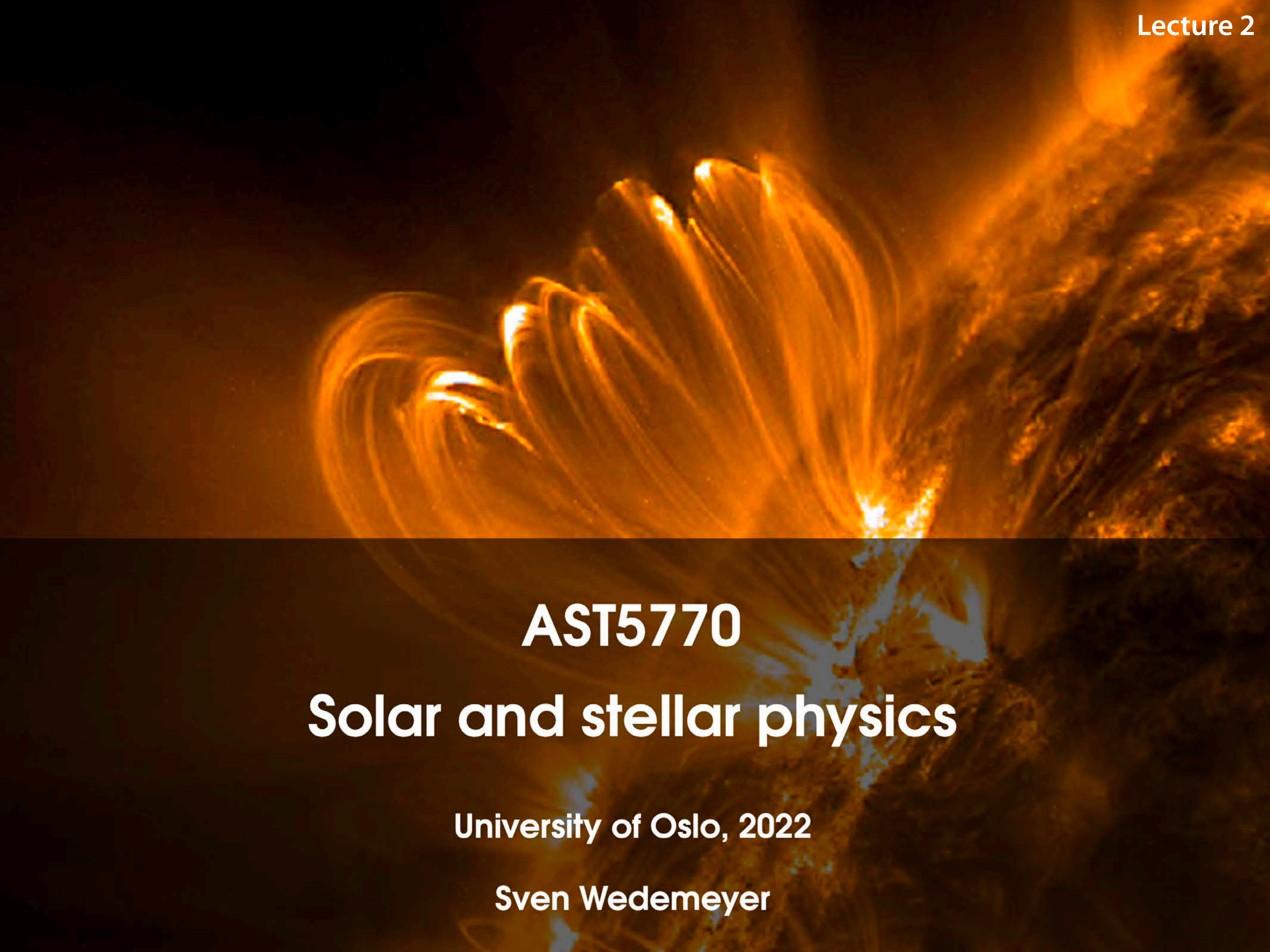


Lecture 2

AST5770

Solar and stellar physics

University of Oslo, 2022

Sven Wedemeyer

Observational stellar parameters

Bolometric brightness

- **Bolometric = <u>all</u> wavelengths**

- Integration of the wavelength-dependent radiation flux F_λ over **all wavelengths,** or equivalently F_ν over **all frequencies**

$$F = \int_0^\infty F_\lambda d\lambda = \int_0^\infty F_\nu d\nu$$

➡ **Apparent bolometric brightness** m_{bol} is a measure for the **total radiative flux F** of a star

Observational stellar parameters

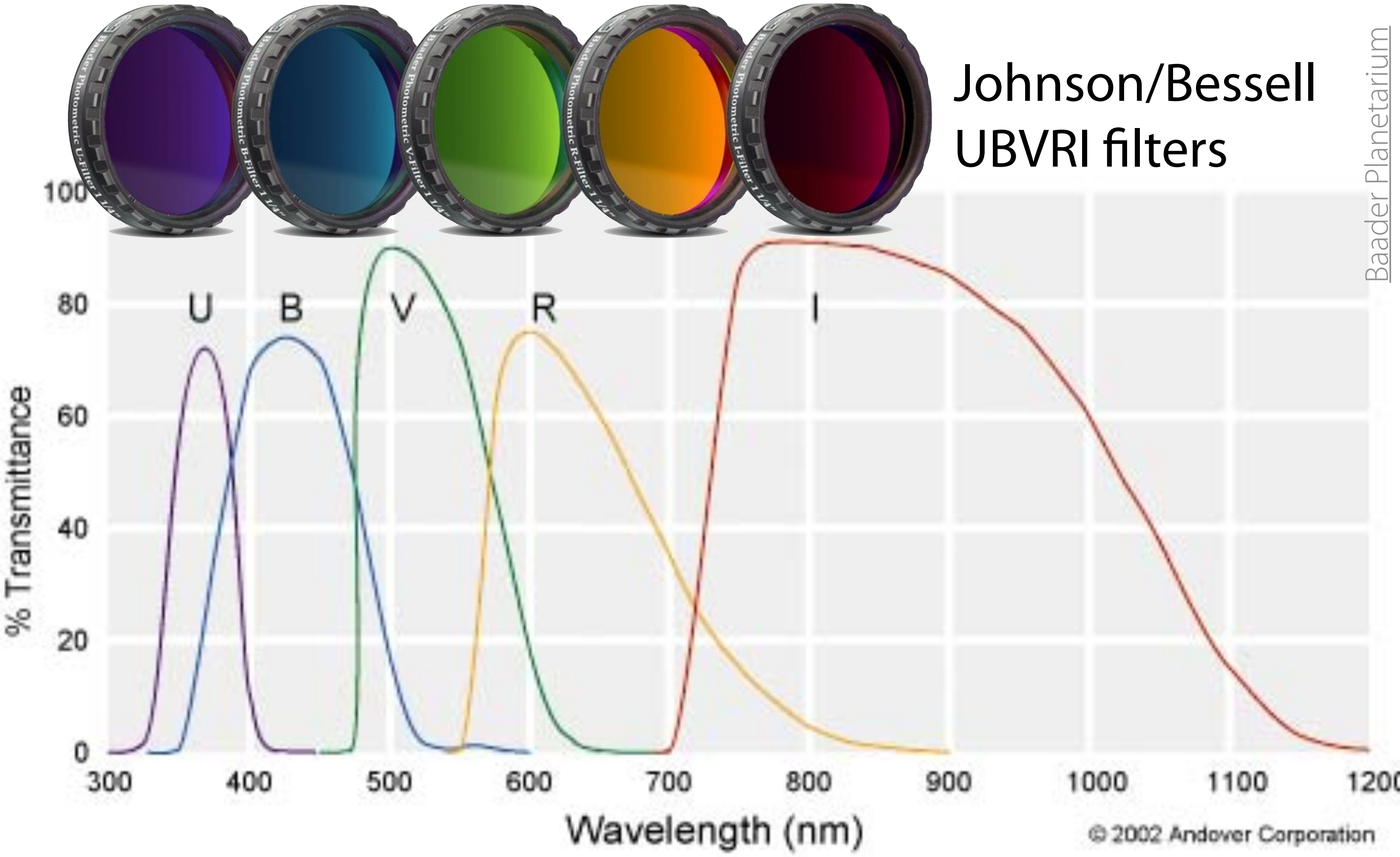
Photometry and colours

- Use of different filters in an observation
- Transmission of only limited wavelength ranges
- Standardised filter system(s)
  - Most common: UBVRI(+)
  - Originally **UBV** (ultraviolet — blue — visual)
  - Extended into the infrared (IR)

- Brightness measured in a selected filter marked with corresponding index
- Example: Visual (**V**)
  - Apparent brightness: $m_V = V$
    (Often only the filter ID is used!)
  - Absolute brightness: M_V

UBVRI filter system + extension

Filter	descript.	λ [nm]	FWHM [nm]
U	UV	365	66
B	blue	440	94
V	visual	548	88
R	red	658	138
I	IR	806	149
J	IR	1220	213
H	IR	1630	307
K	IR	2190	490
L	IR	3450	473
M	IR	4750	460



Johnson/Bessell UBVRI filters

Baader Planetarium

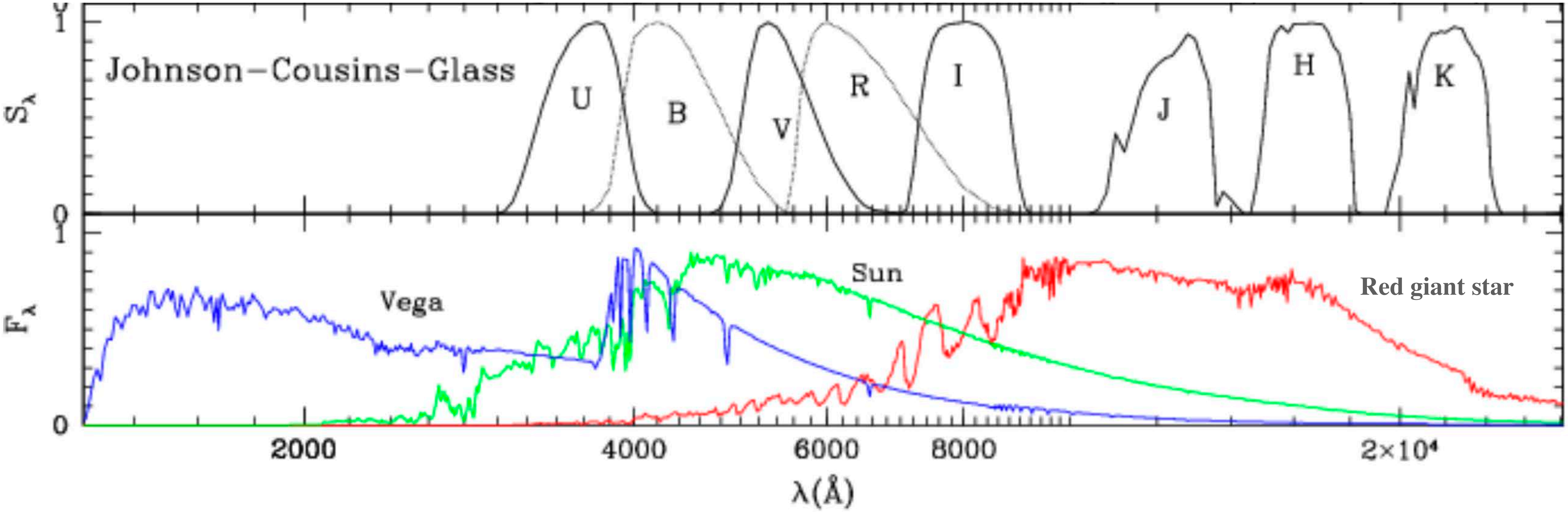
Observational stellar parameters

Photometry and colours, color index

- Measuring brightness with different filters captures
  variation of flux density as function of wavelength (spectrum)
  ➡ Reveals difference between stars

- **Colour index:** difference of two brightness measured in different bands, $m_X - m_Y$.

- Example: $m_B - m_V = B - V$



Observational stellar parameters

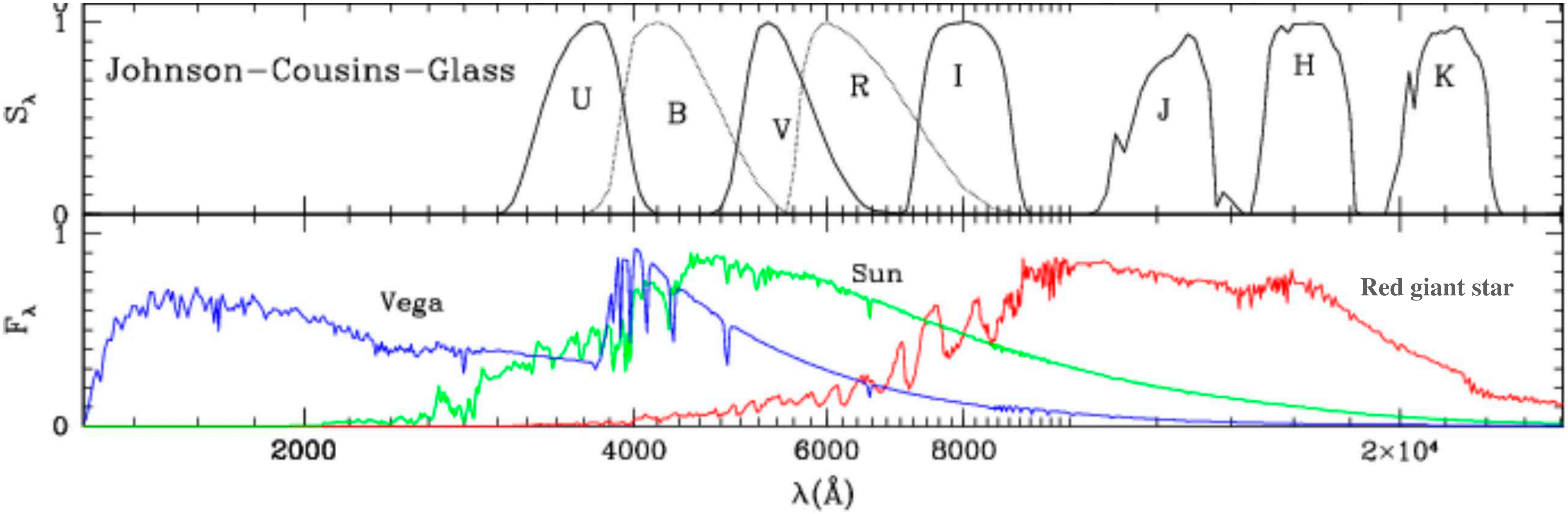
Photometry and colours, color index

- Measuring brightness with different filters captures **variation of flux density as function of wavelength** (spectrum)

  ➡ Reveals difference between stars

- **Colour index:**
  - **Examples**

Star	$U = m_U$	$B = m_B$	$V = m_V$	$(U - B)$	$(B - V)$
Sun	-25.85	-26.03	-26.70	0.18	0.67
Proxima Cen	14.21	12.95	11.13	1.26	1.82
Betelgeuse	~ 0.50	~ -1.6	~ -3.4	+2.06	+1.85



Observational stellar parameters

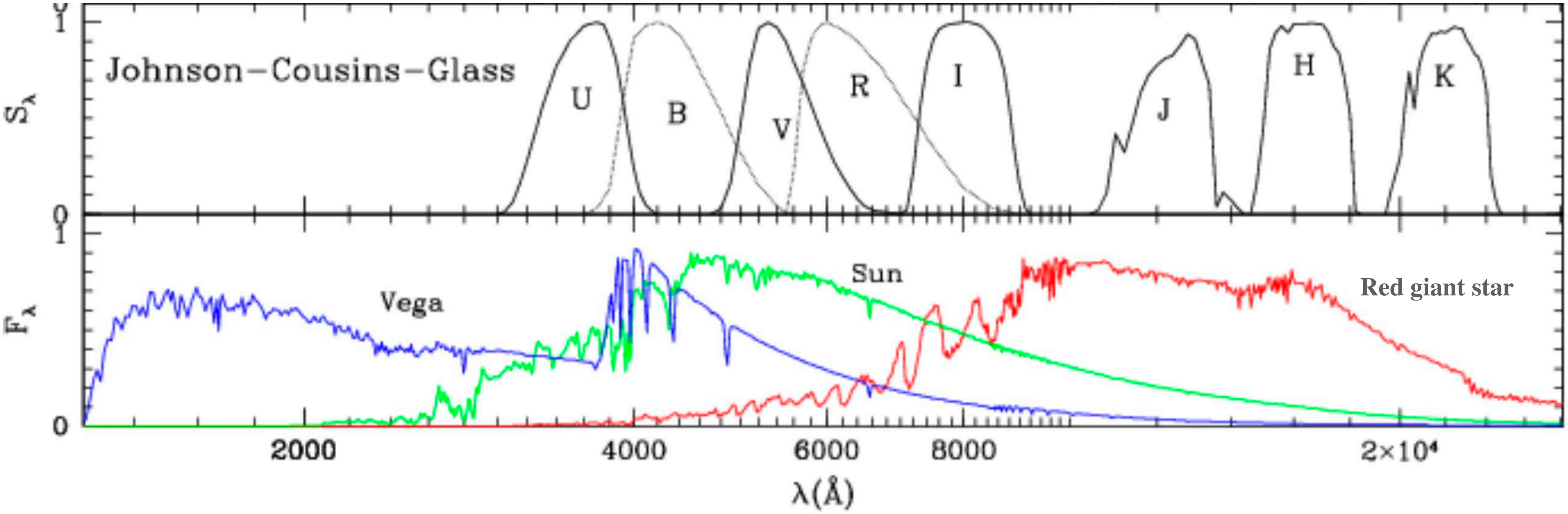
Bolometric correction

- Filters cover only parts of the spectrum
- Especially measuring with few filters may give incomplete picture

