Del 8

exercise 2

9) $t'_A = 0$

$x'_A = 0$

$t'_C = t'_B = \frac{L'}{c} = 1.3\,ms$

$x'_C = 260.661\,km$

$t'_B = \frac{L'}{c} = 1.3\,ms$

$x'_B = c\,\Delta t'_{AB} = c\,\frac{L'}{c} = L' = \underline{400\,km}$

$t'_D = 2\,t'_B = 2\,\frac{L'}{c} = 2.6\,ms$

$x'_D = x'_A = 0$

$\Delta t'_{AB} = t'_B - t'_A = \frac{L'}{c} = \underline{1.3\,ms}$

$\Delta t'_{BD} = t'_D - t'_B = 2\,\frac{L'}{c} - \frac{L'}{c} = \frac{L'}{c} = \underline{1.3\,ms}$

x' i sekund: $x_B'^{\,sek} = \frac{x_B'^{\,meter}}{c}$

$= \frac{400\,km}{c} = \frac{L'}{c} = \underline{1.3\,ms}$

$x_C'^{\,sek} = \frac{260.661\,km}{c} = \underline{0.87\,ms}$

10) $t_A = 0$

$x_A = 0$

$t_C = t_C$

$x_C = 0$

$t_B = t_B$

$x_B = x_B$

$t_D = t_D$

$x_D = -v\,t_D$

$\Delta t_{AB} = t_B - t_A = t_B$

$\Delta t_{BD} = t_D - t_B$

$\Delta x_{AB} = x_B$

$\Delta x_{BD} = -v\,t_D - x_B$

11)
$$\Delta s^2_{AB} = \Delta s'^2_{AB}$$
$$\Delta t^2_{AB} - \Delta x^2_{AB} = \Delta t'^2_{AB} - \Delta x'^{\,sek\,2}_{AB}$$
$$t_B^2 - x_B^2 = \left(\frac{L'}{c}\right)^2 - \left(\frac{L'}{c}\right)^2$$
$$\underline{t_B = x_B} \quad \square$$

12)

$$\Delta s_{AC}^2 = \Delta s'^2_{AC}$$

$$\Delta t_{AC}^2 - \Delta x_{AC}^2 = \Delta t'^2_{AC} - \Delta x'^2_{AC}$$

$$t_C^2 = t'^2_C - x'^{rek\,2}_C$$

$$t_C = \sqrt{t'^2_C - x'^{rek\,2}_C}$$

$$= \sqrt{(1.3\,ms)^2 - (0.87\,ms)^2}$$

$$= \underline{\underline{0.97\,ms}}$$

13)

$$\Delta s_{BC}^2 = \Delta s'^2_{BC}$$

$$\Delta t_{BC}^2 - \Delta x_{BC}^2 = \Delta t'^2_{BC} - \Delta x'^2_{BC}$$

$$(t_C - t_B)^2 - x_B^2 = (t'_C - t'_B)^2 - (x'^{rek}_C - x'^{rek}_B)^2$$

$$(t_C - t_B)^2 - t_B^2 = -(x'^{rek}_C - x'^{rek}_B)^2$$

$$t_C^2 - 2t_Bt_C + t_B^2 - t_B^2 = -(x'^{rek}_C - x'^{rek}_B)^2$$

$$2t_Bt_C = t_C^2 + (x'^{rek}_C - x'^{rek}_B)^2$$

$$t_B = \frac{t_C^2 + (x'^{rek}_C - x'^{rek}_B)^2}{2t_C}$$

$$= \underline{\underline{0.58\,ms}}$$

14)

