AST5770
Solar and stellar physics

University of Oslo, 2022

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Practical information

Updated schedule

mandag 10:15-12:00	Forlesning 1	rom 209 / 304
tirsdag	/	
onsdag 14:15-16:00	Gruppeundervisning	rom ikke bestemt
torsdag **10:00**-11:40	Forlesning 2	rom 304

Practical information

Data / material for assignments

- Data: `/mn/stornext/d9/svenwe/lecture/AST5770/data`

- Templates 4 assignments: `/mn/stornext/d9/svenwe/lecture/AST5770/assignment`

- **Everybody please check: Can you access these directories?**

- Try to open/copy a file, e.g.:

  `/mn/stornext/d9/svenwe/lecture/AST5770/data/assignment/`
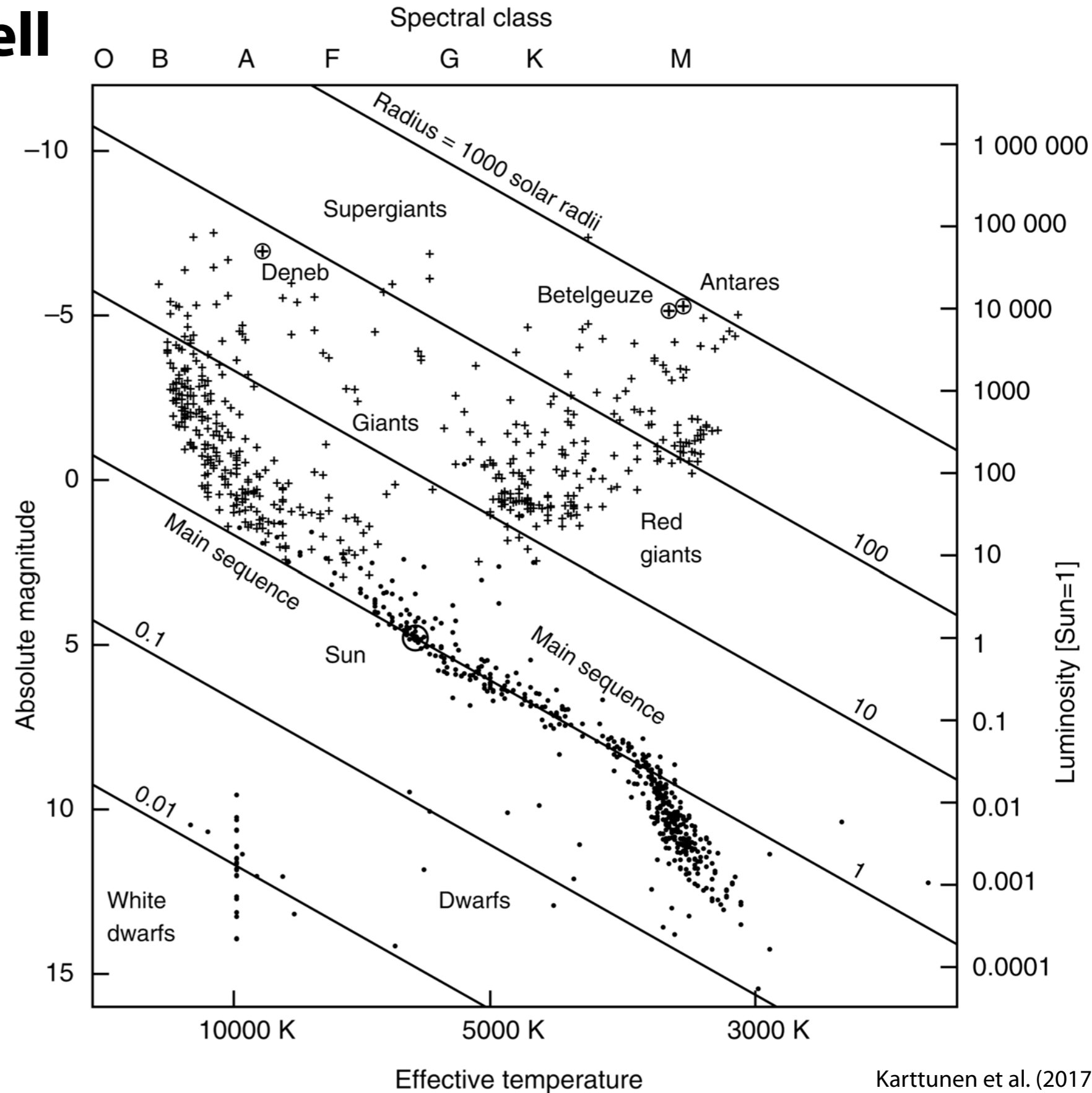 `AST5770_projectassign.pdf`

- More information and practical aspects on Thursday 10:00

Physical stellar parameters

Hertzsprung-Russell diagram

- Stars clearly not distributed randomly

- Connections between

  - T_{eff}/Spectral class

  - Luminosity/absolute brightness

  - Radius

➡ Constraints for models of structure and evolution of stars

Karttunen et al. (2017).

Physical stellar parameters

Effective temperature

- Notes:
  - Spectrum of a star can – in first approximation – be described with a black body spectrum for an effective temperature T_{eff}
  - real stellar spectra deviate from blackbody curve
  - effective temperature of a star is defined over the integral

  - Sun: $T_{eff,\odot} \approx 5770$ K.

  - Typical values for other **main sequence stars** :
    - from ~ 2 200 K for the coolest red dwarf stars
    - to up to ~ 45 000 K for the hottest O-type stars.

  - **White dwarfs** can exhibit much higher temperatures of up to ~ 2×10^5 K (basically "exposed" stellar core remnants)

Physical stellar parameters

Gravity acceleration

$$g = \frac{GM}{R^2}$$

mass M,
radius R,
gravity constant G

- Commonly specified in logarithmic form as **log g** in cgs units.

- Sun: $g_\odot = 274$ ms^{-2} = 2.74×10^4 cm s^{-2} — (log g)$_\odot$ = 4.44

- Large range of stellar radii + dependence $g \propto R^{-2}$,

➡ g varies over 8 orders of magnitude from white dwarfs to supergiants

Giants and supergiants:	$g \leq 1 \times 10^{-2}\,\mathrm{m\,s^{-2}}$	$\log g \sim 0$
Main sequence:	$g \sim 2 \times 10^2\,\mathrm{m\,s^{-2}}$	$\log g \sim 4.3 - 4.5$
White dwarfs:	$g \sim 1 \times 10^6\,\mathrm{m\,s^{-2}}$	$\log g \sim 8$

Physical stellar parameters

Chemical composition

- Most stars (in particular main sequence stars) consist primarily of **hydrogen and helium**.

- The elements heavier than helium are commonly called **metals** in astrophysics, sometimes abbreviated as M.

- **Astronomical abundance scale**: logarithmic scale log ε relative to the Sun

  - Hydrogen as origin of scale with log ε (H) = 12 and all other element relative to hydrogen:

    $$A(\mathrm{El}) = \log \varepsilon = log(n_{\mathrm{El}}/n_{\mathrm{H}}) + 12$$ n: number density of element

- The **metallicity** is the relative content of the metals, **M/H**, with respect to the Sun:

  $$\varepsilon = \frac{(\mathrm{M/H})_{\mathrm{star}}}{(\mathrm{M/H})_{\odot}} \qquad \text{or} \qquad \log \varepsilon = [\mathrm{M/H}]$$
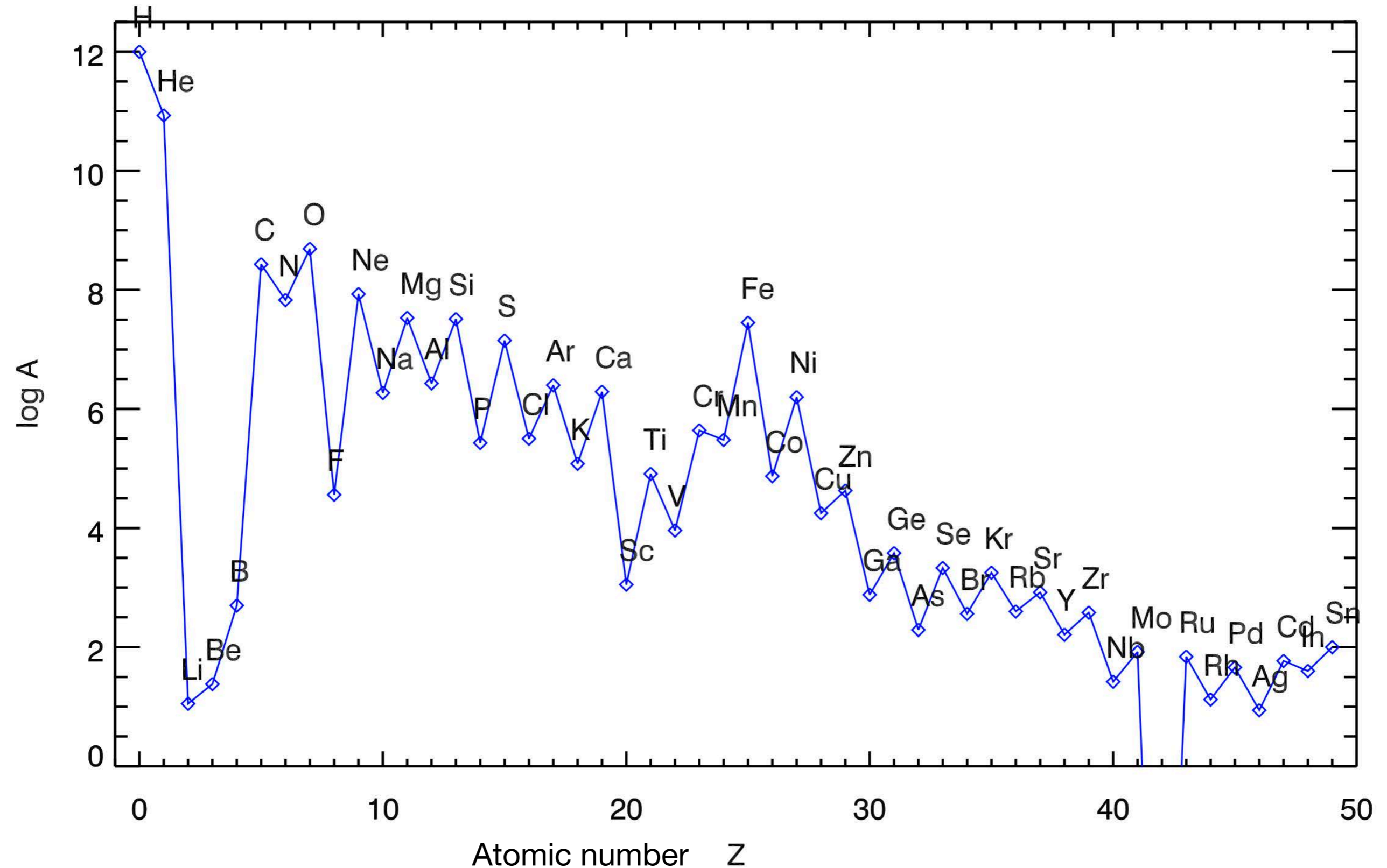
- Iron abundance relatively easy to measure due to large number of spectral lines *(and as an important nucleosynthesis product)* — often used as a *representative metal*

  $$[\mathrm{Fe/H}] = \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{star}} - \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{sun}}$$