➡ Correction for missing wavelength ranges needed when absolute brightness across all wavelengths is wanted

➡ **Bolometric correction (BC):** $M_{\mathrm{bol}} = M_{\mathrm{V}} + \mathrm{BC}$

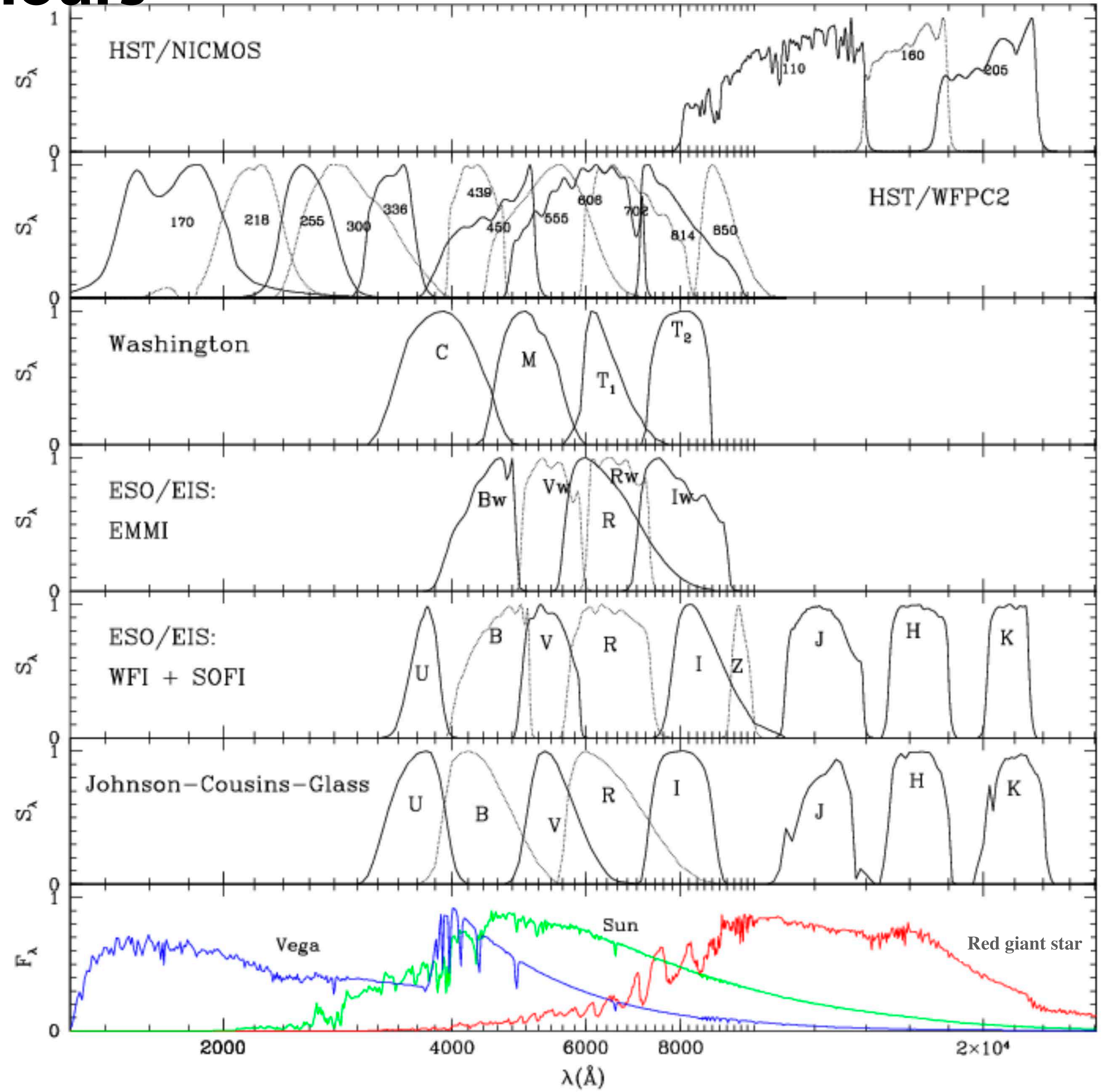
- Note: hot stars radiate much in UV, not well captured by the available filter systems; (very) cool stars better covered with IR filters



Observational stellar parameters

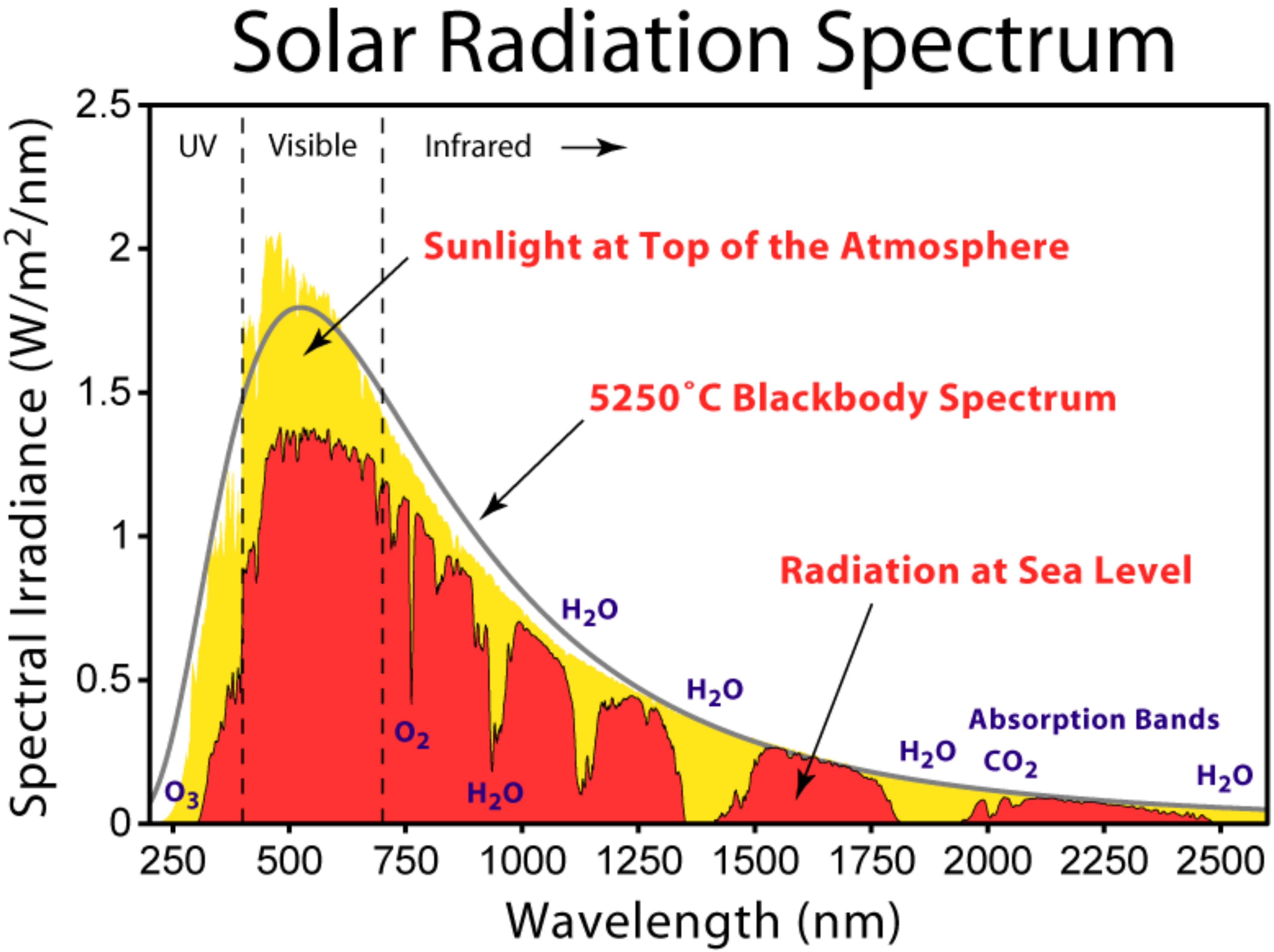
Photometry and colours

- Note that different instruments at different telescopes can have other filter systems



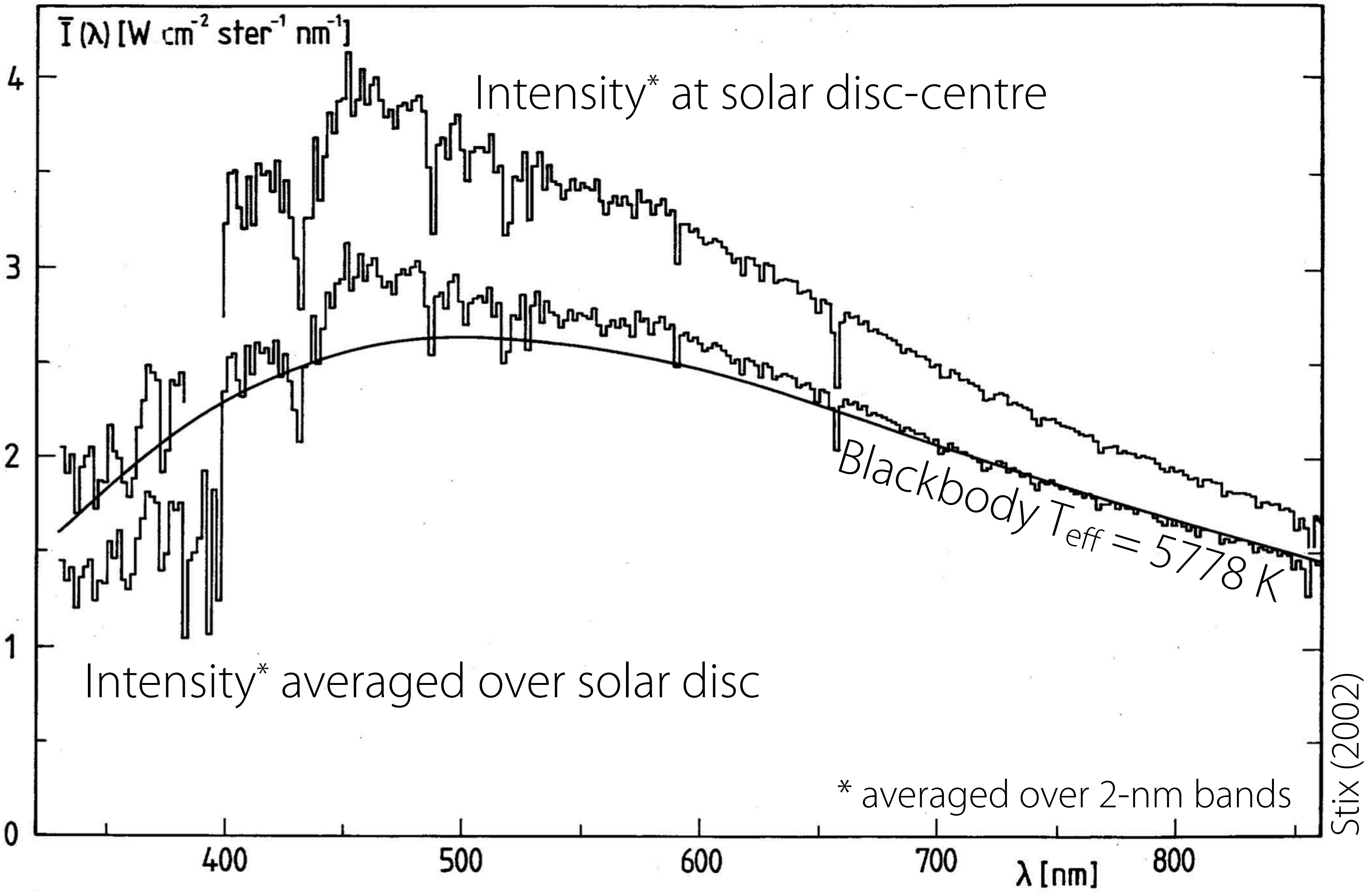
Observational stellar parameters

Stellar spectrum



Observational stellar parameters

Stellar spectrum



Stix (2002)

- Measured Sun's intensity as function of wavelength
- The black body spectrum for an effective temperature of Teff = 5778 K
- Data from Neckel and Labs (1984) - Also provided for your project assignment.

Observational stellar parameters

Blackbody spectrum

- Radiation flux density (spectrum) resembles a **blackbody** spectrum, which is given by the Planck function **B_ν (T)**.
  - B_ν (T): spectral density of electromagnetic radiation emitted by a blackbody in thermal equilibrium at a given temperature T .

➡ The flux density F_ν of a blackbody:

$$F_\nu = \pi B_\nu(T)$$

➡ Bolometric flux by integration over all frequencies:

$$F = \int_0^\infty F_\lambda d\lambda = \int_0^\infty F_\nu d\nu = \int_0^\infty \pi B_\nu(T) d\nu = \sigma T^4 \Rightarrow F = \sigma T_{\mathrm{eff}}^4$$

- Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.67 \times 10^{-8} \mathrm{W\,m^{-2}\,K^{-4}} = 5.67 \times 10^{-5} \mathrm{erg\,s^{-1}\,cm^{-2}\,K^{-4}}$$

Observational stellar parameters

Stefan-Boltzmann law

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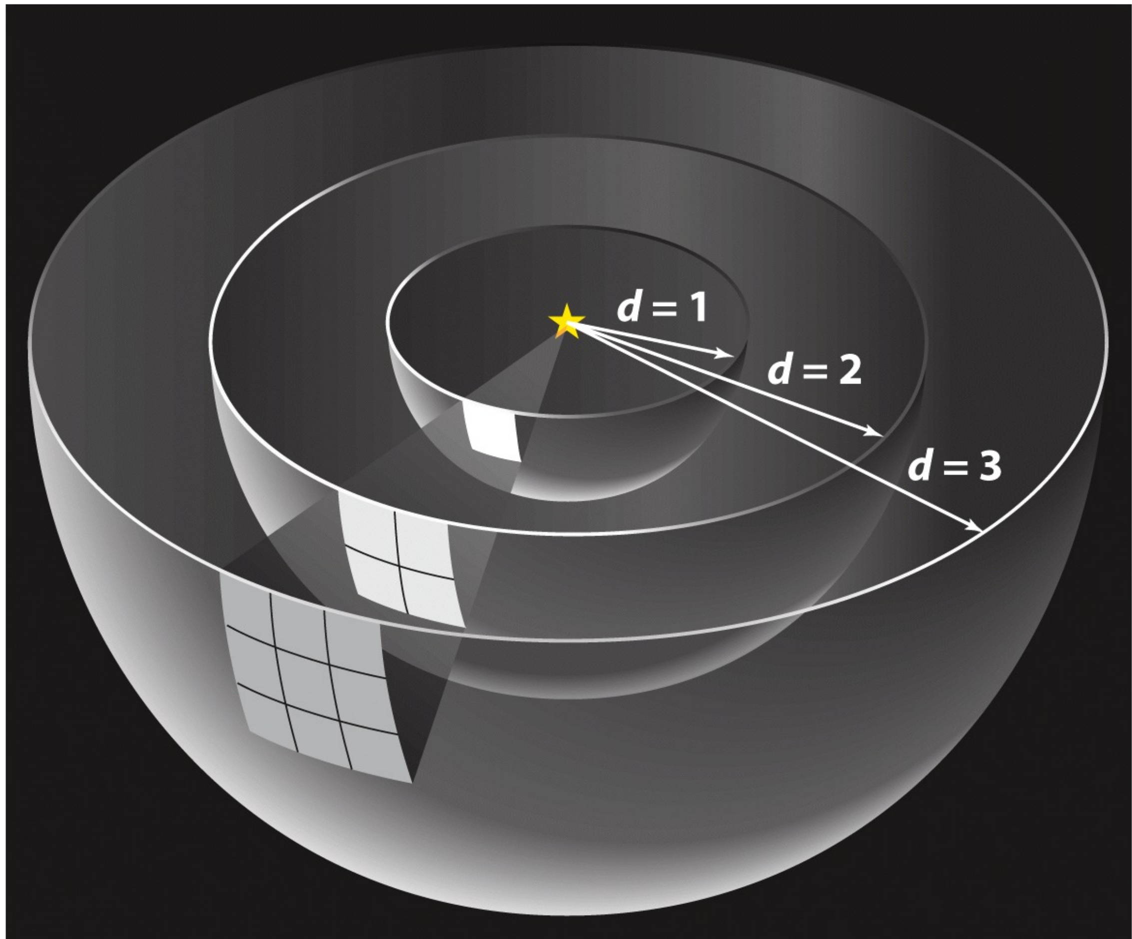
Observational stellar parameters

Stefan-Boltzmann law

- **Example: Sun**
- Bolometric flux density of the Sun measured just outside Earth's atmosphere by a satellite:

➡ **solar constant** = 1.36 kW m^{-2}

  - Radiation emitted from the Sun's "surface" at radius 1 $R_\odot$
  - Diluted over a sphere with radius d = 1 AU with surface area $A = 4\pi d^2$



