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Depletion of fossil fuels and the impacts of global warming

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Abstract

This paper combines the theory of optimal extraction of exhaustible resources with the theory of greenhouse externalities, to analyze problems of global warming when the supply side is considered. The optimal carbon tax will initially rise but eventually fall when the externality is positively related to the stock of carbon in the atmosphere. It is shown that the tax will start falling before the stock of carbon in the atmosphere reaches its maximum. If there exists a non-polluting backstop technology, it will be optimal to extract and consume fossil fuels even when the price of fossil fuels is equal to the price of the backstop. The total extraction is the same as when the externality is ignored, but in the presence of the greenhouse effect, it will be optimal to slow the extraction and spread it over a longer period. If, on the other hand, the greenhouse externality depends on the rate of change in the atmospheric stock of carbon, the evolution of the optimal carbon tax is more complex. It can even be optimal to subsidize carbon emissions to avoid future rapid changes in the stock of carbon, and therefore future damages.

JEL classification: D62; Q32

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1. Introduction

An important environmental challenge during the next decades will be to reduce the impacts of global warming. Carbon dioxide (CO_2) is the main greenhouse gas, and 70–75% of all CO_2 emissions is due to combustion of fossil fuels (see for example Halvorsen et al., 1989). In most papers analyzing the economics of global warming, the supply of fossil fuels is modelled like any other good, and the exhaustibility of these resources is not considered. There have been many studies on externalities from fossil fuel consumption and the depletion of exhaustible resources (see, e.g., Dasgupta and Heal, 1979; Baumol and Oates, 1988; Pearce and Turner, 1990), but few authors have tried to combine these two theories. Among the first studies is Sinclair (1992) who analyzes the impacts on consumer and producer prices of a constant carbon tax rate in addition to the development of an optimal carbon tax. He argues that in steady state, the ad valorem carbon tax should be falling over time. This is followed up in Ulph and Ulph (1994). The authors study the evolution of an optimal carbon tax using quadratic benefit and damage functions. They find that the carbon tax (in both absolute and ad valorem terms) should initially rise but eventually fall. Sinclair's conclusion, they argue, is driven by very special assumptions. Devarajan and Weiner (1991) use a two-period model to analyze the importance of international global warming agreements, assuming that the consumption of fossil fuels in period two is the difference between the initial stock of fossil fuels and period one consumption. Finally, Withagen (1994) compares the optimal rate of fossil fuel depletion without greenhouse externality with the case where this externality is present.

In this paper we combine the theory of greenhouse externalities with the theory of exhaustible resources using the framework of a simple model described in Section 2. In line with Heal (1976), the model is one of economic exhaustion (zero long-term Hotelling rent) rather than physical exhaustion. The problems analyzed are the design of the optimal policy response under different damage functions; one in which the damage is due to the level of global warming, and the other where damage is related to the speed of climate change. As the optimal policy response to global warming may depend on how we specify the damages, it is important to study the optimal carbon tax under the different specifications of the damage function.

Most studies analyzing the damage from global warming, specify the damage as a function of the temperature level or alternatively the atmospheric concentration of greenhouse gases (GHGs). In Section 2, we apply this damage function to derive the optimal carbon tax under the exhaustibility assumption. First, the optimal policy response is analyzed without considering possible substitutes for fossil fuels. However, even today there exist several alternatives to fossil fuels, albeit at higher costs. The traditional result from the theory of a competitive mining industry facing a backstop technology with a constant unit cost of extraction less than the choke price, is that the industry will deplete the resource until the price reaches the cost of the backstop. At this price, the resource is exhausted, and the consumers will switch immediately to the backstop. With costs increasing in accumulated production, the results are similar (see Heal, 1976). Introducing external greenhouse effects, will, however, give some new results described in Section 3.

Because the ecology can adapt to a certain change in the climate, given that the rate of change is not too high, it can be argued that what matters is the speed of climate change (e.g., the rate of change in temperature or atmospheric concentration) and not only the level of temperature itself. Thus, in Section 4 the optimal carbon tax is derived when the damage depends on the speed of climate change.

The damage from global warming probably depends on both the level of the climate as well as the rate of climate change. Therefore, damage functions taking into account only one of these elements must of course be considered as extreme cases. Analyzing the two elements separately, however, points out some main features.

We summarize the conclusions in Section 5.

2. Optimal depletion in the presence of global warming

Let x be the extraction (and consumption) of all fossil fuels in carbon units. The benefits of the society from fossil fuel consumption, $u(x)$, are assumed to increase in current consumption ($u'(x) > 0$ for all x), but the marginal utility is bounded above ($u'(0) < \infty$) and decreasing ($u''(x) < 0$). Define $u(x) = \int_0^x p(s)\mathrm{d}s$, where $p(x)$ is the consumer price. The utility function, $u(x)$, is thus identical to the total willingness to pay, and the marginal utility equals the consumer price ($u'(x) = p$). [1]

We define $A_t = \int_0^t x_\tau \mathrm{d}\tau$ as the accumulated extraction of fossil fuels up to time t. The total extraction cost, $c(A)x$, increases both with the current extraction rate ($c(A) > 0$ for all A) and the cumulative extraction up to date ($c'(A)x > 0$ for $x > 0$). Moreover, we assume that $c'(A)$ is bounded away from zero (i.e., that $c'(A) > \epsilon$ where ϵ is some small positive number). No fixed quantity is assumed for the total availability of the resource (as for instance in Dasgupta and Heal, 1979). However, in line with Farzin (1992), only a limited total amount will be economically recoverable at any time. This is due to the assumption $c'(A)$ is bounded away from zero, which means that increasingly large quantities of the fossil fuel resource can be exploited only at increasingly higher incremental costs. With $c(A) \to \infty$ as $A \to \infty$, it will be optimal to extract a finite amount of the resource since the marginal utility is bounded above. As we shall see, the

[1] Both utility and costs are measured in 'money', i.e., in an unspecified numeraire good.

cumulative extraction along the optimal path reaches an upper limit, $\bar{A}$, defined by $c(\bar{A}) = u'(0)$, as $t \to \infty$.