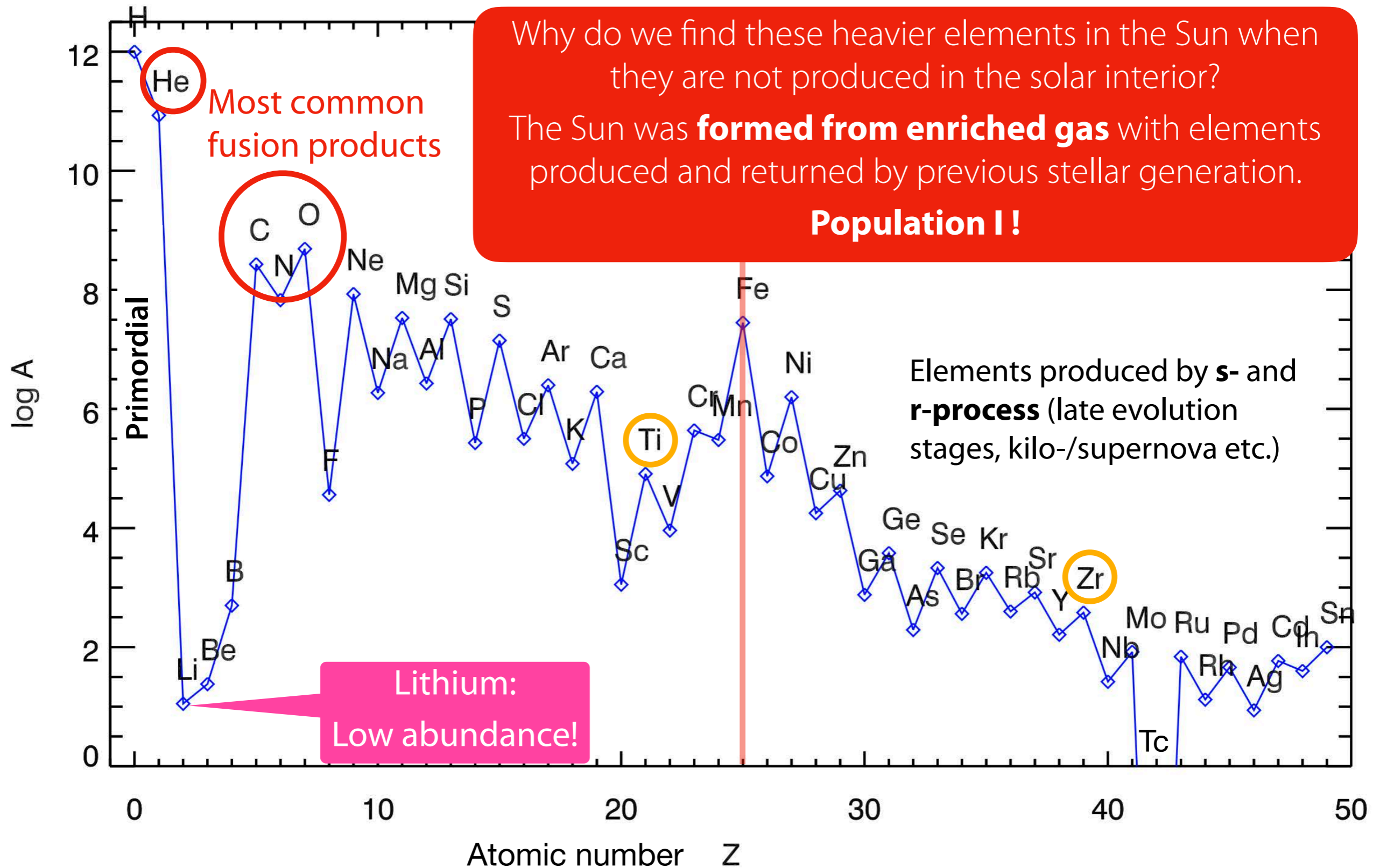- Also: Cosmo-chemical scale with silicon (Si) as reference but less common for stars

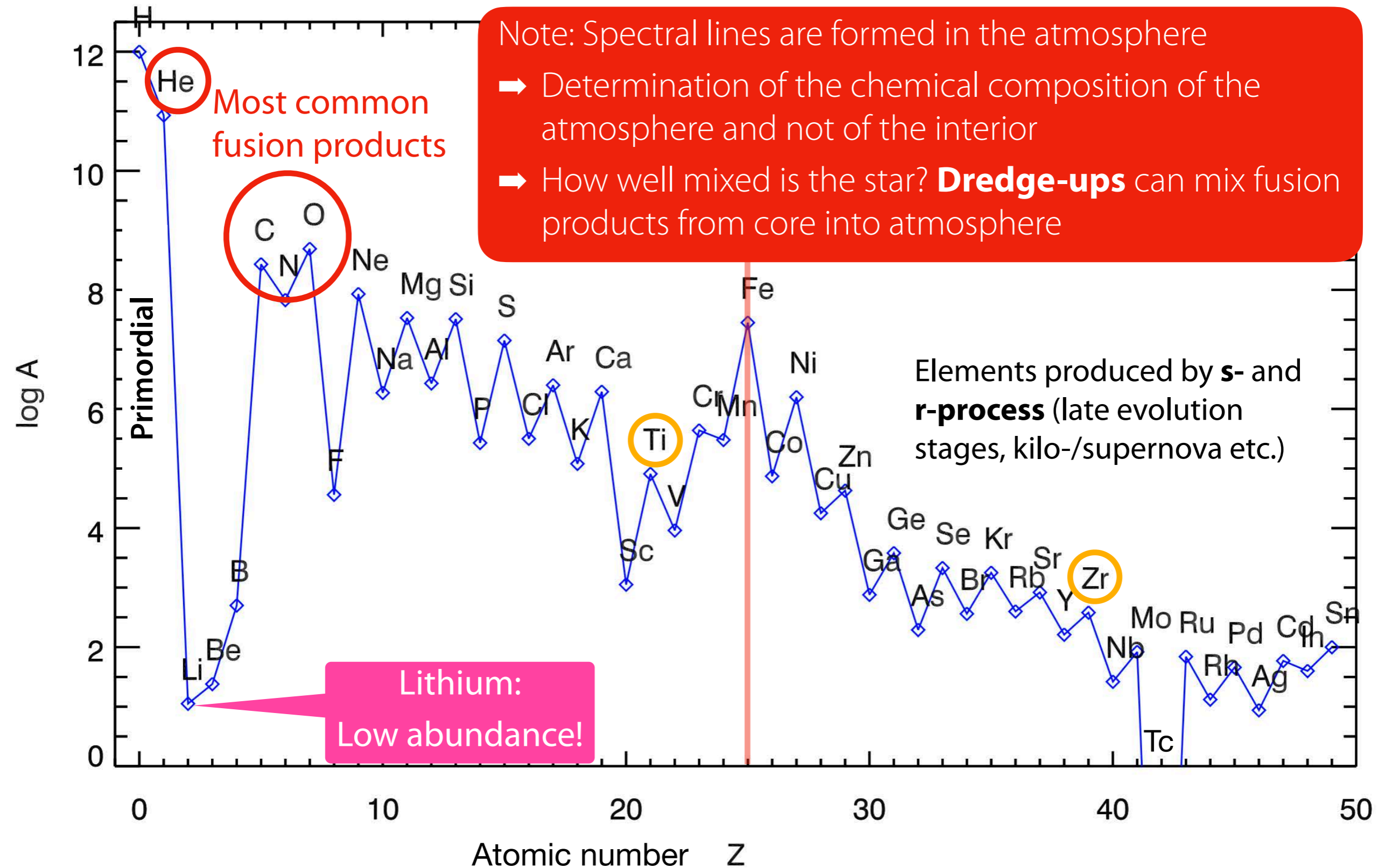# Physical stellar parameters

Chemical composition — Solar abundances

Physical stellar parameters
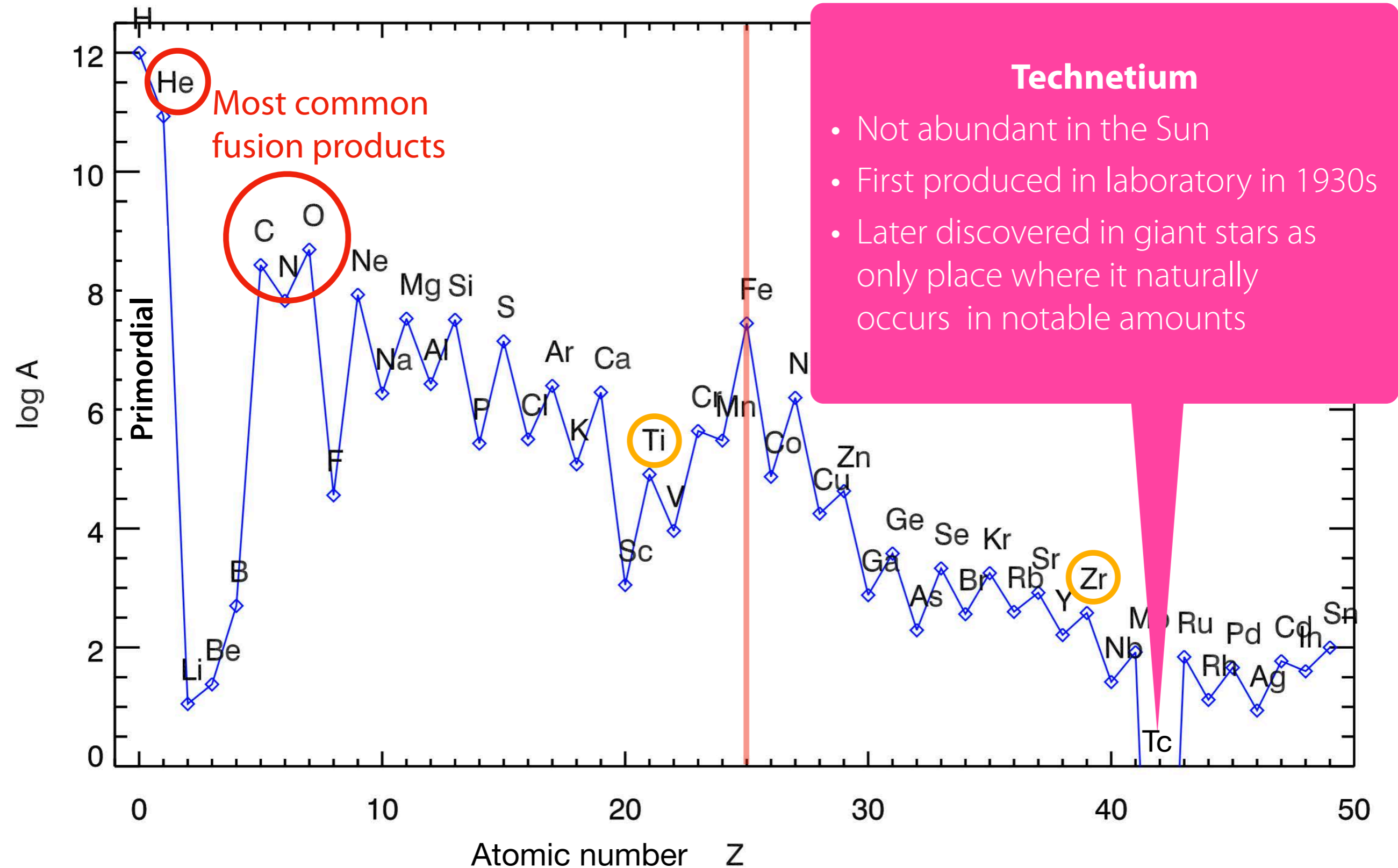
Chemical composition — Solar abundances

Most common fusion products

Why do we find these heavier elements in the Sun when they are not produced in the solar interior?

The Sun was **formed from enriched gas** with elements produced and returned by previous stellar generation.

Population I !

Elements produced by **s-** and **r-process** (late evolution stages, kilo-/supernova etc.)

Lithium: Low abundance!

Physical stellar parameters

Chemical composition — Solar abundances

Most common fusion products

Note: Spectral lines are formed in the atmosphere

➡ Determination of the chemical composition of the atmosphere and not of the interior

➡ How well mixed is the star? **Dredge-ups** can mix fusion products from core into atmosphere

Elements produced by **s-** and **r-process** (late evolution stages, kilo-/supernova etc.)

Lithium: Low abundance!

Physical stellar parameters

Chemical composition — Solar abundances

Most common fusion products

Technetium

- Not abundant in the Sun
- First produced in laboratory in 1930s
- Later discovered in giant stars as only place where it naturally occurs in notable amounts

This is really how astronomers see the periodic table according to astronomer and author Heidi Weissman Kneale (Supplied: Heidi Weissman Kneale (@heidikneale))

