FIL2405/FIL4405 – Philosophical logic and the philosophy of mathematics

Instructor

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Readings

The readings are listed below. You *must* have read the readings marked by '*' before class; otherwise you won't be able to follow the discussion. You should expect to have to read the main readings *several times*. I have listed a number of further optional readings, which are *not* part of the official curriculum.

Students should obtain copies of the following two books:

- Stewart Shapiro, *Thinking about Mathematics* (Oxford UP, 2000)
- Paul Benacerraf and Hilary Putnam, *Philosophy of Mathematics: Selected Readings* 2nd ed. (Cambridge UP, 1983)

The former is an excellent introduction to the subject. The latter is a classic anthology containing most of the articles we will study. All our main readings will be available in these two books, online, or through Fronter.

Course overview

Pure mathematics appears to be *very* different from the empirical sciences: It appears not to rely on experience but to be completely *a priori*; its truths appear to be *necessary* rather than contingent; and it appears to be concerned with *abstract objects* rather than concrete (spatiotemporal, causally efficacious) ones. These three features of pure mathematics—its apparent apriority, necessity, and concern with abstract objects—give rise to some deep and extremely interesting philosophical questions. Are these features to be taken at face value? If so, how are they to be understood? In particular, how are these features to be reconciled with a scientific world view? Alternatively, if the special features of mathematics are *not* taken at face value, can we give an alternative explanation of mathematics which nevertheless does justice to mathematical practice and mathematical experience?

We will discuss a number of classical and contemporary approaches to these questions and related ones. Topics to be discussed include the following.

- Some traditional philosophical views of mathematics (Plato, Kant)
- Is mathematics reducible to "pure logic"? (Frege, Russell)
- Are mathematical truths just useful conventions? (Hempel)
- Is mathematics a science of mental constructions? (Brouwer, Heyting)
- Is mathematics just a formal game with uninterpreted symbols (Curry, Hilbert)
- Is mathematics empirical after all, just unusually general and abstract? (Quine)

- If there are abstract mathematical objects, how can we know about them? (Benacerraf, Gödel, Maddy)
- Can sense be made of mathematics without postulating mathematical objects? (Field)
- Are mathematical objects just points in mathematical structures? (Benacerraf, Resnik)

Programme and readings

1. Mathematics as a philosophical problem

Theme: Mathematics appears to be very different from other sciences in being *a priori*, necessary, and concerned with abstract objects. How might such a science be possible?

Main readings

- *Plato, this excerpt from *Meno*
- Kant's *Critique of Pure Reason*, B-Edition Introduction, sections I–V
- *Shapiro, pp. 51-63, 73-91

Optional further readings

- Shapiro, ch.s 1 and 2
- Kant's *Critique of Pure Reason*, "The Discipline of Pure Reason in Its Dogmatic Use" (Part II, Ch. 1, Section 1; esp. A712/B740-A724/B752)

2. Truth in mathematics

Theme: Mathematics is a science, not just a game or an activity of make-believe.

Main readings

- *Frege, *Basic Laws of Arithmetic* (OUP, 2013), Sections 86-94, 106-9, 113-4, 118-9, 123-5
- Resnik, *Frege and the Philosophy of Mathematics* (Cornell UP, 1980), pp. 54-65

Optional further readings

- Eklund, "Fictionalism", *Stanford Encyclopedia of Philosophy*
- Yablo, "The Myth of the Seven" (available from his home page)
- Yablo, "Abstract Objects: A Case Study", *Philosophical Issues* 12 (2002)
- Yablo, "Go Figure", *Midwest Studies in Philosophy*, 25 (2001), Appendix pp. 93-102

3. Proof

Theme for this session and the following two: Proof is the most important tool for the discovery of mathematical truths. But mathematical truth cannot be reduced to proof.

Main readings

- Frege, Preface to *Begriffsschrift*

- Putnam, "The thesis that mathematics is logic", in his *Mathematics, matter and method*

Optional further readings

- Curry, "Remarks on the Definition and Nature of Mathematics", *Dialectica* 8 (1954) and in Benacerraf and Putnam (1983) [5pp]
- Resnik, *Frege and the Philosophy of Mathematics* (Cornell UP, 1980), pp. 65-75 and 119-130

4. Hilbert's formalism

- Hilbert, "On the Infinite", in Benacerraf and Putnam (1983)
- Shapiro, *Thinking about Mathematics*, ch. 6

Optional further readings

- Detlefsen, *Hilbert's Program* (Reidel, 1986)
- Resnik, *Frege and the Philosophy of Mathematics* (Cornell UP, 1980), pp. 76-104
- Tait, "Finitism", *Journal of Philosophy*, 1981
- Zach, "Hilbert's Program", *Stanford Encyclopedia of Philosophy*

5. Intuitionism

- Heyting, "The Intuitionist Foundations of Mathematics" and "Disputation," both in B&P
- Shapiro, ch. 7

Optional further readings

- The remaining articles from B&P on intuitionism

6. Platonism

Theme: Frege articulates a powerful argument for the existence of mathematical objects. But the argument falls short of establishing a robust form of platonism.

Main readings

- *Frege's *Foundations of Arithmetic*, in B&P, sections 55-61
- Linnebo, "Platonism in the Philosophy of Mathematics," *Stanford Encyclopedia of Philosophy*
- Bernays, "On Platonism in Mathematics," in B&P

Optional further readings

- Dummett, *Frege: Philosophy of Language*, ch. 14

7. Abstraction

Theme: Might an account of abstraction, perhaps inspired by Frege, explain the nature of mathematics and of mathematical knowledge?

