a similar ray, EN, from the expense point, E, to the net curve $f(T) - (W/P)L$.

Maximum sustained gross yield, as here defined, is at a maximum, not when $T = b$. To wait until each tree slowly achieves its top lumber content is to fail to realize that cutting the tree to make the land available for a faster-growing young tree is truly optimal. Ignoring all wage subtractions, sustained yield of gross lumber is maximized at the lower rotation age, T_g, defined by

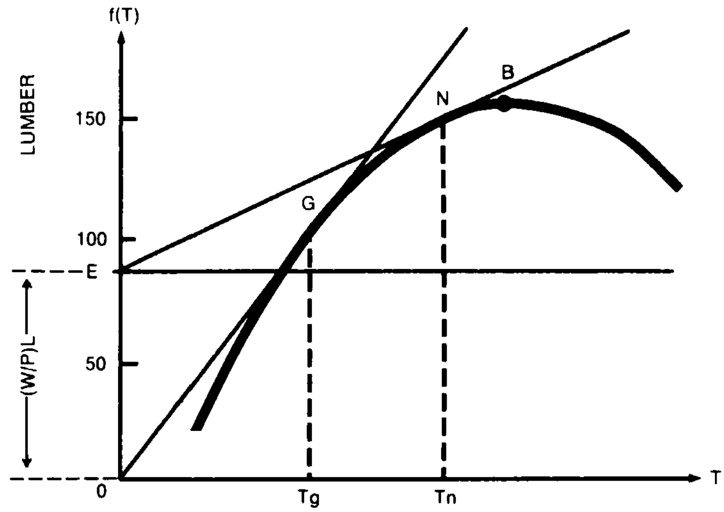
$$\max_{T} \; [f(T)/T] = f(T_g)/T_g \,, \tag{2}$$

where

$$f'(T_g) = f(T_g)/T_g, \quad T_g < b \,.$$

FIGURE 1

Maximum Sustained Yield



LENGTH OF TREE ROTATION (YEARS)

T_g is rotation period for maximum *gross* sustained yield irrespective of initial planting costs. T_n is rotation period for maximum *net* sustained yield.

Since a short rotation period makes us pay the same wages more often, once we introduce wage subtractions, we arrive at a forester's true "maximum sustained (net) yield" at a rotation age somewhat greater than T_g, namely at T_n defined as

$$(3) \qquad \underset{T}{Max}\ [f(T) - (W/P)L]/T = [f(T_n) - (W/P)L]/T_n\ ,$$

where[1]

$$f'(T_n) - f(T_n)/T_n = -(W/P)L/T_n, \quad T_g < T_n < b.$$

This provides us with an unambiguous and useful definition of "maximum sustained net yield." And it is this definition of sustained yield that I shall compare with what will actually emerge as steady-state competitive equilibrium, and with the various optima that one or another economist has proposed when poaching on the territory of forest economics.

THE TRUE COMPETITIVE SOLUTION

The above Jevons model will illustrate the false economic solutions and the correct solution.

First, consider the most popular method which maximizes present discounted value or PDV, calculated over one planting cycle only and involving cash receipts other than land rent. This gives T_1, defined by the equations below as

$$(4) \qquad f(T_1)e^{-rT_1} - (W/P)L \geqslant 0 = \underset{T}{Max}\,[f(T)e^{-rT} - (W/P)L],$$

where

$$f'(T_1)/f(T_1) = r$$

and

$$T_1 < b \quad \text{when} \quad r > 0,$$

where r is the market-given competitive interest rate at which everyone can borrow and lend in unlimited amounts. This is the famous Jevons relation, which had already been glimpsed by Thunen and which Fisher was later mistakenly to apply to a forest growing on limited land. (Only if land is so abundant as to be redundant and rent-free, so that P/W falls to equal $f(T_1)L^{-1}$, will T_1 give the correct competitive rotation period

1. Note that, as $(W/P) \rightarrow f(T)/L$, so that land rent is zero even at $r = 0$, $T_n \rightarrow b$.

of the forest. As we'll see, the correct rotation period, call it T_∞, will be shorter than T_1; but unlike T_1 it must always fall short of T_n.)

A defect in many good economic discussions is to present alternative maximum criteria, as if it were a matter of choice which to adopt. One such is to maximize the so-called internal rate of return, defined by

$$(5) \qquad \underset{T}{Max}\ \varrho = \underset{T}{Max}\ \{T^{-1} \log [f(T)(P/WL)]\} = r_i = f'(T_i)/f(T_i),$$

$$= T_i^{-1} \log [f(T_i)(P/WL)] = r_i$$

where

$$f'(T_i)/f(T_i) = T_i^{-1} \log [f(T_i)(P/WL)]$$

$$T_i < T_1 \quad \text{when} \quad r_i > r.$$

Anyone who misguidedly adopts this foolish T_i rotation period will find that he either goes broke or is permanently sacrificing return on original capital that could be his. (To prove that $T_i < T_1$, note that increasing r can be shown to lower T_1; also note that for $r = r_i$, T_1 and T_i would coincide. Hence, the T_1 for smaller r would be greater than T_i. Q.E.D.)

