Problem 1

You can use $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ where you need to in the following.

a) Find the proper distance to the particle horizon, $d_{\mathrm{PH}}(z)$, as a function of redshift z for the Einstein-de Sitter model.

b) Calculate the mass contained within $d_{\mathrm{PH}}(z)$ as a function of redshift, that is, the mass within a sphere of radius $d_{\mathrm{PH}}(z)$.

c) Explain why the result in b) is not in conflict with conservation of mass.

d) Estimate (no calculator!) at what redshift the mass contained within the particle horizon was equal to the mass of the Local Supercluster, about $10^{15} M_{\odot}$. What was the size of the particle horizon at this redshift? And how old was this EdS universe then?

Problem 2

This problem is from Yu. L. Bolotin and I. V. Tanatarov, arXiv:1310.6329. Consider two widely separated observers, A and B. Suppose they have overlapping particle horizons, but each can see things that the other cannot. Can B communicate to A information that extends A's knowledge of things beyond his horizon? If so, then a third observer C may communicate to B information that extends her horizon, which can then be communicated to A. Hence, an unlimited sequence of observers B, C, D, E,... may extend A's knowledge of the Universe to indefinite limits. According to this argument, A has no true horizon. This is the horizon riddle. Try to resolve it for a static universe of finite age, that is, for a Minkowski spacetime which came into existence a finite time ago. (Hint: The easiest way to do this is to draw a space-time diagram).

Problem 3

Estimate, using just pen and paper, at what time in the history of the Universe the baryon density was equal to the density of air. Use $\Omega_{\mathrm{b0}} = 5 \times 10^{-2}$.

Problem 4

Prove that the temperature of a gas of non-relativistic particles following the ideal gas law varies as a^{-2}, assuming adiabatic expansion.