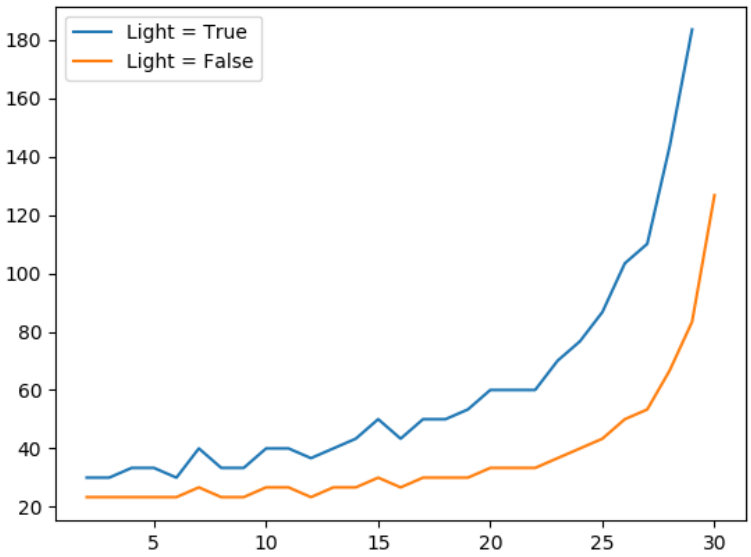
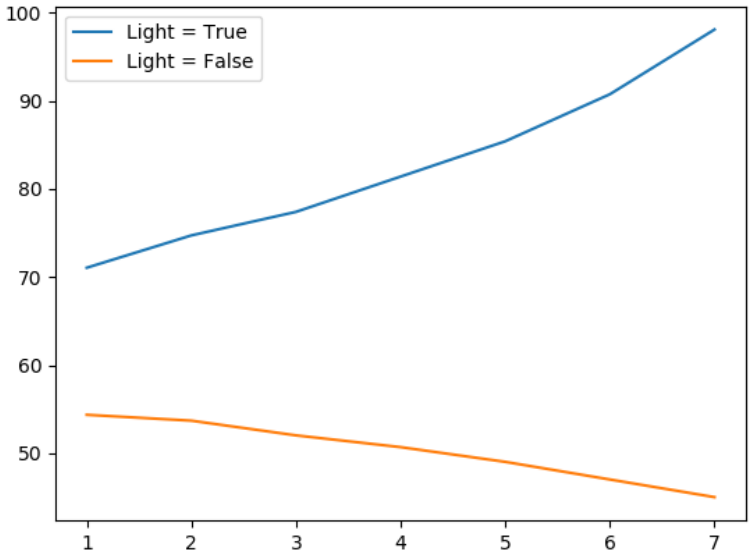
Solutions part 2E

Exercise 2E.2

Plotting the time differences should result in something similarly to figure 1 and 2. You should see a trend where the difference in time interval time increases.



Figur 1: The differences between true light speed and false for the shell observer.



Figur 2: The differences between true light speed and false for the infalling spaceship.

Exercise 2E.3

1. The tangential velocity is given by
$$v_\phi = r\frac{d\phi}{dt}. \tag{0.1}$$
2.
$$(L/E)^2 = \frac{r^2}{\left(1-\frac{2M}{r}\right)}. \tag{0.2}$$
4. Light moves faster tangentially than radially.

Exercise 2E.6

2. At a distance $r = R$ and the velocity is only tangential to the radial vector and the radial velocity is therefore 0.

Exercise 2E.7

2. α
3. $\Delta\phi$

4. $\gamma \to \pi/2$

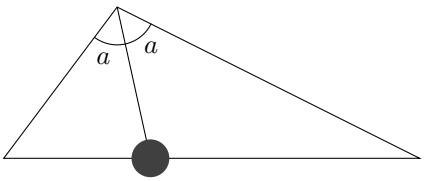
6.
$$\Delta\phi = 4 \cdot \frac{1.4849\text{km}}{695508\text{km}} \cdot 206264.806 = 1.76\text{arc seconds} \tag{0.3}$$

7.
$$\Delta\phi = 4 \cdot \frac{5.485 \cdot 10^{-8}\text{km}}{1737.1\text{km}} \cdot 206264.806 = 2.61 \cdot 10^{-5}\text{arc seconds.} \tag{0.4}$$

Exercise 2E.8

3. It must be located to the left.

4. See figure 3.



Figur 3: A corrected version of figure (6) from the lecture notes but without the curvature.

5. Try adjusting the start radius in the program closer to the black hole. You will then see clearly that the symmetry vector points to the left.

Exercise 2E.9

6.
$$M = 1.35 \cdot 10^{15} M_{\odot} \tag{0.5}$$