Finally, as Faustmann showed in 1849, the correct description of what will emerge in competitive forest-land-labor-investment equilibrium is an optimal rotation period shorter than the forester's T_n and Thunen-Fisher's T_1, but longer than Boulding's T_i, namely T_∞ as defined by either of the following equivalent formulations.

$$R_\infty \ \underset{T}{Max}\ Rs = \underset{T}{Max}\ R \quad \text{for } s = 1, \text{subject to}$$

(6a)

$$0 = \underset{T}{Max}\ \{Pf(T)e^{-rT} - WL - R\int_0^T e^{-rT}dt\} =$$

$$Pf(T_\infty)e^{-rT_\infty} - WL - R_\infty[1 - e^{-rT_\infty}]r^{-1}$$

or,

$$(6b) \qquad \underset{T}{Max}\ [Pf(T)e^{-rT} - WL][1 + e^{-rT} + (e^{-rT})^2 + \dots]$$

$$= \underset{T}{Max}\ [Pf(T)e^{-rT} - WL]/[1 - e^{-rT}] =$$

$$= \underset{T}{Max}\ (R/r) = [Pf(T_\infty)\, e^{-rT_\infty} - WL]/[1 - e^{-rT_\infty}] = R_\infty/r,$$

land's value, where

$$\text{(6c)} \qquad f'(T_\infty) - rf(T_\infty) = +r[Pf(T_\infty)e^{-rT_\infty} - WL]/[1 - e^{-rT_\infty}]$$

$$= r(R_\infty/r) = r \text{ land value} = R_\infty$$

$$T_i < T_\infty < T_1 \text{ and } T_\infty < T_n \text{ for } R_\infty > 0.$$

The first line of (6b) is the correct Faustmann-Gaffney-Hirshleifer formulation. Its equivalence with the maximum-land-rent formulation of (6a) is seen from solving the last relation of (6a) for R and noting its equivalence with the second relation of (6b) except for the extraneous constant r.

Figure 2 shows the familiar relation among the different rotation periods. Note that a reduction in P/W would lower the curve in the figure until at the zero-land-rent state the line through E with slope r would just touch the new curve at the new *coinciding* points T_i and T_1 and T_∞. It can be shown that, as $r \rightarrow 0.$[2]

DIGRESSION: LABOR AND LAND VARIABLE

The general problem recognizes that Q_{t+T} output can, for each T, be affected by how much labor, L_t, one uses initially and how much land, $s_{t;t+T}$, one uses throughout the time interval t to $t+T$. Hence, we replace $f(T)$ in the steady state by

$$\text{(1')} \qquad Q = f(s, L; T) \equiv \lambda^{-1} f(\lambda s, \lambda L; T)$$

and $f(\ )$ concave in (s, L) jointly; $f(\ )$ can be smoothly differentiable in the neoclassical fashion, or it can take the fixed-coefficients form $f(Min[s/\alpha,\ L/\beta];\ T)$ where $(\alpha,\ \beta)$ are positive constants that can be set equal to unity by proper choice of input units. For brevity, I analyze the neoclassical case.

2. This is better brought out by my maximum rent formulation of (6a) than by Faustmann's infinite number of cycles as in (6b) here. Thus for $r = 0$ (6a) becomes equivalent to

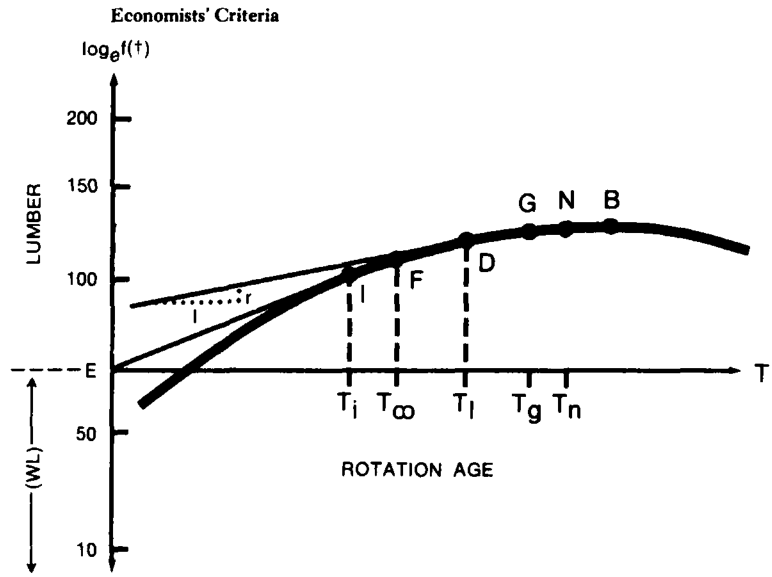
$$\text{subject to } 0 = \underset{T}{Max}\,\{f(T) - (W/P)L - R\int_0^T dt\}$$

Maximize R, namely

$$\underset{T}{Max}\,\{[f(T) - (W/P)L]/T\} = [f(T_n) - (W/P)L]/T_n$$

where T_n is defined by my earlier equation (3). B. Ohlin, I now learn, worked out much of this as a graduate student: cf. Ohlin (1921).