$$\Delta s_{AD}^2 = \Delta s'^2_{AD}$$

$$\Delta t_{AD}^2 - \Delta x_{AD}^2 = \Delta t'^2_{AD} - \Delta x'^2_{AD}$$

$$t_D^2 - v^2 t_D^2 = 4\left(\frac{L'}{c}\right)^2 - 0$$

$$t_D^2(1 - v^2) = 4\frac{L'^2}{c^2}$$

$$t_D = 2\frac{L'}{c}\frac{1}{\sqrt{1-v^2}}$$

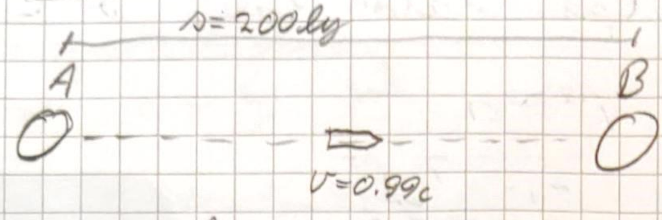
$$= \gamma t'_D$$

$$= \underline{\underline{4.39\,ms}}$$

Del 8

Oppg. 3

part 1.1)



$$t = \frac{s}{v} = \frac{200ly}{0.99c} \approx 202yr = \Delta t$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-(\frac{v}{c})^2}}$$

$$\Delta t' = \Delta t \sqrt{1-(\frac{v}{c})^2}$$
$$= 202yr \cdot \sqrt{1-(\frac{0.99c}{c})^2}$$
$$= 202yr \cdot \sqrt{1-0.99^2}$$
$$\approx \underline{28.5yr}$$

Her er planetane det umerka referanse-systemet, og Lisa er det merka referanse-systemet.
For planetane vil reisa ta 202 år, medan for Lisa vil det ta 28.5 år.

1.2) Ved symmetri ser vi at for planetane vil det ta 202 år å reise attende, medan Lisa vil oppleve 28.5 år.

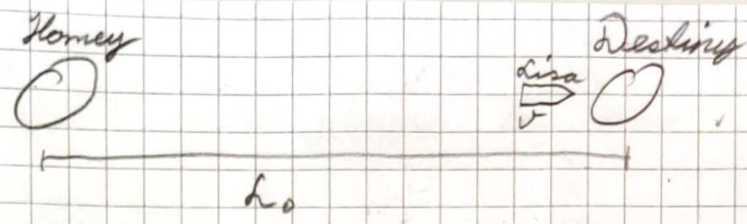
part 2.1) Her er $\Delta t = 28.5yr$. Brukar tidsdilatasjon:

$$\Delta t' = \Delta t \sqrt{1-(\frac{v}{c})^2}$$
$$= 28.5yr \sqrt{1-0.99^2}$$
$$\approx \underline{4yr}$$

Klokka på planeten viser no 4 år.

2.2) Ved symmetri ser vi at det tok 4 år for Homey å nå Lisa.

part 3.1)



Lisa kjem fram ved

$$t_B = \frac{\Delta x}{v} = \underline{\underline{\frac{L_0}{v}}} = 202\,yr$$

3.2) Vi har Lorentztransformasjonane gjeve ved

$$t_B = v\gamma x' + \gamma t_B'$$

og $$t_B' = -v\gamma x + \gamma t_B$$

Dette gjev oss

$$t_B' = -v\gamma L_0 + \gamma \frac{L_0}{v}$$

$$= \gamma L_0 \left(\frac{1}{v} - v\right)$$

$$= \gamma L_0 \left(\frac{1}{v} - v\frac{v}{v}\right)$$

$$= \gamma L_0 \left(\frac{1}{v} - \frac{v^2}{v}\right)$$

$$= \gamma L_0 \frac{1-v^2}{v}$$

Skriv vi så om Lorentzfaktoren, får vi

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\gamma^2 = \frac{1}{1-v^2}$$

$$v^2 = 1 - \frac{1}{\gamma^2}$$

Set dette inn i transformasjonen:

$$t_B' = \gamma L_0 \frac{1-1+\frac{1}{\gamma^2}}{v}$$

$$= \gamma L_0 \frac{1}{v\gamma^2}$$

$$\underline{= \frac{L_0}{\gamma v}}$$

Så set vi inn tal:

$$t_B' = \frac{200\,ly}{\frac{1}{\sqrt{1-0.99^2}} \cdot 0.99c} \approx \underline{\underline{28.5\,yr}}$$