Observational stellar parameters

Stefan-Boltzmann law

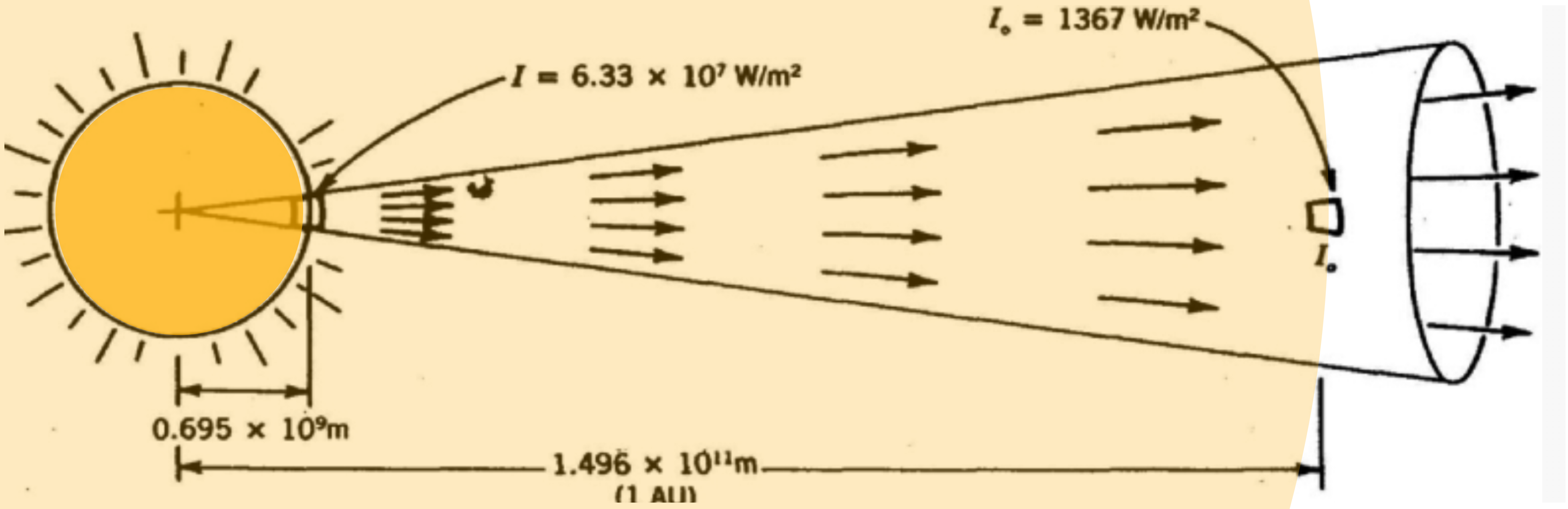
- **Example: Sun**
- Bolometric flux density of the Sun measured just outside Earth's atmosphere by a satellite:

➡ **solar constant** = 1.36 kW m^{-2}

  - Radiation emitted from the Sun's "surface" at radius 1 R$\odot$
  - Diluted over a sphere with radius d = 1 AU with surface area $A = 4\pi\, d^2$
- Correction for "dilution" effect with the factor $d^2/R_\odot^2$ gives flux density at Sun's surface ($1R_\odot$):

$F_\odot = 6.3 \cdot 10^7\ \mathrm{Wm^{-2}}$

- Stefan-Boltzmann law: $F_\odot = \sigma\, T_{\mathrm{eff},\odot}^4$ —> effective temperature of the Sun: **$T_{\mathrm{eff},\odot} \approx 5770$ K**



Stellar parameters

Luminosity

- **Total radiative energy output of a star**

  given by flux that emerges across the total surface of a star.

- Assumption: star is spherical with radius R

➡ Surface area $A = 4\pi R^2$

➡ Luminosity of a star $L = A \, F$

Stellar parameters

Luminosity

- **Total radiative energy output of a star**

  given by flux that emerges across the total surface of a star.

- Assumption: star is spherical with radius R

Stefan-Boltzmann law:

$F = \sigma T_{eff}^4$

➡ Surface area $A = 4\pi R^2$

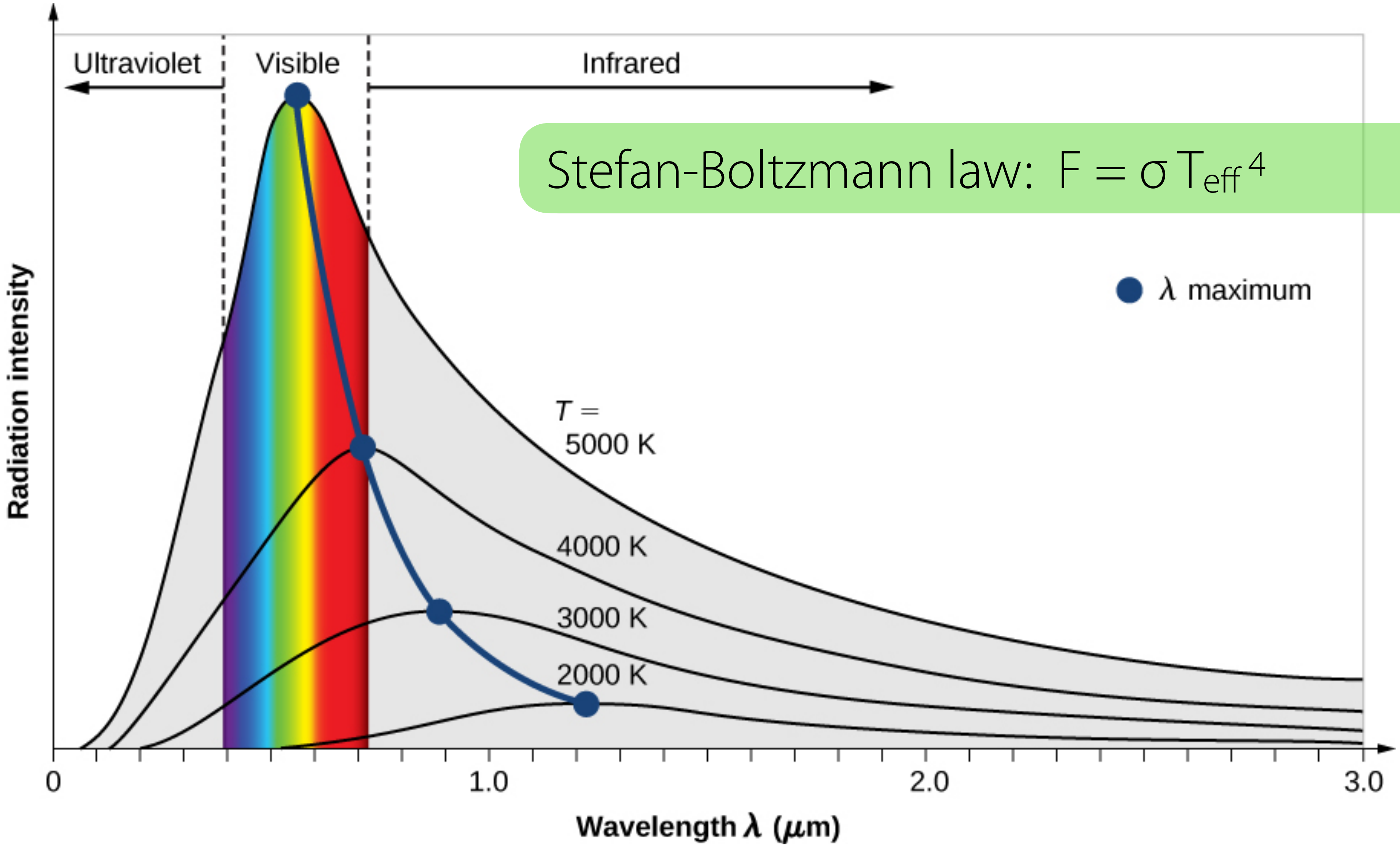
➡ Luminosity of a star $L = 4\pi R^2 \sigma T_{eff}^4$

- Units: W (SI) or erg s–1 (cgs)
- Convenient to use the bolometric luminosity of the Sun as unit

$$L_\odot = 3.84 \times 10^{26}\ \mathrm{W}.$$

Stellar spectra

- Blackbody for different temperatures



Stefan-Boltzmann law: $F = \sigma T_{eff}^{4}$

- Increasing T_{eff}

➡ Bolometric flux F (integral under the curve) increases with T_{eff}^{4}

➡ Wavelength of peak becomes shorter: Wien's displacement law

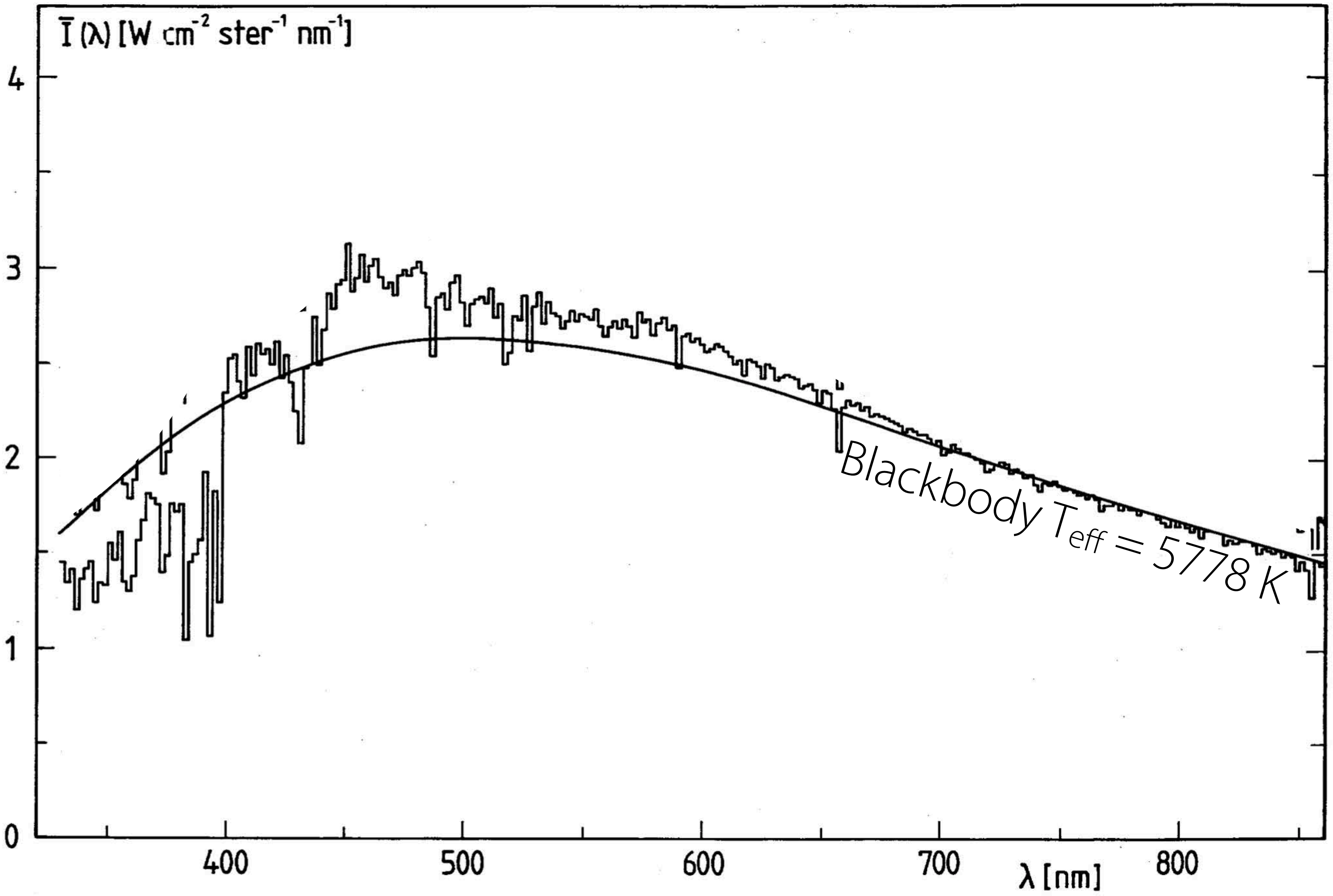
$$\lambda_{\max} = \frac{b}{T}$$

Wien's displacement constant

$(b = 2.89777 \times 10^{-3}\,\mathrm{m \cdot K} \approx 2900\,\mu\mathrm{m \cdot K})$

Interactive app:
https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html

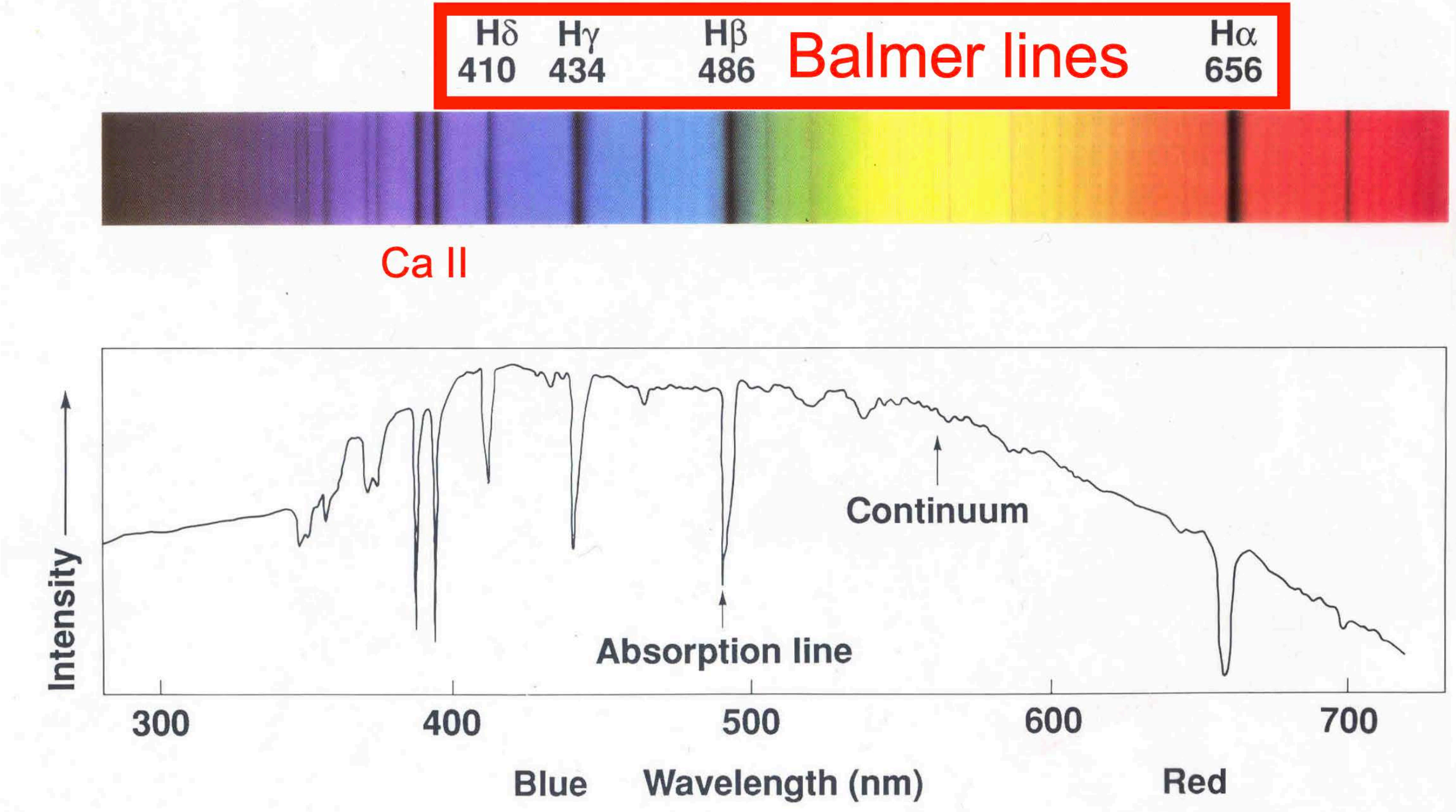
Stellar spectra



Stix (2002)

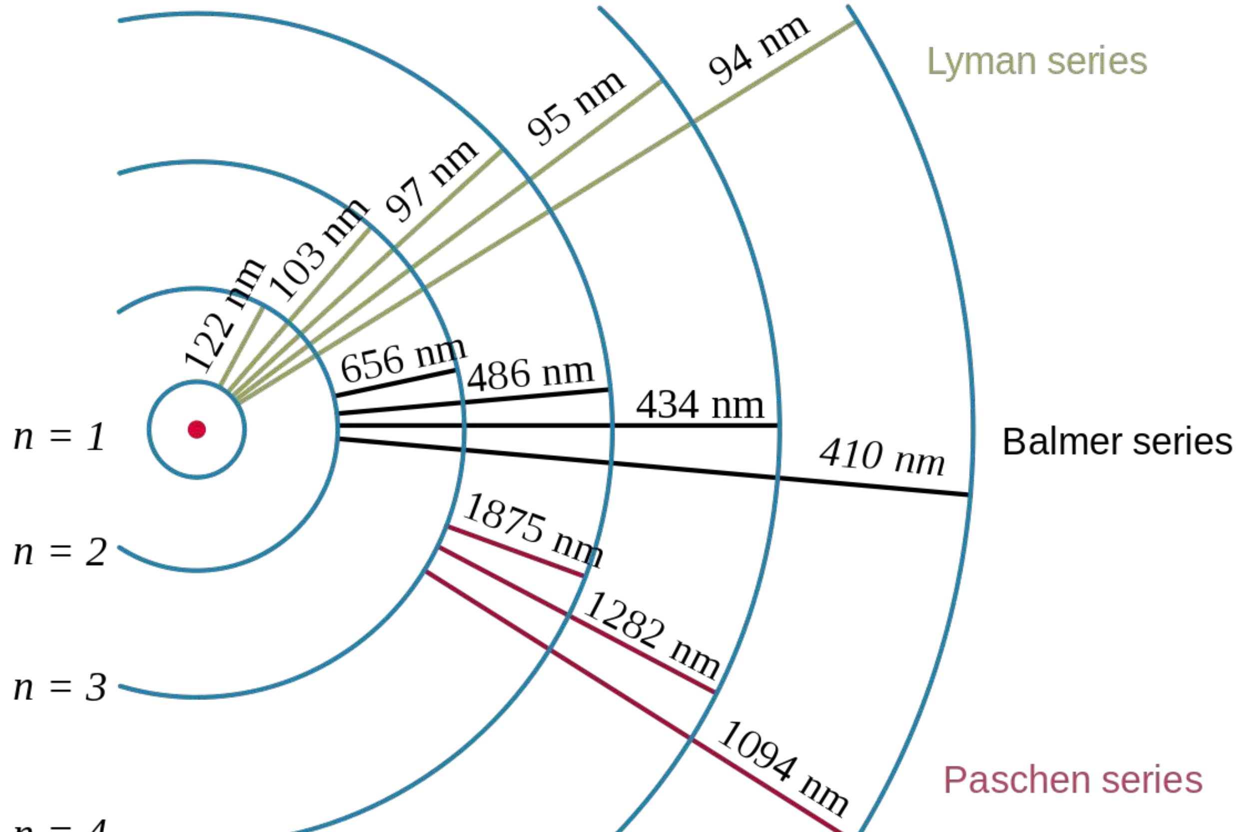
- A real stellar spectrum …
  - … is reasonably well described at longer wavelengths
  - … **deviates** in particular at shorter wavelengths and/or in wavelength regions with many (absorption) lines!