FIGURE 2



Using the rate of interest (r) as a discount factor, economists like von Thünen and Irving Fisher favored cutting trees when percentage growth of gross lumber just equals the interest rate, giving T_1 as optimal age where D's slope just equals r on ratio chart. Boulding and those who say, maximize internal rate of return, select lower T_i where slope of ray through E is at its steepest because of tangency at I. Correct competitive solution is that of Martin Faustmann (1849), which maximizes *present discounted* value over infinity of cycles (not just one cycle as with Fisher): correct T_∞ is between T_i and T_1 and maximizes land rent of steady-state forest. If $r \rightarrow 0$, $T_\infty \rightarrow T_n$, the point at which the net sustained yield is maximized.

Competitive equilibrium requires, for given $(r, W/P)$,

$$(7) \qquad 0 = \underset{T,\, s,\, L}{Max} \; \{f(s, L; T)e^{-rT} - (W/P)L - (R/P)s \int_0^T e^{-rt}\,dt\}$$

$$= f(s_\infty, L_\infty; T_\infty)\, e^{-rT_\infty} - (W/P)L_\infty - (R/P)_\infty s[1 - e^{-rT_\infty}]r^{-1}$$

where $[L_\infty/s_\infty, T_\infty, (R/P)_\infty]$ are roots of

$$(8a) \qquad e^{-rT}\partial f(1, L/s; T)/\partial L = W/P$$

$$(8b) \qquad e^{-rT}\partial f(1, L/s; T)/\partial s = (R/P)[1 - e^{-rT}]r^{-1}$$

$$(8c) \qquad \partial f(1, L/s; T)/\partial T - rf(1, L/s; T) = (R/P)$$

These equations are not all independent; and, of course, even if total land available for forestry, $\bar{s}$, is knowable in advance because such land has no other viable use, we need to know the consumer's demand for labor, and the workers' supply of labor to forestry as against alternative uses, before the extensive scale of (Q_∞, L_∞) are determined. It is worth noting that, in the steady state, there is a fundamental three-variable factor-price frontier of the form

$$r = \psi(W/P, R/P) \tag{9}$$

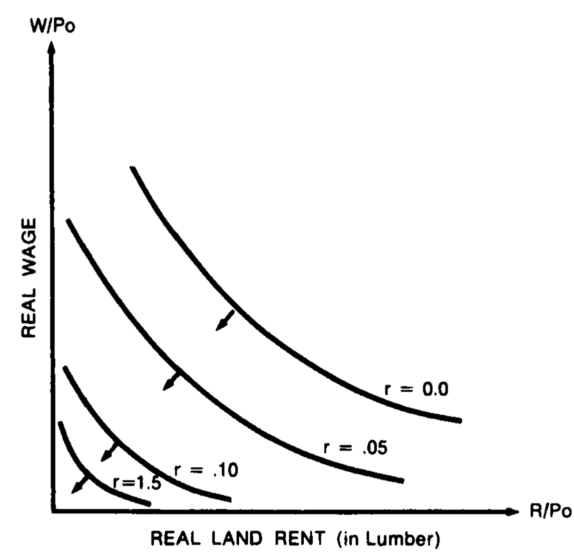
where ψ is a monotone-decreasing function that has contours that are convex. Figure 3 shows such contours for equi-spaced values of r: the fact they are alternately bunched and spread out indicates that, in the Sraffa fashion, the relation between r and any one variable can be wavy, with variable curvature.

SIMPLE GENERAL EQUILIBRIUM: STATIC AND DYNAMIC

An oversimplified case can illustrate the general equilibrium of lumber and other prices, and can show that some efficiency properties are produced by that equilibrium. Suppose the total supply of land is fixed at $s = \bar{s}$, and that it is suitable only for forestry. Suppose the total

FIGURE 3

Factor-Price Frontier



The higher the rate of interest, the lower will be the land rent that can be earned in forestry for each real wage given in terms of lumber price. The tradeoff between such real wage rates and land rents will be the convex contours of frontiers shown here; however, equal increments of profit rate may have quite unequal shift effects upon contours.

steady-state supply of labor is fixed at $\bar{L}$, to be divided between L for forestry and $\bar{L}$-L for the other (composite) good. Let Q be the steady-state output of wood, as produced by the production function in (1′); and let C be the output of the other good, which is producible instantaneously from $\bar{L}$-L alone, by $C = (\bar{L}-L)/c$. Finally, suppose everyone spends his income on lumber and the other good in the same way whether rich or poor, and in the same way as does any other person. Therefore, demand curves can be regarded as generated by the "homothetic" collective utility function, $U[C, Q] \equiv \lambda^{-1} U[\lambda C, \lambda Q]$, where U is a concave first-degree-homogeneous function with standard regularity properties. With the interest rate at which society neither saves nor dissaves given by time preference rate ϱ, so that $r = \varrho > 0$, the full equilibrium is defined by