S is the atmospheric stock of carbon in excess of the preindustrial stock, and $D(S)$ expresses the negative externality of carbon consumption. Thus, the damage from global warming is a function of the atmospheric stock of carbon. We have not taken into account the lags between the atmospheric stock of carbon and the climate, a lagged adjustment process which is due to the thermal inertia of oceans (see Houghton et al., 1990, 1992). The stock is increasing in fossil fuel combustion, but decays with the depreciation rate $\delta > 0$, according to the atmospheric lifetime of CO_2. This is of course a simplification. The depreciation rate is probably not constant, but will decrease with time since a possible saturation of the carbon-sink capacity of the oceans as they get warmer, will give a longer lifetime of CO_2 in the atmosphere (see Houghton et al., 1990, 1992). However, we stick to this assumption, since it is widely employed in economic studies of global warming, see, e.g., Nordhaus (1991, 1993), Peck and Teisberg (1992), Ulph and Ulph (1994) and Withagen (1994). The preindustrial stock is assumed to be an equilibrium stock, meaning that the atmospheric stock will approach the preindustrial level in the long run ($S \to 0$) when fossil fuels are exhausted. We assume that the damage can be described by an increasing and convex function of the atmospheric stock (i.e., $D'(S) > 0$ and $D''(S) > 0$ for all $S > 0$), [2] but that it is negligible for the initial stock increase ($D'(0) = 0$). There is no irreversible damage, which means that $D(0) = 0$.

We assume that the social planner maximizes the present value of welfare to the global society, i.e., he seeks an extraction path of fossil fuels which will

$$\begin{aligned} &\textit{maximize} \int_0^\infty e^{-rt} \cdot \left[u(x_t) - c(A_t)x_t - D(S_t) \right] dt \\ &\text{s.t.} \quad \dot{A}_t = x_t, \\ &\qquad \dot{S}_t = x_t - \delta S_t, \\ &\qquad x_t \ge 0 \end{aligned} \tag{1}$$

where $r > 0$ is a discount rate which is fixed throughout the analysis. Moreover, we set $A_0 = 0$, and from the definition of S it follows that $S_0 > 0$.

[2] In several papers (see, e.g., Peck and Teisberg, 1992, Nordhaus, 1993 and Kverndokk, 1994) the damage is specified as a convex function of the temperature level. We simplify the matters since the temperature is positively related to the atmospheric stock of carbon. However, if the damage is a convex function of the temperature level, it is not necessary a convex function of the atmospheric stock of carbon since the relation between the temperature level and the stock is assumed to be logarithmic (see, e.g., Houghton et al., 1990, 1992).

The optimization problem above is solved using optimal control theory. The current value Hamiltonian is

$$H = u(x_t) - c(A_t)x_t - D(S_t) + \lambda_t x_t + \mu_t \cdot (x_t - \delta S_t) \tag{2}$$

and the necessary conditions for an optimum are

$$\frac{\partial H}{\partial x_t} = u'(x_t) - c(A_t) + \lambda_t + \mu_t \le 0 \quad (=0 \text{ for } x_t > 0), \tag{3}$$

$$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t, \tag{4}$$

$$\dot{\mu}_t - r\mu_t = -\frac{\partial H}{\partial S_t} = D'(S_t) + \delta\mu_t, \tag{5}$$

$$\dot{A}_t = x_t, \tag{6}$$

$$\dot{S}_t = x_t - \delta S_t, \tag{7}$$

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\lambda_t = 0, \tag{8}$$

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\mu_t = 0. \tag{9}$$

From (4) and (5) it follows that λ_t and μ_t must be non-positive, as (8) and (9) otherwise would be violated. From (3) it therefore follows that

$$u_t'(x_t) \ge c(A_t) \quad \text{for all } t. \tag{10}$$

This inequality in turn implies that we must have

$$A_t \le \bar{A} \quad \text{for all } t \tag{11}$$

where $\bar{A}$ is defined by

$$c(\bar{A}) = u'(0). \tag{12}$$

Since A_t is bounded above, x_t must approach zero in the long run. Assuming $c'(A)$ finite for all A we thus have

$$\lim_{t\to\infty} x_t = 0,$$

$$\lim_{t\to\infty} c'(A_t)x_t = 0. \tag{13}$$

The solution to the model can be expressed by the consumer price p. Using $u'(x) = p$ and (3), it is clear that whenever we have positive extraction, the consumer price is equal to the sum of the producer price and a carbon tax θ. The optimal carbon tax at time t, θ_t, is defined as the negative of μ_t, where μ_t (<0) is the shadow cost associated with accumulated atmospheric stock up to t. The producer price is the sum of the marginal extraction cost, $c(A)$, and the scarcity rent π. As λ_t (< 0) is the shadow cost associated with cumulative extraction up to t, the scarcity rent at time t, π_t, is defined as the negative of λ_t. Eq. (3) may thus be rewritten as

$$p_t = c(A_t) + \pi_t + \theta_t \equiv c(A_t) - \lambda_t - \mu_t. \tag{14}$$

To see how the price p_t and extraction rate x_t of fossil fuels develop over time, we first find how π_t and θ_t develop. Consider first the scarcity rent π_t. Using the definition $\pi_t = -\lambda_t$ together with the differential Eq. (4) and the transversality condition (8) we get

$$\pi_t = \int_t^{\infty} e^{-r(\tau - t)} c'(A_\tau) x_\tau \, d\tau. \tag{15}$$

In Appendix A, it is shown that

$$\lim_{t \to \infty} \pi_t = 0. \tag{16}$$

The scarcity rent thus converges to zero for $t \to \infty$. This is due to the increasing marginal extraction cost; The costs will be so high that an additional unit of fossil fuels extracted will not add to the welfare.

As for the optimal carbon tax θ, it follows from the definition $\theta_t = -\mu_t$ together with the differential Eq. (5) and the transversality condition (9) that

$$\theta_t = \int_t^{\infty} e^{-(r+\delta)(\tau - t)} D'(S_\tau) \, d\tau \tag{17}$$

i.e., the optimal carbon tax is always equal to the discounted future negative externalities due to accumulation of carbon in the atmosphere. The expression for the optimal carbon tax is not dependent on exhaustibility. However, the following properties of the tax rest on the exhaustibility assumption (see Appendix A):

$$\lim_{t \to \infty} \theta_t = 0, \tag{18}$$

$$\dot{\theta}_t = \int_t^{\infty} e^{-(r+\delta)(\tau - t)} D''(S_\tau) \cdot \dot{S}_\tau \, d\tau, \tag{19}$$

$$\lim_{t \to \infty} \dot{\theta}_t = 0. \tag{20}$$

According to (18), the optimal carbon tax converges to 0 for $t \to \infty$. The intuition behind this is as follows. As $S \to 0$ for $\delta > 0$, there will be no cost associated with a marginal increase in the atmospheric stock when $t \to \infty$, thus $\mu \to 0$. As the shadow cost reflects the optimal carbon tax, this will converge to zero for $t \to \infty$.

From (17) we see that the carbon tax is positive as long as marginal damage is positive, that is as long as $S > 0$. It will smoothly approach zero in the long run as the stock of carbon in the atmosphere decays and reaches the equilibrium stock (see (18) and (20)). This means that the tax will be positive even if the atmospheric stock declines after a certain time due to low extraction and consumption of fossil fuels; the decay of carbon in the atmosphere is higher than the additional carbon from new extraction ($x < \delta S$).