3.3) Her er $x'_{B'} = -L'_0$ der $L'_0 = \frac{L_0}{\gamma}$ ved lengdekontraksjon.

$$\Rightarrow x'_{B'} = -\frac{L_0}{\gamma}$$

Finn då $t_{B'}$ ved Lorentztransformasjon:

$$t_{B'} = v\gamma x'_{B'} + \gamma t'_{B'}$$
$$= v\gamma\left(-\frac{L_0}{\gamma}\right) + \gamma t'_{B'}$$
$$= -vL_0 + \gamma t'_{B'} \quad \text{der } t'_{B'} = t'_B = \frac{L_0}{\gamma v}$$
$$= -vL_0 + \gamma \frac{L_0}{\gamma v} \quad \text{fordi dei skjer samstundes.}$$
$$= \underline{\frac{L_0}{v} - vL_0}$$
$$= \frac{200\,ly}{0.99c} - 0.99c \cdot 200\,ly$$
$$\approx \underline{\underline{4\,yr}}$$

ynart 4.1)

$$\Delta x_{BD} = x_D - x_B = 2L_0 - L_0 = L_0 = 200\,ly$$
$$\Delta t_{BD} = t_D - t_B = -\frac{L_0}{v} = -202\,yr = 202\,yr$$

$$\Delta x''_{BD} = x''_D - x''_B = 0 - 0 = 0$$
$$\Delta t''_{BD} = t''_D - t''_B = 0 - t'_B = -\frac{L_0}{\gamma v} = -28.5\,yr = 28.5\,yr$$

$$\underline{\underline{t''_B = t'_B}}$$

4.3) og 4.4)

$x_D = 2L_0$ $\quad t_D = 0$

$x''_D = 0$ $\quad t''_D = 0$

$x''_B = 0$ $\quad t_{B''} = t_{B''}$

$x''_{B''} = \frac{L_0}{\gamma}$

$$t''_{B''} = t''_B = t'_{B'} = t_B = \frac{L_0}{\gamma v}$$

$\Delta x_{DB''} = x_{B''} - x_D = \underline{-2L_0}$ $\quad \Delta x''_{DB''} = x''_{B''} - x''_D = \underline{\frac{L_0}{\gamma}}$

$\Delta t_{DB''} = t_{B''} - t_D = \underline{t_{B''}}$ $\quad \Delta t''_{DB''} = t''_{B''} - t''_D = \underline{\frac{L_0}{\gamma v}}$

(4.4 lenger fram)

4.4)

$$\Delta s_{DB''}^2 = \Delta s''^2_{DB''}$$

$$\Delta t_{DB''}^2 - \Delta x_{DB''}^2 = \Delta t''^2_{DB''} - \Delta x''^2_{DB''}$$

$$t_{B''}^2 - (-2L_0)^2 = \frac{L_0^2}{\gamma^2 v^2} - \frac{L_0^2}{\gamma^2}$$

$$t_{B''}^2 = L_0^2\left(\frac{1}{\gamma^2 v^2} - \frac{1}{\gamma^2} + 4\right)$$

$$t_{B''} = L_0\sqrt{\frac{1}{\gamma^2}\left(\frac{1}{v^2}-1\right)+4}$$

$$= L_0\sqrt{(1-v^2)\left(\frac{1}{v^2}-1\right)+4}$$

$$= L_0\sqrt{\left(\frac{1}{v^2}-1-1+v^2\right)+4}$$

$$= L_0\sqrt{\frac{1}{v^2}+v^2+2}$$

$$= L_0\sqrt{\left(v+\frac{1}{v}\right)^2}$$

$$= L_0\left(v+\frac{1}{v}\right)$$

$$= L_0 v + \frac{L_0}{v} \qquad \square$$

part 5.1) $t_{B'} = v\gamma x'_{B'} + \gamma t'_{B'}$

$$\Delta s^2_{BB'} = \Delta s'^2_{BB'}$$

$$\Delta t^2_{BB'} - \Delta x^2_{BB'} = \Delta t'^2_{BB'} - \Delta x'^2_{BB'}$$

her er

$$\Delta t_{BB'} = t_{B'} - t_B = t_{B'} - \frac{L_0}{v}$$

$$\Delta x_{BB'} = x_{B'} - x_B = 0 - L_0 = -L_0$$

$$\Delta t'_{BB'} = t'_{B'} - t'_B = t'_B - t'_B = 0$$ fordi det skjer samtundes.