(10a)
$$C = (\bar{L} - L)/c$$
$$Q = f(\bar{s}, L; T)$$

(10b)
$$W/P_C = c^{-1}$$
$$W/P_Q = e^{-\rho T} \partial f(\bar{s}, L; T)/\partial L$$
$$P_Q/P_C = c^{-1} e^{\rho T} [\partial f(\bar{s}, L; T)/\partial L]^{-1}$$

(10c)
$$\partial f(1, L/\bar{s}; T)/\partial T - \varrho f(1, L/\bar{s}; T) = (R/P_Q)$$

(10d)
$$P_Q/P_C = \frac{\partial U(C, Q)/\partial Q}{\partial U(C, Q)/\partial C}$$

Here (10a) gives the steady-state production functions. (10b) gives the labor and land marginal productivity relations, discounted when necessary, and with the implied steady-state price ratios. (10c) gives the Faustmann optimal-rotation relation to determine T_∞. (10d) gives the needed demand relations. Note that, with T determined, we can use (10a) to express the right-hand side of (10d) in terms of L as the only unknown; substituting the right-hand side of (10b)'s last relation into the left-hand side of (10d), (10d) become one implicit equation for the one unknown, namely L. So an equilibrium does exist (and, under strong sufficiency conditions, it may well be unique). However, is there anything at all socially optimal about this positivistic competitive solution? Is there ever any "intertemporal efficiency" to this market equilibrium? The answer can be shown to be yes in a certain definable sense.

Specifically, imagine a Ramsey planner who maximizes an integral of discounted social utility, with discounting at an exponential rate of time preference, $r = \varrho$, in $\exp(-\varrho t)$. His steady-state optimality relations will be of the exact same form as the steady-state competitive relations.

The following simple example can help to illustrate the general principles involved.

OPTIMAL PROGRAMMING AND FORESTRY ROTATION

Thus, restrict T to only integral values — say to either $T = 1$ or $T = 2$; and replace the equality in (10c) that determines T_∞ by a corresponding inequality condition. And suppose a planner for this society acts to solve a Ramsey (1928) optimal-control problem, namely for t restricted to integral values,

$$(11) \qquad Max \sum_{t=0}^{\infty} U[C(t), Q(t)]e^{-\rho t} \text{ subject to}$$

$$C(t) \leqslant c^{-1}[L - L_1(t) - L_2(t)]$$

$$\bar{S} \geqslant S_1(t) + S_2(t) + S_3(t)$$

$$L_i(t) \geqslant 0, S_i(t) \geqslant 0$$

$$Q(t) = f[S_1(t-1), L_1(t-1); 1] + f[Min[S_2(t-2), S_3(t-1)], L_2(t-2)],$$

with specified initial conditions

$$L_1(-1), S_1(-1), S_2(-2), S_2(-1), S_3(-1).$$

Such a problem is known to have a determinate solution, with implied optimal T_∞ rotation periods that can prevail at each time. And normally it will have the property that as $t \rightarrow \infty$, the optimal solution approaches the "turnpike" defined by the steady-state equations (10) above, once they are modified for discrete time periods. Of course, for still lower ϱ, a different solution will be optimal, and presumably the optimal T_∞, call it $T_\rho = 1$, becomes optimal in the steady state. See Samuelson (1973) for indicative analysis concerning the dynamic aspects of profit-including prices.

NON-STEADY-STATE CONSIDERATIONS

The forester's notion of sustained yield is a steady-state notion. The economist's shorter rotation period for the forest, due essentially to a positive interest rate, is also a steady-state notion. But life is not now in a steady-state. It never was. It never will be. Incessant change is the law of life. You might correctly infer from this that economists' simple notion of stationary equilibrium needs to be generalized and replaced by the notion of a perpetual Brownian motion, as dramatized by the perpetual dance of the colloidal particles one sees in the microscope as they are buffeted to and fro around an average position of equilibrium by ever-present molecules, numerous and random, but unseen. A beginning has been made at the frontier of modern economics toward replacing equilibrium by an *ergodic probability distribution*. Since my time here is limited, I shall only refer to the works of Mirrlees (1965), Brock and Mirman (1972), R. C. Merton (1973), Samuelson (1971). But it is not the purely probabilistic perturbation of equilibrium that is important for the great debate on sustained yield. What is important is the realistic presence of *systematic trends* or *transients*, which move away from one steady-state equilibrium and which need not settle down to a newer one. It is no paradox that steady-state analysis is useful in the understanding of realistic trend analysis.

Foresters are concerned with sustained yield precisely because they have lived in a world where virgin stands have been decimated. It is only too easy to understand why, with new technologies and consumer tastes and with the cheapening of transport of exports to affluent North America and Western Europe, much land that was once devoted here to trees is transferred to other uses.

