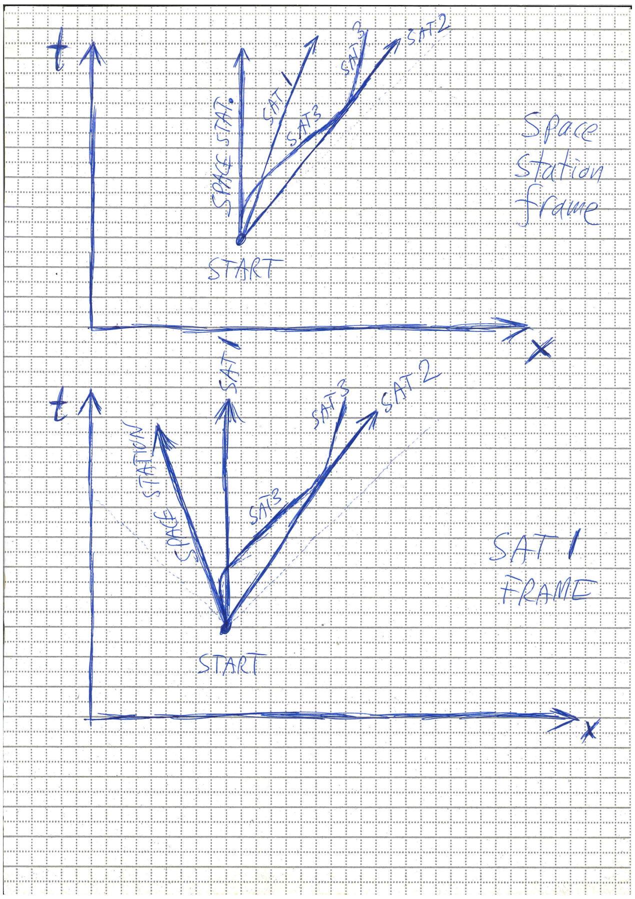
Solutions part 2B

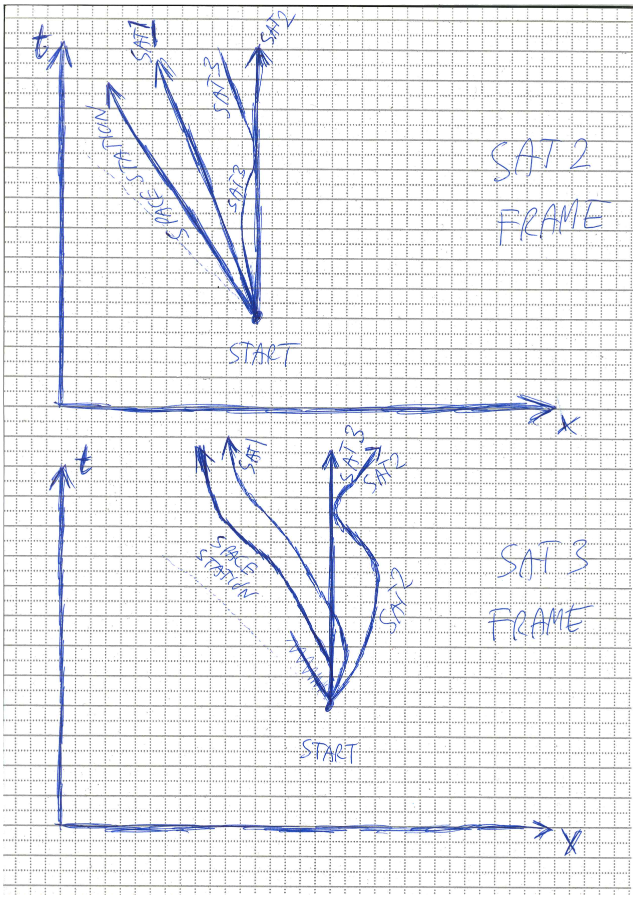
The following are short solutions to many exercises in part 2B. In addition to this document there is also a solution file (pure text file) which can be found in the solutions folder where the xml is downloaded. Note that your numerical solutions will be very close to, but seldom exactly equal to the numbers found in the solution file. The reason for this is that the solution file is created with the same code generating the xml for the video: since the video has a limited time resolution, the times of events will not be exact in the video and for this reason there will be small errors (of the order 1% or less) in most numbers given in the solution file.