$$\Delta x'_{BB'} = x'_{B'} - x'_B = -L'_0 - 0 = -\frac{L_0}{\gamma}$$

$$\Rightarrow \Delta t^2_{BB'} - \Delta x^2_{BB'} = \Delta t'^2_{BB'} - \Delta x'^2_{BB'}$$

$$\left(t_{B'} - \frac{L_0}{v}\right)^2 - (-L_0)^2 = 0 - \left(-\frac{L_0}{\gamma}\right)^2$$

$$\left(t_{B'} - \frac{L_0}{v}\right)^2 - L_0^2 = -\frac{L_0^2}{\gamma^2}$$

$$\left(t_{B'} - \frac{L_0}{v}\right)^2 = L_0^2 - \frac{L_0^2}{\gamma^2}$$

$$= L_0^2\left(1 - \frac{1}{\gamma^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\gamma^2 = \frac{1}{1-v^2}$$

$$\frac{1}{\gamma^2} = 1 - v^2$$

$$t_{B'} - \frac{L_0}{v} = L_0\sqrt{1 - \frac{1}{\gamma^2}}$$

$$t_{B'} = L_0\sqrt{1 - \frac{1}{\gamma^2}} + \frac{L_0}{v}$$

$$= L_0\sqrt{1 - (1 - v^2)} + \frac{L_0}{v}$$

$$= L_0(\pm v) + \frac{L_0}{v}$$

$$= \frac{L_0}{v} \pm L_0 v$$

For hastigheit i positiv retning får vi

$$t_{B'} = \frac{L_0}{v} - L_0 v$$

$$= L_0\left(\frac{1}{v} - v\right)$$

$$= L_0\left(\frac{1-v^2}{v}\right)$$

$$= L_0\left(\frac{1}{\gamma^2}\frac{1}{v}\right)$$

$$= \frac{L_0}{v\gamma^2} \quad \square$$

5.2) $t_{turn} = t_B + t$

der t er tida frå hending B til skipet har stoppa opp, altså nå $v = 0$:

$v = gt + v_0$

$0 = gt + v_0$

$t = -\frac{v_0}{g}$

$$\Rightarrow \underline{t_{turn} = t_B - \frac{v_0}{g}} \quad \square$$

5.3) Sidan vi antek at skipet held fram med den same negative akselerasjonen etter å ha nådd $v = 0$, vil det byrje å få ei negativ hastigheit

$\Rightarrow$ skipet vil gå i motsett retning.

Sidan skipet har same akselerasjon og same distanse å reise attende, vil det bruke like lang tid attende som fram.

5.4) Akselerert system $\rightarrow$ ikkje Lorentz!

$x_Y = L_0 + v_0(t - t_B) + \frac{1}{2}g(t - t_B)^2$ $\qquad$ $x'_Y = 0$

$t_Y = t_Y$ $\qquad$ $t'_Y = t'_Y$

$x_{Y'} = 0$ $\qquad$ $x'_{Y'} = -\frac{x_Y}{\gamma(t_Y)}$ lengdekontraksjon

$t_{Y'} = t_{Y'}$ $\qquad$ $t'_{Y'} = t'_Y$

fordi dei skjer samstundes i same ref. system.