Before considering the detailed development of the carbon tax, we now consider how the fuel price p_t, and thus fuel extraction x_t, develop over time. Consider first total cumulative extraction over our infinite time horizon. From (3), (16) and (18) we have

$$\lim_{t\to\infty} \left[u'(x_t) - c(A_t)\right] \leq 0. \tag{21}$$

Together with (10) and (13) this implies that

$$\lim_{t\to\infty} A_t = \bar{A} \tag{22}$$

where $\bar{A}$ is defined by (12). In other words, total cumulative extraction over the infinite time horizon is equal to $\bar{A}$, independent of how much environmental harm the extraction causes. The environmental cost function thus only effects *when* extraction takes place.

Consider next the price of the resource. From (14) and (6) we see that

$$\dot{p}_t = c'(A_t)x_t + \dot{\pi}_t + \dot{\theta}_t. \tag{23}$$

Inserting (4) and $\pi_t = -\lambda_t$ thus gives

$$\dot{p}_t = r\pi_t + \dot{\theta}_t. \tag{24}$$

In the absence of a carbon tax (implying $\dot{\theta}_t = 0$), the fuel price must therefore rise over time whenever we have positive extraction (since $\pi_t > 0$). With a carbon tax, however, we cannot rule out the possibility that p_t will decline, i.e., that there will be increasing extraction, over some intervals. For this to be the case, the carbon tax must be falling at a sufficiently rapid rate during such intervals. [3]

Whenever resource extraction is positive, it moves in the opposite direction of the fuel price. From the result above it is thus clear that in the absence of a carbon tax, resource extraction must always be declining. With a carbon tax, however, we cannot rule out the possibility that there will be increasing extraction over some intervals. We cannot rule out an initial phase of $x_t = 0$, which may occur if the

[3] To have a better understanding of the conditions under which p might decline over time, we can study the importance of r and δ. If $r = 0$, we see from (24) that $\dot{p} = \dot{\theta}$, thus the price increases when the tax increases and falls when the tax falls. For $(r + \delta) = 0$, i.e., if there is no discounting and the lifetime of CO_2 in the atmosphere is infinite, Eq. (19) is no longer valid. In that case we can use the condition $\dot{\theta} = (r+\delta)\theta - D'(S)$, which is derived from Eq. (5) and $\theta = -\mu$, and is valid for all values of $(r + \delta)$. For $(r + \delta) = 0$, we get $\dot{\theta} = -D'(S)$, that is, the carbon tax is always falling and, therefore, has its maximum value at $t = 0$. The intuition is the following. Since we have no discounting and the carbon will stay in the atmosphere forever, a unit of carbon emitted today is more damaging than a unit emitted in the future, simply because it stays around causing damage for a longer period. In this case $\dot{p} = -D'(S)$, and the price will therefore fall over the entire time horizon.

initial stock of carbon in the atmosphere is sufficiently high. However, in Appendix A it is shown that once x_t is positive, it will be positive for ever after.

We now consider the development of the optimal carbon tax in more detail. The behaviour of the carbon tax over time depends on whether the carbon stock is increasing or decreasing (see (19)). From (19) and $D'' > 0$ it is clear that if S_t is declining from a time t_2 onwards, then θ_t will be declining from a time t_1 onwards, where $t_1 < t_2$ if $t_2 > 0$ and $r + \delta < \infty$. Moreover, it is shown in Appendix A that once θ_t starts to decline (as we know from (18) that it will sooner or later), it will continue to decline for ever.

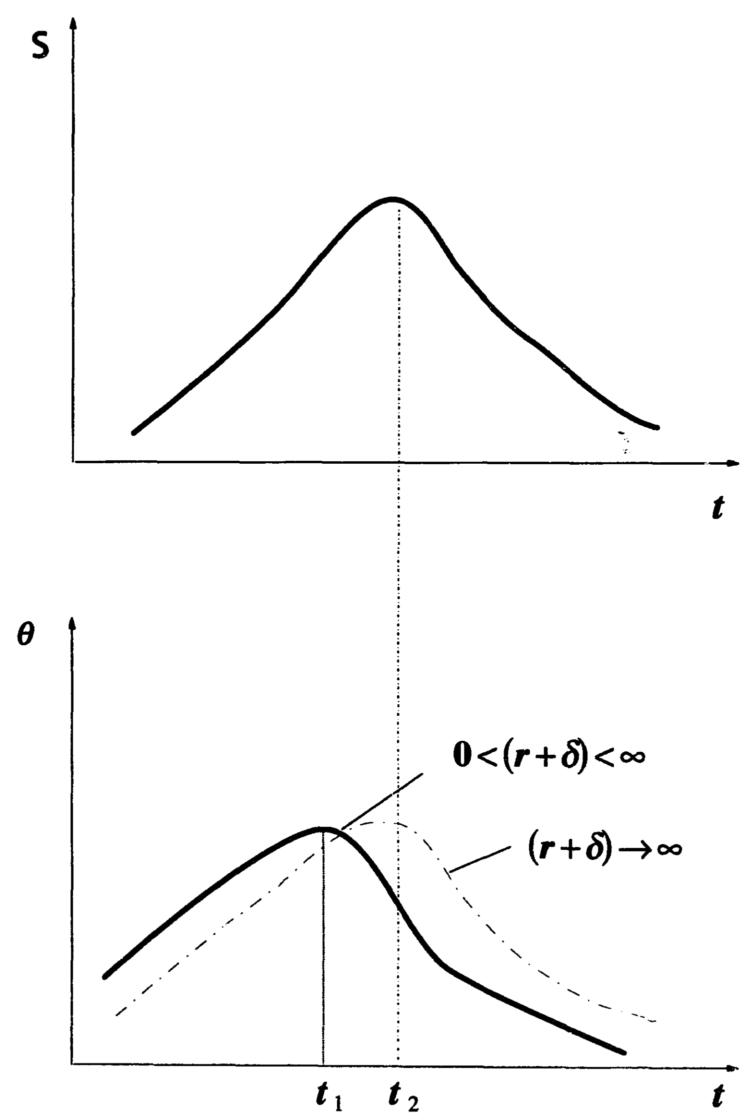

Fig. 1. The atmospheric stock of carbon and the optimal carbon tax when damage is related to the level of the stock.