Physical stellar parameters

Addition: Broadband photometry with GAIA

BP   -   RP   =   Color index

blue   Red

G passband

Instrument

Passbands

Transmitted wavelength ranges

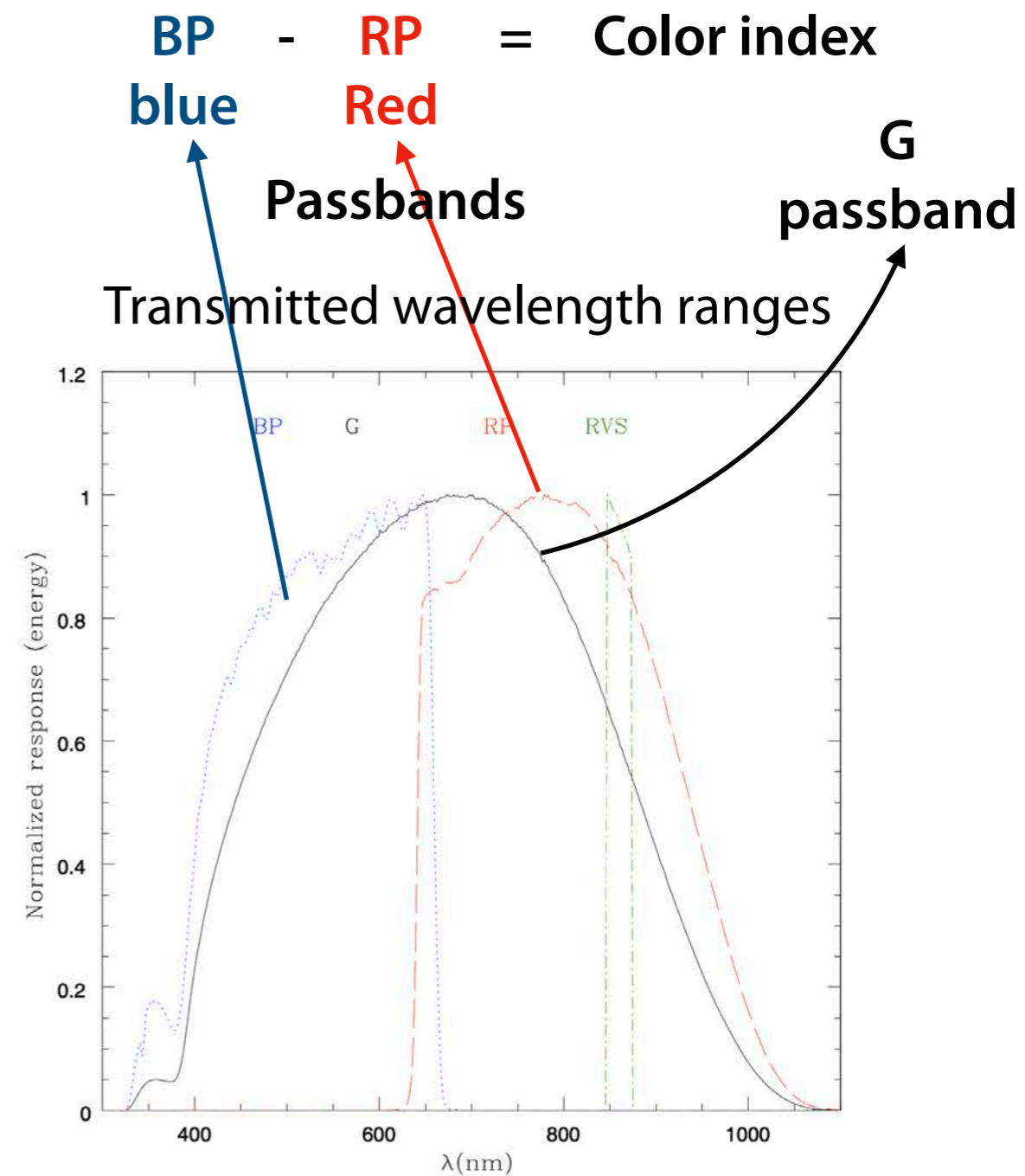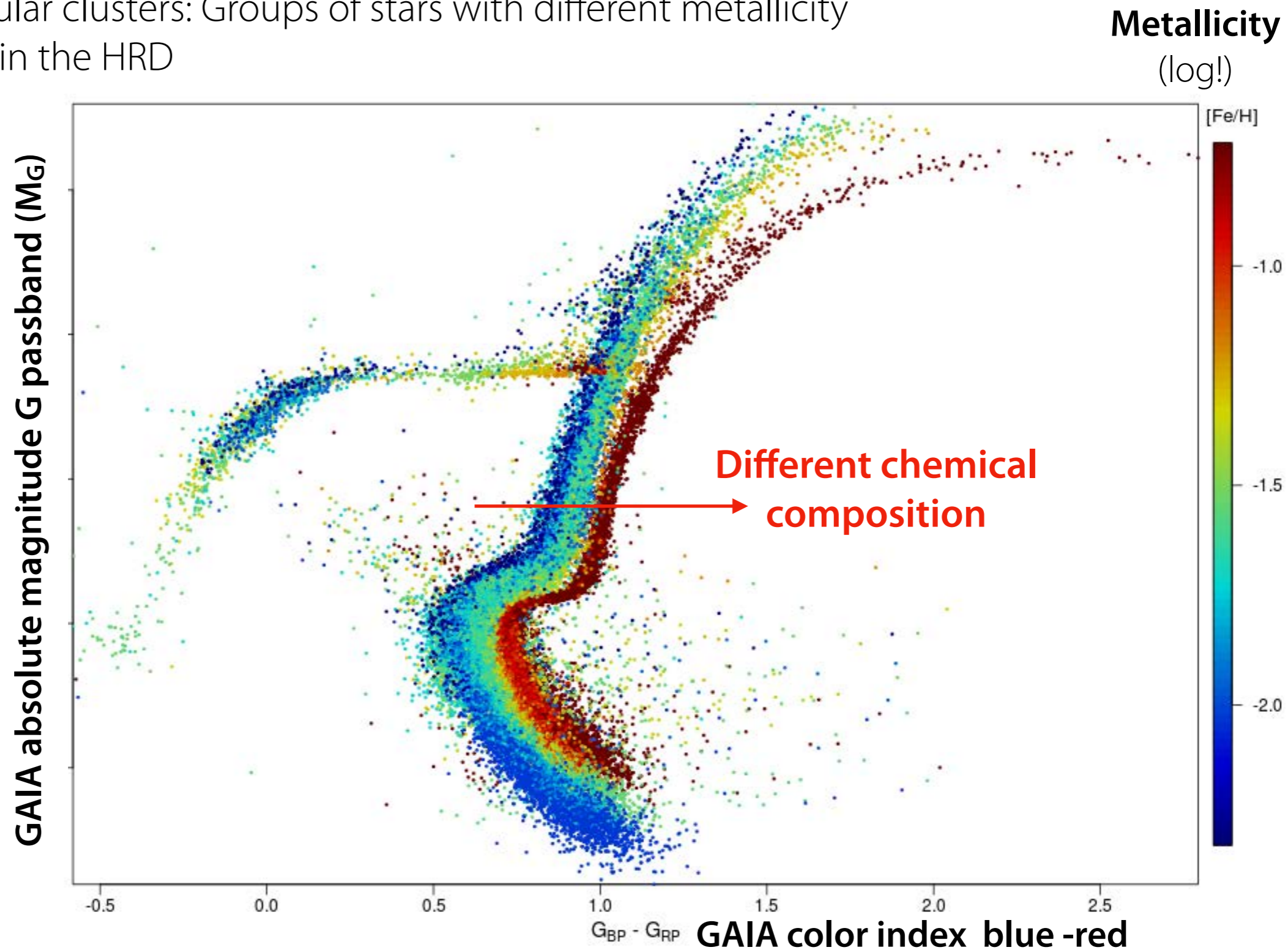Fig. 2. On their way to the BP/RP and RVS sections of the focal plane, light from the two *Gaia* telescopes is dispersed in wavelength. Picture courtesy of EADS-Astrium.

Fig. 3. *Gaia G* (solid line), G_{BP} (dotted line), G_{RP} (dashed line) and G_{RVS} (dot-dashed line) normalised passbands.

Jordi et al. (2010)

Physical stellar parameters

Chemical composition

- HRD for 14 globular clusters: Groups of stars with different metallicity
➡ Distinct subsets in the HRD

Metallicity
(log!)

Credits: ESA/Gaia/DPAC, Carine Babusiaux et al. A&A 616, A10 (2018) "Gaia Data Release 2: Observational Hertzsprung-Russell diagrams"

Physical stellar parameters

Chemical composition

- HRD for 14 globular clusters: Groups of stars with different metallicity
- ➡ Distinct subsets in the HRD

Metallicity
(log!)

Different chemical composition

- In general (all stars) range in metallicity: **-4.5 < [Fe/H] < +1.0**

- Note the logarithm

  - **[Fe/H] = 0.0: Solar metallicity**, relative abundance of metals (Fe) like in the Sun
  - [Fe/H] = +1.0: metals (Fe) 10 times more abundant in this star than in the Sun
  - [Fe/H] = -2.0: metals (Fe) 100 times less abundant in this star than in the Sun

Physical stellar parameters

Chemical composition

- Chemical composition of **stellar interior not directly observable**

- Specified only as mass fractions of hydrogen (X), helium (Y), and all heavier elements (Z):
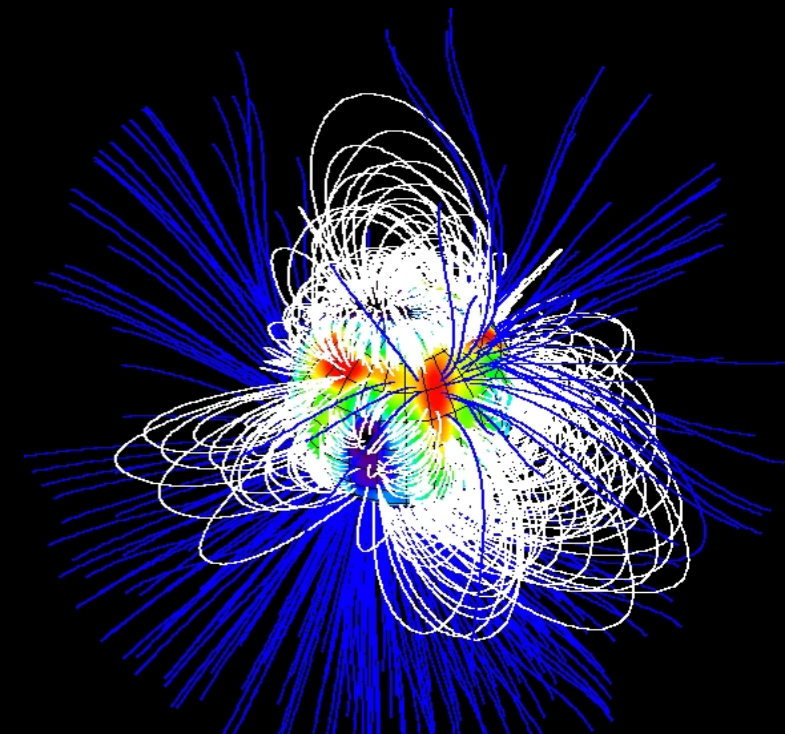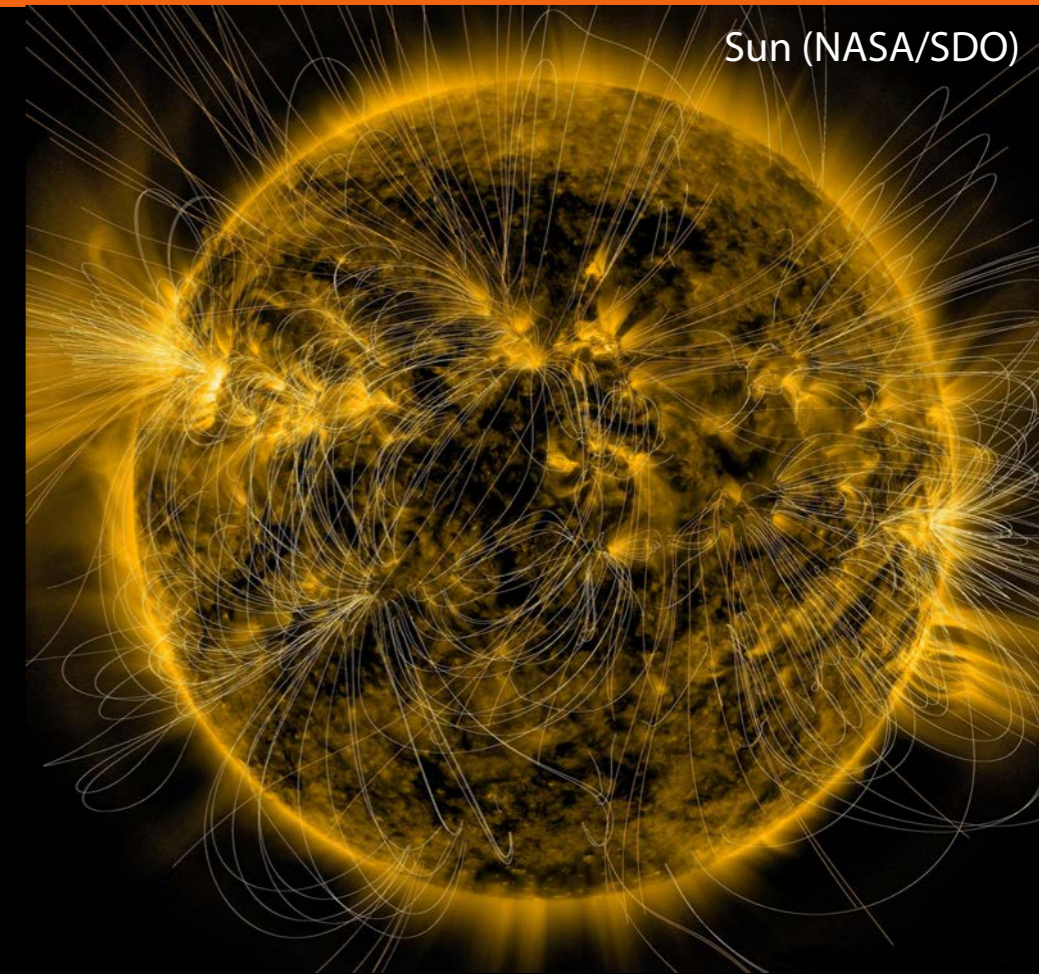
$$X+Y+Z=1$$

- Mass fractions vary with locations, in particular in radial direction from the core to the atmosphere (element segregation, fusion, etc.)

- Sun (based on models): X = 0.73, Y = 0.25, Z = 0.02.

- Numbers above refer to **number densities**, i.e. the number of particles per volume.

- Different atomic weight of elements/nuclei result in different values for the mass fraction!

- In terms of mass, the Sun consists of ~ 90 % H, ~ 10 % He, and only 0.1 % for the remaining elements.

