Fractals Everywhere

What is a fractal

- A subset of a Euclidean space for which the Hausdorff dimension (roughness or chaos) strictly exceeds the topological dimension (Mandelbrot & Benoît B. ,1983).

- Fractals exhibit similar patterns at increasingly small scales called self similarity. As finer details are revealed, the form of the details is similar to the whole.

$$\mathbf{L}(\boldsymbol{ar}) = \boldsymbol{k}\ \mathbf{L}(\boldsymbol{r})$$

Fractal physiology

- Physiological signals are commonly irregular. Healthy physiological dynamics exhibit complex temporal fluctuations that are not simply due to uncorrelated random errors. **Bassingthwaighte et al., 1994**

- Physiological systems possess “memory,” such that the change from one output event to the next displays a subtle “hidden” temporal structure.

- Such temporal fluctuations possess interesting properties, one of which is the property of long- range, fractal organization.

- Such dynamics are thought to exhibit the combination of critical properties of a control system: stability, flexibility, and adaptability.

- The presence of fractal properties in physiological and behavioral outputs is one critical marker of physiological complexity in humans

Fractal physiology (Quiet Standing)

- The postural control system does not behave chaotically, but rather it can be modeled as a correlated *random walk*, with almost perfect scaling behavior.

- Balance of a normal human being is the result of a most likely nonlinear superposition of sensory signals originating from the vestibular, visual, and somatosensory systems:
  - Visual system - 0-0.1 Hz
  - Vestibular system - 0.1-0.5 Hz
  - Somatosensory system - 0.5-1 Hz
  - Central nervous system - >1Hz

- In case of damages of one component of the balance system, certain characteristic frequency bands will change their relative weights

Thurner et al., 2000; Blázquez et al, 2008; Sample et al., 2016;

DFA

- **Detrended Fluctuation Analysis**: One of the most popular methods to estimate the self-similarity and scaling property in fractal signals:

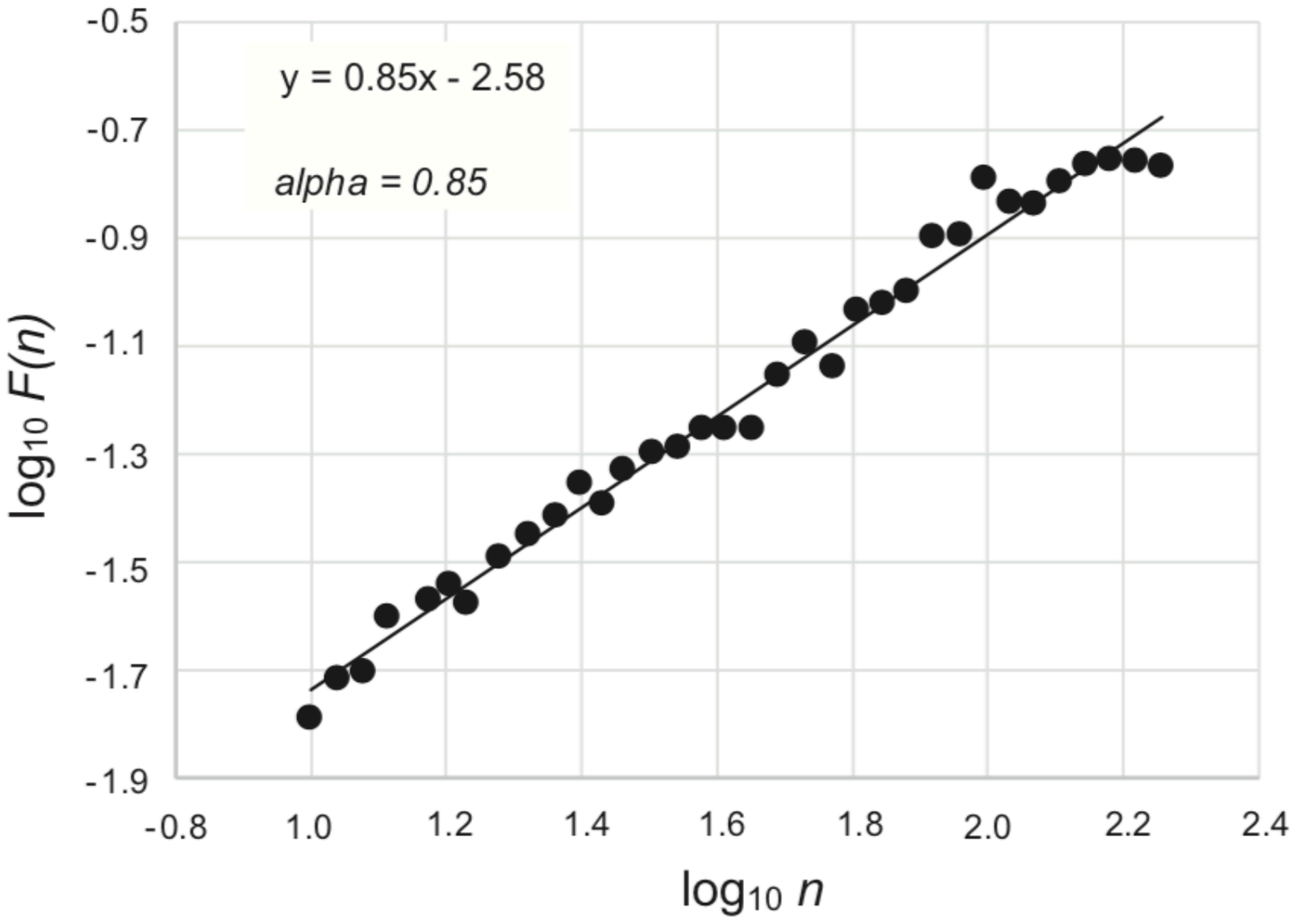
  - DFA computes the RMS error of linear fits over progressively larger bins (non- overlapped “boxes” of similar size, the scale) of an integrated time series.