Observational stellar parameters

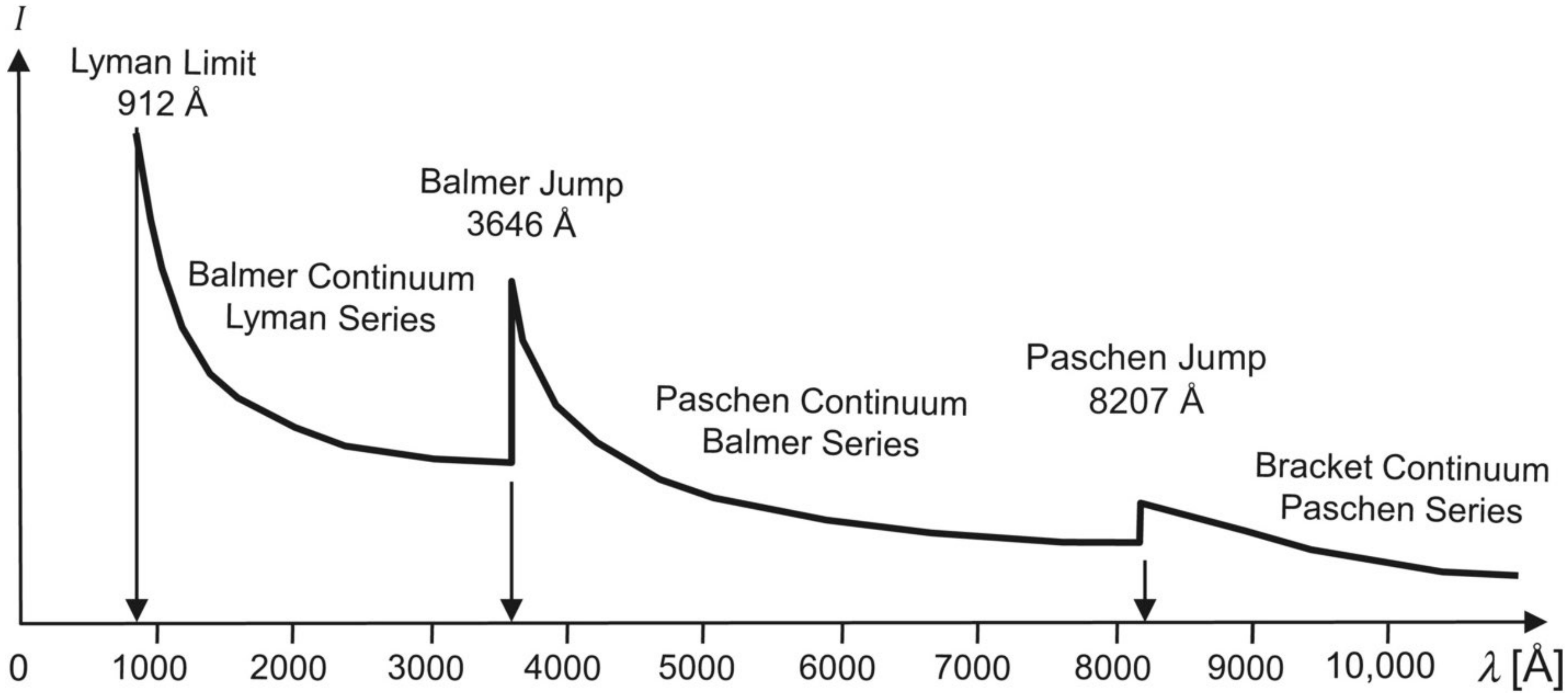


Stellar spectra

Hydrogen lines and continua



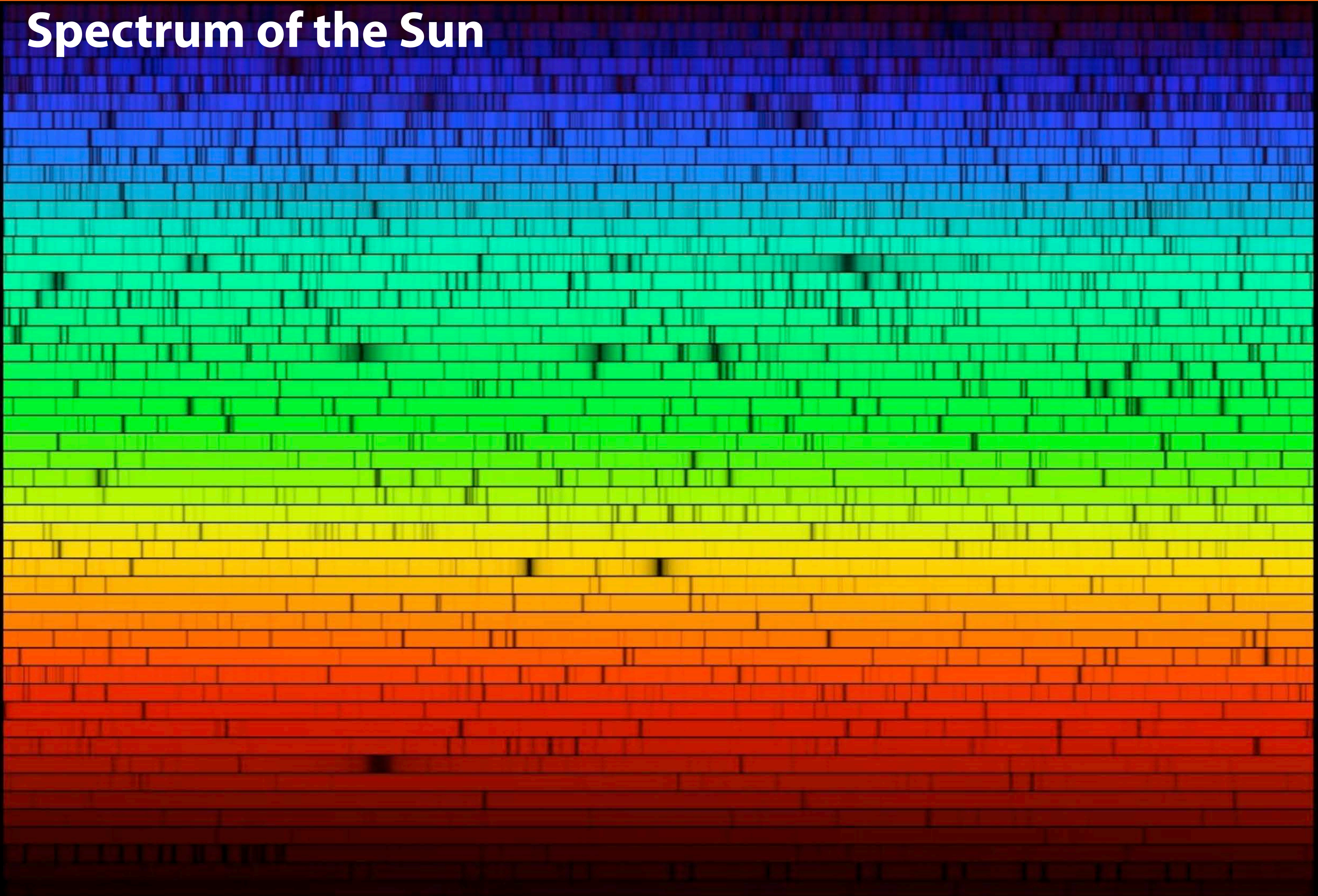
- Energy of photon E = h ν
- Photon absorbed at matching energy differences in the atom
- Hydrogen energy levels: $E_n \propto n^{-2}$
- Rydberg formula $\lambda^{-1} \propto (n_1^{-2} - n_2^{-2})$



➡ Notable deviations from blackbody!

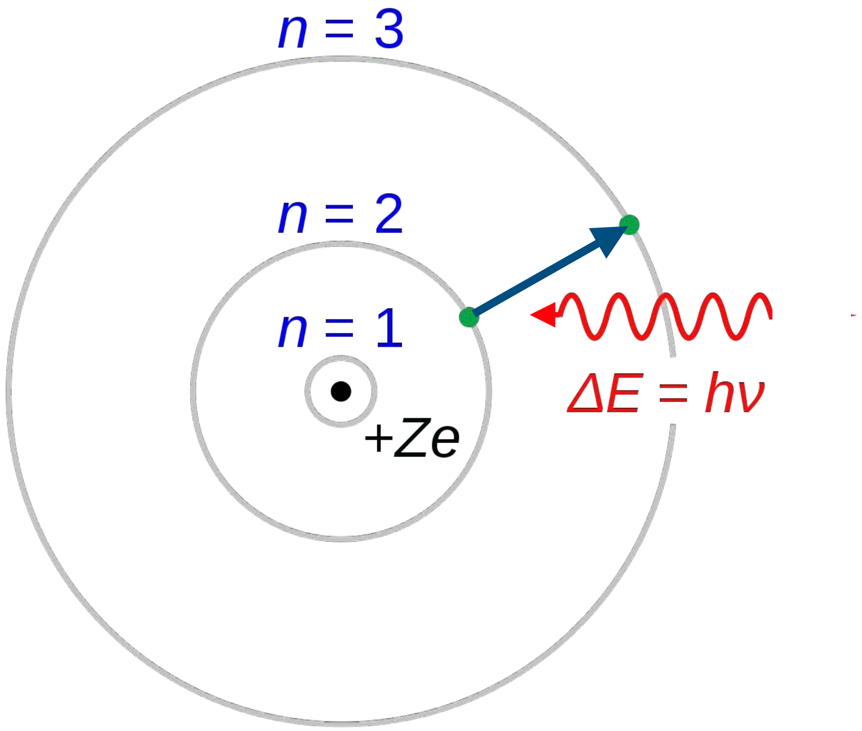
Stellar spectra

Spectrum of the Sun



Spectral classification

Line strength



- **Strength of a spectral line** depends on the number of atoms/molecules with electrons in the starting orbit for the spectral line under consideration

- **Example: H-α** line at 656.3nm in absorption, n=2 → n=3

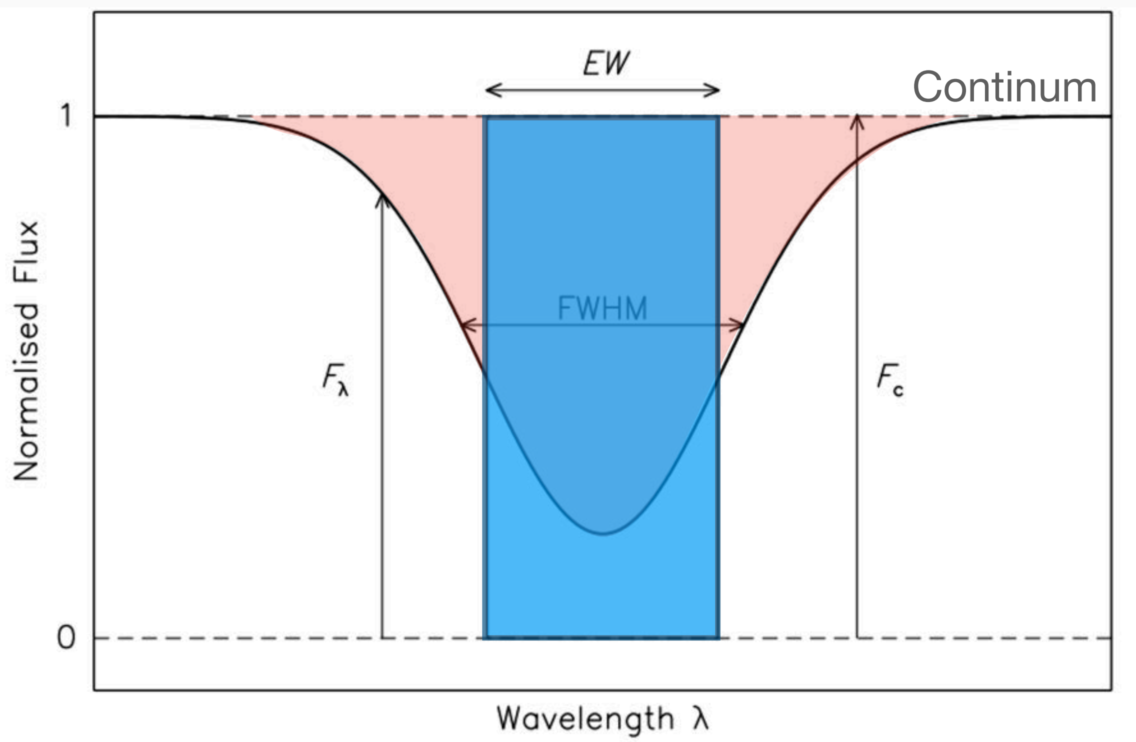
  ➡ H atoms needed that have electrons in level n=2,

  ➡ Must already have absorbed a photon to raise n=1 → n=2

  ➡ Requires high enough temperature

- Level populations and thus strength of a spectral line depends thermodynamic properties of the gas (and chemical abundance)

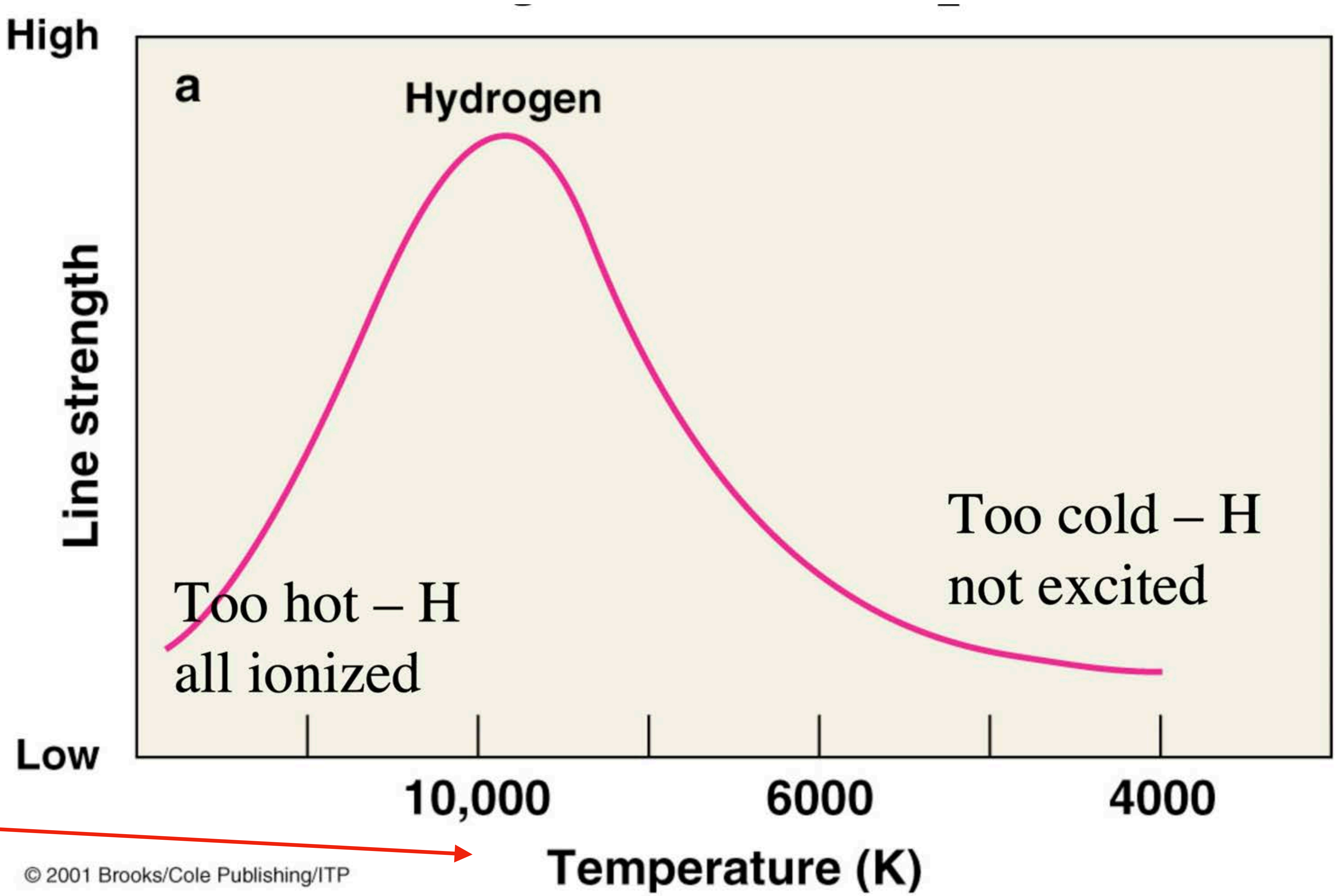
- Line strength measured as **equivalent width**
  - Area between spectral line and continuum and reshape into rectangle that extends from 1 to 0
  - Width of covered area



Spectral classification

Line strength

- Strength of a spectral line depends on chemical abundance of the element and thermodynamic properties such as the gas **temperature**



Too hot – H all ionized

Too cold – H not excited

Reversed axis!