We have seen that the rotation age in the virgin forest is greater than what competitive enterprise will countenance. Indeed, were it not that, so to speak by accident, historical governments own much timber land, there would be even fewer trees in North America today. Our analysis warns that applying what is sound commercial practice to government's own utilization of public forests, or what is the same thing, renting out public land to private lumbering interests at the maximum auction rent competition will establish — this is a sure prescription for future chopping down of trees.

NO TREES LEFT?

Is this prospect a good or bad thing? That cannot be decided in advance of lengthy discussion. Surely, from the vantage point of the final third of the twentieth century, few will agree with the beginning-of-the-century claim of Dean Fernow (1913) that wood is our most important necessity, second only to food in societal importance. Wood is only wood, just as coal is only coal, plastics are only plastics, and, some would say, as bubblegum is only bubblegum. Proper transient analysis does not justify the implied fear that, once forests cease to be cultivated at maximum sustained yield, the descent is inevitable to the hell of zero timber anywhere in the world where we engage in trade. As wood becomes scarce, it will become more expensive. As it becomes expensive, people will economize on its use. But so long as there remain important needs for wood that people will want to satisfy, the price of wood will rise to the level necessary to keep a viable supply of it forthcoming. This in a sense is a "doctrine of sustained yield," but of course not the traditional forester's doctrine of maximum sustained yield. Indeed, by contrast with deposits of oil, coal peat, and high-concentration ores, all of which are constantly being irreversibly mined, trees bottle up sunshine into cellulose in a reversible cycle.

Nothing said so far should rule out that, in a world where preparation for war is only prudent, governments may have some interest in subsidizing activities that will lower the probability that in emergency times the nation will find itself bereft of steel, uranium, food, energy, and certain kinds of wood. This is not ruled out by sound economics; but it is only fair to mention that economists have a good deal to say by way of criticism of the efficiency with which governments program their subsidies for national defense purposes.

CONSERVATION AND FLOOD CONTROL

When people in a poor society are given a choice between staying alive in lessened misery or increasing the probability that certain species of flora and fauna will not go extinct, it is understandable that they may reveal a preference for the former choice. Once a society achieves certain average levels of well-being and affluence, it is reasonable to suppose that citizens will democratically decide to forego some calories and marginal private consumption enjoyments in favor of helping to preserve certain forms of life threatened by extinction. It is well-known that clearcutting forests is one way of altering the Darwinian environment. Therefore, pursuit of simple commercial advantage in forest management may have as a joint product reversible or irreversible effects upon the environment. When information of these tradeoffs is made available to the electorate, by that same pluralistic process which determines how much shall be spent on defense and other social goods, and how much shall be taxed for interpersonal redistributions of income, the electorate will decide to interfere with laissez-faire in forest management. This might show itself, for example, in forest sanctuaries of some size located in some density around the nation: the optimal cutting age there and indeed the whole mode of timber culture will have little to do with Faustmann copybook algorithm. Or, putting the matter more accurately, I would have to say the future vector of real costs and real benefits of each alternative will have to be scrutinized in terms of a generalization of the spirit and letter of the Faustmann-Fisher calculus.

Everything said about species conservation can, with appropriate and obvious modifications, be said about the programming of a nation's geographical resources to provide benefits for vacationers, campers, sportsmen, and tourists. Even the unspeakable fox hunter is an endangered species, and it is part of the political decision-making process to decide at what sustained level he is to be permitted to flourish.

Beyond pointing out these simple truisms, I need only mention in the present connection that, when a sophisticated cost-benefit calculus is applied to each of these areas, it is unlikely the optimal solution will include many virgin forests located in inaccessible places. Land use is shot through with externalities. Zoning, public regulations, and various use taxes will presumably be the rational way recommended by economists who study these matters. The organization of land use activities is likely, in the good society, to fall heavily inside the walls of government and regulatory authorities; but there seems reason to believe part of the problem can be effectively franchised out to enterprisers motivated by the hope of financial return. So far, in awarding television licenses or gasoline and restaurant franchises on public highways, governments have been disappointing in the efficiency with which they have worked out such arrangements. But this does not necessarily mean that turning over our landscape to the untender mercies of push-shove laissez-faire is better, or more feasible, than improving the efficiency with which the public sector organizes these activities.

Earlier I accepted the denial that externality problems which crop up in fisheries are equally applicable to forest economics. But that was in connection with the forest merely as a producer of cellulose. Ecologists know that soil erosion and atmospheric quality at one spot on the globe may be importantly affected by whether or not trees are being grown at places some distance away. To the degree this is so, the simple Faustmann calculus and the bouncings of the futures contracts for plywood on the organized exchanges need to be altered in the interests of the public (i.e., the interests of both Pareto-optimality and interpersonal-equity). Again, when the implied optimal sustained yield pattern comes to be estimated, it might well involve numerous clusters of trees planted hexagonally over much of the nation's terrain, rather than huge isolated forest reserves.