Fig. 1 shows the evolution of the optimal carbon tax over time for the case when the stock of carbon in the atmosphere initially is increasing, after which it declines monotonically. As long as $0 < (r + \delta) < \infty$, the optimal tax will start to decline before the stock of carbon in the atmosphere reaches its peak. This means that if the carbon stock in the atmosphere is ever increasing, there exists a period with a falling carbon tax and an increasing stock of carbon in the atmosphere. When the stock of carbon declines ($\dot{S} < 0$), the optimal carbon tax will definitely be falling.[4] The intuition behind this is as follows. We know that the marginal damage will be higher the larger is the stock of carbon in the atmosphere. Assume first that $r \to \infty$. Then, only damage in the current period counts, and the tax will therefore be equal to the current marginal damage. Thus, the tax will increase as long as $\dot{S} > 0$ and fall for $\dot{S} < 0$. For $r < \infty$ (and $\delta < \infty$), that is, the social planner is not totally myopic and the lifetime of CO_2 in the atmosphere is not zero, future damages will also count. Since the marginal damage will start falling in the future due to a decreasing carbon stock, a unit emitted immediately before S reaches its maximum creates more damage than a unit emitted when S is at its maximum point. On the other hand, a unit of carbon emitted in the beginning of the planning period stays in the atmosphere when the carbon stock is low. Thus, this unit creates less damage than a unit emitted when S is at its maximum. The optimal carbon tax, which reflects all damages made by a unit carbon emitted, will therefore reach its maximum before the time giving the largest stock of atmospheric carbon. This is illustrated in Fig. 1. In this figure it is assumed that the carbon tax rises initially, while it falls from time t_1 onwards when the stock of carbon is still increasing. The evolution of the carbon tax is consistent with the results of Ulph and Ulph (1994), who argue that the optimal carbon tax might initially rise but will eventually fall. However, they do not relate the behaviour of the optimal tax to the evolution of the stock of carbon in the atmosphere, but only claim that the carbon tax should definitely be falling once fossil fuels are exhausted.

3. The optimal depletion with a non-polluting backstop technology

So far, we have not explicitly considered the existence of substitutes for fossil fuels. However, substitutes such as nuclear power, hydro power, biomass, solar and wind power already exist. Assume that there exists a non-polluting perfect substitute for fossil fuels, y, with an unlimited stock and a constant unit cost, $\bar{c}$, where $c(0) < \bar{c} < u'(0)$. In a competitive market, the price will be equal to this cost since there are no stock constraints. By definition, a backstop source is available in unlimited quantities at a constant marginal cost. The traditional result from the

[4] For $(r + \delta) = 0$, see footnote 2.

theory of a competitive mining industry facing a backstop technology with the constant unit cost less than the choke price, but higher than the initial marginal extraction cost, is that the industry depletes the resource until the price reaches the cost of the backstop. At this price, the consumers will switch immediately to the backstop, cf. Appendix B. (This result is also derived in for example Heal, 1976, and Dasgupta and Heal, 1979. But Dasgupta and Heal, 1979, assume a fixed quantity of the exhaustible resource.) The extraction path will be different in the presence of external greenhouse effects. By introducing the non-polluting backstop into the model from Section 2, the social planner seeks to

$$\begin{aligned} &\text{maximize} \int_0^\infty e^{-rt} \cdot \left[u(x_t + y_t) - c(A_t)x_t - \bar{c}y_t - D(S_t) \right] dt \\ &\text{s.t.} \quad \dot{A}_t = x_t, \\ &\qquad \dot{S}_t = x_t - \delta S_t, \\ &\qquad x_t \geq 0, \\ &\qquad y_t \geq 0. \end{aligned} \tag{25}$$

The current value Hamiltonian for this optimization problem is thus

$$H = u(x_t + y_t) - c(A_t)x_t - \bar{c}y_t - D(S_t) + \lambda_t x_t + \mu_t \cdot (x_t - \delta S_t). \tag{26}$$

Hence, the necessary conditions for an optimum are

$$\frac{\partial H}{\partial x_t} = u'(x_t + y_t) - c(A_t) + \lambda_t + \mu_t \leq 0 \quad (= 0 \text{ for } x_t > 0), \tag{27}$$

$$\frac{\partial H}{\partial y_t} = u'(x_t + y_t) - \bar{c} \leq 0 \quad (= 0 \text{ for } y_t > 0), \tag{28}$$

$$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t, \tag{29}$$

$$\dot{\mu}_t - r\mu_t = -\frac{\partial H}{\partial S_t} = D'(S_t) + \delta\mu_t, \tag{30}$$

$$\dot{A}_t = x_t, \tag{31}$$

$$\dot{S}_t = x_t - \delta S_t, \tag{32}$$

$$\lim_{t \to \infty} e^{-rt}\lambda_t = 0, \tag{33}$$

$$\lim_{t \to \infty} e^{-rt}\mu_t = 0. \tag{34}$$

In accordance with Section 2, we define $u(z) = \int_0^z p(s) ds$, $z = x + y$, where $p(z)$ is the consumer price. Then, $u'(x + y) = p$. Using (27), (28), $\pi = -\lambda$ and

$\theta = -\mu$, we see that along the optimal path, the consumer price has to satisfy the following conditions:

$$p(x_t + y_t) \leq c(A_t) + \pi_t + \theta_t \quad (= \text{for } x_t > 0), \tag{35}$$

$$p(x_t + y_t) \leq \bar{c} \quad (= \text{for } y_t > 0). \tag{36}$$

The detailed development of π_t and θ_t are much the same as what we found in Section 2. In particular, by proceeding as we did in Section 2, it can be shown that Eq. (13) and (15)–(20) are valid also for the present case. The same is true for Eq. (22), except that $\bar{A}$ is now defined by

$$c(\bar{A}) = \bar{c}. \tag{37}$$

It follows from these equations that the total amount extracted is independent of whether or not there is an environmental externality. However, the optimal extraction path depends on the externality. It is shown in Appendix B that the transition from resource extraction to substitute production in the presence of an environmental externality is gradual. Moreover, it follows from this proof that although $x_t \to 0$ as $t \to \infty$, extraction will be positive for all finite t (except possibly during an initial period, when it will be zero if S_0 is sufficiently high).

During the period of transition, i.e., when both x_t and y_t are positive, it follows from (35) and (36) that

$$c(A_t) + \pi_t + \theta_t = \bar{c}. \tag{38}$$

Differentiating and using (6) and (29) gives

$$\dot{\theta}_t = -r\pi_t < 0, \tag{39}$$

that is, the optimal tax is falling with the same rate as the producer price is increasing, namely the discount rate times the scarcity rent. The intuition of the gradual transition is as follows. If we stop depleting fossil fuels when $p_t = \bar{c}$, the stock of carbon in the atmosphere will decrease ($\dot{S}_t = -\delta S_t < 0$), and the potential optimal carbon tax will fall (see (19)). This means that the consumer price of fossil fuels will fall below $\bar{c}$, which again makes fossil fuels economically viable. Therefore, it will not be optimal to stop the extraction of fossil fuels. Thus, while the scarcity rent reaches zero when the extraction costs reaches the unit cost of the backstop in absence of externalities (see, e.g., Heal, 1976), the scarcity rent is positive over the entire time horizon in the presence of externalities.

