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NOTE: There might be errors in the solution. If you find something which doens't look right, please let me know

Partial solutions to problems: Part 3C

Exercise 3C.1

1. We use supplied formula for *pp*-chain:
$$\epsilon_{pp} \approx \epsilon_{0,pp} X_H^2 \rho T_6^4 \approx 0.001$$
where $\epsilon_{0,pp} = 1.08 \cdot 10^{-12}$, $X_H = 0.33$, $\rho = 1.5 \cdot 10^5$ and $T = 15.7$. Similarly,a
$$\epsilon_{cno} \approx 3.4 \cdot 10^{-4}$$
and
$$\epsilon_{3\alpha} \approx 0$$
as $0.157^{41} \approx 0$.
2. We see that $\epsilon_{pp}/\epsilon_{cno} \approx 3$, which is way off the 1% expectation. This answer is wrong because we assumed that the temperature is constant in the core, which is not true: the high temperatures are only evident in the center of the core. This means that most of the energy is created at lower temperatures, where the *pp* chain dominates. The 3α is practically non-existing.
3. With $T = 13 \cdot 10^6 K$, we obtain a ratio $\epsilon_{pp}/\epsilon_{cno} \approx 65$, which is more closer to reality (that is, *pp* dominates CNO by approx 1.5%).
4. At what temperature T is $\epsilon_{pp} = \epsilon_{CNO}$? We equate:
$$\epsilon_{0,pp} X_H^2 \rho T_6^4 = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20}$$
Solving for T_6:
$$T = \Big(\frac{\epsilon_{0,pp} X_H}{\epsilon_{0,CNO} X_{CNO}} \Big)^{\frac{1}{16}} \approx 17$$
such that CNO dominates from 17 million K (assuming the temperature in the core is homogeneous).
5. The total energy L_{sun} emmitted from the sun must equal the total mass inside the core of the sun multiplied with the reaction rate:
$$L_{sun} = \frac{4}{3}\pi R^3 \cdot \rho \cdot \epsilon$$
Solving for R:
$$R = \Big(\frac{3L_{sun}}{4\pi\rho\epsilon} \Big)^{1/3} \approx 0.15 R_{sun}$$
when the values have been inserted.
6. Using the same equation as in the previous question, we obtain that $R \approx 0.6 R_{sun}$.