THE CLAIMS OF POSTERITY

My time is almost up. Yet I've only been able to scratch the surface of what needs to be explored in depth by catholic men of good will. At the least I must conclude by touching on an issue that goes to the heart of the controversy. Suppose that the competitive interest rates which will guide commercial forestry practices turn out to be very far from zero — say, 10 percent or even more. Must one necessarily accept this penalty on the use of time as the untouchable correct rate of discount a good society will want to recognize in its capital and intergenerational decision-making? This is not an easy question to answer. My earlier equation systems (7) and (8) show there are indeed theoretical models from which market solutions emerge which *also agree* with a technocrat's computation of a society's welfare optimum. So perhaps there is some presumption in favor of the market solution, at least in the sense that a vague burden of proof can be put against those who argue for interferences. At least many of today's mainstream economists would so argue.

I personally think the issue is more open. But I do not wish to pronounce on a matter that time does not permit us to do full justice to. Let me simply conclude therefore with some overly brief comments.

1. Economists like Cambridge's Pigou and Ramsey, or such Rawlsian writers as Phelps and Riley (1974), have asserted that we ordinary citizens in our day-to-day and lifetime decision-making about spending, consuming, and saving actually act in *too myopic* a way. If we display time-preference rate of 6 or 10 percent per annum, those rates are not the law of Moses and the Prophets. When we gather together periodically to form social compacts, set down constitutions, and elect representative legislators, a democracy may well decide that government coercion (involving taxing, fiscal changes in the public debt, and control by the central bank of the money supply) ought to alter the trends of capital formation and the amounts of capital bequeathed by each generation to subsequent generations. This is an argument for having lower interest rates at some future date when the policies described have become effective. It is not necessarily an argument for programming the use of publicly-owned forests now with a hypothetical interest rate much lower than interest rates that prevail elsewhere. The latter rates may very well be needed to ration optimally the supply of capital in its actual limited state.

2. There is still some debate among economists as to whether the interest rates appropriate for a government to use should be at all lower than those of private enterprise, and in particular, of the smaller private enterprises and corporations. Marglin (1963) has argued in this fashion, and so in a sense Arrow and Lind (1970) seem to have argued. Hirshleifer (1966) has given arguments against such a dichotomy; Diamond and Mirrlees (1971) have applied the powerful techniques of Ramseyian second-best analysis to analyze the problem. Pending the ultimate verdict of the informed jury in this matter, it seems safe to guess that no simple historical notion of "maximum sustained yield" will be likely to be recommended as optimal.

Whatever else my analysis today may have accomplished, I daresay it will provide corroboration to the old theorem that, when economists and forecasters meet to reason together, economists are likely stubbornly to act like economists. This is an indictment to which I would have to plead guilty, and throw myself on the mercy of your indulgent sentence. Let the penalty fit the crime!

BIBLIOGRAPHICAL NOTES

Rather than burden my text with footnotes, I include here some sketchy comments on previous writings. The notion is ancient that wood is so important and the time periods of forestry are so long that the state cannot leave the matter to commercial laissez-faire. See, for example, the

Roman and German background as discussed in Fernow (1902, 1913); Fernow takes for granted that the ideas of Adam Smith are pernicious when applied to long-lived forests.

The foresters' notion of "sustained yield," with allowable cut to be regulated by how much the average tree age is above or below the optimal age that maximizes steady-state lumber yield per acre is already present in the 1788 "Austrian Cameral Valuation Method." With a little charity, we might interpret this as an attempt to reduce cut of trees below the age T_g at which $f'(T_g) = f(T_g)/T_g$ (as in my Equation (2)), and to encourage cut at older ages.

In the early nineteenth century discounting future receipts at compound interest had reared its head. A momentary 1820 flash of insight by Pfeil, which he later regretted, called for "a rotation based on maximum soil rent" (Fernow 1913, p. 139), as in my Equation (6). Von Thunen (1826, 1966, Hall English edition, p. 121) seems to anticipate the (incorrect!) Jevons-Fisher relation $f'(T_1)/f(T_1) = r$ (of my Equation (4)) in his statement: "When the right methods are adopted, only trees of the same age will stand together; and they will be felled (just?) before the relative increment in their value sinks to (r =) 5 percent — the rate of interest I have assumed to prevail throughout the Isolated State."

The highwater mark comes in 1849 when Martin Faustmann corrects an attempt by E. F. von Gehren to use present discounted values to put a fair price on (1) forest land taken by eminent domain for alternative agricultural uses, and (2) existing forest stands on that land. Because von Gehren uses too long a rotation age for his postulated interest rate, applies bad approximations to true compound interest, and mistakenly values unripe trees at their then-current wood value rather than at their best future value properly discounted, he arrives at wrong and inconsistent results. He concluded, for example, that land value is negative when he subtracted too high a stand value from total land-cum-stand value.

Faustmann corrects all this, applying the infinite cycle formula — maximum $[Pf(T)e^{-rT} - WL][1 + e^{-rT} + \dots]$ for our idealized case — as in my Equation (6b). He shows that evaluating each tree or age-cohort, or evaluating a synchronized forest, must always lead to the *same* result, a truth denied as late as 1951 by Lutz and Lutz (1951, p.33). I rely on the excellent translation of Faustmann and von Gehren given in Gane (1968). In my quick reading, I judge Faustmann to know how to calculate the correct optimal rotation age; but I cannot recall exactly where he has done so, if he has indeed done so.