Before $p_t = \bar{c}$ is reached, it follows from (6), (29) and (35) that

$$\dot{p}_t = r\pi_t + \dot{\theta}_t. \tag{40}$$

Just as in the case without any backstop technology, we may have periods of a declining resource price, provided the carbon tax declines at a sufficiently rapid rate.

The optimal consumer price path is illustrated in panel A of Fig. 2 (in this figure the consumer price is assumed never to decline). The consumer price of

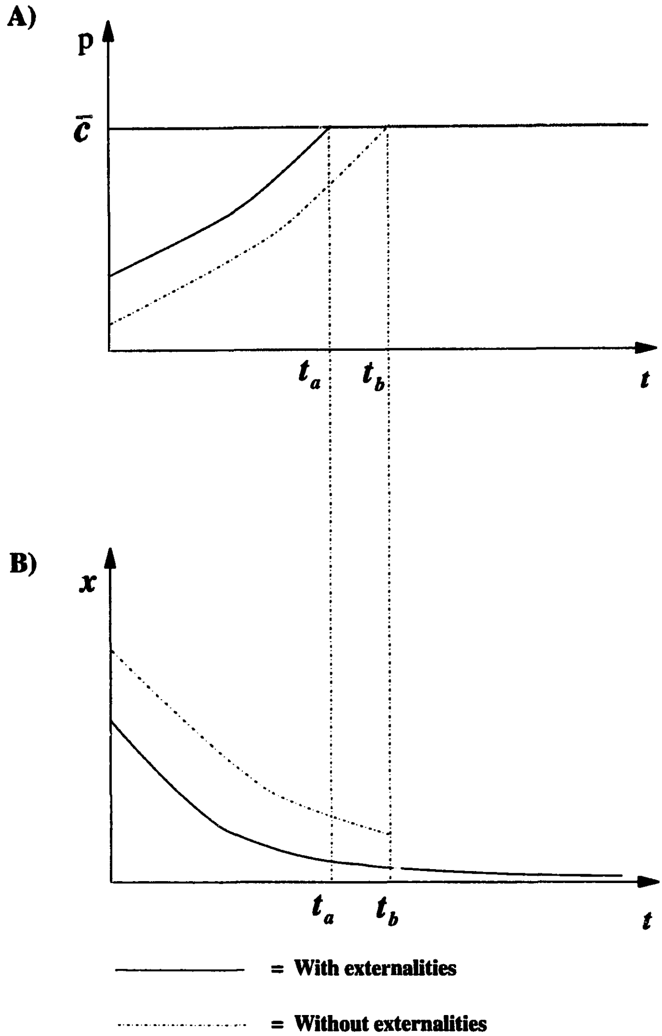

Fig. 2. Optimal extraction and consumer prices with a backstop technology.

fossil fuels is constant and equal to the consumer price of the backstop technology (which is the unit cost) from time t_a onwards (see Fig. 2.A).

Fig. 2.B illustrates the optimal extraction path of fossil fuels. With the greenhouse externality, both fossil fuels and the backstop technology will be consumed from time t_a onwards. The advantage of consuming fossil fuels instead of the backstop is the lower production costs. The disadvantage is, however, the external effects. As the optimal tax approaches zero as time goes to infinity, the optimal fossil fuel extraction will also fall and approach zero in the long run.

In Fig. 2, we have also compared the extraction path and the consumer price with the corresponding paths in the absence of external effects. As the total extraction is the same with the external effect as without, and the consumer price reaches $\bar{c}$ in finite time in both cases, accumulated extraction at time t_b is higher in the absence of external effects than with the external effects. Therefore, the extraction path without externalities will be higher than the path were greenhouse externalities are present (see Fig. 2.B). In the same way, in Fig. 2.A it is seen that the price path without externalities will be lower than the corresponding path with externalities.

4. Environmental damage as a function of the rate of change in the atmospheric stock of carbon

Most studies analyzing the damage from global warming assume that the damage is related to the temperature level (see, e.g, Nordhaus, 1991, 1993; Peck and Teisberg, 1992; Kverndokk, 1994). However, it can be argued that the damage will depend as much on the rate of temperature change as on the absolute value itself, because the ecology is able to adapt to a certain temperature change. There are however costs of adapting to a new stage (see e.g. Tahvonen, 1995). If the rate of climate change is high, there may be a period of large damage until the original species have been replaced by more resistant ones. While for instance agriculture may be quite adaptable to climate over the long term, unmanaged ecosystems are much less adaptable to change in temperature. Another example is human beings, who are capable of adjusting to climatic variations, and can live under more or less every climatic condition existing on earth. However, rapid changes in climate have impacts on human amenity, morbidity and mortality (see Fankhauser, 1992; Cline, 1992). Some first attempts in this direction have been made by Tahvonen (1995), Tahvonen et al. (1994) and Hoel and Isaksen (1995). These papers, however, do not take into account the exhaustibility of fossil fuels.

It is reasonable to believe that there are costs of adapting to a colder ($\dot{S} < 0$) as well as a warmer climate ($\dot{S} > 0$), and that these costs increase the higher is the rate of climate change. Therefore, we assume that the damage is convex in the rate of atmospheric carbon accumulation. Further, there are no damage costs for a constant climate ($\dot{S} = 0$), independent on the level of atmospheric concentration. Hence, if $d(\cdot)$ is the damage function, we have $d(\dot{S}) > 0$ for $\dot{S} \neq 0$, and $d(0) = 0$. We also assume that the damages are negligible for marginal changes in the stock under a constant climate ($\dot{S} = 0$), giving $d'(0) = 0$. The convexity of damages means:

$$\begin{aligned} &d'(\dot{S}) > 0 \text{ for } \dot{S} > 0 \ \wedge \ d'(\dot{S}) < 0 \text{ for } \dot{S} < 0 \\ &d''(\dot{S}) > 0 \text{ for all } \dot{S} \end{aligned} \tag{41}$$

It should be noted that our way of modelling the fact that climate *change* is important to the environment is rather crude: It could be argued that it is not the current rate of temperature change at any particular moment (represented by $\dot{S}$) which is important, but rather the speed at which the climate has been changing over several decades. Our approach is particularly dubious for the beginning of a period of $\dot{S} < 0$ after a long period of $\dot{S} > 0$. In this case, one could argue that since the environment has not yet adapted to the warmer climate, it is better for the environment to have a declining S_t over a moderate period of time than having S_t constant. For an attempt to introduce climate change in a more realistic (and more complicated) way than we have done here, see Hoel and Isaksen (1995). [5]