x_Y: L_0 pluss avstanden mellom Megard og Y, vanleg avstandslikning

$x(t) = L_0 + \int v(t)\,dt$ $\quad$ der $v(t) = v_0 + g(t - t_B)$

5.5)

$$\Delta s_{\gamma\gamma'}^2 = \Delta s'^2_{\gamma\gamma'}$$

$$\Delta t_{\gamma\gamma'}^2 - \Delta x_{\gamma\gamma'}^2 = \Delta t'^2_{\gamma\gamma'} - \Delta x'^2_{\gamma\gamma'}$$

der vi har at

$$\boxed{v(t_\gamma) = \frac{g}{c}(t_\gamma - t_B) + v_0}$$

$$\Delta t_{\gamma\gamma'} = t_{\gamma'} - t_\gamma$$

$$\Delta x_{\gamma\gamma'} = x_{\gamma'} - x_\gamma = -x_\gamma$$

$$\Delta t'_{\gamma\gamma'} = t'_{\gamma'} - t'_\gamma = t'_\gamma - t'_\gamma = 0$$

$$\Delta x'_{\gamma\gamma'} = x'_{\gamma'} - x'_\gamma = -\frac{x_\gamma}{\gamma(t_\gamma)}$$

$$\Rightarrow (t_{\gamma'} - t_\gamma)^2 - (-x_\gamma)^2 = 0 - \left(-\frac{x_\gamma}{\gamma(t_\gamma)}\right)^2$$

$$(t_{\gamma'} - t_\gamma)^2 = x_\gamma^2 - \left(\frac{x_\gamma}{\gamma(t_\gamma)}\right)^2$$

$$= x_\gamma^2\left(1 - \frac{1}{\gamma^2(t_\gamma)}\right)$$

$$= x_\gamma^2\left(1 - (1 - v^2(t_\gamma))\right)$$

$$= x_\gamma^2 v^2(t_\gamma)$$

$$t_{\gamma'} - t_\gamma = \pm x_\gamma v(t_\gamma)$$

$$\underline{t_{\gamma'}(t_\gamma) = t_\gamma \pm \left(L_0 + v_0(t_\gamma - t_B) + \frac{1}{2}\frac{g}{c}(t_\gamma - t_B)^2\right)v(t_\gamma)}$$

5.6) $$g(\text{nat}) = \frac{\Delta x(\text{sek})}{\Delta t^2(\text{sek})} = \frac{\frac{\Delta x(SI)}{c}}{\Delta t^2(SI)} = \underline{\underline{\frac{1}{c}\frac{\Delta x(SI)}{\Delta t^2(SI)}}}$$

5.7) Her er då $v_0 = v = 0.99c$

$$\Rightarrow t_{turn} = t_B - \frac{v_0}{g}c$$

$$= 202\,yr - \frac{0.99c}{(-0.1\,m/s^2)}c$$

$$= 202\,yr + 94\,yr = \underline{\underline{296\,yr}}$$

5.8) $$t_{\gamma'}(t_{turn}) = t_{turn} - \left(L_0 + v_0(t_{turn} - t_B) + \frac{1}{2}\frac{g}{c}(t_{turn} - t_B)^2\right)v(t_{turn})$$

Men i turningpoint vil $v = 0$

$$\Rightarrow t_{\gamma'} = t_{turn} - 0$$

$$\underline{t_{\gamma'} = t_{turn} = t_\gamma}$$

Kva skjer med Lisa under aks. ?

12) $\Delta T = \frac{\Delta T'}{\sqrt{1-v^2}}$

$\Delta T' = \sqrt{1-v^2}\,\Delta T$

$= \sqrt{1-g^2T^2}\,\Delta T$

Dette kan vi bruke i DET gjeve referansesystemet

$\Rightarrow T' = \int \sqrt{1-g^2T^2}\,dT$

$= 74$ år

Lisa: 28.5 år fram, 74 deaks.,
74 år oppaks., 28.5 tilbake
$= \underline{205 \text{ år}}$