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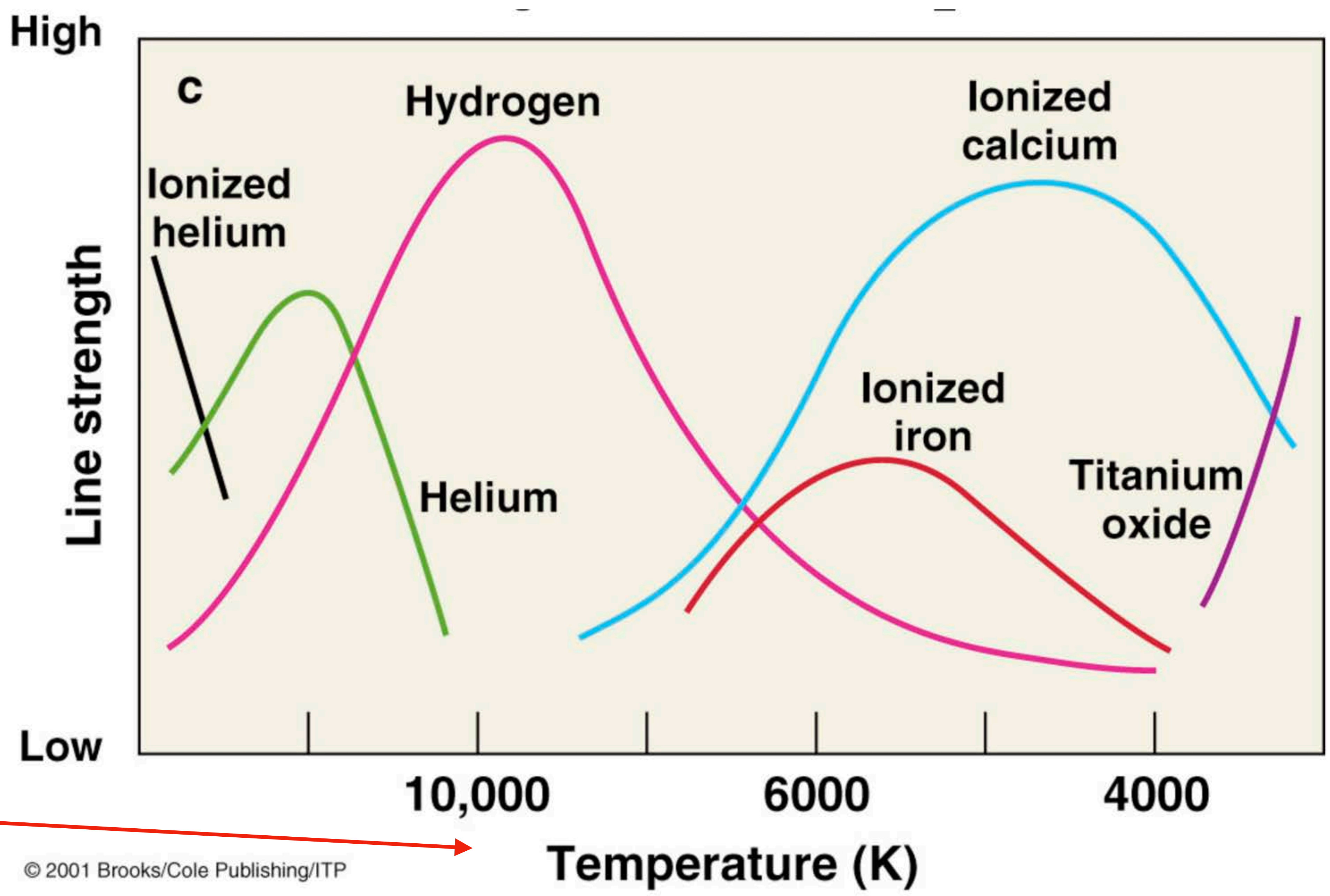
Spectral classification

Line strength

- Analysis of many spectral lines of different elements (and ionisation stages) allows to determine the gas temperature



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Reversed axis!

Spectral classification

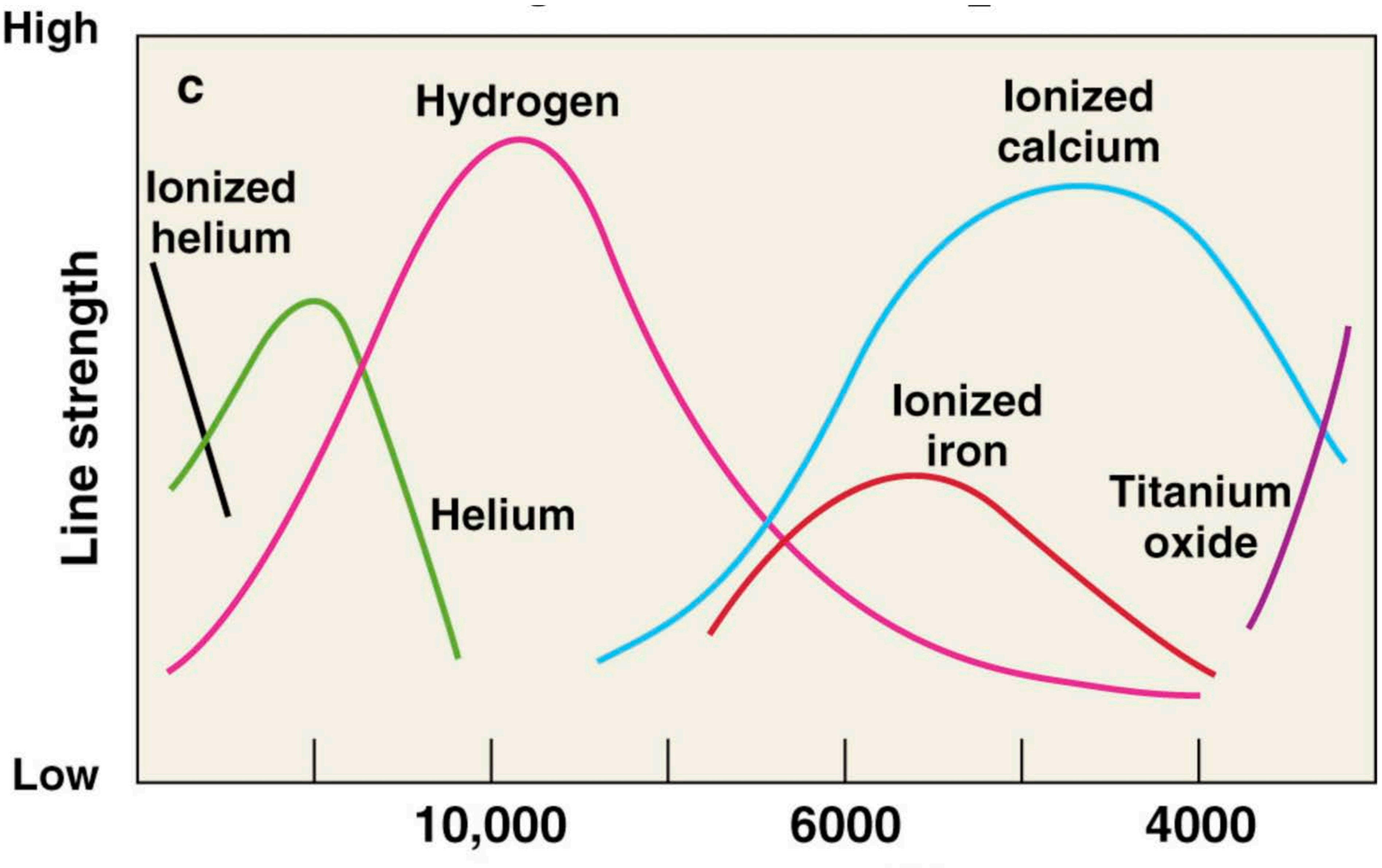
Line strength

- Saha equation: Ionization degree for any gas in thermal equilibrium

$$\frac{n_{i+1} n_e}{n_i} \propto \exp\left[-\frac{(\epsilon_{i+1} - \epsilon_i)}{k_B T}\right]$$

n: Number density of atoms in the i-th ion state (i electrons removed)

ε: ionisation energy



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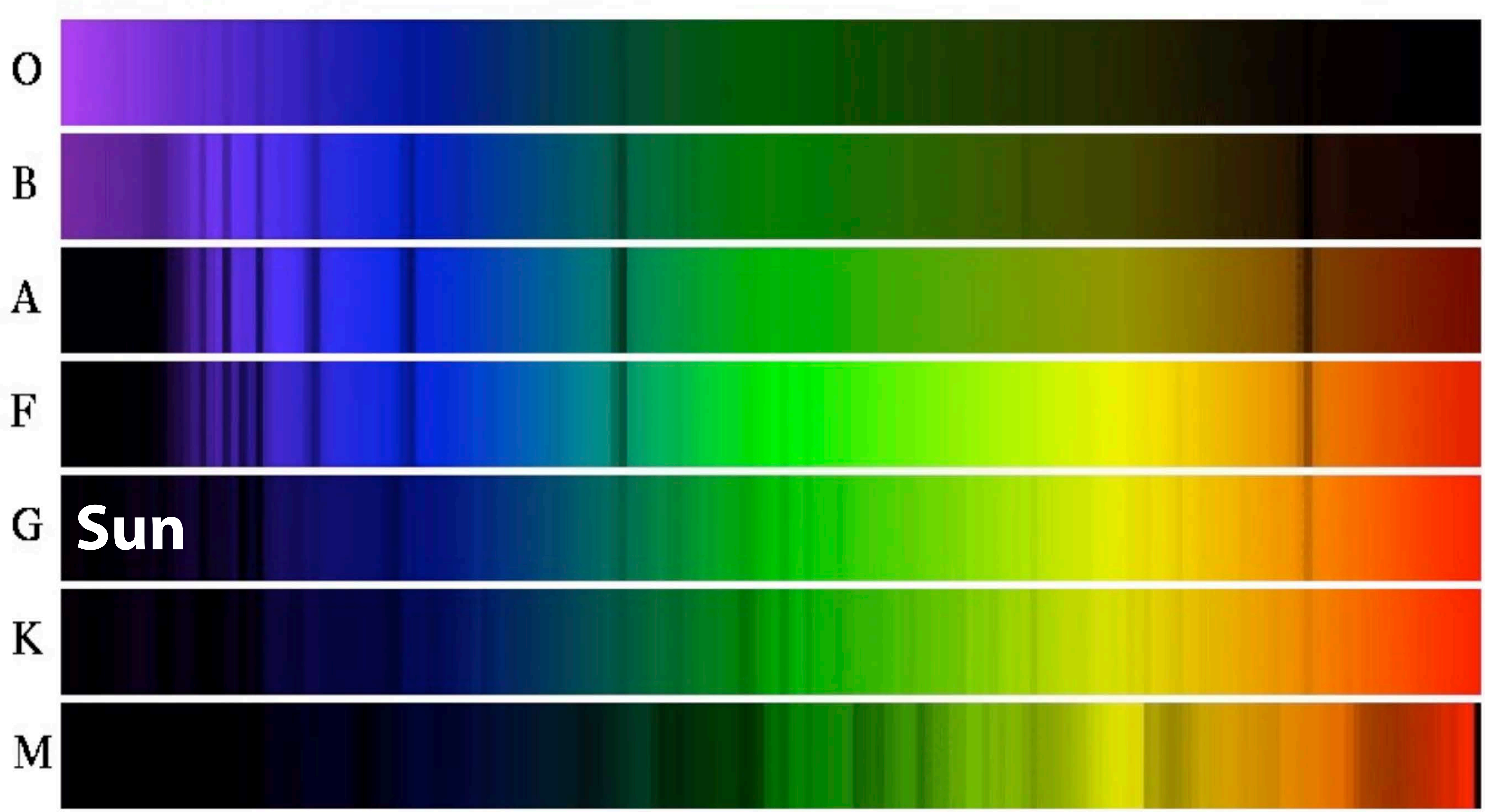
Spectral classification

- Spectral types defined according to occurrence and strength of spectral lines

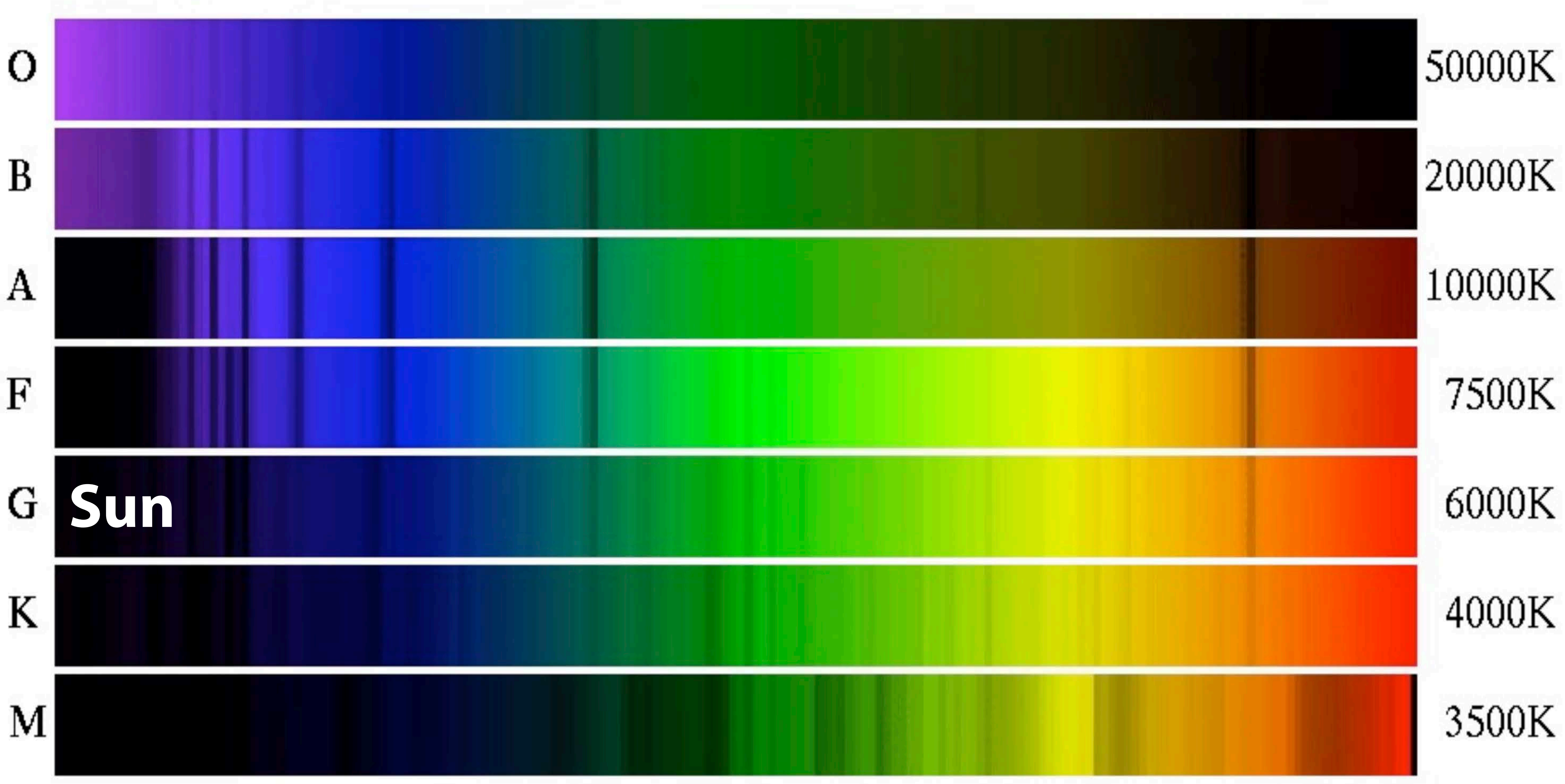
➡ Sorting the different types into a sequence