With the damage function above, the optimization problem is

$$\begin{aligned} &\text{maximize} \int_0^{\infty} \mathrm{e}^{-rt} \cdot \left[u(x_t) - c(A_t)x_t - d(\dot{S}_t)\right] \mathrm{d}t \\ &\text{s.t.} \quad \dot{A}_t = x_t, \\ &\qquad \dot{S}_t = x_t - \delta S_t, \\ &\qquad x_t \geq 0. \end{aligned} \tag{42}$$

The current value Hamiltonian for this problem is

$$H = u(x_t) - c(A_t)x_t - d(x_t - \delta S_t) + \lambda_t x_t + \gamma_t \cdot (x_t - \delta S_t). \tag{43}$$

The necessary conditions for an optimum are:

$$\frac{\partial H}{\partial x_t} = u'(x_t) - c(A_t) - d'(\dot{S}_t) + \lambda_t + \gamma_t \leq 0 \quad (= 0 \text{ for } x_t > 0), \tag{44}$$

$$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t, \tag{45}$$

$$\dot{\gamma}_t - r\gamma_t = -\frac{\partial H}{\partial S_t} = -\delta d'(\dot{S}_t) + \delta\gamma_t, \tag{46}$$

$$\dot{A}_t = x_t, \tag{47}$$

$$\dot{S}_t = x_t - \delta S_t, \tag{48}$$

$$\lim_{t \to \infty} \mathrm{e}^{-rt}\lambda_t = 0, \tag{49}$$

$$\lim_{t \to \infty} \mathrm{e}^{-rt}\gamma_t = 0. \tag{50}$$

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[5] The considerations above may to some extent be taken care of if one assumed that small changes in the climate do not cause any damage. This can be specified as $d(\dot{S}) = d'(\dot{S}) = 0$ for $|\dot{S}| \leq K$, where K is a constant. If $K \geq \delta S_t$ for all t, the decline in the atmospheric concentration of CO_2 will never be large enough to make $d'(\dot{S})$ negative (as $|\dot{S}| = |\delta S| \leq K$ for $x = 0$). This will simplify some of the results below, since in this case, the shadow price of the atmospheric stock, γ, will always be non-negative (see (55)). However, for $\dot{S} > K$, we will still have the two contradictory effects described below (see (52)). This gives similar dynamic effects as if we assume that the ecological systems have not adjusted, and there are therefore no adjustment costs when S falls back to its long-term level (i.e., $d(\dot{S}) = d'(\dot{S}) = 0$ for $\dot{S} < 0$, but $d(\dot{S}) > 0$ and $d'(\dot{S}) > 0$ for $\dot{S} > 0$). In Section 2 above, we could in a similar way argue that $D(S) = D'(S) = 0$ for $S \leq M$, where M is a constant. This would not change the main results, however, the carbon tax would reach zero in finite time when $S_t \leq M$ for all t.

As in the previous sections, we define the scarcity rent as $\pi_t = -\lambda_t$. Using $u'(x_t) = p_t$, we find from (44) that when $x_t > 0$

$$p_t = c(A_t) - \pi_t + d'(\dot{S}_t) - \gamma_t. \tag{51}$$

Since p_t is the consumer price and $c(A_t) - \pi_t$ is the producer price, the optimal carbon tax at time t, σ_t, is:

$$\sigma_t = d'(\dot{S}_t) - \gamma_t \tag{52}$$

where γ_t is the shadow price associated with accumulated atmospheric stock up to t.

Proceeding precisely as we did to find (15) and (17), and in Appendix A in deriving (A.1)–(A.6), and using $\dot{S}_t \to 0$ as $t \to \infty$ and $d'(0) = 0$, we now find

$$\pi_t = \int_t^\infty e^{-r(\tau - t)} c'(A_\tau) x_\tau \, d\tau, \tag{53}$$

$$\lim_{t \to \infty} \pi_t = 0, \tag{54}$$

$$\gamma_t = \delta \int_t^\infty e^{-(r+\delta)(\tau - t)} d'(\dot{S}_\tau) d\tau, \tag{55}$$

$$\lim_{t \to \infty} \gamma_t = 0, \tag{56}$$

$$\dot{\gamma}_t = \delta \int_t^\infty e^{-(r+\delta)(\tau - t)} d''(\dot{S}_\tau) \cdot (\dot{x}_\tau - \delta \dot{S}_\tau) d\tau, \tag{57}$$

$$\lim_{t \to \infty} \dot{\gamma}_t = 0. \tag{58}$$

Using these properties and defining $\dot{\sigma}$ as the time derivative of σ, we find that

$$\lim_{t \to \infty} \sigma_t = 0, \tag{59}$$

$$\dot{\sigma}_t = d''(\dot{S}_t) \cdot (\dot{x}_t - \delta \dot{S}_t) - \dot{\gamma}_t, \tag{60}$$

$$\lim_{t \to \infty} \dot{\sigma}_t = 0. \tag{61}$$

Consider first the situation with an increasing stock of carbon in the atmosphere ($\dot{S} > 0$). Fossil fuel consumption (and extraction) increases the damage from global warming via accelerated buildup of the atmospheric stock (represented by $d'(\cdot)$ in Eq. (52)), but on the other hand, this leads to a larger stock in the atmosphere and therefore higher decay in the future. A high decay will reduce the rate of change in the atmospheric stock, and hence the damage from global warming. Note therefore, that while the shadow price of accumulated atmospheric stock is negative for the damage functions used in Section 2 and Section 3, where damage was determined by the stock level (i.e., $\mu = -\theta < 0$), it is positive in the present case ($\gamma > 0$) if $\dot{S} > 0$ for a sufficiently long period.[6] This is also shown in Tahvonen (1995). Thus, a larger stock of carbon in the atmosphere represents a cost if the damage is positively related to the level of this stock, while it represents a benefit if the damage is positively related to the rate of change in the stock as long as this stock is increasing for a sufficiently long period.

For $\dot{S} < 0$, an increase in fossil fuel consumption (and extraction) will reduce the absolute value of $\dot{S}$, $|\dot{S}|$, and therefore the adaption costs. This effect gives a lower optimal carbon tax, and is represented by $d'(\dot{S}) < 0$ in (52). However, increasing fossil fuel consumption gives a larger stock of carbon in the atmosphere, and therefore a larger decay of this stock in the future. This leads to even lower values of $\dot{S}$, and higher adaption costs, in the future. Thus, while the shadow price of accumulated atmospheric stock may be positive for $\dot{S} > 0$, it is negative when $\dot{S} < 0$.

Increasing fossil fuel consumption may therefore give two contradictory effects. From (52) we see that the carbon tax can be negative or positive depending on which effect is the strongest. This is also a different result compared to the model expressed in Section 2, where the optimal carbon tax is always non-negative.[7]

As seen from (57) and (60), the behaviour of the optimal carbon tax depends on the evolution of the marginal damage over time (which is $\partial d'(\dot{S})/\partial t = d''(\dot{S}) \cdot (\dot{x} - \delta \dot{S})$). The behaviour of σ is rather complex, due to the two contradictory effects described above. However, it is clear from (59) that the optimal carbon tax will approach zero as time goes to infinity, as it did for the case treated in Section 2.