$r = \frac{1}{2} g T^2$

$T = \sqrt{\frac{2r}{g}}$

$\Rightarrow \Delta T' = \sqrt{1-g^2T^2}\,\Delta T$

$= \sqrt{1-g^2\frac{2r}{g}}\,\Delta T$

$= \sqrt{1-2rg}\,\Delta T$ der $g = \frac{\cancel{G}M}{r^2}$

$= \sqrt{1-2r\frac{M}{r^2}}\,\Delta T$

$= \underline{\sqrt{1-\frac{2M}{r}}\,\Delta T}$

Exercise 6

1) $P'_\mu(e) = c_{\mu\nu} P_\nu(e)$

$$\begin{pmatrix} E' \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma_{rel} & -v_{rel}\gamma_{rel} & 0 & 0 \\ -v_{rel}\gamma_{rel} & \gamma_{rel} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$\Rightarrow \quad E' = \gamma_{rel} E - v_{rel}\gamma_{rel}\, p_x$

$p'_x = \gamma_{rel}\, p_x - v_{rel}\gamma_{rel}\, p_x$

siden vi berre har rørsle i x-retning.

$P'_\mu = m V'_\mu \qquad$ der $V'_\mu = \gamma(1, \vec{v}')$

$= m\gamma(1, \vec{v}')$

$= \gamma(m, m\vec{v}')$

$P'_\mu(e) = \gamma_e(m_e, m_e\vec{v}'_e)$

$= \frac{1}{\sqrt{1-v_e'^2}}(m_e, m_e\vec{v}'_e)$

$P'_\mu(p) = \frac{1}{\sqrt{1-v_p'^2}}(m_p, m_p\vec{v}'_p)$

4)

$$P'_\mu(n) = P'_\mu(p) + P'_\mu(e)$$
$$(m_n, 0) = \gamma'_p(m_p, m_p\vec{v}'_p) + \gamma'_e(m_e, m_e\vec{v}'_e)$$

$$\Rightarrow \quad m_n = \gamma'_p m_p + \gamma'_e m_e \qquad 0 = \gamma'_p m_p \vec{v}'_p + \gamma'_e m_e \vec{v}'_e$$

$$\gamma'_p = \frac{m_n - \gamma'_e m_e}{m_p} \qquad \gamma'_e = -\frac{\gamma'_p m_p \vec{v}'_p}{m_e \vec{v}'_e}$$

$$= \frac{1}{m_p}\left(m_n - \left(-\frac{\gamma'_p m_p \vec{v}'_p}{m_e \vec{v}'_e}\right)\right)$$

$$= \frac{1}{m_p}\left(m_n + \frac{\gamma'_p m_p \vec{v}'_p}{m_e \vec{v}'_e}\right)$$

$$= \frac{m_n}{m_p} + \frac{\gamma'_p \vec{v}'_p}{m_e \vec{v}'_e}$$

$$\gamma'_p\left(1 - \frac{\vec{v}'_p}{m_e v_e}\right) = \frac{m_n}{m_p}$$

$$\gamma'_p =$$

$$m_n = \gamma'_p m_p + \gamma'_e m_e$$

$$\gamma'_e = \frac{m_n - \gamma'_p m_p}{m_e}$$

$$0 = \gamma'_p m_p \vec{v}'_p + \gamma'_e m_e \vec{v}'_e$$
$$= \gamma'_p m_p \vec{v}'_p + \left(\frac{m_n - \gamma'_p m_p}{\cancel{m_e}}\right)\cancel{m_e}\vec{v}'_e$$
$$= \gamma'_p m_p \vec{v}'_p + m_n \vec{v}'_e - \gamma'_p m_p \vec{v}'_e$$

$$\gamma'_p(m_p \vec{v}'_p - m_p \vec{v}'_e) = -m_n \vec{v}'_e$$

$$\gamma'_p = -m_n \vec{v}'_e$$

$$\gamma'_p = \gamma'_e \frac{m_e \vec{v}'_e}{m_p \vec{v}'_p}$$
$$= \frac{m_n - \gamma'_p m_p}{m_e} \frac{m_e \vec{v}'_e}{m_p \vec{v}'_p}$$
$$= \frac{m_n \vec{v}'_e - \gamma'_p m_p \vec{v}'_e}{m_p \vec{v}'_p}$$
$$= \vec{v}'_e$$