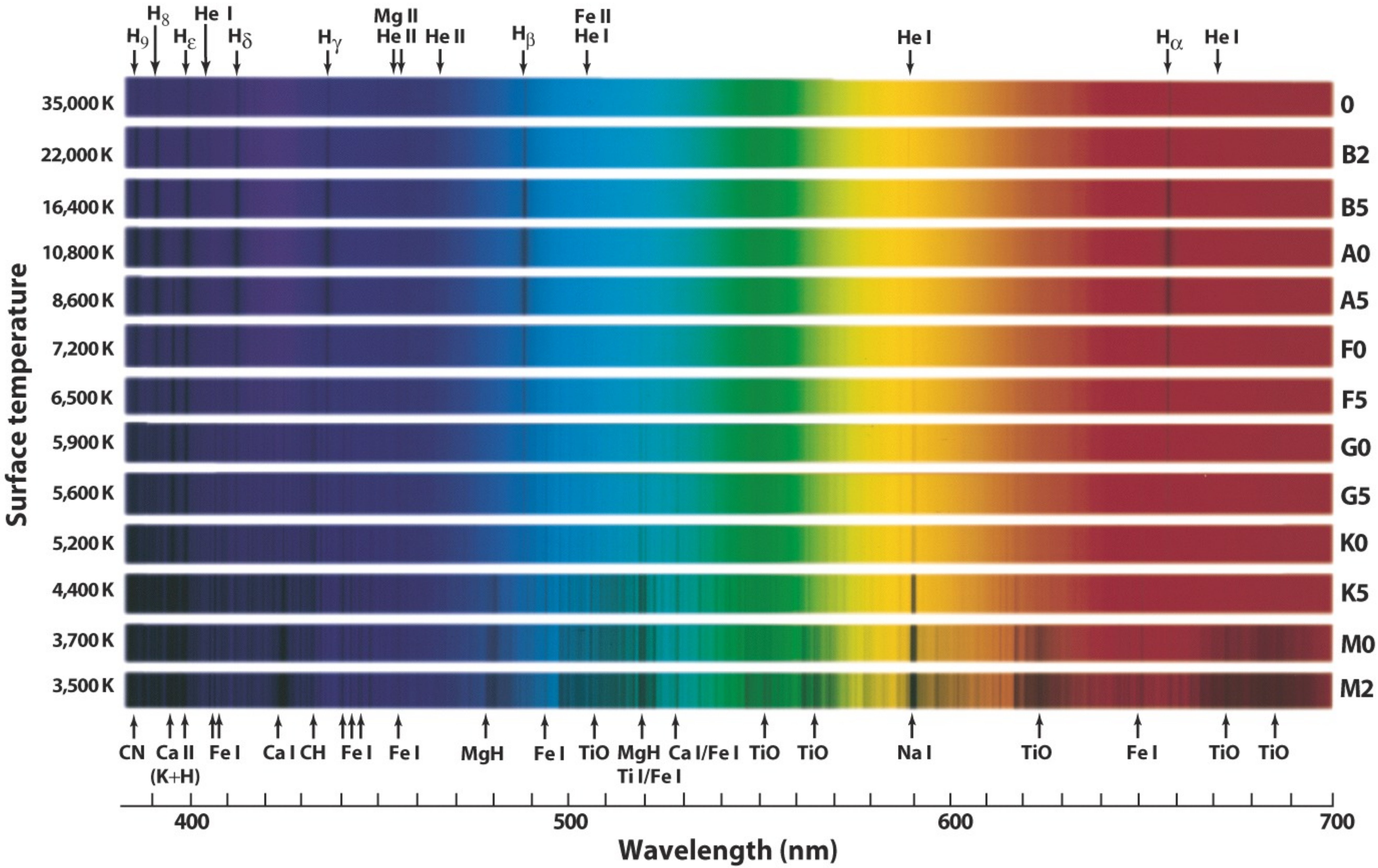
Spectral classification

- Measuring brightnesses, colour indices, spectral lines (+Stefan-Boltzmann law)

➡ Spectral types can sorted into a sequence as function of temperature



Stellar spectra

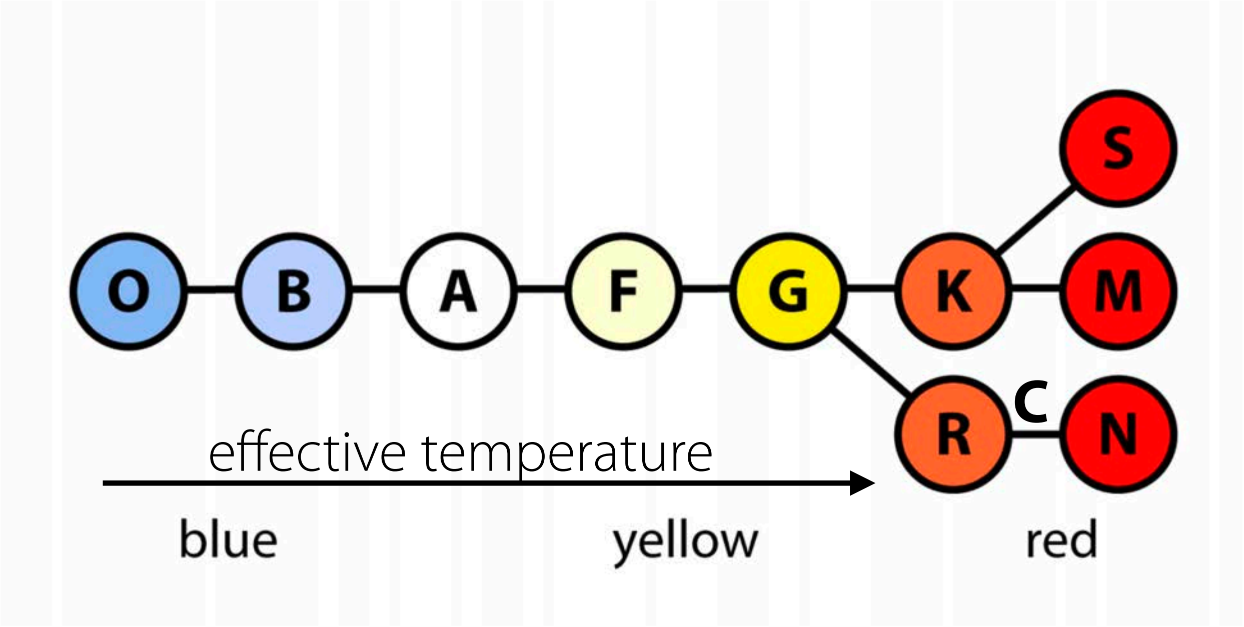


Spectral classification

- Different classification schemes
- **Harvard spectral classification**
- Further developed and extended

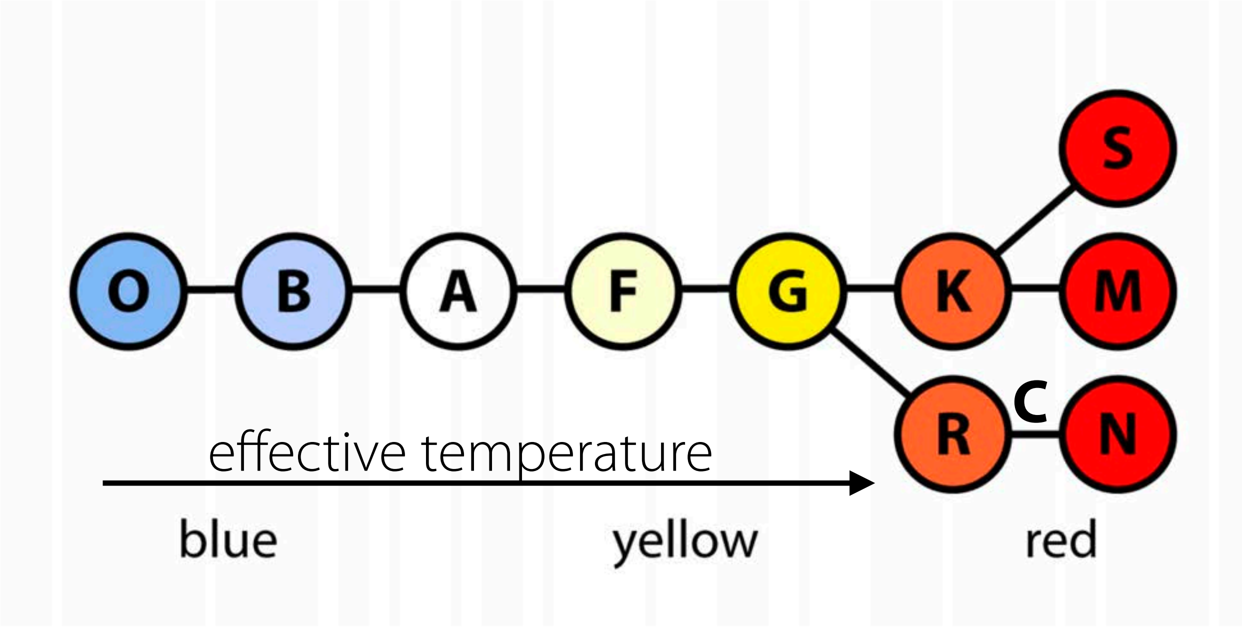
➡ **Morgan–Keenan system**

- A star is classified by a
  - Spectral class
  - Decimal sub-division (0-9) with effective temperature decreasing with in increasing digit
  - Luminosity class



Spectral classification

Main spectral classes			
O	violet	> 28 000 K	less than few visible absorption lines, weak Balmer lines, ionised helium lines
B	blue	10 000 – 28 000K	neutral hydrogen lines, more prominent Balmer lines
A	blue	7 500 – 10 000K	strongest Balmer lines, other strong lines
F	blue-white	6 000 – 7 500K	weaker Balmer lines, many lines including neutral metals
G	white-yellow	5 000 – 6 000K	Balmer lines weaker still, dominant ionised calcium lines
K	orange-red	3 500 – 5 000K	neutral metal lines most prominent
M	red	< 3 500 K	strong neutral metal lines and molecular bands



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Supplementary classes of cool stars			
R (C)	red	< 3 000 K	Carbon compounds, S-process elements
N (C)	red		Carbon compounds, S-process elements
S	red	∼ 3 000 K	s-process elements, molecular bands (especially ZrO and TiO)

- **R,N or C-type: “carbon stars”** — red giant stars and Asymptotic Giant Branch (AGB) stars.
  - regular M-type giant stars have more oxygen and carbon -> referred to as “oxygen-rich” stars.
- **S-type stars:** carbon and oxygen are approximately equally abundant
  - prominently spectral features due to the s-process elements (e.g. zirconium monoxide (ZrO).

Spectral classification

Main spectral classes			
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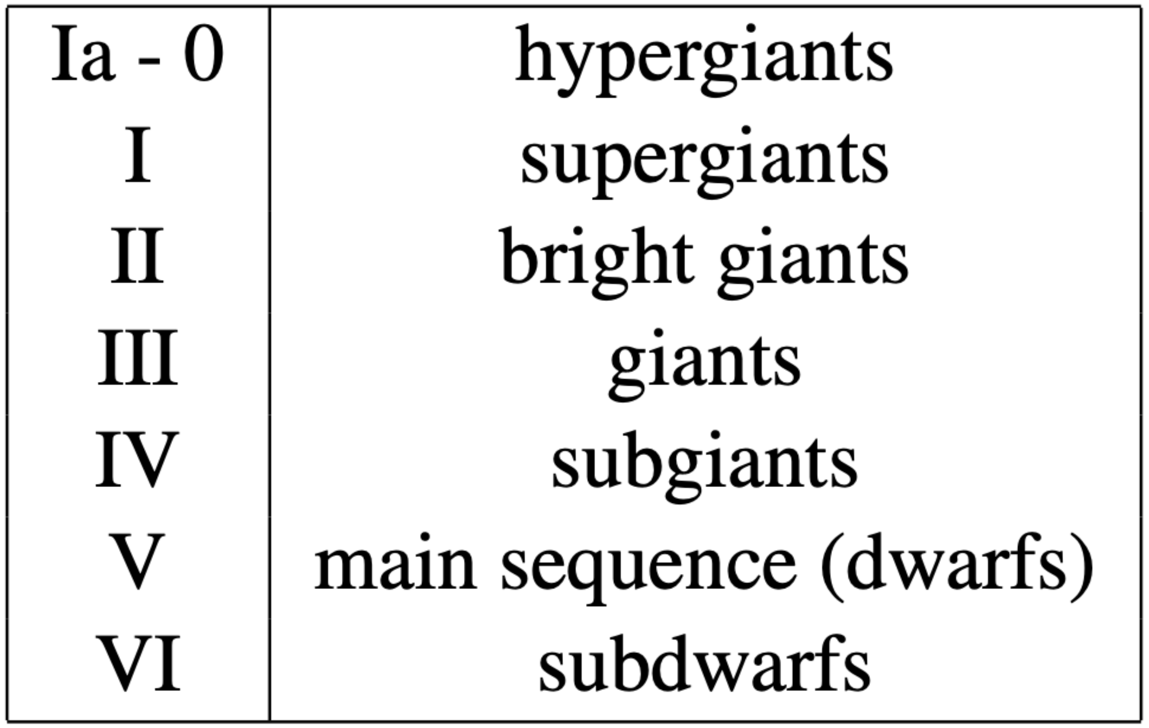
S	red	∼ 3 000 K	s-process elements, molecular bands (especially ZrO and TiO)
Very low mass /sub-stellar spectral classes (mostly brown dwarfs)			
L	IR	1 500 – 2 500 K	lines of alkali metals (e.g. N) and metallic compounds (e.g. FeH)
Y	IR	800 – 1 500 K	methane absorption lines
T	IR	< 800 K	water and ammonia lines

Spectral classification

Luminosity class

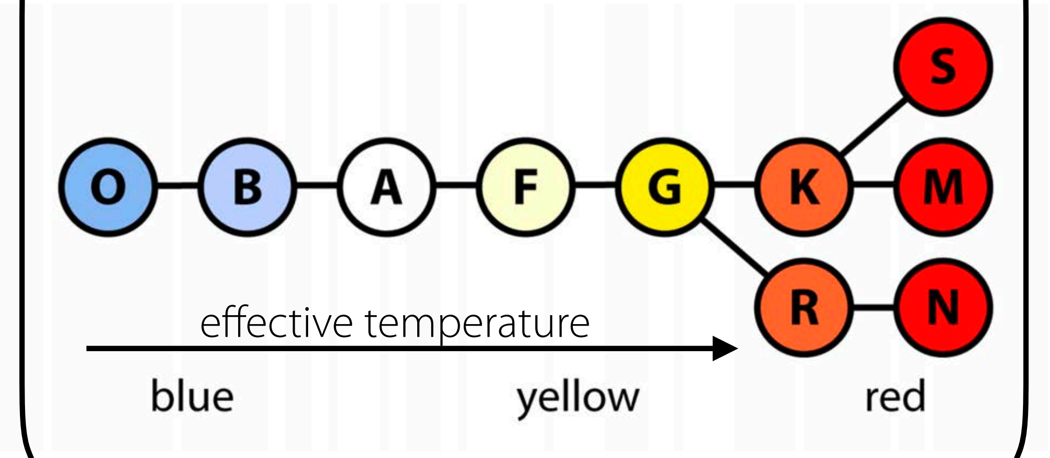


Ia - 0	hypergiants
I	supergiants
II	bright giants
III	giants
IV	subgiants
V	main sequence (dwarfs)
VI	subdwarfs

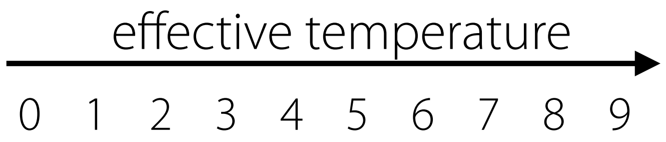
Spectral classification

- A star is classified by a

Spectral class



Decimal sub-division



One decimal possible!

Luminosity class

Ia - 0	hypergiants
I	supergiants
II	bright giants
III	giants
IV	subgiants
V	main sequence (dwarfs)
VI	subdwarfs

- **Examples**

Sun	G2V
Sirius A	A0V
Proxima Cen	M5.5V
Betelgeuse	M1I
Aldebaran	K5III

Spectral classification

Additional classification

- Special spectral types
    - **W** for Wolf-Rayet stars with no hydrogen lines in their spectra
    - **D** for white dwarfs
    - …

- **Extra information** can be added to the spectral type of a star if it differs from the other regular types / show **peculiarities**
    - e: presence of pronounced emission lines
    - v: variable spectral features.
    - …
    - Example: M5.5Ve

- Please note that
    - the accuracy of a spectral classification depends on the quality of the available data
    - a spectral classification can change as a star changes

Spectral classification

Stellar populations

- Observations show that decreasing **metal content** correlated with increasing age of stars.

- Stars (in our galaxy) can be further divided into populations according to their chemical composition or metallicity

  - **Population I:** "recent" stars, high metallicity

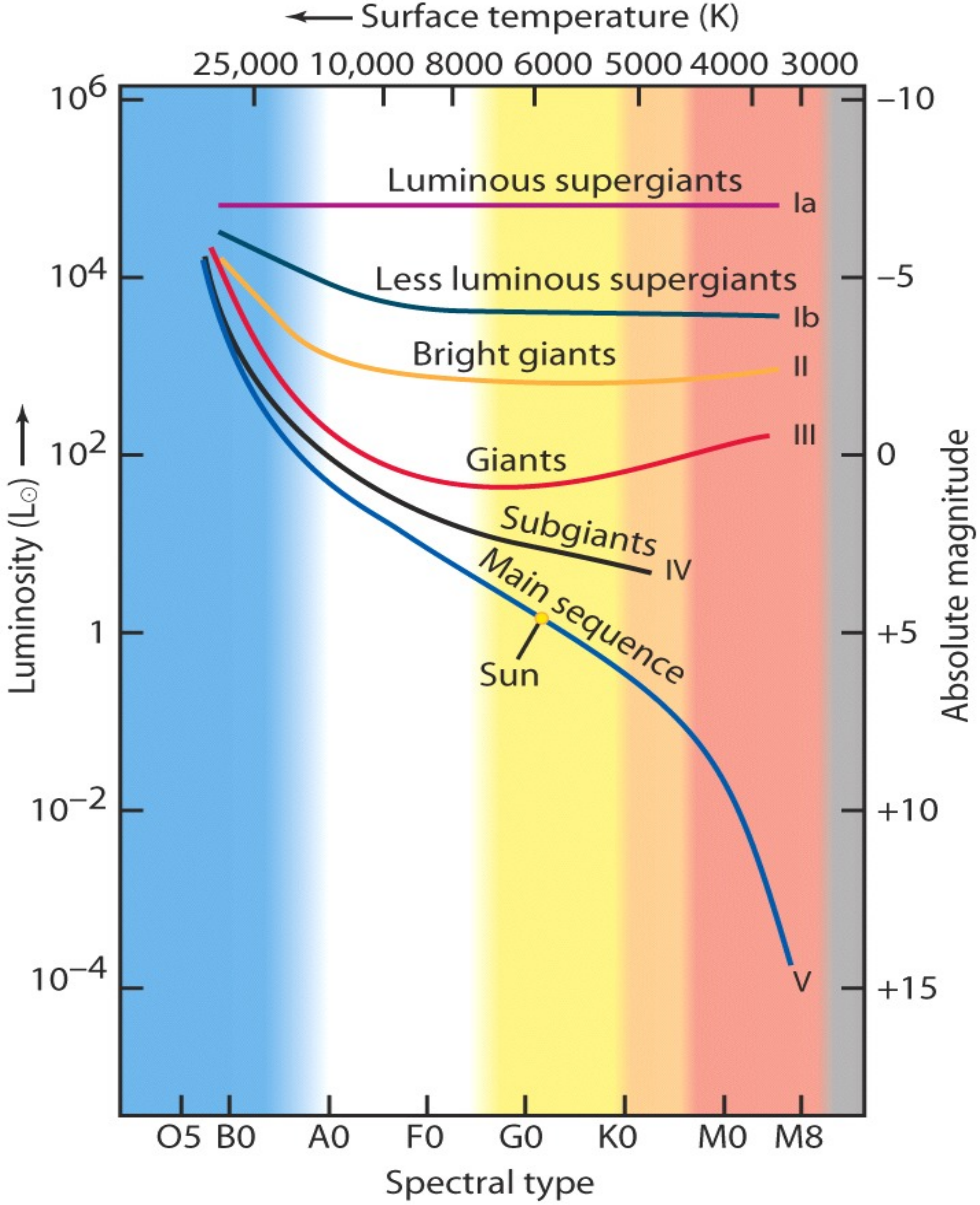
  - **Population II:** old stars, low metallicity