  - Relationship between overall RMS error and the scale is the fluctuation function

  - When the fluctuation function follows a linear trend, the slope of this trend is scaling parameter α, which gives information concerning the correlation properties of a signal

PhysioToolkit (Goldberger et al., 2000)

DFA


y = 0.85x - 2.58
alpha = 0.85
-0.5
-0.7
-0.9
-1.1
-1.3
-1.5
-1.7
-1.9
log10 F(n)
-0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
log10 n

DFA

- **α = 0.5 is characteristic of an uncorrelated random series (or white noise)**
- **α < 0.5 the signal presents negative correlations**
- **α > 0.5 positive correlations**
- **DFA correlation means that the displacement of CoM/CoP depends not only on the value of the most recent values, but also on the values of those at relatively remote times, thus indicating a memory effect in the behavior of the control system. ---> Very complex dynamics, whose optimal functioning seems to be chaotic.**
- **DFA allows us to discover the presence of long-range correlations, even in seemingly non-stationary time series**
- **DFA is insensitive to the length of the series**

Warning

- **Choice of measurement and DFA parameters (box size, number of scales, etc) has a distinct influence on the computed metric**
- **Signal-to-noise ratios always influence outcomes, and time signals always contain noise (SNR has to be specified for assessing results)**

Müller et al., 2016

References

Arsac, L. M., & Deschodt-Arsac, V. (2018). Detrended fluctuation analysis in a simple spreadsheet as a tool for teaching fractal physiology. Advances in physiology education, 42(3), 493-499.

Bassingthwaighte, J. B., Liebovitch, L. S., & West, B. J. (2013). Fractal physiology. Springer.

Blázquez, M. T., Anguiano, M., de Saavedra, F. A., Lallena, A. M., & Carpena, P. (2009). Study of the human postural control system during quiet standing using detrended fluctuation analysis. Physica A: statistical Mechanics and its Applications, 388(9), 1857-1866.

Eke, A., Herman, P., Bassingthwaighte, J., Raymond, G., Percival, D., Cannon, M., & Ikrényi, C. (2000). Physiological time series: distinguishing fractal noises from motions. Pflügers Archiv, 439(4), 403-415.

Goldberger, A. L., Amaral, L. A., Hausdorff, J. M., Ivanov, P. C., Peng, C. K., & Stanley, H. E. (2002). Fractal dynamics in physiology: alterations with disease and aging. Proceedings of the national academy of sciences, 99(suppl 1), 2466-2472.

Müller, W., Jung, A., & Ahammer, H. (2017). Advantages and problems of nonlinear methods applied to analyze physiological time signals: human balance control as an example. Scientific reports, 7(1), 2464.