Exercise 2B.1

2. See figure 1 and the upper plot of figure 2.

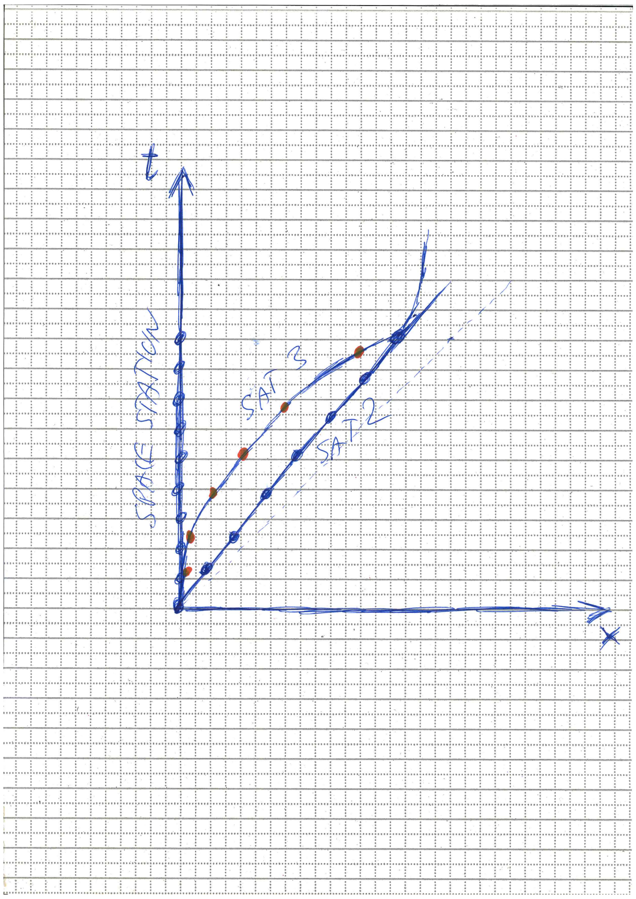


Figur 1: Spacetime diagram for the space station and ship 1.



Figur 2: Spacetime diagram for ship 2 and ship 3.

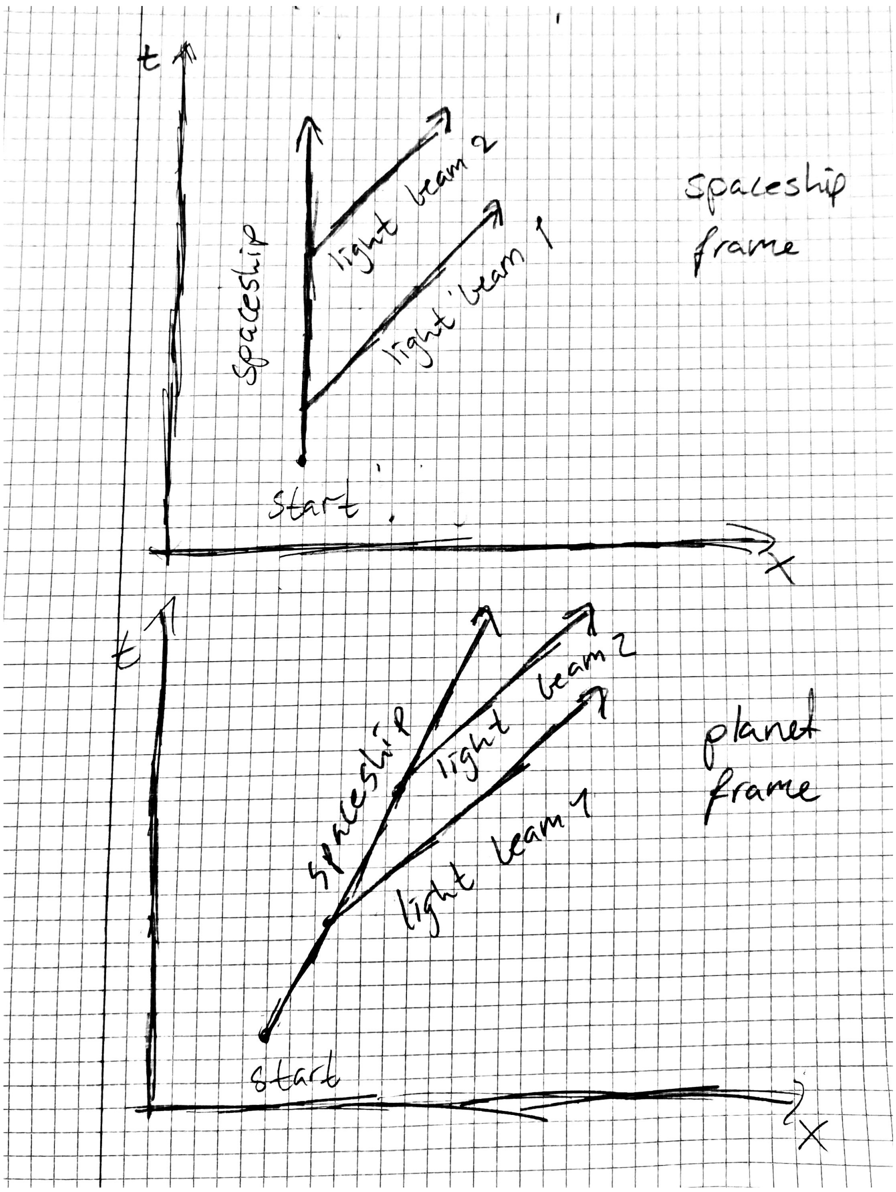
5. See lower plot of figure 2.
6. See figure 3.
7. See figure 3, note that the spacing between ticks for satellite 2 is supposed to be equal for all ticks.
8. less
9. See figure 3.



