AST 2000 - Part 10
Studying Your Star

This is the final part of the project! You are going to investigate the star in your solar system: its origins, its main sequence life and its inevitable death.

ABOUT PART 10

In this part of the project you are going to investigate your star using theory from the 3rd section of the course, that is, Lecture Notes 3A, 3B, 3C, 3D and 3E.

Part 10 consists of 3 Sections:

1. The origin and current state of your star.
2. Nuclear reactions in your star's core.
3. The death of your star.

As in part 8 and 9, this is supposed to be more similar to and educational text more than a scientific paper. You may write and structure you document as you wish, but you will be evaluated in the following way:

- Each of the 3 parts will count 40% (if you do all perfectly you get 120%)
- In each part we will weight the grading in the following manner:
  - The explanation of the problem with you own words. (20%)
  - The explanation of the idea behind your way of solving the problem. As always, make sure to explain the principles behind the methods and formulae you use, and justify any assumptions you make. Make sure you show that you understand the physics. You are, of course, allowed to base your thinking on theory from the lecture notes. (30%)
  - The way you actually solve the problems, including any math of derivations. You will also be evaluated on the results here. (30%)
  - The way you discuss you assumptions and results. You are not expected to make long discussions, but show that you have thought critically about your results and assumptions in the same way as in the first parts of the project. (30%)

I. THE ORIGIN AND CURRENT STATE OF YOUR STAR

A. Main Sequence Star

1. Is your star a main sequence star?

   First, determine the luminosity of your star using its radius and surface temperature. Second, classify your star by plotting your star in an HR-diagram. If you really want to, you can draw the HR-digram by hand, but you can also use the `StarPopulation` class of `ast2000tools`, see documentation and example here.

2. In Lecture Notes 3D you find a relationship between the mass of a star and its remaining time on the main sequence. Explain the principles behind this relationship without the use of mathematics.

   Next, use this relation to estimate the time your star is expected to live on the main sequence.

3. Is your star a well-behaved main sequence star?

   Investigate how your star behaves according to the mass-temperature and mass-luminosity relations relative to another known main sequence star.

B. Giant Molecular Cloud

Let us begin by assuming that your star started out as a spherically symmetric Giant Molecular Cloud (GMC) with temperature 10 K that consisted of 75% Hydrogen and 25% Helium atoms. Furthermore, assume your GMC began collapsing in on itself without the help of shock waves from supernovae. Also assume that the mass of the gas cloud equals the current mass of your star.

1. What is the smallest possible radius your GMC could have had? Explain and dicuss the main principles behind any formulae you use.
2. Say the radius of your GMC is just above its minimum, find its luminosity and place it in the HR-diagram.

II. NUCLEAR REACTIONS IN YOUR STAR'S CORE

Use the following assumptions as a base for modelling your star's interior in all the following challenges:

→ Assume the density of your star is uniform.

→ Approximate the pressure inside your star with the gaseous pressure of an ideal gas. You can ignore radiation pressure, etc.

→ Model the interior of your star by assuming hydrostatic equilibrium.

→ Assume your star consists entirely of protons with a mass of $m_H = 1.673 \cdot 10^{-27}$ kg (this value is available in the `constants` module).

A. Core Temperature

1. Derive an expression for $M(r)$: the spherical mass profile of your star. Recall that the mass profile is the total mass inside a sphere with radius r.

2. Combine the relevant assumptions and derive

$$\frac{\mathrm{d}}{\mathrm{d}r}T(r) = -\frac{4\pi}{3}G\rho_0\frac{\mu m_H}{k}r \tag{1}$$

3. Integrate (1) from $r = 0$ to $r = R$ in order to show that

$$T_c = T(R) + \frac{2\pi}{3}G\rho_0\frac{\mu m_H}{k}R^2 \tag{2}$$

   where $T_c = T(0)$ is your star's core temperature.

   Finally, estimate the core temperature of your star.

B. Energy Production and Luminosity

The goal of this task is to estimate the luminosity of your star, based on a model that describes the nuclear reactions in its core. Assume that all nuclear reactions take place within a sphere of radius $0.2R$. In addition, assume the density of the star is uniform and that the core temperature you calculated above is the same throught the core.

Depending on the core temperature you found in the previous task, choose one of the following options:

$T_c < 90 \cdot 10^6$ K:

- Assume energy production in the core occurs via the pp-chain and CNO-cycle.
- Assume the core consists of 74.5% Hydrogen, 25.3% Helium and 0.2% Carbon, Oxygen and Nitrogen.

$T_c > 90 \cdot 10^6$ K:

- Assume energy production in the core occurs solely via the 3α reaction.
- Assume the core consists of 20% Hydrogen and 80% Helium.

How accurate is your estimated luminosity compared to the luminosity you calculated in Section I A? Using your estimated luminosity, provide an estimate for the surface temperature of your star. Discuss the assumptions made in your model in light of how your new luminosity estimate compared with the luminosity from Section I A.

III. THE DEATH OF YOUR STAR

A. Leaving the main sequence

Your next task is to describe the life of your star as it leaves the main sequence by explaining the principles behind each step of the process. In addition, you will also need to plot the position of your star in the HR-diagram as it progresses through its different stages. Plots and figures are very welcome! Scanned hand drawings are also welcome, you may for instance print an HR-diagram and draw on top before you scan. Use maximum one page of text for the description.

B. The End

The last task is to describe the final stage of your star's life cycle. Depending on the original mass of your star M, choose one of the following options:

$M < 8M_\odot$:
Your star will end as a white dwarf, you can assume that the mass of your white dwarf is:

$$M_{\mathrm{WD}} = \frac{M}{8M_\odot}M_{\mathrm{Chandresekhar}} \tag{3}$$

$M > 8M_\odot$:
Your star will end as a neutron star, you can assume that the mass of your neutron star is:

$$M_N = 1.5M_\odot + \frac{M - 8M_\odot}{17M_\odot}1.3M_\odot \tag{4}$$

1. First, explain the principles and assumptions behind the radius approximation in Lecture Notes 3E. Second, use this expression in conjunction with reasonable assumptions regarding A and Z in order to estimate the radius of your white dwarf/neutron star.

   Note that if you have neutron star you will need to modify the expression slightly, explain why you are justified in doing so and why you are justified in using the radius approximation in the first place.

2. Calculate:
   a) the weight of one litre of white dwarf/neutron star material
   b) the gravitational acceleration on the surface of your white dwarf/neutron star.

3. Without the use of mathematics, explain briefly (maximum 1/2 to 1 page in total):
   a) what degenerate matter is
   b) what the end state of your star consists of.

FIG. 1. Relativistically star-born, orbitally motivational, atmospherically oriented, non-Lorentzian, trajectory-planning duck. Launched & landed.