We shall conclude this section by considering the limiting case of $\delta = 0$. In this case it follows from (48) that $\dot{S} = x$ and $d(\dot{S}) = d(x)$, i.e., in this case it is only the flow of the emissions which matters for the environment. From (52) and (55) we thus have

$$\sigma_t = d'(x_t) > 0, \tag{62}$$

$$\dot{\sigma}_t = d''(x_t) \cdot \dot{x}_t. \tag{63}$$

[6] The stock of carbon in the atmosphere may initially increase but eventually it will decline. Therefore, the shadow price associated with accumulated atmospheric stock, γ, may consist of both positive and negative elements as $d'(\cdot) > 0$ for $\dot{S} > 0$ and $d'(\cdot) < 0$ for $\dot{S} < 0$ (see Eq. (55)). This means that $\dot{S}$ has to be positive for a sufficiently long period for γ to be positive.

[7] Karlsen (1995) gives some simulation results for this model over a time horizon of 250 years, where $d(\dot{S}) = \alpha \dot{S}^{\beta}$. β is set equal to 2, while α is calibrated such that the optimal tax initially (year 1995) corresponds to the marginal costs of CO_2 emissions in Hoel and Isaksen (1995), i.e., \$60 per ton of carbon, and Fankhauser (1994), i.e., \$20 per ton of carbon, respectively. In both cases, the optimal tax falls monotonically and reaches negative values after 140 and 130 years. An increase in the initial value of the tax prolongs the period with positive taxation.

The price development for this case follows from (51) and (52) in combination with (45) and (47):

$$\dot{p}_t = r \cdot \pi_t + \dot{\sigma}_t. \tag{64}$$

Inserting (63) into (64) yields

$$\dot{p}_t - d''(x_t)\dot{x}_t = r \cdot \pi_t. \tag{65}$$

Since $\pi_t > 0$ for $x > 0$ from (53), and $\dot{p}$ and $\dot{x}$ must have opposite signs, we see from (65) that

$$\begin{aligned} &\dot{p}_t > 0, \\ &\dot{x}_t < 0, \\ &\dot{\sigma}_t < 0, \end{aligned} \tag{66}$$

where the last inequality follows from (63) and $d'' > 0$. For the special case of $\delta = 0$, in which only the emission flow is of importance to the environment, the optimal carbon tax should decline monotonically. The reason for this is that the extraction will decline monotonically even in the absence of the externality, implying that the marginal environmental cost of carbon emissions declines over time.

5. Conclusions

Most papers on the economics of global warming concentrate on the external effects from fossil fuels combustion without taking into account the exhaustibility of these resources. This paper combines the theories of greenhouse externalities and non-renewable resources, to analyze several aspects of global warming.

The model presented in Section 2, defines the negative greenhouse externalities as positively related to the stock of carbon in the atmosphere. The exhaustibility of fossil fuels is modelled by increasing extraction costs in accumulated extraction. A carbon tax is used to implement the optimal solution to this model. This tax should initially be increasing but eventually fall and approach zero as time goes to infinity. It should start decreasing before the stock of carbon in the atmosphere reaches its maximum point.

The next problem analyzed is the depletion of fossil fuels if there exists a non-polluting backstop technology. If we ignore the external effects, the traditional theory gives the result that the resource should be depleted until the price reaches the cost of the backstop. At this price, consumers will switch immediately to the backstop. Taking into account the greenhouse effect will give different time paths for prices and extraction. When the consumer price of fossil fuels reaches the price of the backstop, it will be optimal to consume both the backstop and fossil fuels. This is due to a falling carbon tax of fossil fuels, and therefore a fall in the consumer price if fossil fuels are not consumed. Total extraction will be the same as for no external effects, but the greenhouse effect makes it optimal to slow down the extraction and spread it over a longer period.

Changing the specification of the externalities to depend on the rate of change in the atmospheric stock of carbon, changes the model quite significantly. While the shadow price of the atmospheric stock was negative in the basic model, indicating a cost of increasing the stock, it is positive in this new model if the stock of carbon rises over a sufficiently long period. This is due to an increase in the depreciation of carbon in the atmosphere when the stock of carbon increases, which gives a lower rate of change in the future stock. This effect can make the optimal carbon tax negative even for high concentrations of carbon in the atmosphere.

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Appendix A. Extraction of fossil fuels in the absence of a backstop technology

From (15) it follows that

$$\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \int_t^{\infty} \mathrm{e}^{-r\tau} c'(A_\tau) x_\tau \, \mathrm{d}\tau / \mathrm{e}^{-rt}. \tag{A.1}$$

Applying L'Hôpital's rule and using (13), we get

$$\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \frac{c'(A_t) x_t}{r} = 0. \tag{A.2}$$

Using the same techniques as above (replace $c'(\cdot)x_t$ by $D'(\cdot)$ and r by $(r+\delta)$), and given $D'(0)=0$, it follows from (17) that

$$\lim_{t\to\infty} \theta_t = \lim_{t\to\infty} \frac{D'(S_t)}{(r+\delta)} = 0. \tag{A.3}$$

Consider (17). Integrating by parts yields, after some manipulation (as $(r+\delta) \neq 0$):

$$\theta_t = \frac{D'(S_t)}{(r+\delta)} + \frac{1}{(r+\delta)} \int_t^{\infty} \mathrm{e}^{-(r+\delta)(\tau-t)} \frac{\partial D'(S_\tau)}{\partial \tau} \, \mathrm{d}\tau. \tag{A.4}$$

Applying $\pi_t = -\lambda_t$ and substituting (A.4) into (5) gives

$$\dot{\theta}_t = \int_t^{\infty} \mathrm{e}^{-(r+\delta)(\tau-t)} \frac{\partial D'(S_\tau)}{\partial \tau} \mathrm{d}\tau = \int_t^{\infty} \mathrm{e}^{-(r+\delta)(\tau-t)} D''(S_\tau) \cdot \dot{S}_\tau \, \mathrm{d}\tau. \tag{A.5}$$

Then, inserting $\pi_t = -\lambda_t$ and (A.4) into (5), and applying (A.3) and $S \to 0$, we get

$$\lim_{t\to\infty} \dot{\theta}_t = \lim_{t\to\infty} (r+\delta)\theta_t - \lim_{t\to\infty} D'(S_t) = 0. \tag{A.6}$$

To show that once x_t is positive, it will remain positive for ever, assume the opposite: If $x_t > 0$ for some interval $[t_1, t_2)$ and $x_t = 0$ for some interval $[t_2, t_3]$, then it follows from (3) and the definitions of π_t and θ_t that

$$\begin{aligned} &u'(0) - c(A_{t2}) = \pi_{t2} + \theta_{t2} \\ &u'(0) - c(A_{t2}) \leq \pi_t + \theta_t \quad \text{for} \quad t \in [t_2, t_3]. \end{aligned} \tag{A.7}$$

But during the period $[t_2, t_3]$ it follows from (15) and (19) that π_t is constant and θ_t is declining. This contradicts (A.7), and thus proves that x_t cannot become zero in finite time once it has been positive.

