Modelling energy production

Term project 1

This project involves calculating the energy production at the center of a star given a temperature and density. The goal is to write a class/function that accepts a temperature and density, and then calculates the amount of energy produced by the fusion chains discussed in Chapter 3 of the lecture notes. The class/function is going to be used for your later model of a star in Project 2. Read through the entire project description before you start coding.

Assumptions

- You will assume that the mass fraction of each atomic species is independent of location in the star, and given by

$$
\begin{aligned}
X &= 0.7 \\
Y_{^3_2\mathrm{He}} &= 10^{-10} \\
Y &= 0.29 \\
Z_{^7_3\mathrm{Li}} &= 10^{-7} \\
Z_{^7_4\mathrm{Be}} &= 10^{-7} \\
Z_{^{14}_7\mathrm{N}} &= 10^{-11}
\end{aligned}
$$

- When calculating the electron density you can assume that all elements are fully ionized, e.g. a Helium atom consists of a core and two electrons.

- You need to include the processes making up the PP chains and the

CNO[1] cycle. Assuming that produced $^2_1$D is immediately consumed by the next step to produce $^3_2$He, you may combine the first two steps of the PP chain into a single step with the reaction rate of proton-proton fusion. Similarly, you can combine the entire CNO cycle into a single step with the limiting reaction rate.

- You do not need to consider changes over time, as you are looking at a snapshot of a star at a particular moment in time. In other words, the elemental abundances do not evolve as described in Ch. 3.5 in the lecture notes.

Tasks

- Before you begin coding, identify the SI units of the following variables in the lecture notes:

$$T_9 \qquad \lambda_{ik} \qquad r_{ik} \qquad Q_{ik} \qquad \varepsilon \tag{1}$$

  For some of these, you may want to use "number of reactions" (or just "reactions") as one of the units involved, making it clearer what the variables represent. Note that the reaction rates listed in Table 3.1 are reactions per second per (mole per cubic cm) or $\mathrm{s}^{-1}\ \mathrm{mol}^{-1}\ \mathrm{cm}^3$.

- Write a class/function for calculating the energy production (in SI units) in the center of a star, including the reactions in the PPI, PPII, and PPIII branches and the dominant CNO cycle. You should include the upper limit of the $^7_4$Be electron capture[2].

- Adjust your class/function so that no step consumes more of an element than the previous steps are able to produce. Take extra care when several reactions consume the same element.

- Calculate the energy released that goes to heating up the star, both the total energy ε and the energy from completed PPI, PPII, and PPIII branches and the dominant CNO cycle. Note first that some of the reactions in the PP branches are shared between several branches –

[1] The dominant CNO cycle, given in Eqs. (3.13)-(3.18) in the lecture notes.

[2] See table 3.1 text in the lecture notes. Note that the number density of electrons n_e, needed to calculate the upper limit of the $^7_4$Be electron capture, is not in units of cm^{-3}. Hence, this rate should be treated differently from the rest of the rates in Table 3.1.

the energy of these reactions must be appropriately shared between the different PP branches. Note also that not all branches are completed, in many cases the first reactions have higher rates than the last ones. In these cases, the energy from the incompleted reactions should not be included in the energy produced by individual branches, but should still be included in the total energy ε.

Sanity check

In order to verify your code, you need to implement a sanity check. Using the temperature and density of the solar core (see Appendix B in the lecture notes) and calculating the number densities by using element masses as an integer (the number of nucleons) times the atomic mass unit (e.g. $m_{^1_1\mathrm{H}} = 1m_\mathrm{u}$), you should get the following results:

$$
\begin{aligned}
r_{^1_1\mathrm{H},^1_1\mathrm{H}}(Q'_{^1_1\mathrm{H},^1_1\mathrm{H}} + Q'_{^1_1\mathrm{H},^2_1\mathrm{D}})\rho &= 4.05 \cdot 10^{2}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^3_2\mathrm{He},^3_2\mathrm{He}}Q'_{^3_2\mathrm{He},^3_2\mathrm{He}}\rho &= 8.69 \cdot 10^{-9}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^3_2\mathrm{He},^4_2\mathrm{He}}Q'_{^3_2\mathrm{He},^4_2\mathrm{He}}\rho &= 4.87 \cdot 10^{-5}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^7_4\mathrm{Be},\mathrm{e}^-}Q'_{^7_4\mathrm{Be},\mathrm{e}^-}\rho &= 1.50 \cdot 10^{-6}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^7_3\mathrm{Li},^1_1\mathrm{H}}Q'_{^7_3\mathrm{Li},^1_1\mathrm{H}}\rho &= 5.30 \cdot 10^{-4}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^7_4\mathrm{Be},^1_1\mathrm{H}}(Q'_{^7_4\mathrm{Be},^1_1\mathrm{H}} + Q'_{\mathrm{decay}})\rho &= 1.64 \cdot 10^{-6}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^{14}_7\mathrm{N},^1_1\mathrm{H}}Q'_{\mathrm{CNO}}\rho &= 9.18 \cdot 10^{-8}\ \mathrm{J\ m^{-3}\ s^{-1}}
\end{aligned}
$$