- Relative content (i.e. the **abundance**) of heavier elements in the Sun is roughly the same as for Earth (formed from the same material in the protostellar cloud)

Physical stellar parameters

Magnetic field (B)

Sun (NASA/SDO)

- Measurement via Zeeman effect in spectral lines
- **Sun**
  - Magnetic field very **inhomogeneous**/structured
  - In atmosphere, strongest: **Sunspots** B =2000–3000 G
  - Photosphere on average B ≈ 100–300 G
  - Min. in Quiet Sun and *coronal holes*: B < 1 G
  - Magn. field strength lower in upper atmosphere

- **Stars**
  - Only observed as point source but time series allow reconstruct of magnetic field as the star rotates
  - ➡ Presence of starspots can be inferred
  - <u>Average</u> field strengths of several 1000 G are detected (up to 6000 G and higher in M-type dwarf stars, compare to White Dwarfs: 10^4 — 10^9 G)

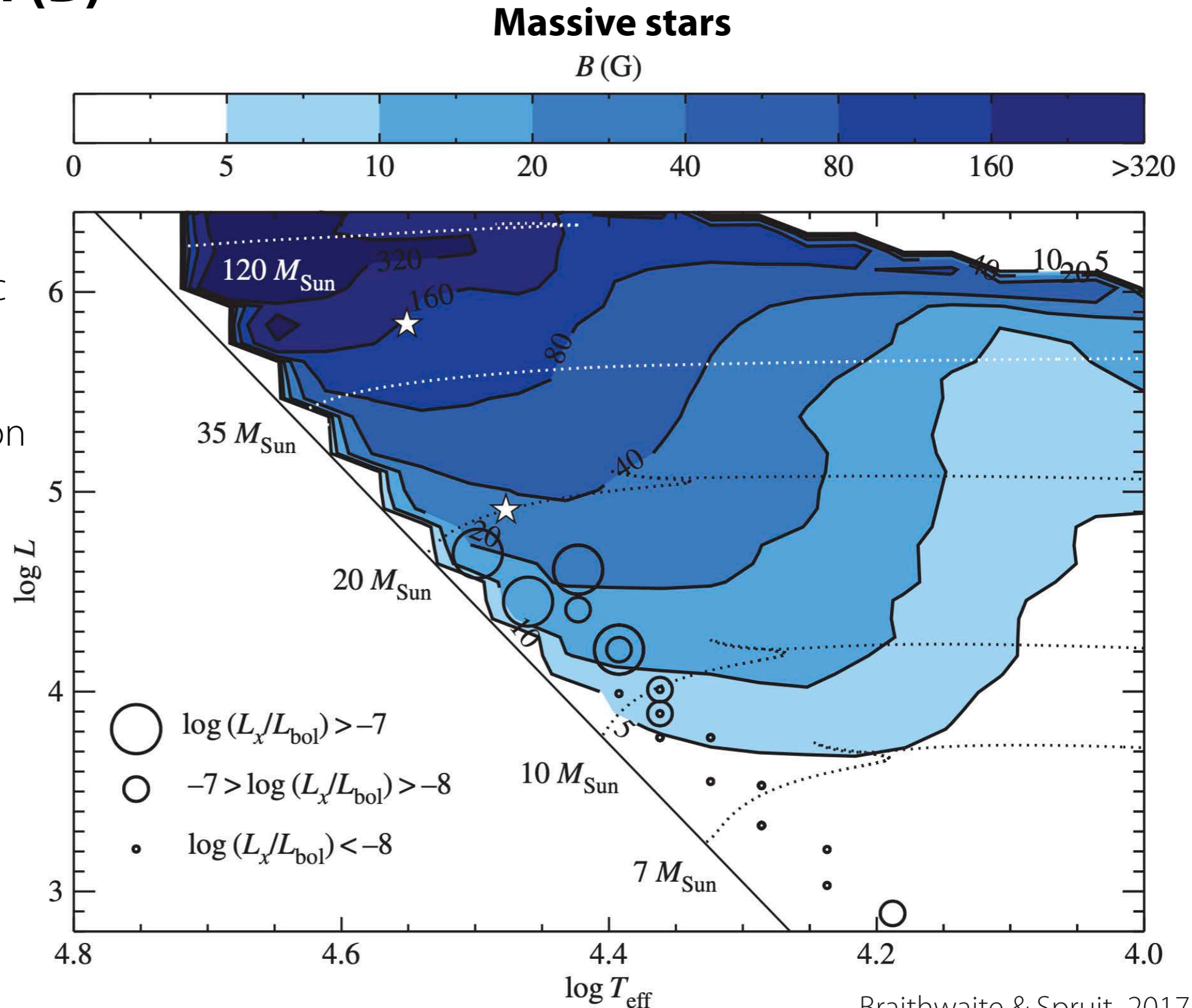- **Solar/stellar cycles:** Magnetic field changes

Zeeman Doppler imaging of SU Aur (P. Petit)

Physical stellar parameters

Magnetic field (B)

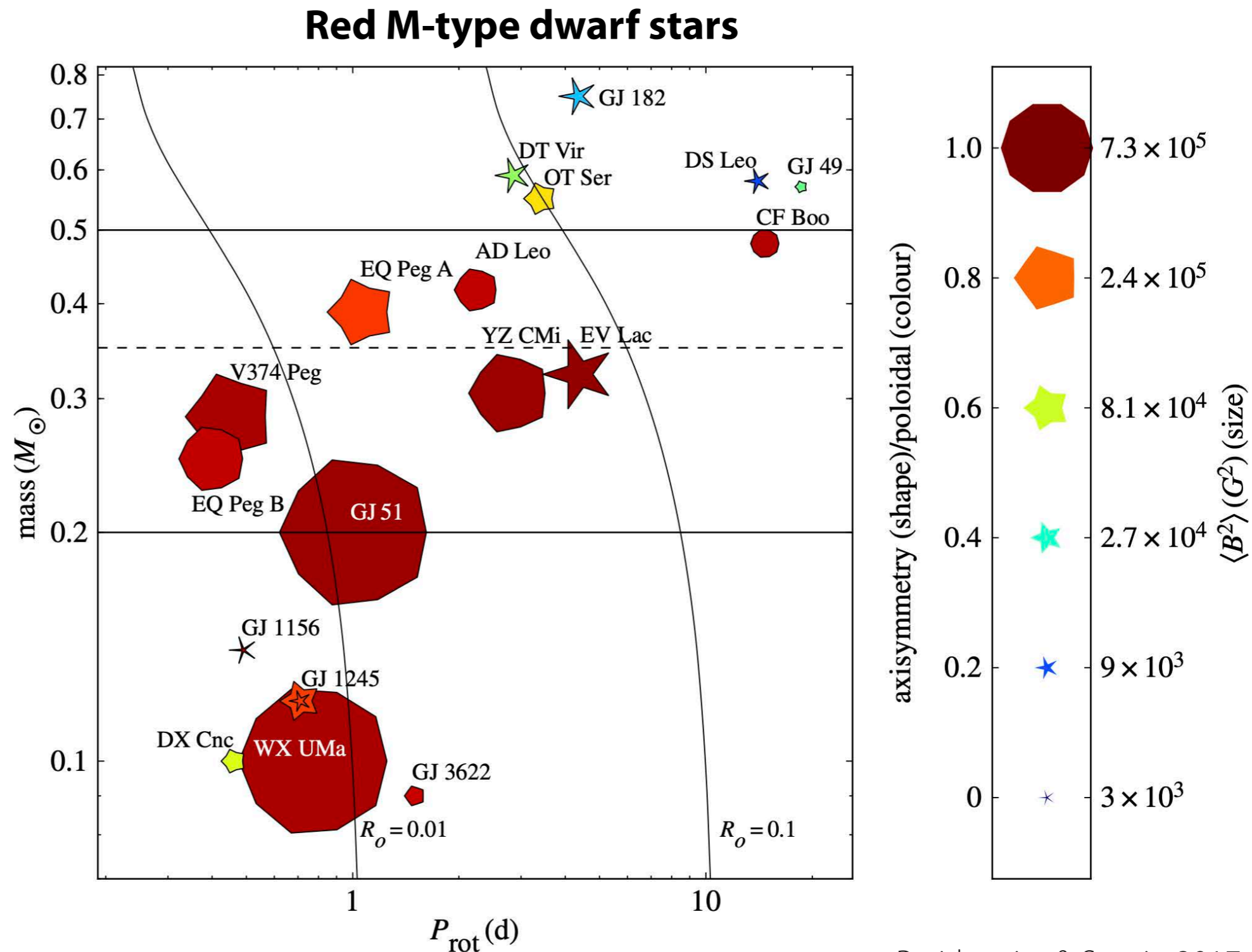- Magnetic field generation via dynamo

➡ We expect magnetic field strengths to depend on stellar structure and rotation

Braithwaite & Spruit 2017

Physical stellar parameters

Magnetic field (B)

- Rotation rate of a star important for magnetic field generation via dynamo

Red M-type dwarf stars

Braithwaite & Spruit 2017

Relations between fundamental parameters

Mass-luminosity relation

- Observations of main-sequence stars imply a relation of L and mass M of a star is related as n
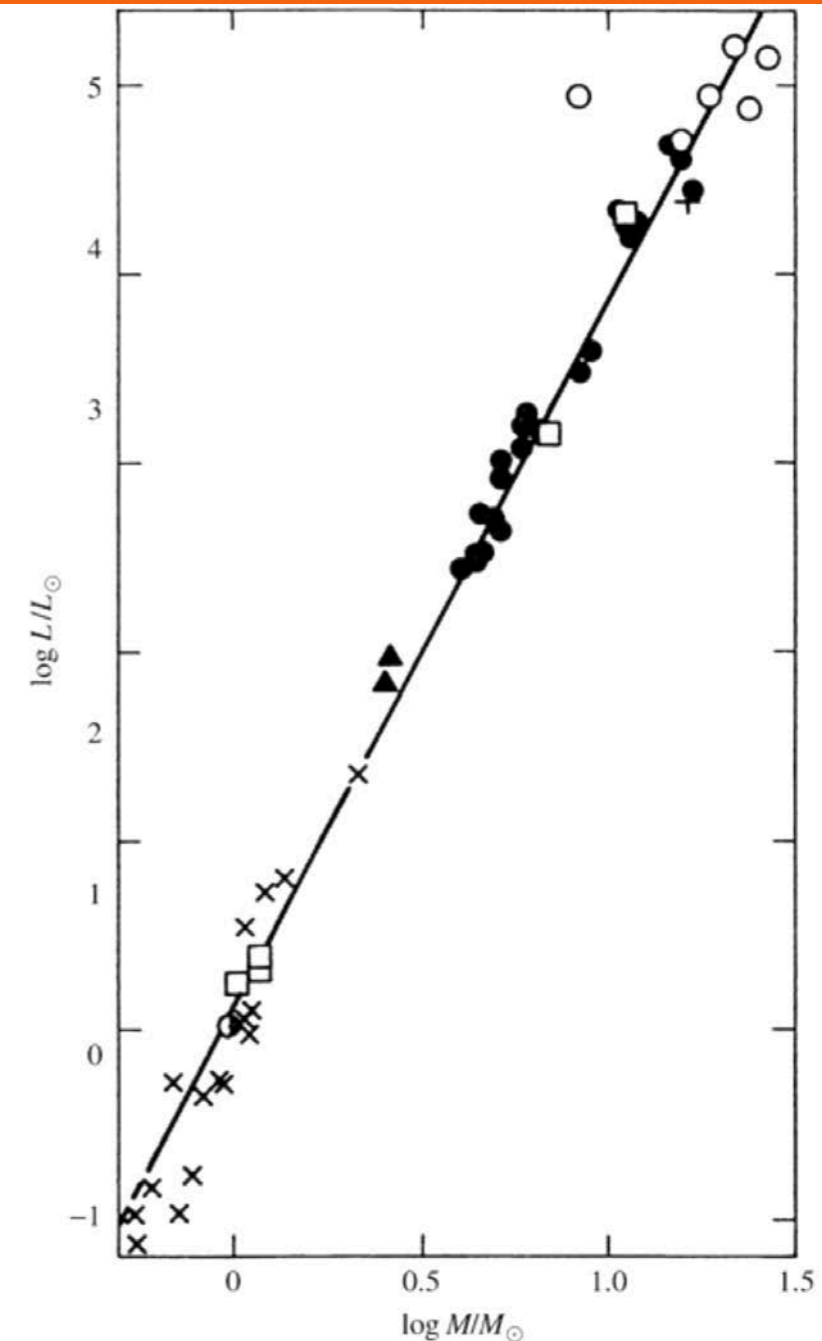
    L / L$_\odot$ = a (M/M$_\odot$)^n

- Exponent n *(and factor a)*

    - same for subsets of stars with similar internal structure / nuclear power source; varies for different stellar types accordingly.