Main readings

- Frege's *Foundations of Arithmetic*, in B&P, sections 62-91 and 106-9
- Shapiro, pp. 107-115, 133-138

Optional further readings

- Dummett, *Frege: Philosophy of Mathematics*, pp. 111-119 and ch. 11 [23 pp]
- R. Heck, "An Introduction to Frege's Theorem," *Harvard Review of Philosophy* 7 (1999), pp. 56-73
- Wright, "On the Philosophical Significance of Frege's Theorem", in *Reason's Proper Study* (OUP 2001)
- Bob Hale and Crispin Wright, *Reason's Proper Study* (OUP, 2001)
- G. Boolos, "Gottlob Frege and the Foundations of Arithmetic," in his *Logic, Logic, and Logic* (Harvard UP, 1998)

8. Set theory

Theme: The iterative conception of sets. Does this conception provide a justification for the axioms of standard ZFC set theory?

Main readings

- *Boolos, "The Iterative Conception of Set", in B&P

Optional further readings

- Parsons, "What is the Iterative Conception of Set?", in B&P
- Boolos, "Iteration Again"
- Bernays, "On Platonism in Mathematics," in B&P

9. Problems with platonism

Theme: There is a genuine and hard philosophical question about how knowledge of abstract objects is possible. But an answer might still be possible.

Main readings

- *Paul Benacerraf, "Mathematical Truth," *Journal of Philosophy* 70 (1973) and in B&P
- Shapiro, pp. 24-33 and 201-11
- Gödel, "What is Cantor's Continuum Problem", the Supplement, in B&P

Optional further readings

- Field, *Realism, Mathematics and Modality*, pp. 25-30
- Penelope Maddy, *Realism in mathematics* (Oxford UP, 1990), pp. 1-5, 28-35, 58-75, 150-9
- Shapiro, pp. 220-4

- Parsons, Charles, "Platonism and mathematical intuition in Kurt Gödel's thought", *Bulletin of Symbolic Logic*, 1 (1995): 44–74

10. Quine's empiricist platonism

Theme: Is Quine right that there is no difference of kind between the truths of mathematics and other scientific truths?

Main readings

- *Quine, "Two Dogmas of Empiricism" (esp. the final two sections), in his *From a Logical Point of View* (Harvard UP, 1953);
- Quine, *Pursuit of Truth* (Harvard UP, 1990), Section 40
- Quine, *From Stimulus to Science* (Harvard UP, 1995), ch. 5
- Shapiro, pp. 212-20

Optional further readings

- Colyvan, "Indispensability Arguments in the Philosophy of Mathematics," *Stanford Encyclopedia of Philosophy*
- A.J. Ayer, "The *A Priori*," in B&P
- Shapiro, pp. 124-133
- Rudolf Carnap, "Empiricism, Semantics, and Ontology," in B&P
- Quine, "Carnap on Logical Truth," in B&P
- Parsons, "Quine and the Philosophy of Mathematics"

11. Nominalism

Theme: Is Field right that science can and should be rewritten in a way that eliminates all reference to mathematical objects? Or can nominalism be established in some easier way?

Main readings

- *Field, "Realism and Anti-Realism about Mathematics," in his *Realism, Mathematics, and Modality* (Blackwell, 1989)
- Shapiro, pp. 226-237, 243-249
- Melia, "On What There's Not", *Analysis* 55 (1995): 223-229

12. Structuralism

Theme: It is often asserted that mathematics is the science of abstract structures. What might this mean, and might it help us explain the nature of mathematics and mathematical knowledge?

Main readings

- *Resnik, "Mathematics as a Science of Patterns: Ontology and Reference," *Nous* 15 (1981), pp. 529-550
- Shapiro, *Thinking about Mathematics*, ch. 10

Optional further readings

- Benacerraf, "What Numbers Could Not Be," in B&P
- Parsons, "The Structuralist View of Mathematical Objects", *Synthese* 84 (1990), pp. 303-46.
- Shapiro, *Philosophy of Mathematics: Structure and Ontology* (OUP, 1997), pp. 71-106
- Resnik, *Mathematics as a Science of Patterns* (OUP, 1997), chapters 10-11
- Shapiro, *Philosophy of Mathematics: Structure and Ontology* (OUP, 1997), ch. 4
- MacBride, "Structuralism Reconsidered", *Oxford Handbook of Philosophy of Mathematics and Logic* (OUP, 2005), Section 3-4
- Linnebo, "Structuralism and the Notion of Dependence", *Philosophical Quarterly* 58 (2008), pp. 59-79
- Hellman, "Three Varieties of Mathematical Structuralism", *Philosophia Mathematica* 9 (2001), pp. 184-211
- MacBride, "Can Structuralism Solve the 'Access' Problem?", *Analysis* 64 (2004), pp. 309-17

13. Intuition and construction

Theme: Is there such a thing as intuition of abstract objects? If so, which mathematical truth can be known on the basis of intuition?

Main reading

- *Parsons, "Mathematical intuition"

Optional further reading

- Parsons, "Platonism and mathematical intuition in Kurt Gödel's thought", *Bulletin of Symbolic Logic*, 1 (1995): 44–74

14. Objectivity and the quest for new mathematical axioms

Theme: Cantor's continuum hypothesis and other mathematical questions are left open by our current axioms. Do such questions have objective answers? If so, can we find new axioms that enable us to *prove* these answers?

Main readings

- Russell, "The Regressive Method in Philosophy," repr. in Lackey ed. *Essays in Analysis by Bertrand Russell* (George, Allen & Unwin, 1973), pp 272-83
- *Gödel, "What is Cantor's Continuum Problem", the Supplement, in B&P

Optional further readings

- Field, "Which Undecidable Mathematical Sentences have Determinate Truth-Values?", in his *Truth and the Absence of Fact* (2001)
- Koellner, "The Question of Absolute Undecidability"