Solutions to exercises in part 2A

The following are short solutions to many exercises in part 2A. In addition to this document there is also a solution file (pure text file) which can be found in the solutions folder where the xml is downloaded. Note that your numerical solutions will be very close to, but seldom exactly equal to the numbers found in the solution file. The reason for this is that the solution file is created with the same code generating the xml for the video: since the video has a limited time resolution, the times of events will not be exact in the video and for this reason there will be small errors (of the order 1% or less) in most numbers given in the solution file.

Exercise 2A.1

1. B passes through A.
2. A passes through B.
3. The observations made in the earlier tasks shows that whichever cylinder passes through which depends on the referencesystem. Therefor z=z' and y=y' must be true to avoid contradictions.

Exercise 2A.2

Note: numerical solutions can be found in the solutions folder for the xml-files.

1. Find answer in MCast
2. Find answer in MCast
3. Find answer in solution file
4. Find answer in solution file
5. Find answer in solution file

7. $\Delta t \approx 5.005 \cdot 10^{-5}\, s$. Hence the muon uses approximentely $50\,\mu s$ to reach the ground (in the planet frame).
8. Since
$$50\,\mu s = \Delta t_{AG} > \Delta t_{\text{Muon}} = 2\,\mu s,$$
where Δt_{Muon} is the mean lifetime of a muon, most muons will never reach the surface of the earth (when we do not take into account relativistic effects).

9. Thinking of the muon as the spaceship from above, we see that this situation is equivalent with the one above. $\Delta t' \approx 2.2\,\mu s$

Exercise 2A.3

Note: numerical solutions can be found in the solutions folder for the xml-files.

- (1a) and (1b): see numbers in videos and in solution file!
- (2c)/(3c): $\Delta t'_{YB} = \frac{\Delta t_{YB}}{\gamma}$
- (2d)/(3d): see solution file
- (4) From length contraction we know that $L = L_0/\gamma$, hence the length between the two lightenings should be largest in the rest frame. You should therefore observe a larger distance in the planet frame.

Exercise 2A.4

Note: numerical solutions can be found in the solutions folder for the xml-files.

Part 1

3. The left laser must be emitted first.
4. The explosions are simultaneous in the spaceship frame and not in the planet frame.
5. The left spaceship must have exploded first.
6. The order of the events in the planet frame must be A, B, C and D.
7. See the video and solution file.
8.

Time or position	Event A	Event B	Event C	Event D
t	$t_A = 0$	t_B (unknown)	t_C (unknown)	t_D (unknown)
t'	$t'_A = 0$	$t'_B = t'_A = 0$	t'_C (known)	$t'_D = t'_C$
x	$x_A = 0$	$x_B = L + vt_B$	$x_C = vt_C$	$x_D = L + vt_D$
x'	$x'_A = 0$	$x'_B = L' = t'_C$	$x'_C = 0$	$x'_D = L' = t'_C$

9.

$$t_D = \frac{L}{1-v}$$

$$x_D = \frac{1}{1-v}$$

10. See solution file for numbers.

11.
$$t_C = \frac{L'}{\sqrt{1-v^2}}$$

$$t_B = t_C - \frac{L}{v+1}$$

$$x_B = vt_C + \frac{L}{v+1}$$

12. See solution file

Part 2

4. $\Delta t = \frac{L}{1-v^2}$, $\Delta t' = L$

Exercise 2A.5

1. $\Delta t'_{AB} = \Delta t'_{BD}$
3. $\Delta t_{BD} > \Delta t_{AB}$
6. Event B must occure before event C.
9. $L' = x'_B = t'_B$, $\Delta t'_{AB} = t'_B - t'_A$ and $\Delta t'_{BD} = t'_D - t'_B$. Find the values in MCast.
10.

Time or position	Event A	Event B	Event C	Event D
t	$t_A = 0$	t_B (unknown)	t_C (unknown)	t_D (unknown)
t'	$t'_A = 0$	$t'_B = t'_C$	t'_C (known)	t'_D (known)
x	$x_A = 0$	x_B (unknown)	$x_C = 0$	$x_D = -v \cdot t_D$
x'	$x'_A = 0$	$x'_B = t'_B$	$x'_C = v \cdot t'_C$	$x'_D = 0$

12. See solution file.
13. See solution file.
14. See solution file.
15. See solution file.
16. See solution file.
17. Watch the video.

Exercise 2A.6

Note: numerical solutions can be found in the solutions folder for the xml-files.

1. (This table will be used throughout the entire exercise. If you wonder where the numbers or relations in later tasks are coming from, check with this table.)

Time or position	Event G	Event P	Event B	Event Y
t	$t_G = 0$	t_P	$t_B = t_P$	t_Y
t'	$t'_G = 0$	t'_P	t'_B	$t'_Y = t'_P$
x	$x_G = 0$	x_P	x_B	$x_Y = 0$
x'	$x'_G = 0$	x'_P	$x'_B = 0$	x'_Y

Table 1: (You should look up the values in MCast). NB! The person with the spaceship frame should not need event B and the person with the planet frame should not need event Y. Thus there is no reason to have the respective event in your own table.

2. (This and the next task are almost identical, therefore we have combined them here. The frame will be specified when needed.)

3. (a) **Planet frame:**

$$t'_B = \sqrt{t_B^2 - x_B^2} \tag{1}$$

Spaceship frame:

$$t_Y = \sqrt{(t'_Y)^2 - (x'_Y)^2} \tag{2}$$

(b) **Planet frame:**

$$x'_P = \sqrt{(t'_P)^2 - t_P^2 + x_P^2} \tag{3}$$

$$\tag{4}$$

Spaceship frame:

$$x_P = \sqrt{t_P^2 - (t'_P)^2 + (x'_P)^2} \tag{5}$$

(c) **Planet frame:**

$$t'_P = \frac{t_P^2 - x_B x_P}{\sqrt{t_B^2 - x_B^2}} \tag{6}$$

Spaceship frame:

$$t_P = \frac{(t'_P)^2 - x'_Y x'_P}{\sqrt{(t'_Y)^2 - (x'_Y)^2}} \tag{7}$$

(d) Numerical solution

4. (Check with your partner)

5. You might need this relation:

$$\gamma^2 - 1 = \frac{1}{1-v^2} - \frac{1-v^2}{1-v^2} = \frac{v^2}{1-v^2} = \gamma^2 v^2$$

$$\gamma^2 = \gamma^2 v^2 + 1 \tag{8}$$