By this century, Irving Fisher (1906, 1907, 1930) has made present discounted value calculations standard in the economics literature. However, Fisher (1930, p. 161-165) still incorrectly calculates over one cycle rather than over an infinity of repeated cycles, deducing in effect as mentioned the relation for T_1 of my Equation (2), rather than the right Faustmann-Ohlin-Preinrich-Bellman-Samuelson relation for T_∞ of

Equation (6c). Hotelling (1925) also concentrates on one cycle of a machine; and Goundrey (1960) claims the economist writers on forestry like Scott (1955) are still concentrating on one cycle. Lutz and Lutz (1951) give numerous alternatives including the correct infinite-cycle, but they fail to deduce just *when* this correct method is mandatory (and, as noted, they become confused on the synchronized-forest case). Preinrich (1938), Alchian (1952), Bellman (1957) provide more accurate discussion of the infinite-cycle case: but until this present paper, I have not seen an adequate elaboration of the maximum-land-rent aspect of the forestry problem. Hirshleifer (1970, p. 88-90) has the correct Faustmann solution and refers in his work to Gaffney (1957). I have to agree with Gaffney that Fisher is wrong, even though Hirshleifer is right in thinking that his principle of maximizing a *proper* PDV is not wrong. Ohlin (1921), I belatedly discover, gives an exactly correct analysis.

The "internal rate of return," r_i, today quite properly associated with Boulding (1935, 1941 and 3 later editions), was already explicitly or implicitly in Bohm-Bawerk (1889), Fisher (1907), Keynes (1936). Samuelson (1937) and Hirshleifer (1958) have debunked "maximizing" this r_i as a proper goal for decision-making by either a perfect or an imperfect competitor, but the corpse will not stay buried. Under free-entry and perfect competition, maximizing proper PDV *happens when PDV is zero also* to make r_i by tautology at a maximum. The only other possible defense for maximum-r_i is farfetched in any application, but has to my knowledge independently been glimpsed or proved by Boulding, Samuelson, Solow, Gale, and Chipman. If there is available to you a time-phased vector of net algebraic cash receipts, which you can initiate at *any* intensity *with no diminishing returns* (as you force down lumber prices, force up wage rates, run out of free forest lands, and bid up the land rent you must pay!), then any dollars that you initially have can ultimately be made, by investment and reinvestment into the postulated golden goose, to grow proportionally to $e^{r_i t}$; hence, having a higher r_i will ultimately come to dominate any lower r_i. However, under these unrealistic assumptions, r_i will come to form r, the market interest rate, itself; for if one could borrow at $r < r_i$, infinite scale would be optimal for this activity, a "meaningless" situation in a finite world; or, in the present application, the fact that trees grow on finite land will require positive rent payments that undermine any excess of r_i over r.

I found Goundrey (1960) a valuable survey, even if in the end he mistakenly comes out in favor of maximum internal rate of return. For the forestry literature on sustained yield, see items like Shepard (1925) and Waggener (1969).

On the proper discount rate to be used for governmental welfare decisions, see Ramsey (1928), Marglin (1963), Arrow and Lind (1970), Hirshleifer (1966), and Diamond and Mirlees (1971).

REFERENCES

Alchian, A. A., 1952, *Economic Replacement Policy*. The Rand Corporation, Santa Monica, Calif.

Arrow, K. J. and Lind, R. C., 1970, "Uncertainty and the evaluation of public investments," *American Economic Review*, *60*, 354-378.

Austrian Government, 1965, "The Austrian command valuation method," *The Forestry Chronicle*, *41*, 84-92. English translation of the 1896 republication.

Bellman, R., 1957, *Dynamic Programming*. Princeton University Press, Princeton.

Boulding, K. E., 1935, "The theory of a single investment," *Quarterly Journal of Economics*, *49*, 475-494.

__________, 1941, 1948, 1955, 1966. *Economic Analysis*. Harper & Bros., New York.

Brock, W. and L. Mirman, 1972. "The stochastic modified golden rule in a one sector model of economic growth with uncertain technology," *Journal of Economic Theory* (June).

Chipman, J. A., 1972, "A renewal model of economic growth," *SIAM* (January). Also p. 43-83 in Day, R. H. and Robinson, S. M., eds., *Mathematical Topics in Economic Theory and Computation*, Society for Industrial and Applied Mathematics, Philadelphia, 1972.

Crutchfield, J. A. and Zellner, A., 1962, "Economic aspects of the Pacific halibut fishery," *Fishery Industrial Research I.*, U.S. Dept. of the Interior, Washington, D.C.

Diamond, P. A. and Mirrlees, J., 1971, "Optimal taxation and public production I-II," *American Economic Review*, *61*, 8-27, 261-278.

Faustmann, M., 1849, "On the Determination of the Value Which Forest Land and Immature Stands Possess for Forestry," English edition edited by M. Gane, *Oxford Institute Paper 42*, 1968, entitled "Martin Faustmann and the Evolution of Discounted Cash Flow," which also contains the prior 1849 paper by E. F. von Gehren.