M/M$_\odot$	< 0.43	0.43 - 2	2 — 55	> 55
a	0.23	1	1.4	3.2 10^4
n	2.3	4	3.5	1

- The mass-luminosity relation can be derived by using aforementioned equations for g and L:

$$g = \frac{GM}{R^2} \quad \Rightarrow \quad M = \frac{gR^2}{G} \quad \text{and} \quad L = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad \Rightarrow \quad \frac{M}{L} = \frac{1}{4\pi G\sigma} \frac{g}{T_{\text{eff}}^4}$$
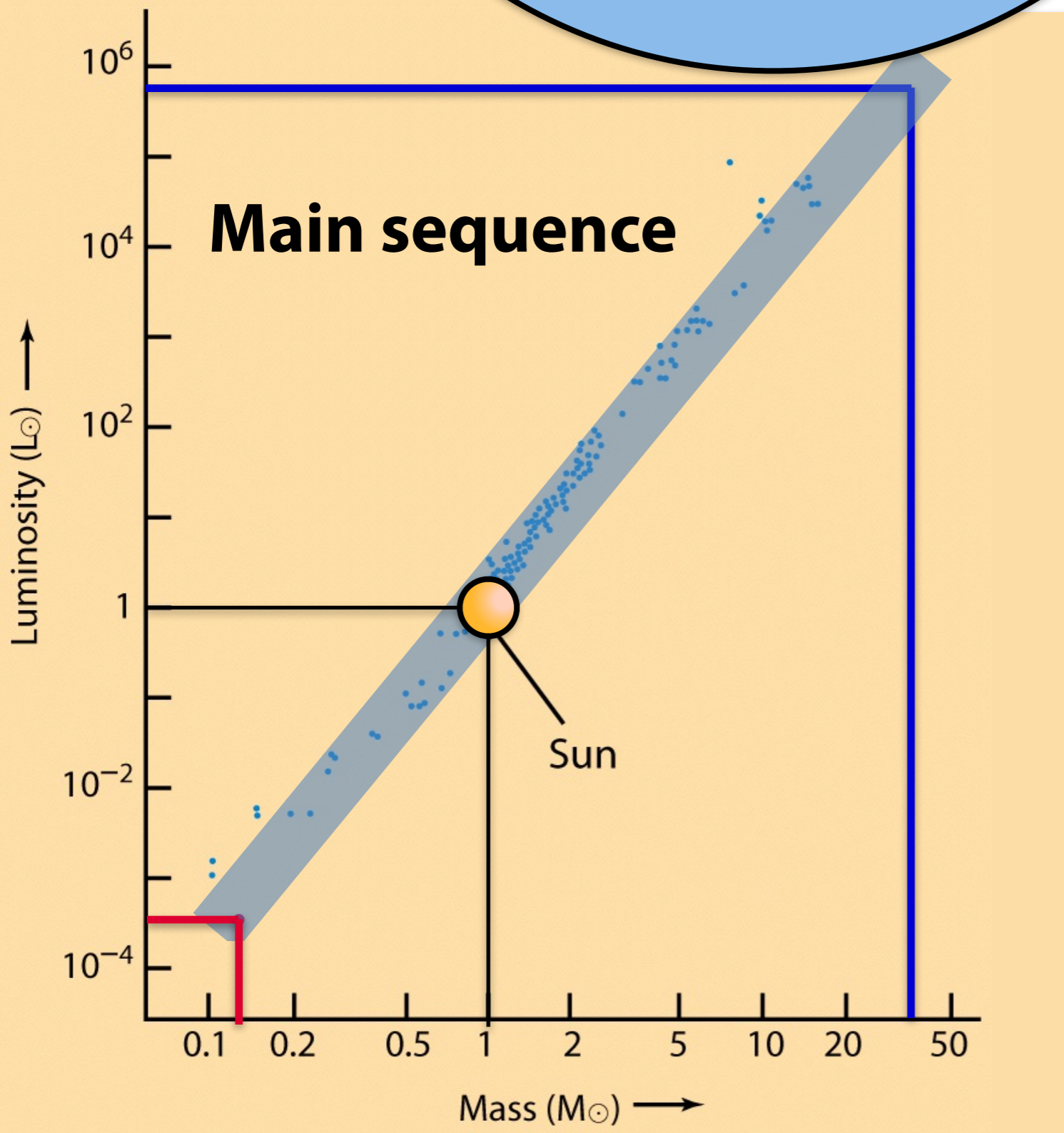
➡ The ratio M/L can thus be derived from the parameters Teff and g, which can be determined from stellar spectra.

Relatio   ntal parameters

Mass-lumi

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^n$$

Main sequence

Sun

Spectral class O6

R = 18 $R_\odot$

T_{eff} = 38 000 K

    = 6.3 $T_{eff,\odot}$

➡ L = 520 000 $L_\odot$

➡ M = 40 $M_\odot$

Red dwarf star

R = 0.1 $R_\odot$

T_{eff} = 0.5 $T_{eff,\odot}$ = 2885 K

➡ L = 6 10^{-4} $L_\odot$

➡ L = 0.06 % $L_\odot$

➡ M = 0.12 $M_\odot$

Relations between fundamental parameters
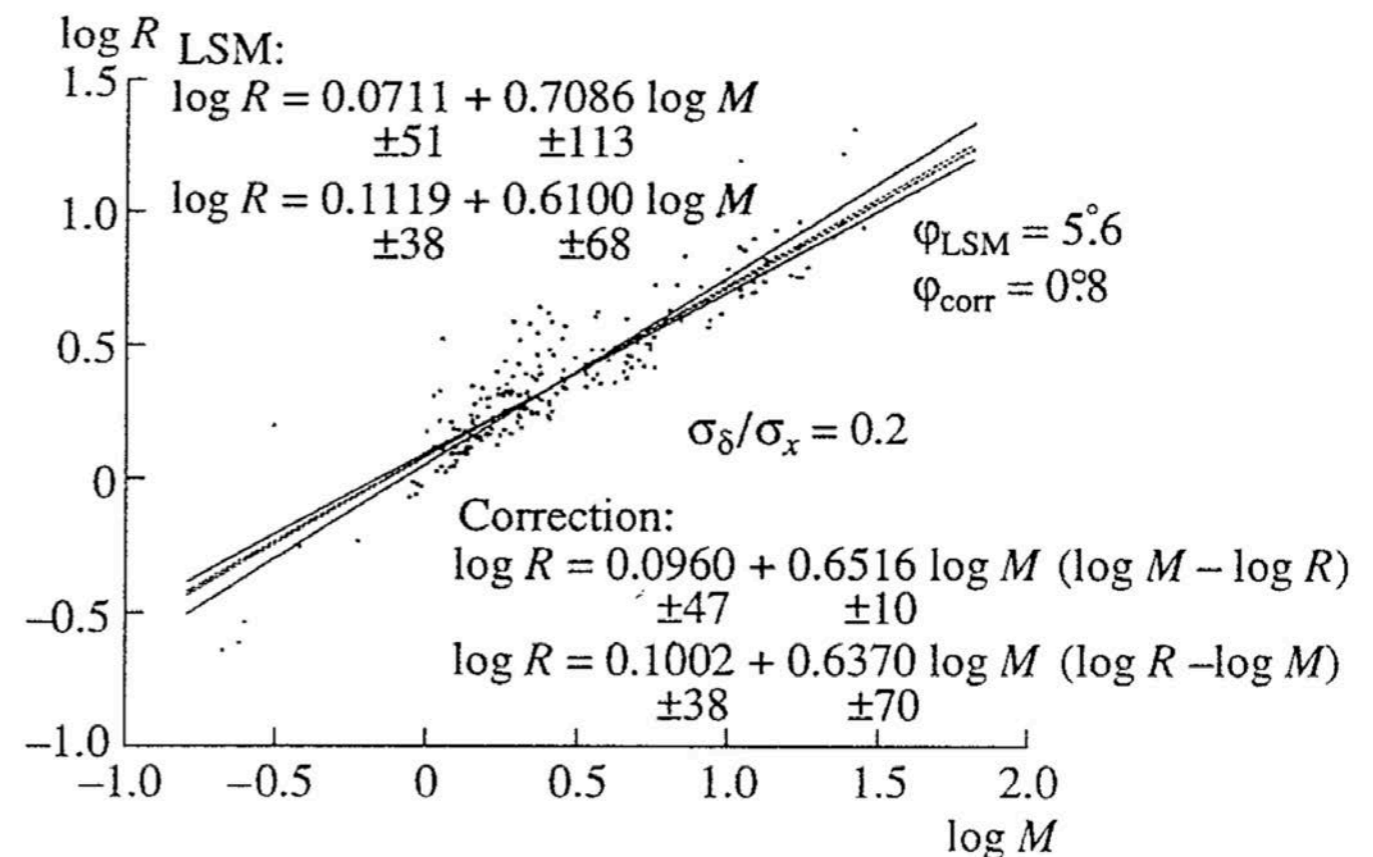
Mass-radius relation

- Mass and radius related via the gravitational acceleration at the surface (which can be estimated from spectral lines)

$$g = \frac{GM}{R^2}$$

- Empirically for main sequence stars with $M \geq M_\odot$

$$R/R_\odot = (M/M_\odot)^{0.6}$$

- More precise relations can be found when considering internal structure of stars in detail

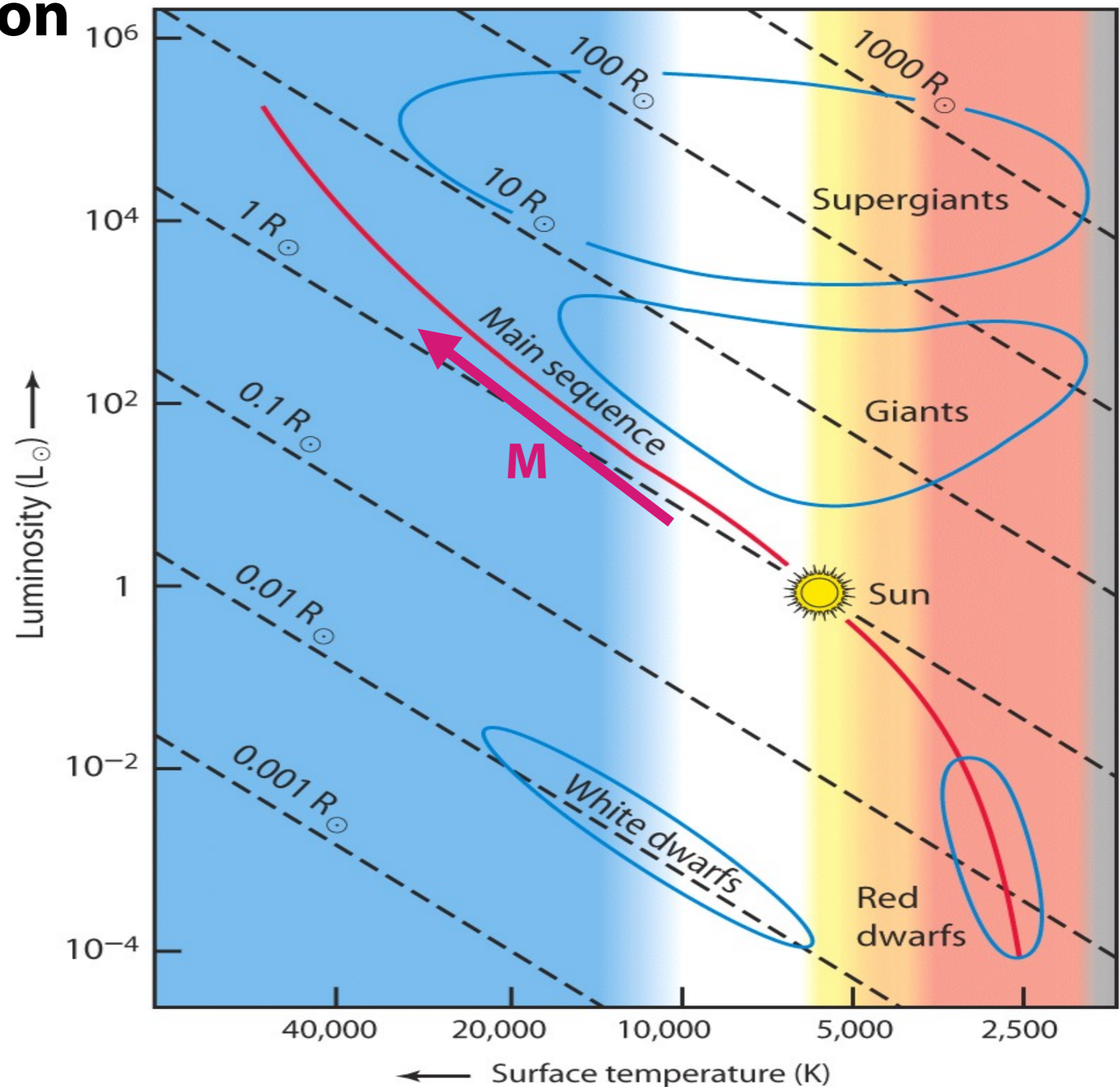LSM:
$\log R = 0.0711 + 0.7086 \log M$
$\pm 51 \quad \pm 113$
$\log R = 0.1119 + 0.6100 \log M$
$\pm 38 \quad \pm 68$

$\varphi_{LSM} = 5°.6$
$\varphi_{corr} = 0°.8$

$\sigma_\delta/\sigma_x = 0.2$

Correction:
$\log R = 0.0960 + 0.6516 \log M \; (\log M - \log R)$
$\pm 47 \quad \pm 10$
$\log R = 0.1002 + 0.6370 \log M \; (\log R - \log M)$
$\pm 38 \quad \pm 70$

Gorda & Svechnikov (1998)