You also need to make sure that the code works when the reaction rate of the common step in the PP chain is lower than the subsequent steps. Using the density of the solar core (see Appendix B) and the temperature $T = 10^8$ K,

you should get the following results:

$$
\begin{aligned}
r_{{}^1_1\mathrm{H},{}^1_1\mathrm{H}}(Q'_{{}^1_1\mathrm{H},{}^1_1\mathrm{H}} + Q'_{{}^1_1\mathrm{H},{}^2_1\mathrm{D}})\rho &= 7.34 \cdot 10^4 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^3_2\mathrm{He},{}^3_2\mathrm{He}}Q'_{{}^3_2\mathrm{He},{}^3_2\mathrm{He}}\rho &= 1.10 \cdot 10^0 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^3_2\mathrm{He},{}^4_2\mathrm{He}}Q'_{{}^3_2\mathrm{He},{}^4_2\mathrm{He}}\rho &= 1.75 \cdot 10^4 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^7_4\mathrm{Be},\mathrm{e}^-}Q'_{{}^7_4\mathrm{Be},\mathrm{e}^-}\rho &= 1.23 \cdot 10^{-3} \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^7_3\mathrm{Li},{}^1_1\mathrm{H}}Q'_{{}^7_3\mathrm{Li},{}^1_1\mathrm{H}}\rho &= 4.35 \cdot 10^{-1} \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^7_4\mathrm{Be},{}^1_1\mathrm{H}}(Q'_{{}^7_4\mathrm{Be},{}^1_1\mathrm{H}} + Q'_{\mathrm{decay}})\rho &= 1.27 \cdot 10^5 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^{14}_7\mathrm{N},{}^1_1\mathrm{H}}Q'_{\mathrm{CNO}}\rho &= 3.45 \cdot 10^4 \text{ J m}^{-3} \text{ s}^{-1}
\end{aligned}
$$

The sanity check needs to be implemented in your code and it must be possible to turn it on and off so that the instructors can check if the sanity check is correctly implemented. The numbers need not be put in the report, but the expected values above, the values calculated by your code, and the relative errors between them should be printed when running the code. The abolute relative errors should be smaller than ~ 0.01 to pass the sanity check. If the code does not pass the sanity check, it should stop! 10 points

Code

The code has to be written using the `python 3` programming language. The way your code is written impacts the number of points you get for this project. It should be easy to read, well commented, and logically structured. The instructors should be able to run your code easily. 10 points

Report

You are required to write a report in this project. The report should include an introduction, theory, results, discussion, and conclusion, as is standard, as well as a reflection in the end (see point 8 below). The maximum number of pages is 10 (excluding a title page, if you choose to include it, and the references) – the report will not be evaluated past the page limit. How your report is written impacts the amount of points you get on this project. It is recommended that you write the report using LaTeX. Use a spellchecker to avoid spelling mistakes and typos. Your figures should have a clear layout with proper axis labels and units, with a caption explaining what the figure

shows. All figures should have a reference in the main text and their content should be discussed.

You should include answers and explanations to the following bullet points. Try to include the answers to the points in the report, in the order they are given here. Some of the bullet points require you to complete the previous ones, but some are possible to complete independently. This is important to remember if you start to run out of time.

1. Explain the different equations that goes into solving this project (you do not need to include the expressions in Table 3.1). Include also the SI units of the variables in Eq. (1) of this document. 5 points

2. Calculate the energy output from each individual reaction in the PP chain and CNO cycle, using the mass difference between the reactants and the products. You can compare your results to the values given in the lecture notes to check if you are close (you do not have to include this check in the report). For each of the branches (PPI, PPII, PPIII, and CNO), how large a percentage of the released energy is lost to neutrinos per produced ${}^4_2\mathrm{He}$? Do these calculations in your code and print the results, so they are easily readable, as well as including them in the report. 10 points

3. Explain why we only need to use the reaction rate from

$$ {}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{15}_{8}\mathrm{O} + 7.30 \ \mathrm{MeV} $$

   for the entire CNO cycle. Explain also where the ${}^{12}_{6}\mathrm{C}$ in the first CNO reaction (Eq. (3.13) in the lecture notes) comes from – both historically and how its amount is not being reduced as it is consumed in the Sun. 5 points

4. Explain how you make sure that no step consumes more of an element than the previous steps are able to produce. 10 points

5. Explain how you calculate the energy production from each of the PP branches and the CNO cycle, individually. Include equations. Remember to take care when a reaction occurs in several PP branches, as well as only including completed branches. 10 points

6. Plot the relative energy production from each of the PP branches and the CNO cycle as functions of temperature, where $T \in [10^4, 10^9]$. Explain what the plot shows. Include also the total energy released ε, relative to the maximum value $\varepsilon_{\max} = \max(\varepsilon(T))$, to see that more energy is produced at higher temperatures. 10 points

7. Calculate the Gamow peak for all the relevant reactions of the PP chain and CNO cycle in the range $E \in [10^{-17}, 10^{-13}]$ J. Include in the report an equation for the curve of the Gamow peak and explanation for why it produces a peak. For each reaction, plot the probability curves normalized to the maximum value. Explain your figure. 15 points

8. Your report should be well structured and easy to read. After the conclusion, you should write a "Reflection" section where you explain what you have learned from this exercise and what you have struggled with. 15 points

The maximum score of this project is 100 points. The report and code is delivered at `https://devilry.ifi.uio.no/`. There will be no extension to the deadline, except in the case of documented medical circumstances. If you cannot solve the project, write the report anyway, explaining your problems and what you have tried in order to solve them.