Figur 3: Spacetime diagram for the space station with the dots drawn in.

Exercise 2B.3

1. See solution file
2. The spacetime diagrams are shown in figure 4.



Figur 4: Spacetime diagrams for both the spaceship and planet frame.

4. $v_x' = 1$ (corresponding to light speed)
6. Sees solution file.

Exercise 2B.4

1. $$P_\mu(e) = m_e \gamma'_e \left(1, v'_e\right) \tag{0.1}$$

2. $$P_\mu(p) = m_p \gamma'_p \left(1, v'_p\right) \tag{0.2}$$

3. $$P(n) = (m_n, 0) \tag{0.3}$$

4. $$v'_e = \sqrt{\frac{C}{C+1}}$$

   where

   $$C = \left(\frac{v'_p \gamma'_p m_p}{m_e}\right)^2$$

   .

5. $$E = \gamma_n E' + v_n \gamma_n p'_x \tag{0.4}$$
   $$p_x = v_n \gamma_n E' + \gamma_n p'_x \tag{0.5}$$

   where v_n is the velocity of the neutron taken in the planet frame and $\gamma_n = 1/\sqrt{1 - v_n^2}$. Here $E' = m\gamma$ and $p'_x = m\gamma v$ are the energy and momentum of the electron or the proton taken in the neutron frame.

   We get the answer in kg.

6. $$v = \sqrt{1 - \left(\frac{m}{E}\right)^2}. \tag{0.6}$$

7. We will use the equation for γ_B obtained in exercise 4 and insert $m_n = m_e + m_p$ and this gives

   $$\gamma'_p = 1 \tag{0.7}$$

   and $v'_p = v'_e = 0$. The electron and proton would remain at rest at exactly the same position.

Exercise 2B.5

1. See Solution file

2. The momenergy vector in the spaceship frame is

   $$P'_\mu(A) = m \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad P'_\mu(B) = m\gamma'_B \begin{bmatrix} 1 \\ v'_B \end{bmatrix}. \tag{0.8}$$

   The momenergy vector in the planet frame is

   $$P_\mu(A) = m\gamma_A \begin{bmatrix} 1 \\ v_A \end{bmatrix} \qquad P_\mu(B) = m\gamma_A \begin{bmatrix} 1 \\ -v_A \end{bmatrix}. \tag{0.9}$$

3. The momenergy vector in the spaceship frame transformed from the planetframe is given by

$$P'_\mu(A) = m\gamma_A^2 \begin{bmatrix} 1 - v_A^2 \\ 0 \end{bmatrix} = m \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad P'_\mu(B) = m\gamma_A^2 \begin{bmatrix} 1 + v_A^2 \\ -2v_A \end{bmatrix}. \tag{0.10}$$

The momenergy vector in the planet frame transformed from the spaceship frame is therefore given by

$$P_\mu(A) = m\gamma_A \begin{bmatrix} 1 \\ v_A \end{bmatrix} \qquad P_\mu(B) = m\gamma'_B\gamma_A \begin{bmatrix} 1 + v_A v'_B \\ v_A + v'_B \end{bmatrix}. \tag{0.11}$$

Note that these expressions are of course equal to the expressions in the previous answer if you use relations between v'_B, γ'_B and v_A, γ_A. You are not expected to do this.

8. See solution file

11. See solution file

13.

$$v = \frac{K - 1}{K + 1}$$

where

$$K = \left(2 - \frac{\lambda \gamma_A m_e c}{h}\right)^2$$

in SI units. Here λ is the wavelength you observe.