Relations between fundamental parameters

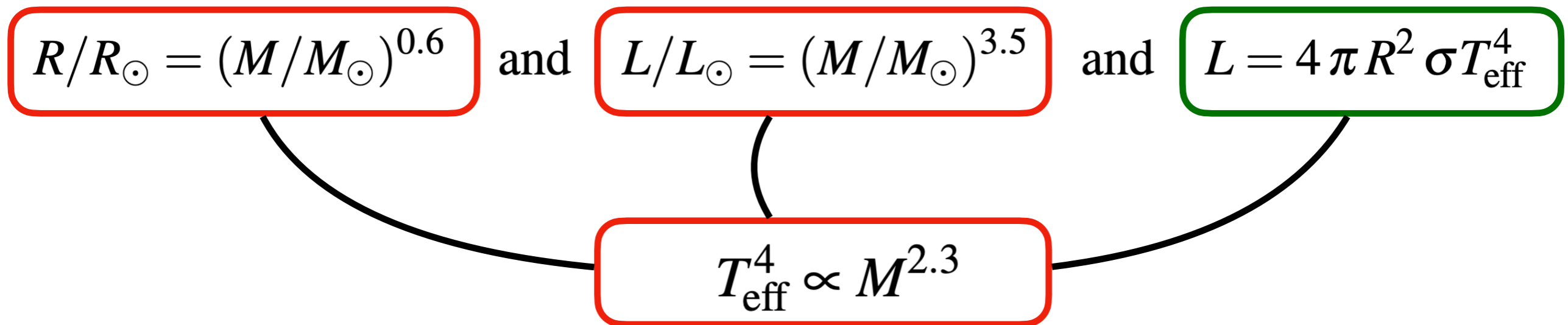## Mass-radius relation

$$R/R_\odot = (M/M_\odot)^{0.6}$$

$$M \geq M_\odot$$

- Careful: Masses are not typically given in HRD as relations dependent on stellar structure in more complicated way than for main sequences

Relations between fundamental parameters

Mass-temperature relation

$$R/R_\odot = (M/M_\odot)^{0.6}$$ and $$L/L_\odot = (M/M_\odot)^{3.5}$$ and $$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$
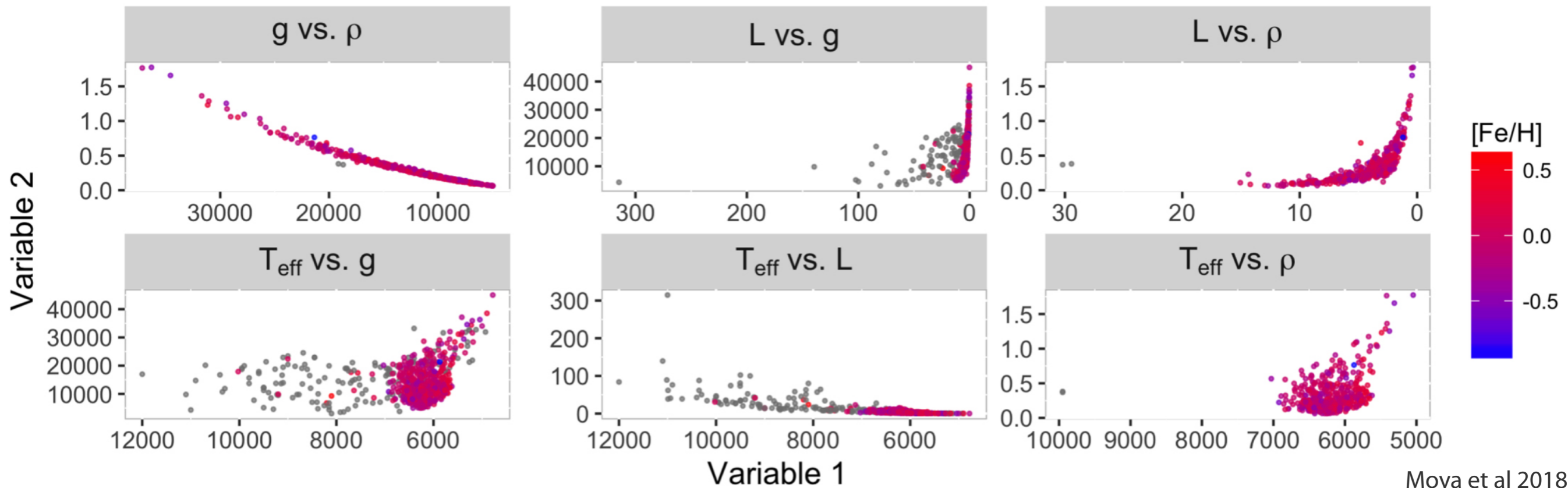
$$T_{\text{eff}}^4 \propto M^{2.3}$$

➡ Along main sequence: **Effective temperature(roughly) proportional to the stellar mass**

- Please note: Empirical relations based on subgroups of stars

  ➡ To be applied and interpreted within limits

  ➡ Detailed physical consideration of stellar structure superior.

Relations between fundamental parameters

Empirical relations

Correlations between independent variables

Moya et al 2018

- Please note: Empirical relations based on subgroups of stars

  ➡ To be applied and interpreted within limits

  ➡ Detailed physical consideration of stellar structure superior.

  ➡ Often it is more than just two parameters that are related!
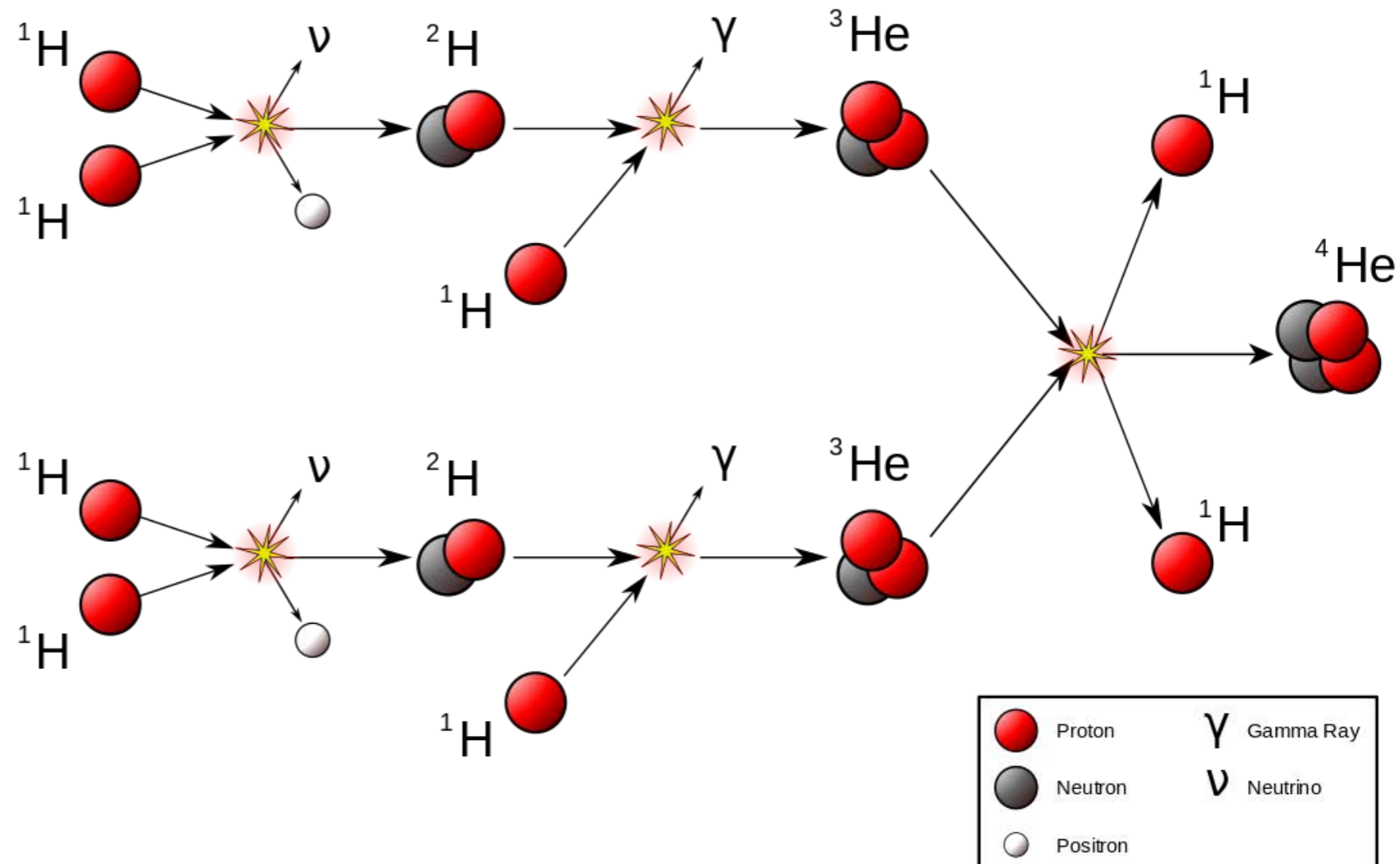
Physical stellar parameters

Lifetime of the Sun on the main sequence

- In solar interior: Thermonuclear fusion (here pp chain)

- 4 hydrogen nuclei (protons) → 1 helium nucleus (alpha particle)

- Mass difference $\Delta m = 4\,m_p - m_\alpha > 0$ corresponds then to released energy

$$\Delta E = \Delta m\, c^2$$

- **Energy released per fusion reaction:**

$$E_{FUS} \sim 4.3 \times 10^{-12}\ Ws$$

Physical *stellar* parameters

Lifetime of the Sun on the main sequence

- In solar interior: Thermonuclear fusion (here pp chain)

- 4 hydrogen nuclei (protons) $\rightarrow$ 1 helium nucleus (alpha particle)

- Mass difference $\Delta m = 4\,m_p - m_\alpha > 0$ corresponds then to released energy

  $\Delta E = \Delta m\,c^2$

• Energy released per fusion reaction:

$$E_{FUS} \sim 4.3 \times 10^{-12}\ \text{Ws}$$

Solar luminosity

$$L_\odot \sim 3.9 \times 10^{26}\ \text{W}$$

- Fusion reactions (pp) in the Sun per second: $N_{FUS} \sim L_\odot / E_{FUS} \sim \underline{10^{38}\ s^{-1}}$

Physical stellar parameters

Lifetime of the Sun on the main sequence

- Mass "lost" via fusion per second

  $\Delta M = N_{FUS} \times 4\, m_p \sim 6.7 \times 10^{11}$ kg/s   (m_p: proton mass)

- Solar mass $M_\odot = 2 \times 10^{30}$ kg

- Solar lifetime if **all mass** is "used" at constant rate via H→He fusion (pp chain)

- $\tau = M_\odot / \Delta M = 2 \times 10^{30}$ kg $/ 6.7 \times 10^{11}$ kg/s $= 3 \times 10^{18}$s $= 9 \times 10^{10}$ år
  90 billion year
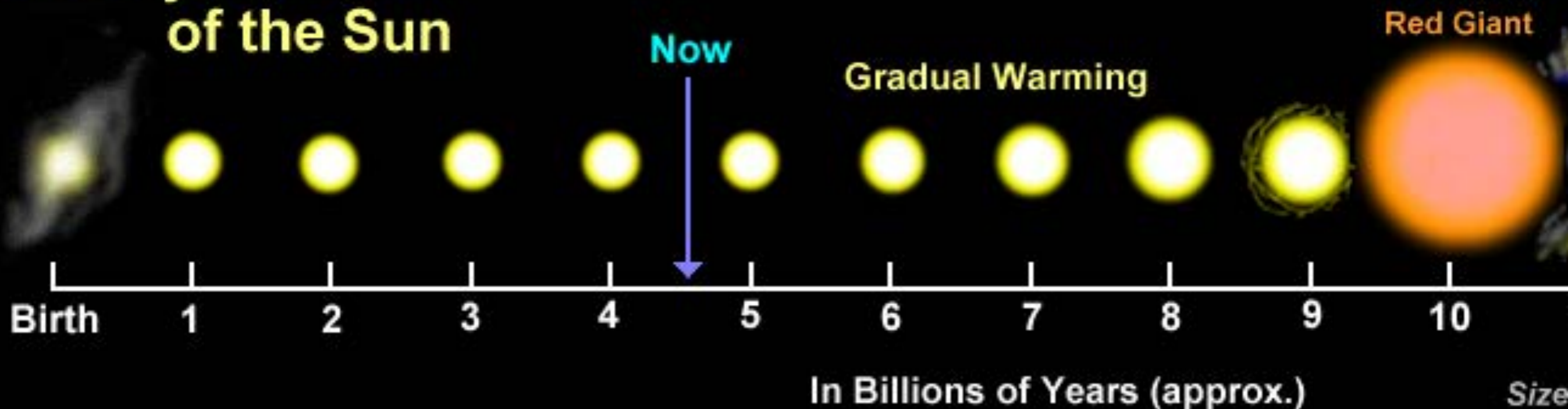
But we know: **Sun's lifetime ~ <u>10^{10} yr!</u>**

➡ Corresponds to 10% of solar mass.

➡ Explanation: H-He fusion only in solar core (and also not constant rate)

➡ More detailed evolution models needed.