  - **Population III:** first stars in the universe (very low metal content)

- Originally, pop I+II, pop III added in 1978

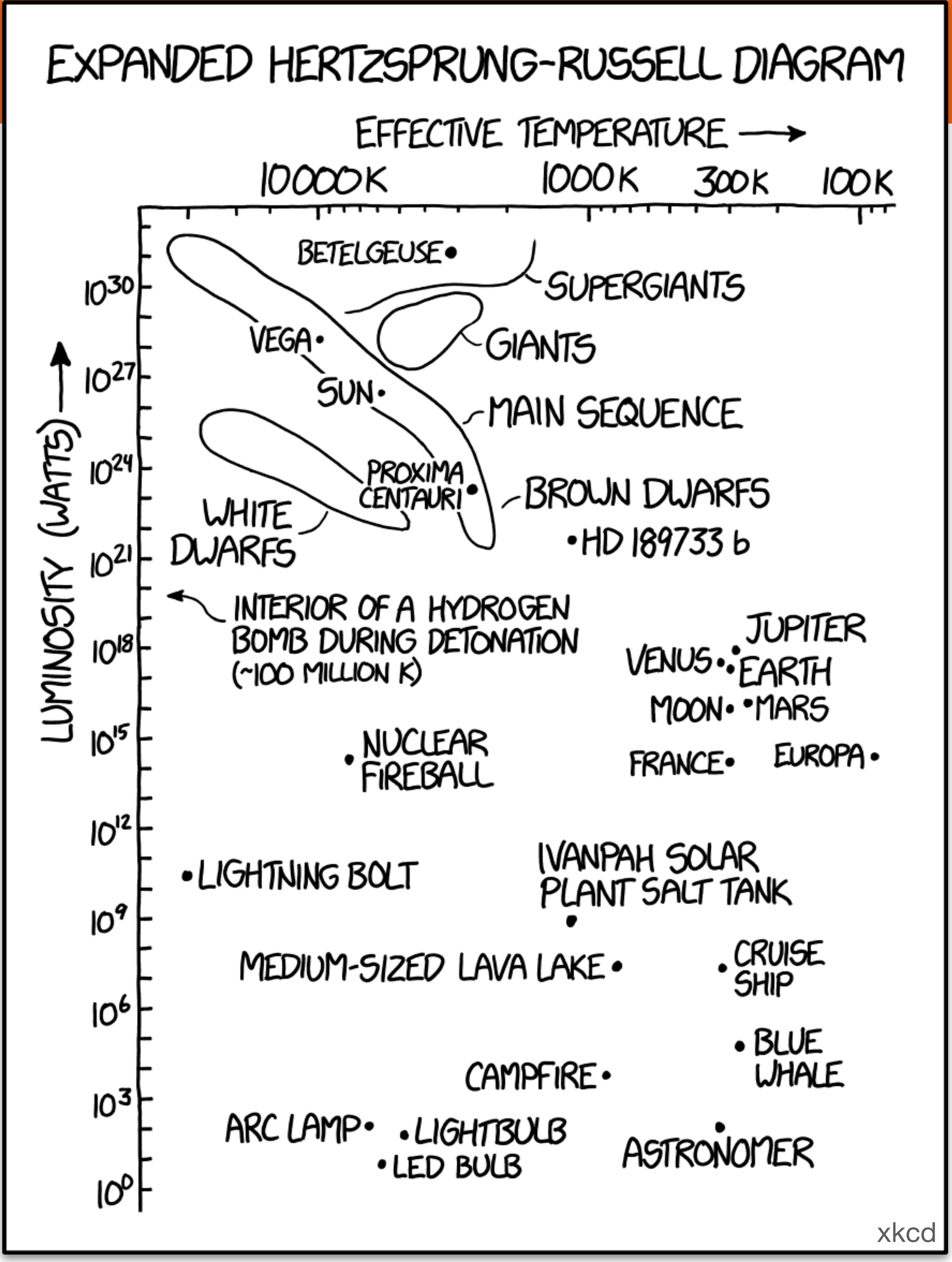
Stellar classification

Hertzsprung-Russell diagram



Stellar classification

Hertzsprung-Russell diagram



Physical stellar parameters

- The fundamental (global) parameters that describe a star are
  - **mass M,**
  - **radius R,**
  - **luminosity L.**
- They are commonly expressed in units of the solar values $M_\odot$, $R_\odot$, and $L_\odot$

- **Stellar atmosphere** (layer from where we receive most of the observable information) is characterised by the following parameters:
  - **effective temperature T_{eff}**
  - **gravity acceleration g**
  - **chemical composition** *(expressed as metallicity)*
  - **magnetic field strength** *(although the magnetic field is typically difficult to be expressed by just one parameter)*

- Often stellar properties can only be derived with **significant uncertainties**,

Physical stellar parameters

Mass

- According to our definition of a star, nuclear fusion in its interior is required.

  ➡ Minimum mass of a star $M_{min} \approx 0.08 M_\odot$.

  ➡ Objects with $M_{min} < 0.08 M_\odot$ (but more mass than planets): brown dwarfs ($M_{bd} < 0.08 M_\odot$).
- Highest masses $M > 100 M_\odot$
  - Known examples with up to ~ $250 M_\odot$

- Number of stars with a certain mass decreases strongly with mass!

  ➡ only few very massive stars but very many low-mass stars.

  ➡ very massive stars are therefore typically far away
- Strong stellar winds and outflowing gas result in clouds surrounding these stars can make the determination of the stellar mass less reliable.

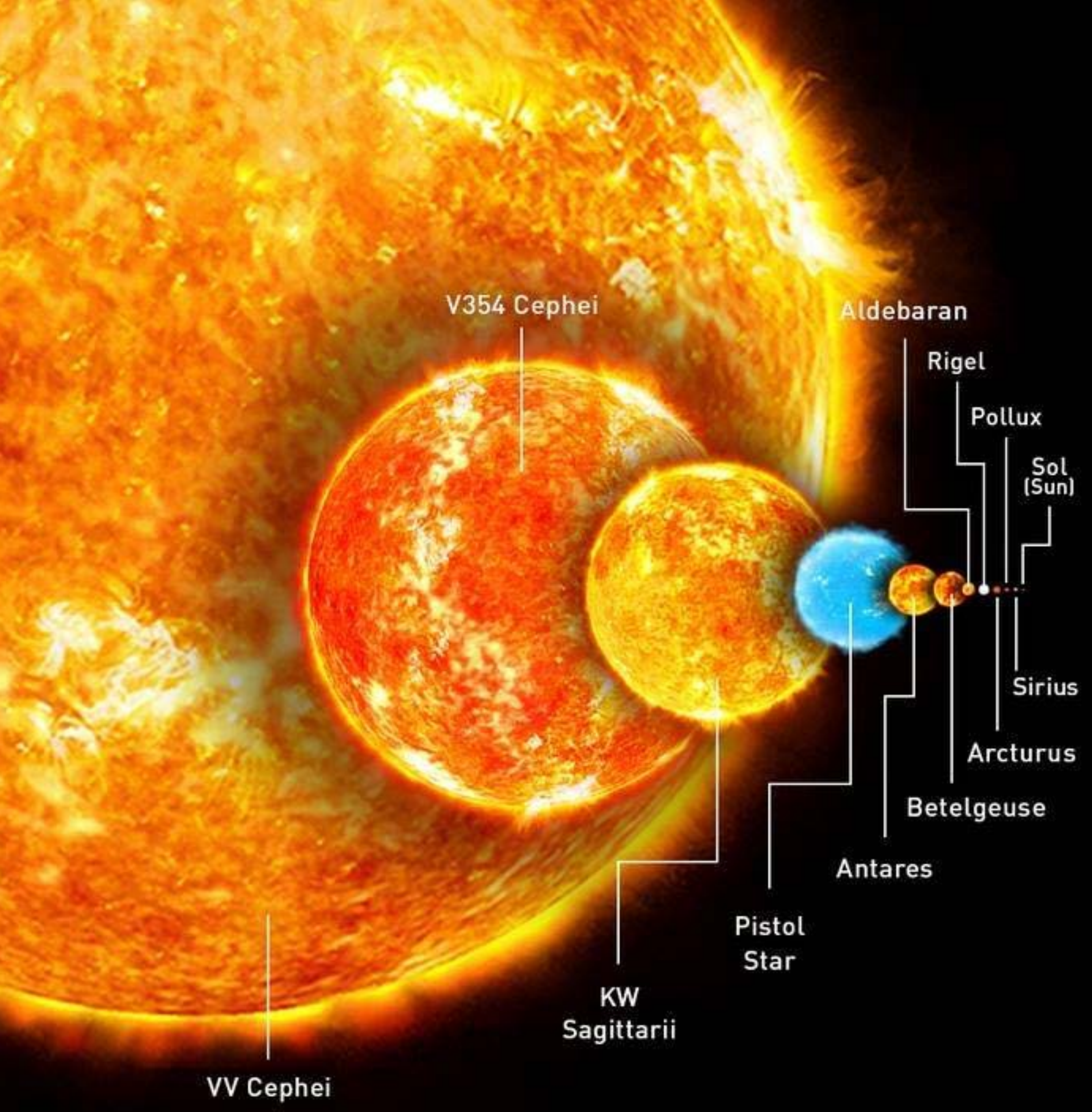
Physical stellar parameters

Radius

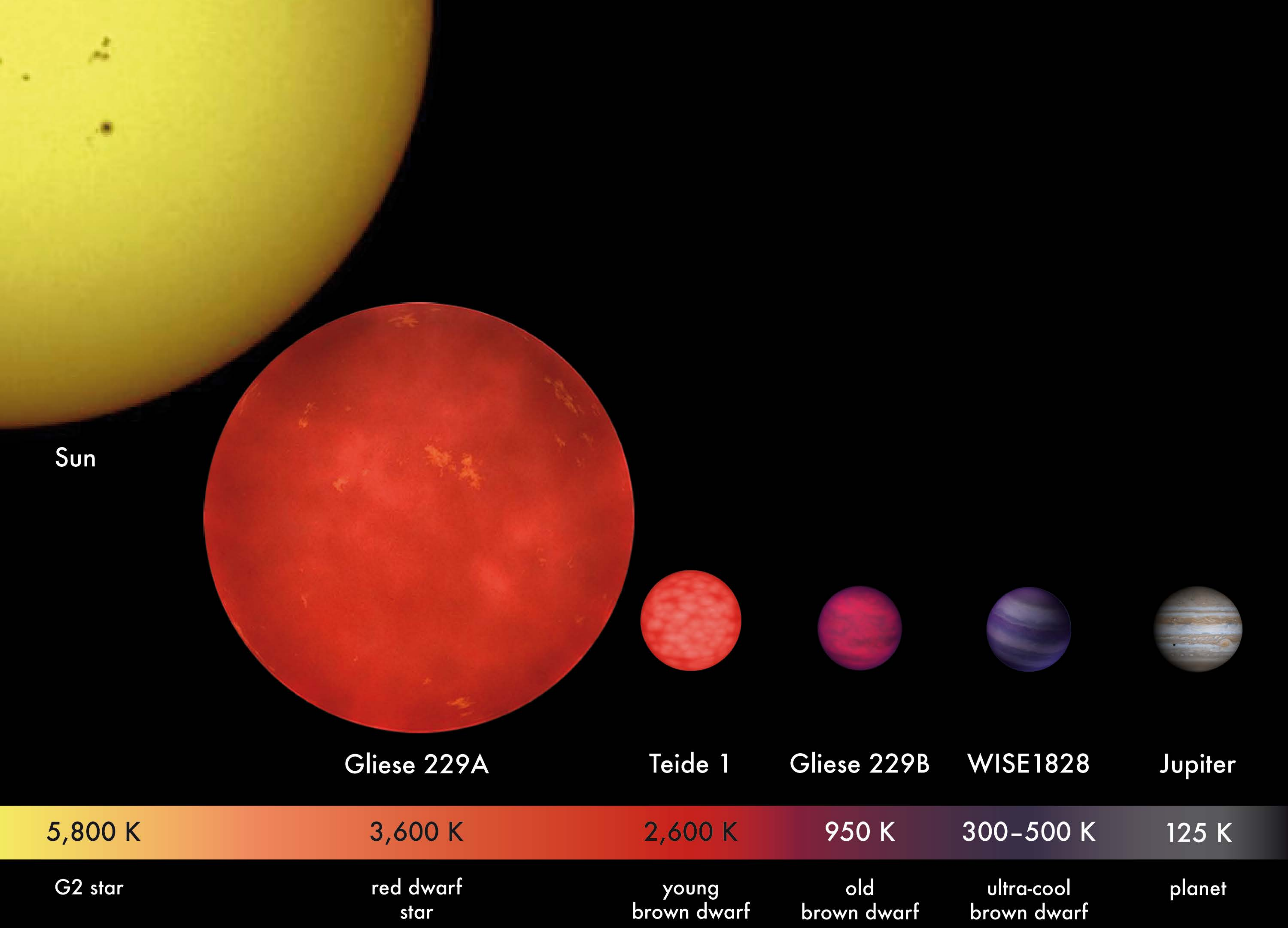
- **Main sequence stars**
  - Typical values: 0.1 $R_\odot$ to ~ 25 $R_\odot$.
  - Radii increase as function of effective temperature along the main sequence
  - Red dwarfs at the cool end being much smaller than the Sun
  - Hot main sequence stars being much larger than the Sun.

- **Red giants, supergiants, …**
  - Diameters larger than the orbit of Mars.
  - Examples: Antares (680 – 800 $R_\odot$), Betelgeuse (900 $R_\odot$), and Mu Cephei (972 – 1,260 $R_\odot$).
  - Largest stars: radii currently estimated to up ~ 2000 $R_\odot$.

- **White dwarfs**
  - $R < 0.02\ R_\odot$



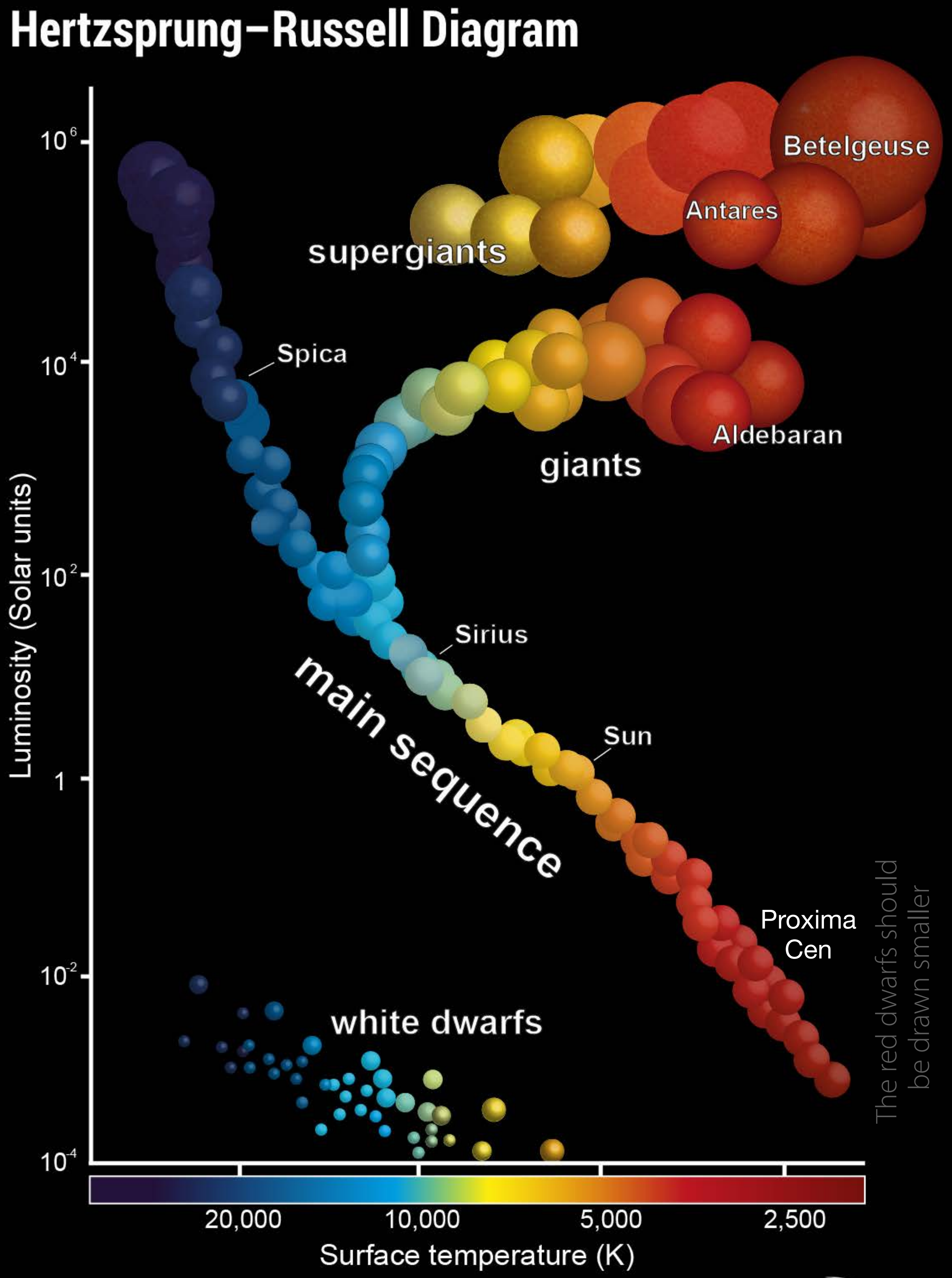
Careful: Scale might not be accurate (anymore)