Fernow, B. E., 1902, *Economics of Forestry*, Thomas Y. Crowell & Co., New York.

__________, 1913, *A Brief History of Forestry*, 3rd Edition, University Press Toronto, Toronto.

Fisher, I., 1906, *The Nature of Capital and Income*, Macmillan, New York.

__________, 1907. *The Rate of Interest*, Macmillan, New York.

__________, 1930, *The Theory of Interest*, Macmillan, New York, particularly p. 161-165.

Gaffney, M., 1957, "Concepts of financial maturity of timber and other assets," *Agricultural Economics Information Series* 62, North Carolina State College, Raleigh, N.C., September.

Gordon, H. S., 1954, "The economic theory of a common-property resource: The fishery," *Journal of Political Economy*, *62*, 124-142.

Goundrey, G. K., 1960, "Forest management and the theory of capital," *Canadian Journal of Economics*, *26*, 439-451.

Hirshleifer, J., 1958, "On the theory of optimal investment decision," *Journal of Political Economy*, *66*, 198-209.

__________, 1966, "Investment decision under uncertainty: Applications of the state-preference approach," *Quarterly Journal of Economics*, *80*.

__________, 1970, *Investment, Interest and Capital*, Prentice-Hall, Inc., Englewood Cliffs.

Hotelling, H., 1925, "A general mathematical theory of depreciation," *Journal of the American Statistical Association*," *20*, 340-353.

Koopmans, T. C., 1967, "Intertemporal distribution and optimal aggregate economic growth," pp. 95-126 in *Ten Economic Studies in the Tradition of Irving Fisher*, John Wiley & Sons, New York.

Lutz, F. and Lutz, V., 1951, *The Theory of Investment of the Firm* (particularly Chs. 2 & 8). Princeton University Press, Princeton.

Marglin, S. A., 1963, "The social rate of discount and the optimal rate of investment," *Quarterly Journal of Economics*, *77*, 95-111.

Merton, R. C., 1975, "An asymptotic theory of growth under uncertainty," *Review of Economic Studies*, *42*, 375-394.

Mirrlees, J. A., "Optimum accumulation under uncertainty," Unpublished MS, December 1965.

Nordhaus, W. D., 1974, "The falling share of profits," *Brookings Papers on Economic Activity*, Okun, A. M. and Perry, G. L., eds., pp. 167-217. The Brookings Institution, Washington, D.C.

Ohlin, B., 1921, "Till fragen om skogarnas omloppstid," *Ekonomisk* Tidskrift, "Festschrift to Knut Wicksell."

Phelps, E. S., and Riley, J. G., "Rawlsian growth: Dynamic programming of capital and wealth for intergenerational maxi-min justice," Columbia University and UCLA. (Paper for private circulation, early 1974).

Preinrich, G. A. D., 1938, "Annual survey of economic theory: The theory of depreciation," *Econometrica*, *6*, 219-241.

Ramsey, F. P., 1928, "A mathematical theory of saving," *Economic Journal*, *38*, 543-559.

Samuelson, P. A., 1937, "Some aspects of the pure theory of capital," *Quarterly Journal of Economics*, *51*, 469-496. Also reproduced in Stiglitz, J. E., ed., *Collected Scientific Papers of Paul A. Samuelson*, *1*, pp. 161-188, M.I.T. Press, Cambridge, Mass. 1966.

__________, 1964, "Tax deductibility of economic depreciation to insure invariant valuations," *Journal of Political Economy* (December) 604-606. Also pp. 571-573 in Merton, R. C., ed., *Collected Scientific Papers of Paul A. Samuelson*, 3, M.I.T. Press, Cambridge, Mass.

__________, 1971, "Stochastic speculative price," *Proceedings of the National Academy of Sciences*, U.S.A., *68*, 335-337. Also pp. 894-896 in Merton, R. C., ed., *Collected Scientific Papers of Paul A. Samuelson*, M.I.T. Press, Cambridge, Mass. 1972.

__________, 1973, "Reply on Marxian matters," *Journal of Economic Theory*, *11*, 64-67.

__________, 1973, "Optimality of Profit-Including Prices Under Ideal Planning," *Proc. Nat. Acad. Sci. USA*, *70*, 2109-2111.

Scott, A., 1955, "The fishery: The objectives of sole ownership," *Journal of Political Economy*, *63*, 116-124.

__________, 1955, *Natural Resources: The Economics of Conservation*, University of Toronto Press, Toronto.

Shepard, W., 1925, "The bogey of compound interest," *Journal of Forestry*, 23.

Smith, V. L., 1968, "Economics of production from natural resources," *American Economic Review*, 58, 409-431.

Thunen, J. H. von, 1826, *Isolated State*, English edition edited by Peter Hall, 1966. Pergamon Press, London.

Waggener, T. R., 1969, "Some economic implications of sustained yield as a forest regulation model," University of Washington, Contemporary Forestry Paper 6.