Physical stellar parameters

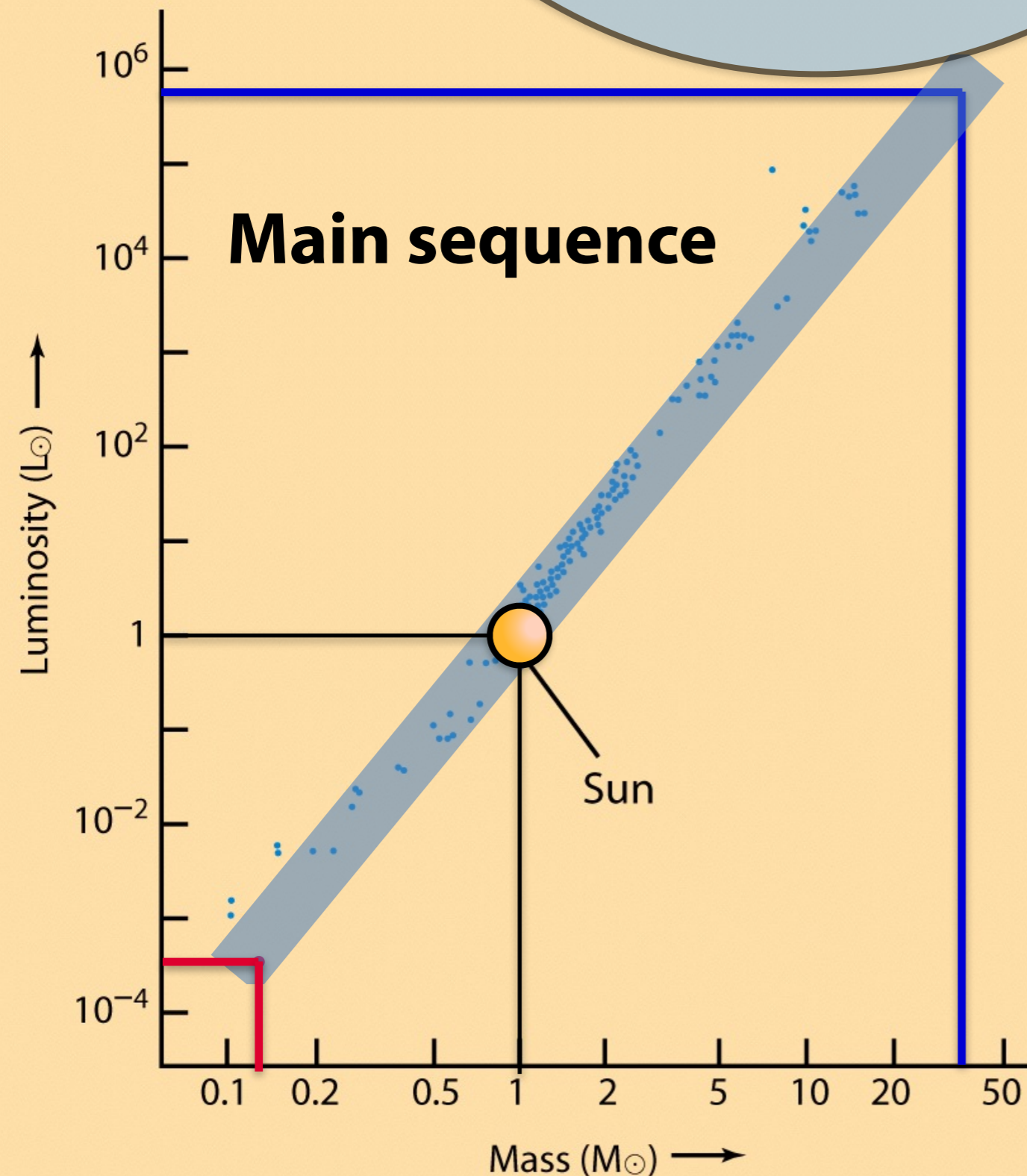## Lifetime of the Sun on the main sequence

But we know: **Sun's lifetime ~ 10^{10} yr!**

➡ Corresponds to 10% of solar mass.

➡ Explanation: H-He fusion only in solar core (and also not constant rate)

➡ More detailed evolution models needed.

Fundamental parameters

Lifetime on main sequence

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

Main sequence

Sun

- Energy set free by fusion
- Stellar **mass** sets absolute limit
- **Luminosity** gives energy loss rate

- Simplifying assumption for an *order of magnitude estimate*: Energy conversion via fusion with constant rate set at constant luminosity
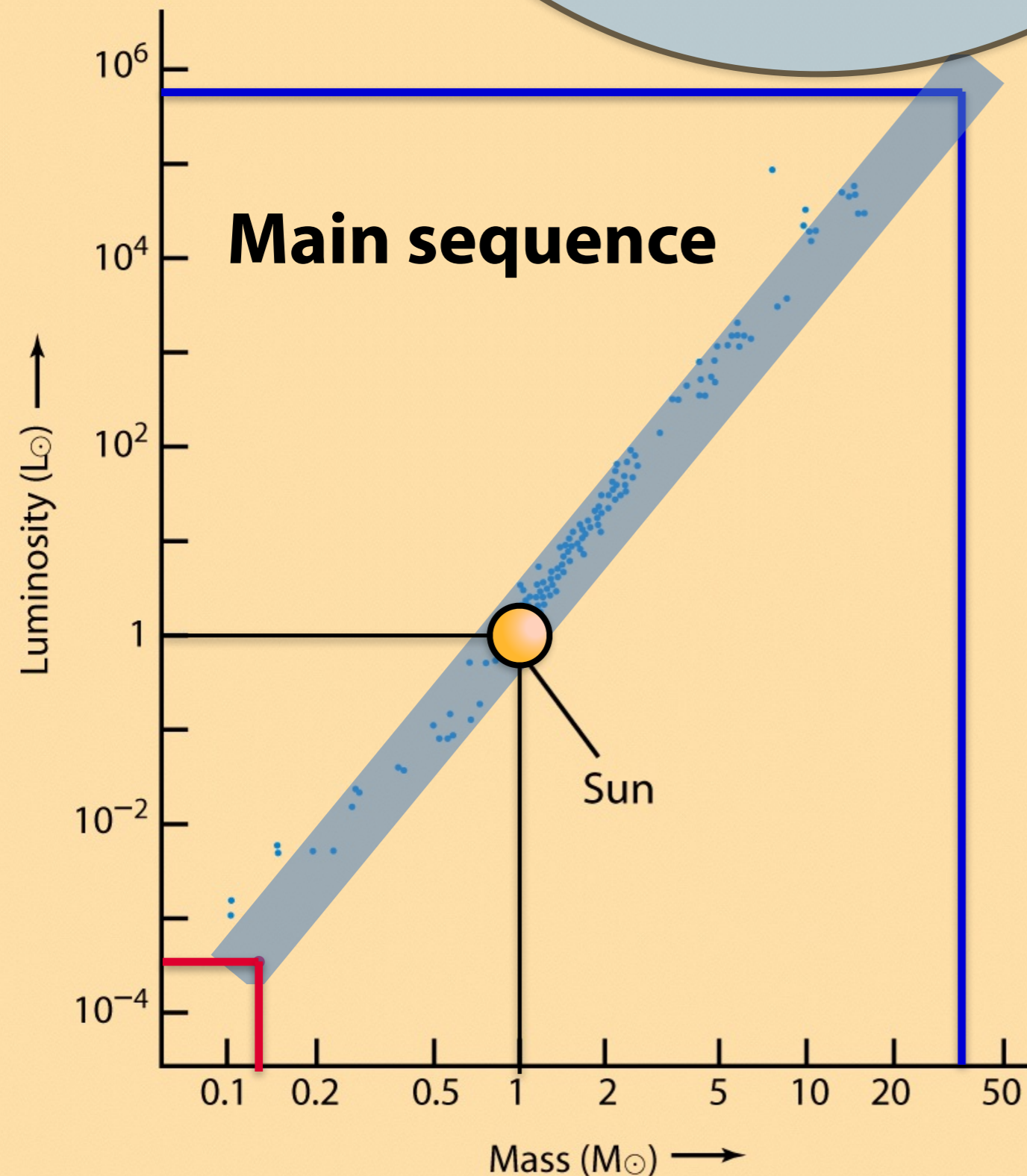
➡ Max. time at "constant burn"

$$\tau \propto M / L = \quad M^{n-1}$$

Fundamental parameters

Lifetime on main sequence

$$\tau \propto M/L$$

Main sequence

Sun

- Energy set free by fusion
- Stellar **mass** sets absolute limit
- **Luminosity** gives energy loss rate
➡Max. time at "constant burn" M / L

Spectral class O6
R = 18 $R_\odot$, T_{eff} = 38 000 K , M = 40 $M_\odot$
➡ L = 520 000 $L_\odot$
➡ Compared to Sun
  M[$M_\odot$]/L[$L_\odot$] = 40 / 520 000 = 8 10^{-5}
➡ 13000 shorter time

Red dwarf star
R = 0.1 $R_\odot$, T_{eff} = 2885 K, M = 0.12 $M_\odot$
➡L = 6 10^{-4} $L_\odot$
➡ Compared to Sun
  M[$M_\odot$]/L[$L_\odot$] = 0.12 / 6 10^{-4} = 200
➡ 200 times longer

Luminosity ($L_\odot$)

Mass ($M_\odot$)

Fundamental parameters

Lifetime on main sequence

$$\tau \propto M / L$$

Main sequence

MS lifetime

"Short"

Sun

Long

- Lifetime of Sun on main sequence on the order of 10^{10} yr

Spectral class O6
➡ $\tau \sim 8 \ 10^{-5} \ 10^{10}$ yr $\sim 800 \ 000$ yr
➡ "Only" less than 1 million years.

Red dwarf star
➡ $\tau \sim 200 \ 10^{10}$ yr $\sim 2 \ 10^{12}$ yr yr
➡ 2000 billion years
➡ Age of universe "only" ~14 billion years!

Evolutionary Time Scales

Overview

- **Time scales are a fundamentally important in physics as they highlight which physical process is important in which context.**

  ➡ **Allows to focus on what is important and derive on "lean" and yet accurate models**

- **Changes in a star** occur on different time scales at different evolutionary phases.

  - Most important:

    - Nuclear time scale t_n

    - Thermal time scale t_t

    - Dynamical or freefall time scale t_d.

  - Usually: $t_d \ll t_t \ll t_n$

Evolutionary Time Scales

Nuclear Time Scale

- **Definition:** time in which a star radiates away all the energy that can be released by nuclear reactions

- **Estimate** (as already done for the Sun) for **hydrogen burning**

  - Time in which all available hydrogen is turned into helium.

  - Theoretical considerations / evolutionary model calculations:

  ➡ ~10 % of total mass of hydrogen in a star can be consumed before other, more rapid evolutionary mechanisms set in.

  ➡ Of these 10% a mass fraction 0.7 % is turned into energy in

$$t_n \approx \frac{0.007 \times 0.1\, M c^2}{L}$$

- For the Sun:

$$t_n \approx \frac{M/M_\odot}{L/L_\odot} \times 10^{10}\ a$$

- Remember: Mass-luminosity relation with high exponent

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot} \right)^n$$

➡ M = 30 M$_\odot$: t$_n$ ~ 2 million yr

Evolutionary Time Scales

Thermal Time Scale

- **Definition:** time in which a star would radiate away all its thermal energy if the nuclear energy production were suddenly turned off

- **Estimate:** time it takes for **radiation from the centre to reach the surface**.

  - Virial theorem: $$\langle T \rangle = -\frac{1}{2}\langle U \rangle$$ T: total kinetic energy of the system
U: potential energy of the system.

  ➡ Kinetic energy of the thermal motion of the gas particles equals half of the potential energy.

$$t_t \approx \frac{0.5\,G M^2/R}{L}$$

G: constant of gravity
R: stellar radius

$$\approx \frac{(M/M_\odot)^2}{(R/R_\odot)(L/L_\odot)} \times 2 \times 10^7 \ \text{a}$$

- For the Sun: $t_t \sim 20$ million yrs $\sim 1/500\ t_n$ (nuclear time scale)

Evolutionary Time Scales

Dynamical Time Scale

- **Definition:** the time it would take a star to collapse if the pressure supporting it against gravity were suddenly removed.

- **Estimate:** time for a particle to **fall freely from the stellar surface to the centre**

  - Equals half of period given by Kepler's third law with stellar radius R corresponding to semimajor axis of the orbit:

$$t_{\mathrm{d}} = \frac{2\pi}{2}\sqrt{\frac{(R/2)^3}{GM}} \approx \sqrt{\frac{R^3}{GM}}$$

- Shortest time scale of the three!