Careful: Scale might not be accurate (anymore)

Stellar classification

Hertzsprung-Russell diagram



The red dwarfs should be drawn smaller

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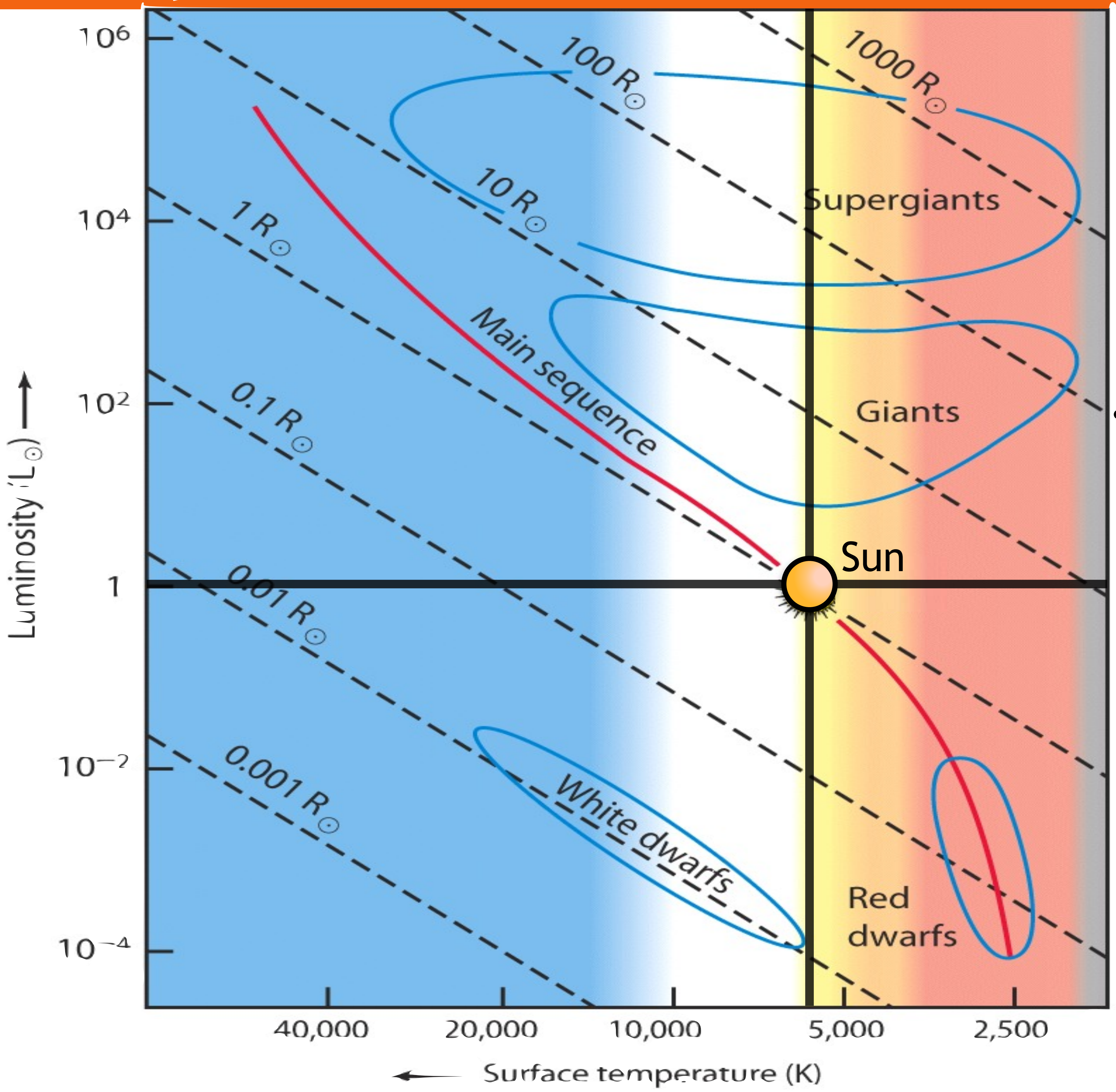
Physical stellar parameters



- Lines of constant radius

Physical stellar parameters

Luminosity ➡ $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$

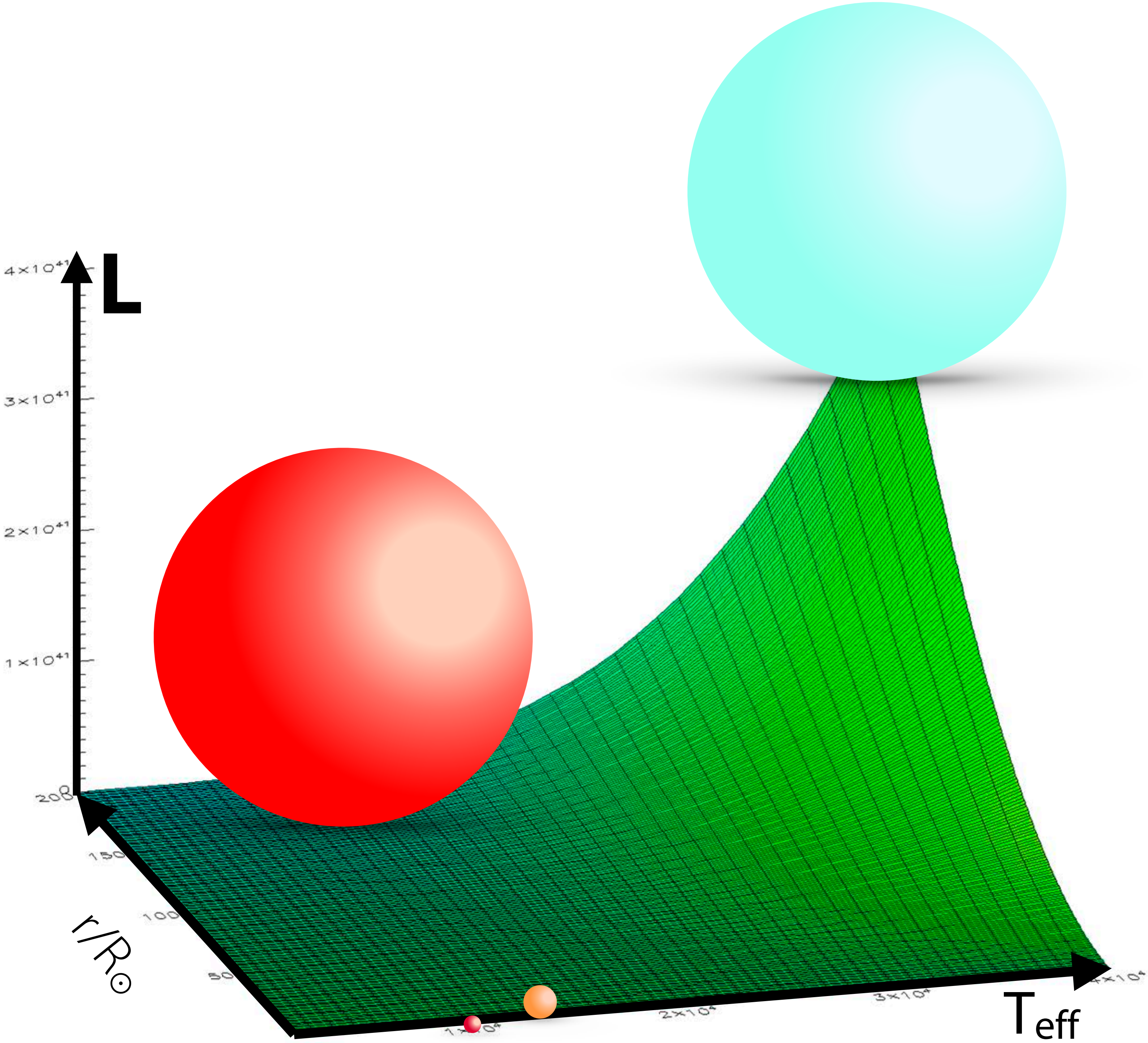
- The bolometric luminosity of stars spans r many orders of magnitude: $10^{-4}\,L_\odot$ — $10^6\,L_\odot$

- Depends to **4th power** on T_{eff}

➡ Small difference in T_{eff} results in a large change in L!
(Same true for uncertainties)

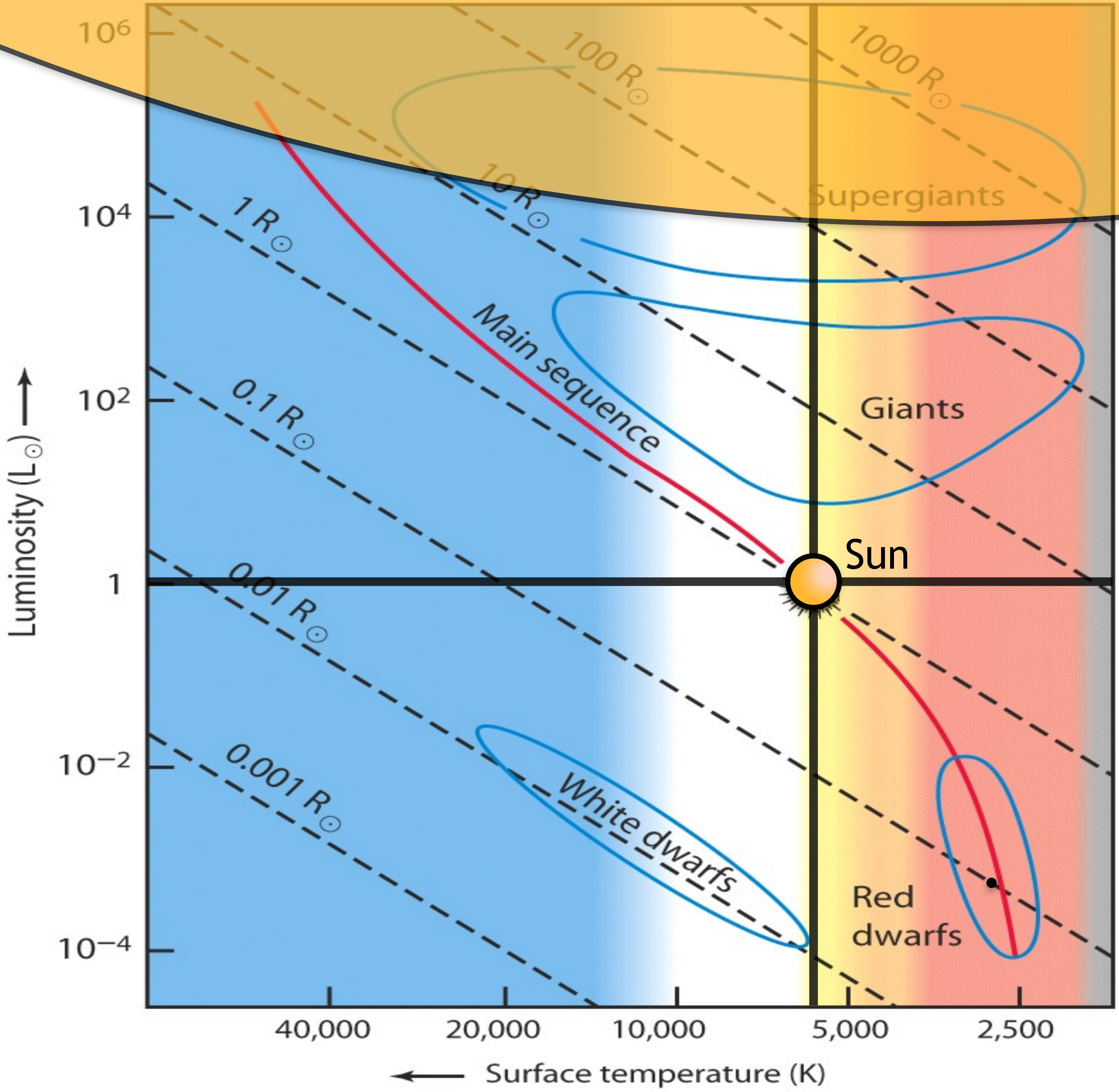
- **Example 1**: blue-white supergiant Deneb (α Cyg) — one of the brightest stars in the sky:
$L \sim 60\,000 — 200\,000 L_\odot$.
  ➡ Large uncertainty is due to the poorly known distance!

- **Example 2**: Red supergiant Betelgeuse $L \approx 100\,000 L_\odot$



Physical stellar parameters

$L \propto R^2\ T_{eff}^{\ 4}$



Super giant

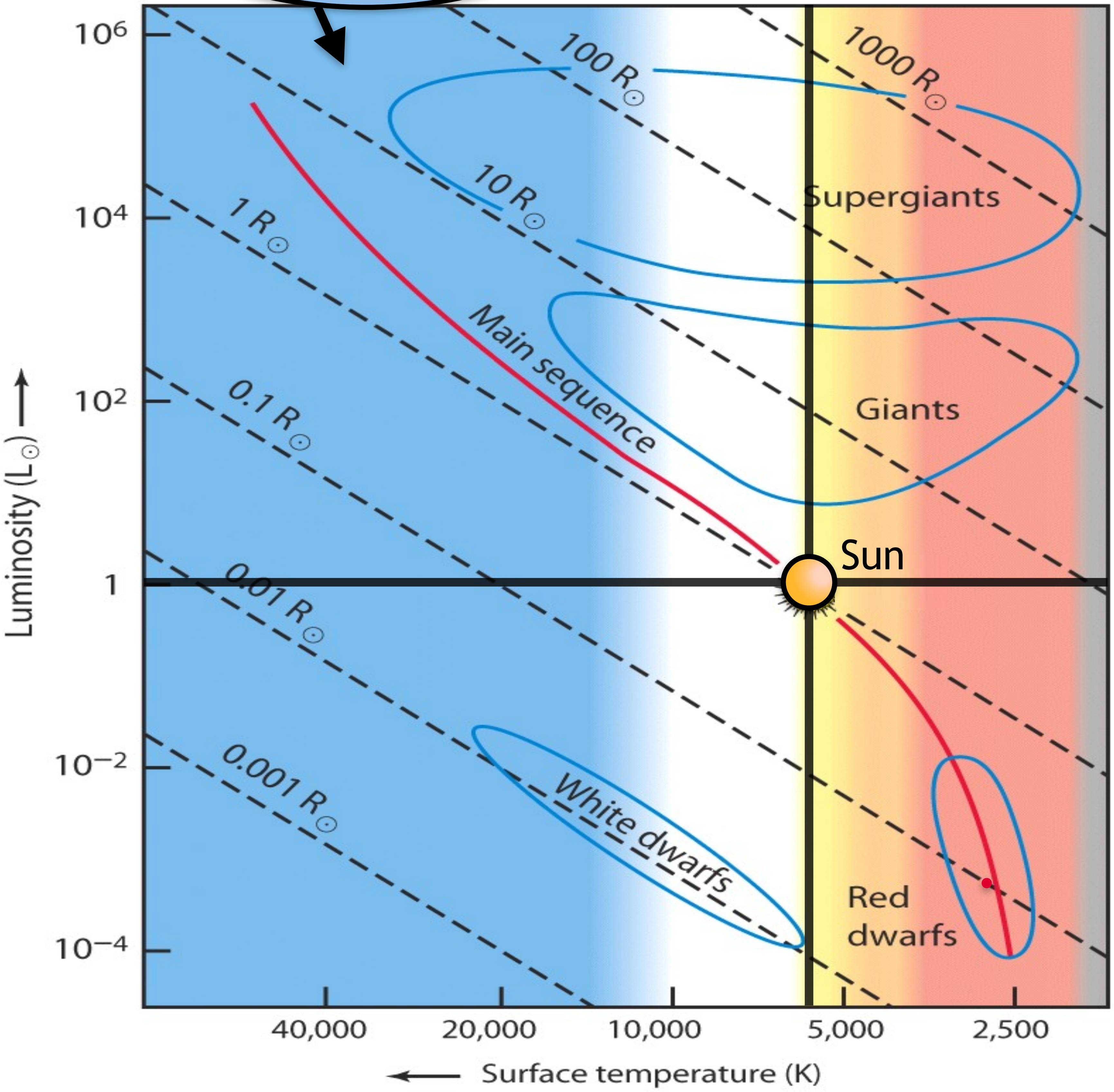
$R = 100\ R_\odot$

$T_{eff} = T_{eff,\odot} = 5770\ K$

➡ $L = 100^2\ L_\odot = 10\,000\ L_\odot$

Ph lar parameters

$L \propto R^2\, T_{\mathrm{eff}}^{4}$



Spectral type O6

$R = 18\,R_\odot$

$T_{\mathrm{eff}} = 38\,000$ K

$= 6.3\,T_{\mathrm{eff},\odot}$

➡ $L = 520\,000\,L_\odot$

Red dwarf star

$R = 0.1\,R_\odot$

$T_{\mathrm{eff}} = 0.5\,T_{\mathrm{eff},\odot} = 2885$ K

➡ $L = 0.1^{2}\,0.5^{4}\,L_\odot$

$= 0.0006\,L_\odot$

$= 0.06\,\%\,L_\odot$