To study the evolution of the tax, consider the condition

$$\dot{\theta}_t = (r+\delta)\theta_t - D'(S_t) \tag{A.8}$$

which is derived from Eq. (5) and $\pi_t = -\lambda_t$. Suppose that there exist $t_2 < t_3$ (see Fig. 3) such that

$$\begin{aligned} &\dot{\theta}_t = 0 \quad \text{for } t = t_2 \wedge t = t_3 \\ &\qquad\quad \wedge \\ &\dot{\theta}_t > 0 \quad \text{for } t_2 < t < t_3. \end{aligned} \tag{A.9}$$

Thus $\dot{\theta}$ is changing from negative to positive at t_2 and back to negative at t_3 since $\dot{\theta}$ is continuous. This means that the tax has a local minimum at t_2 and a local maximum at t_3. Let $\ddot{\theta}$ be the time derivative of $\dot{\theta}$. From (A.8) we find

$$\ddot{\theta}_t = (r+\delta)\dot{\theta}_t - D''(S_t)\dot{S}_t. \tag{A.10}$$

As $D''(S_t) > 0$ for all t, and $\ddot{\theta}_{t2} > 0$ we see from (A.10) that $\dot{S}_{t2} < 0$. We also see that $\ddot{\theta}_{t3} < 0$ gives $\dot{S}_{t3} > 0$. Assuming $\dot{S}$ is continuous, there is a t^* such that $t_2 < t^* < t_3$ and $\dot{S}_{t^*} = 0$. Thus, S has a local minimum at t^*. Since $S_{t3} > S_{t^*}$ and $\dot{S}_{t3} > \dot{S}_{t^*}$, we must have $x_{t3} > x_{t^*}$. Thus $\dot{x}_t > 0$ for at least some value of $t \in [t_2, t_3]$. This contradicts our result in Section 2 that $\dot{p}_t > 0$ and $\dot{x}_t < 0$ for $\theta_t \geq 0$. Therefore, the optimal carbon tax cannot first decrease and later increase. *The optimal carbon tax will therefore either be monotonicallly declining, or it will initially increase and eventually decline. In both cases it will approach zero asymptotically.*

Appendix B. Extraction of fossil fuels given a non-polluting backstop technology

Consider first the case with no environmental externality. To see that we get an instantaneous transition from resource extraction to substitute production in this case, assume the opposite, i.e., that there exists a period with $x_t > 0$ and $y_t > 0$. From (35) and (36) it follows that during this period

$$p(x_t + y_t) = c(A_t) + \pi_t, \tag{B.1}$$

$$p(x_t + y_t) = \bar{c}, \tag{B.2}$$

since $\theta_t = 0$ in the absence of an environmental externality (cf. (17)). Since $c(A_t)$ is increasing over time when $x_t > 0$, it follows from (B.1) and (B.2) that π_t must be declining during this period. However, whenever x_t and y_t both are positive (40) is valid, and together with (B.2) and $\dot{\theta}_t = 0$ it follows that we must have $\pi_t = 0$, i.e., π_t constant. We thus have a contradiction, proving that we cannot have both $x_t > 0$ and $y_t > 0$ during any time interval as long as there are no environmental externalities.

Consider next the case in which we have an environmental externality, i.e., $D'(S) > 0$ for all $S > 0$. To see that it cannot be optimal to have $x_t = 0$ after a

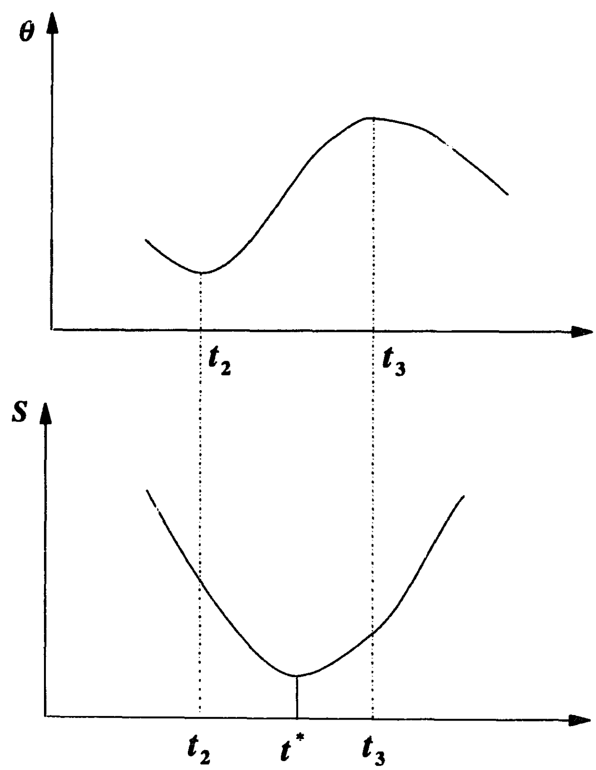

Fig. 3. An impossible evolution of the optimal carbon tax.

period of $x_t > 0$, assume the opposite, i.e., that there exists a t' such that $x_t > 0$ for $t < t'$ and $x_t = 0$ for $t > t'$. In this case $y_t > 0$ for $t > t'$, and (35) and (36) imply that

$$p(x_t + y_t) = c(A_t) + \pi_t + \theta_t \quad \text{for} \quad t < t', \tag{B.3}$$

$$p(x_t + y_t) = \bar{c} \quad \text{for} \quad t > t'. \tag{B.4}$$

By continuity of A_t, π_t and θ_t it follows that both of these two equations holds for $t = t'$. From (15) we have $\pi_t = 0$ for $t \geq t'$, Together with $x_t = 0$ for $t > t'$, this implies that $c(A_t) + \pi_t$ is constant for $t \geq t'$. Moreover, from (7) it follows that $S_t > 0$ for all t. (17) and (19) therefore imply that $\theta_t > 0$ and $\dot{\theta}_t < 0$ for all $t \geq t'$. For all $t \geq t'$ it thus follows that $c(A_t) + \pi_t + \theta_t$ is declining. But this means that the inequality in (35) is violated for $t > t'$. It is therefore not possible to find a t' such that $x_t = 0$ for all $t > t'$.

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