- For the Sun: $t_{\mathrm{d}} \sim$ 30min

Evolutionary Time Scales

Stellar lifetimes
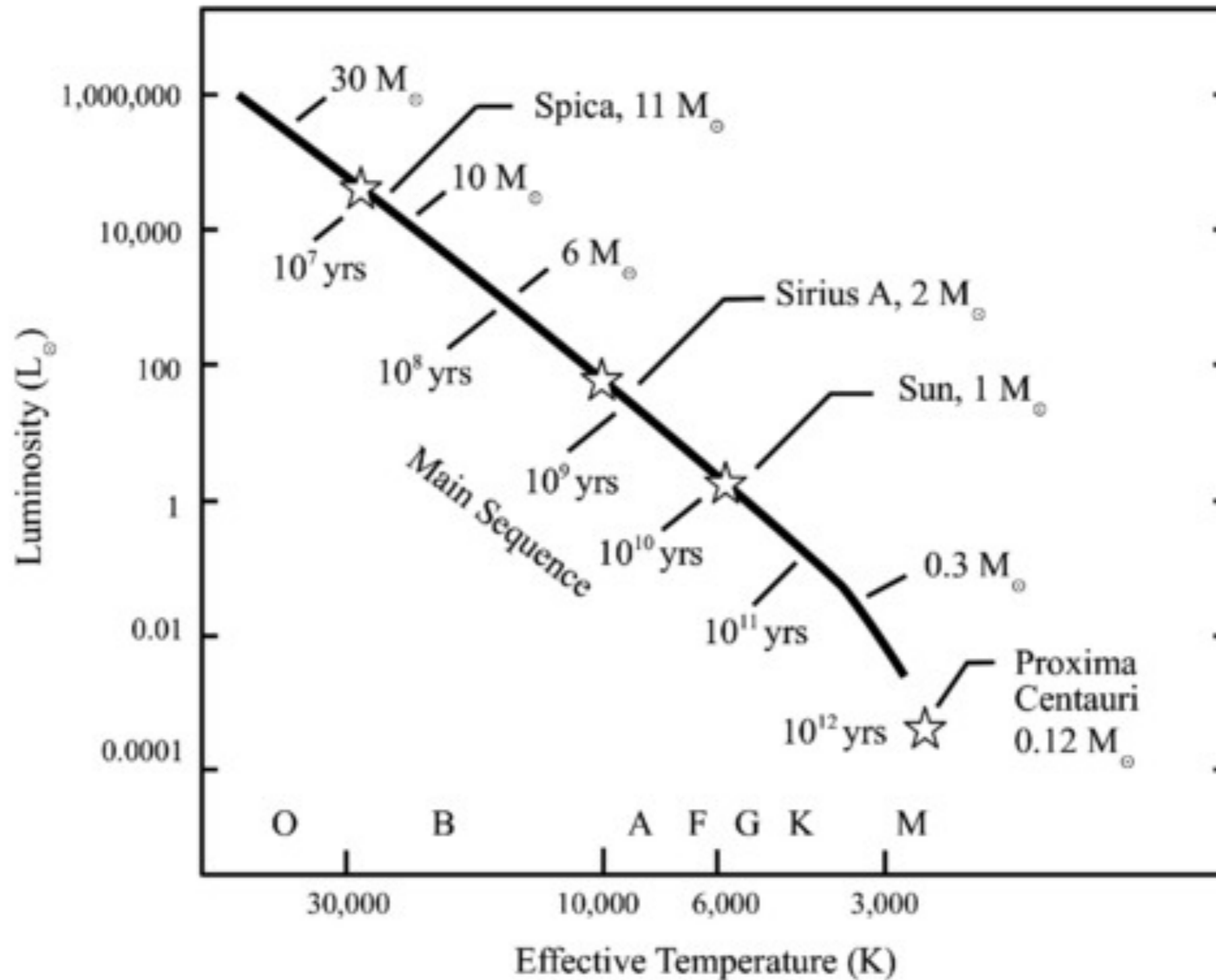
Dynamical
Time scale
t_d

Nuclear
time scale
Hydrogen
burning
t_n

Nuclear
time scale
He burning etc
$t_{n'}$

Stellar lifetimes (unit 10^6 years)

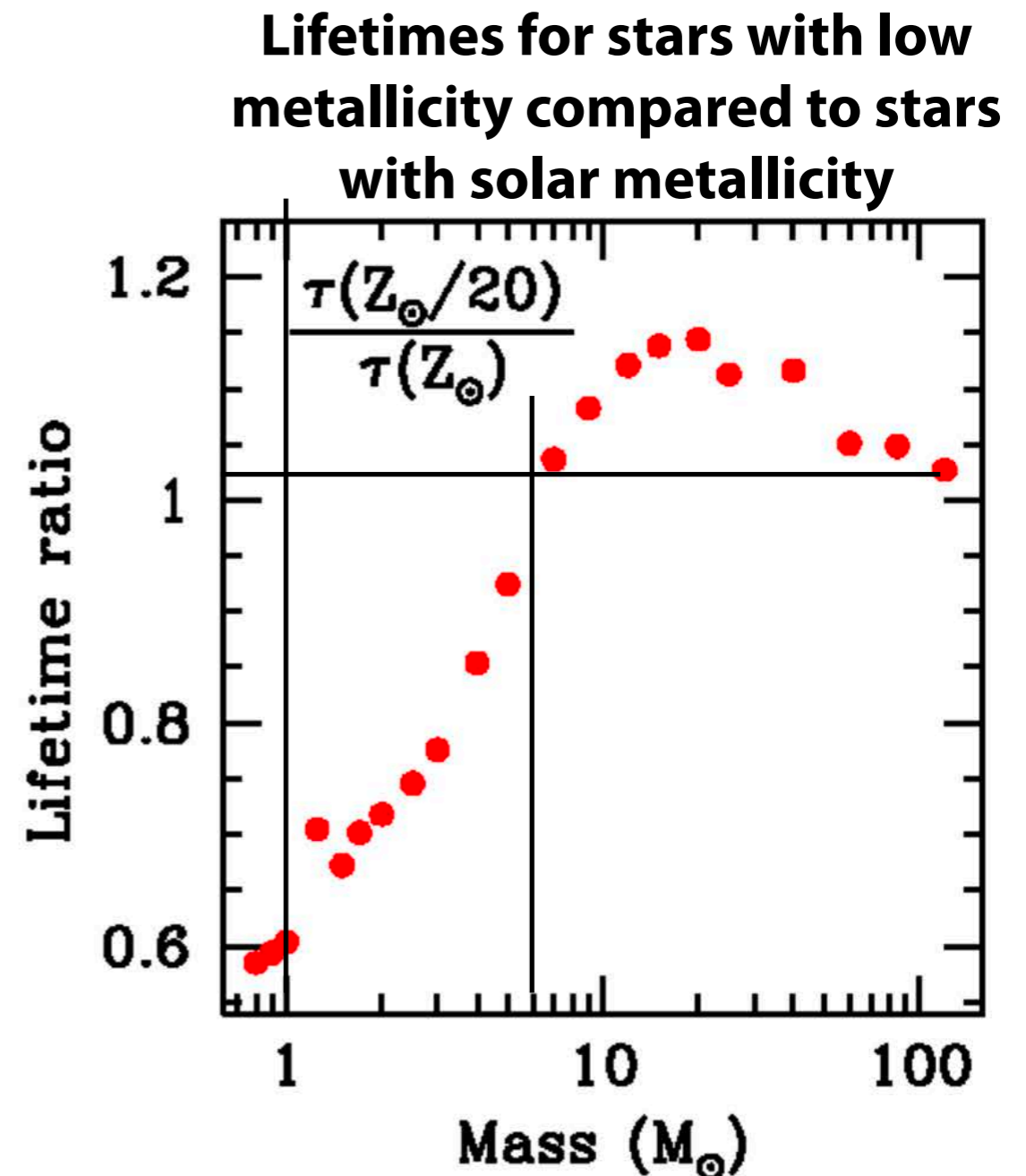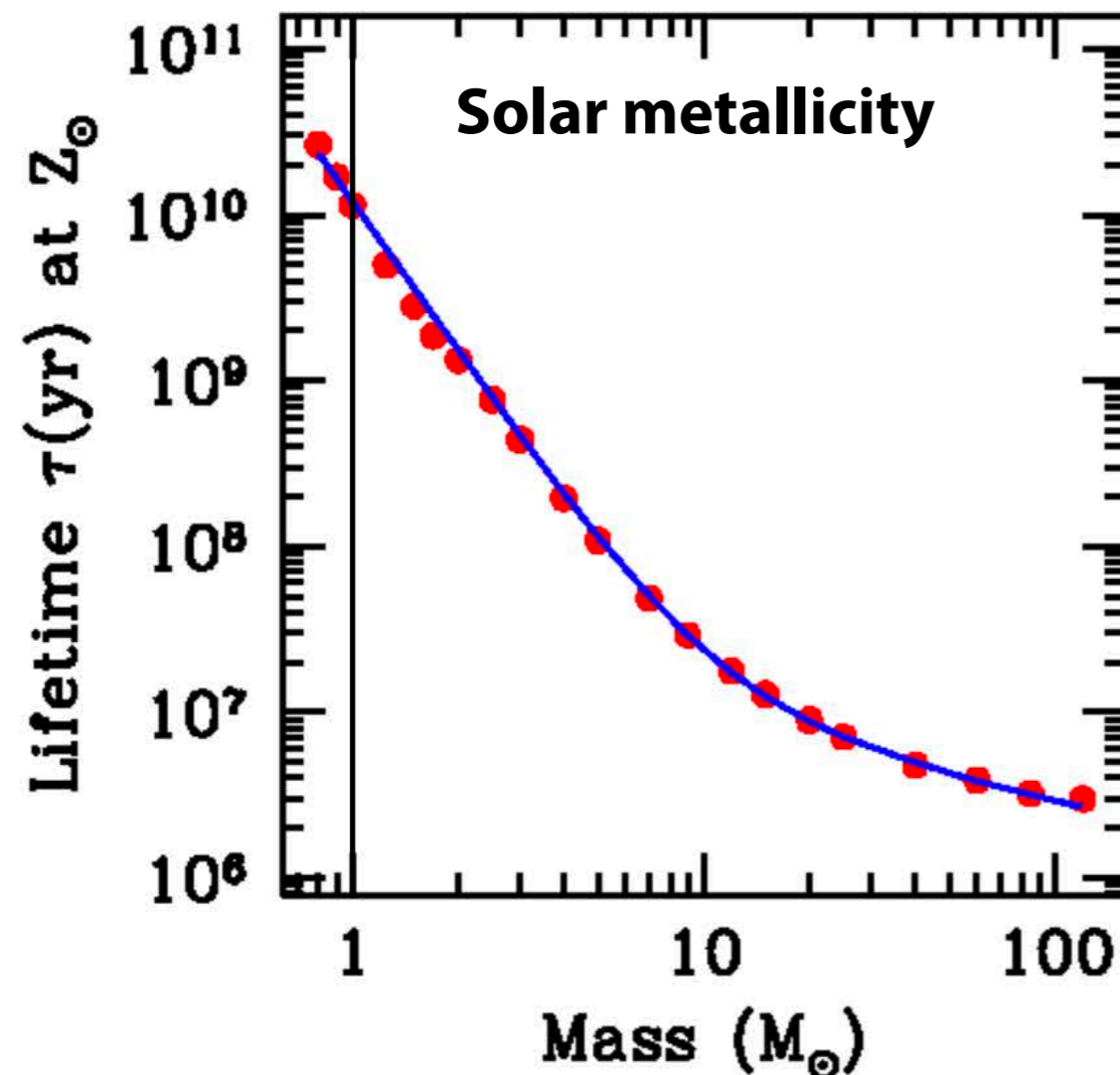| Mass [$M_\odot$]	Spectral type on the main sequence	Contraction to main sequence	Main sequence	Main sequence to red giant	Red giant
30	O5	0.02	4.9	0.55	0.3
15	B0	0.06	10	1.7	2
9	B2	0.2	22	0.2	5
5	B5	0.6	68	2	20
3	A0	3	240	9	80
1.5	F2	20	2000	280	
1.0	G2	50	10,000	680	
0.5	M0	200	30,000		
0.1	M7	500	10^7		

Stellar lifetimes

Stellar lifetimes

Impact of metallicity

- Metallicity (chemical composition) of a star influences its evolution (time scales)

- For $Z = 0.05\ Z_\odot$ (lower metallicity):

- $M < 6\ M_\odot$: lifetimes get shorter

- $M > 6\ M_\odot$: lifetimes get longer

Lifetimes for stars with low metallicity compared to stars with solar metallicity

Stellar lifetimes

Type	Mass	Temp.	Luminosity	Life time [10^6yr]	Occurrence
O	50	40 000 K	100 000	10	0.00001 %
B	10	20 000 K	1000	100	0.1 %
A	2	8500 K	20	1 000	0.7 %
F	1.5	6500 K	4	3 000	2 %
G	1	5700 K	1	10 000	3.5 %
K	0.8	4500 K	0.2	50 000	8 %
M	0.3	3200 K	0.01	200 000	80 %

- **Main sequence stars** with same mass have typically similar radius, luminosity and temperature (only small slow changes during the time of the main seq. and chemical composition)

- **Low mass stars by far most abundant!**

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Stellar lifetimes

Type	Mass	Temp.	Luminosity	Life time [10^6yr]	Occurrence
O	50	40 000 K	100 000	10	0.00001 %
B	10	20 000 K	1000	100	0.1 %
A	2	8500 K	20	1 000	0.7 %
F	1.5	6500 K	4	3 000	2 %
G	1	5700 K	1	10 000	3.5 %
K	0.8	4500 K	0.2	50 000	8 %
M	0.3	3200 K	0.01	200 000	80 %

- NOTE: This is the total lifetime of the main series (these stars are not older than the universe)

- No K or M-type dwarf has left the main sequence yet!

Preview - The solar atmosphere

Interactive exploring — helioviewer

- web-based version: https:// helioviewer.org/

- downloadable application: https://www.jhelioviewer.org/ (has more functions)