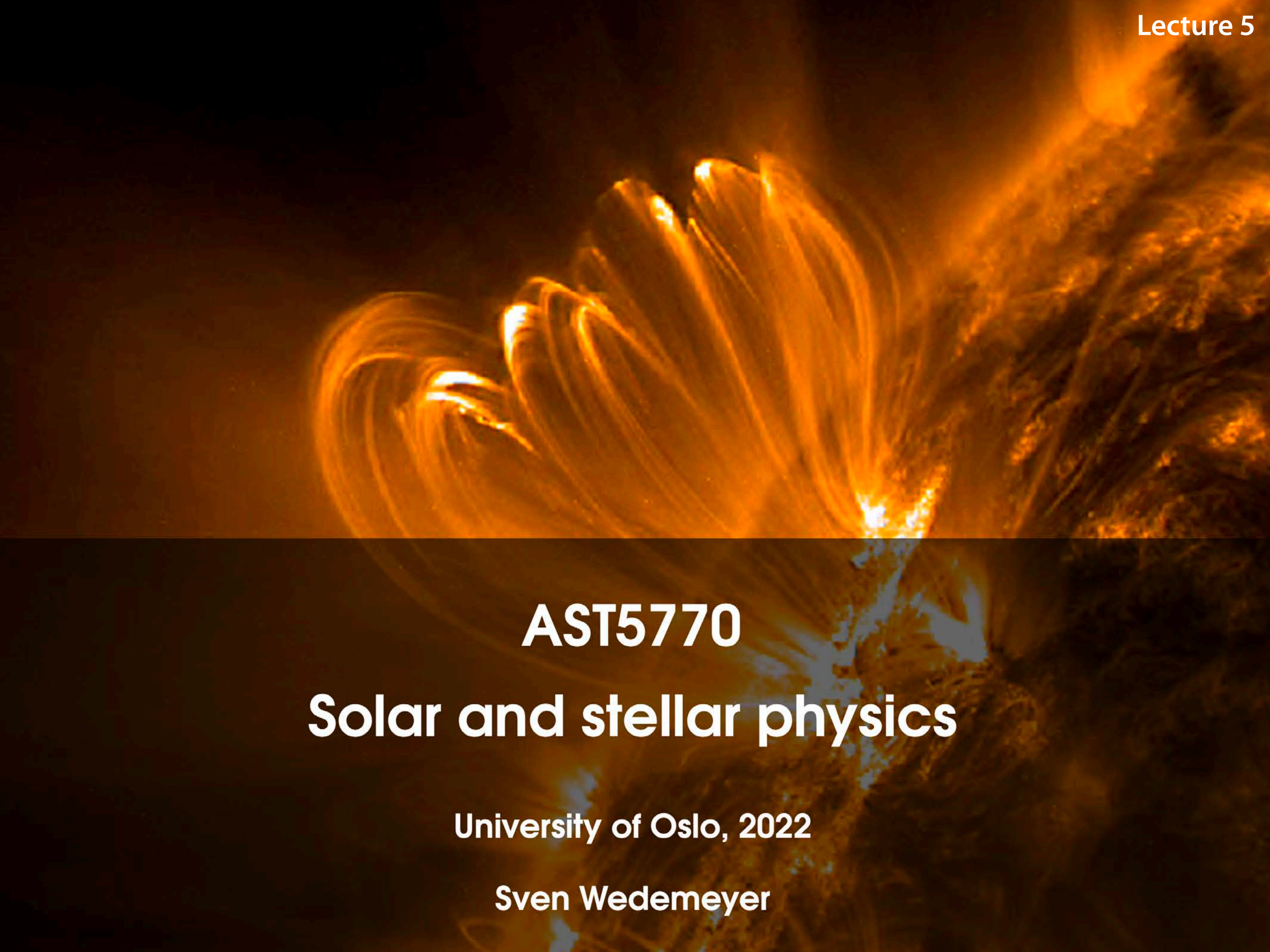


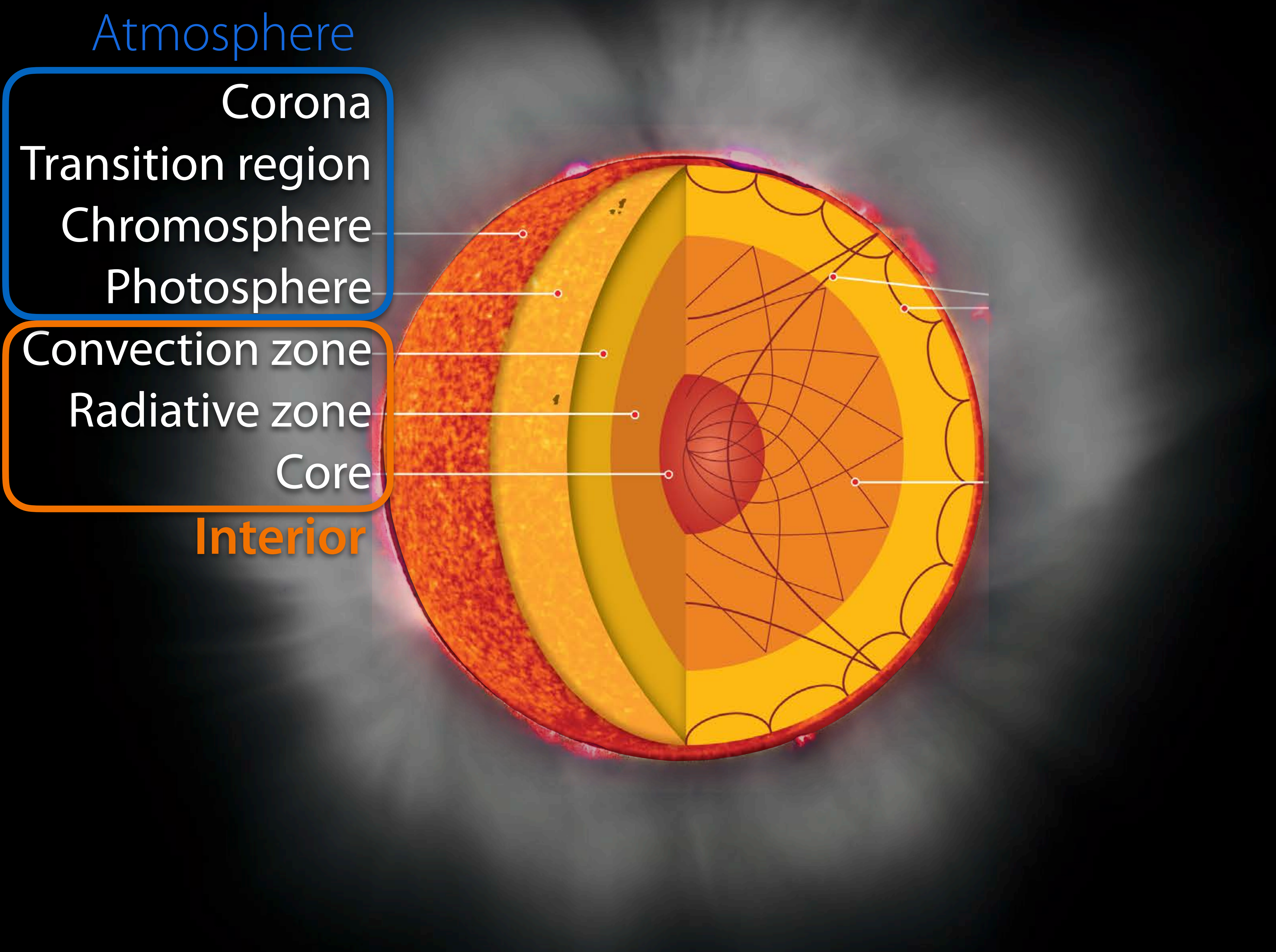
AST5770

Solar and stellar physics

University of Oslo, 2022

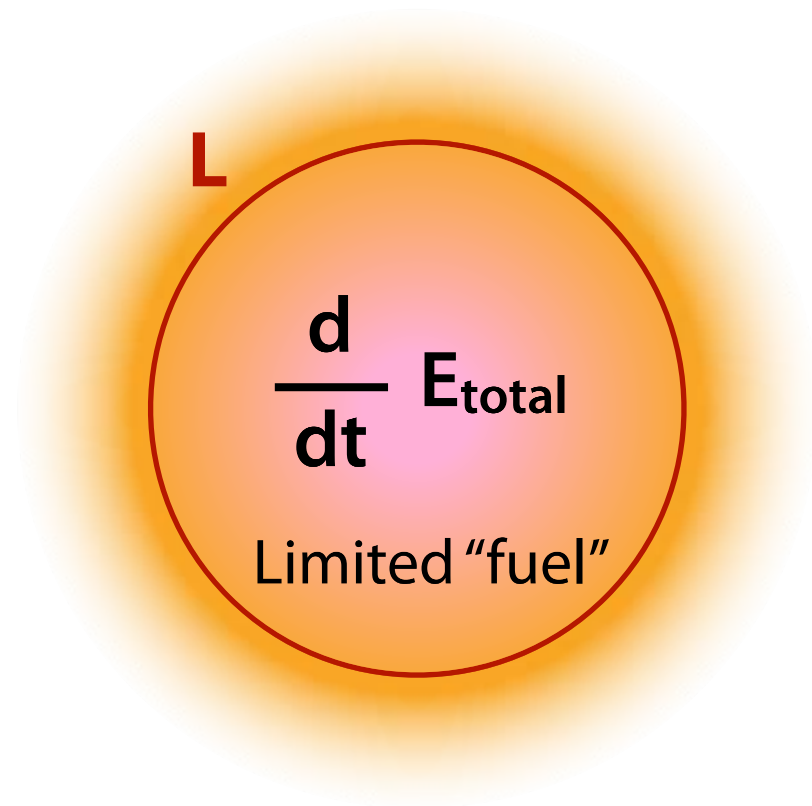
Sven Wedemeyer

Stellar structure — The Sun



Stellar interior

Overview



- Stars radiate (luminosity)
  - **Conservation of energy** (whole system)
  - ➡ Total energy of a star decreases with time
  - ➡ How is the required energy set free?
  - ➡ How is the energy transported to the surface?

- The physical properties of interior structure determined by
  - Temperature T
  - Pressure P
  - Chemical composition μ

  Equation of state

  Opacity

- Gradients d/dr decisive for energy transport and in return the stratification

- Stellar structure due to **balance** and will change as function of time

Energy reservoirs

Overview

- There are different **energy reservoirs** available:
  - Potential energy E_p (all mass elements of a star).
  - Thermal energy E_t (kinetic energy of all particles)
  - Nuclear energy E_n (set free by nuclear reactions, e.g., fusion)
  - *Chemical energy (set free during chemical reactions, atoms combing, typically insignificant under stellar conditions)*

Remember

- Dynamical time scale t_d
- Thermal time scale t_t
- Nuclear time scale t_n

- **Total energy** $E_{total} = E_n + E_t + E_p$

➡ Decreases as a star emits energy as radiation at the surface.

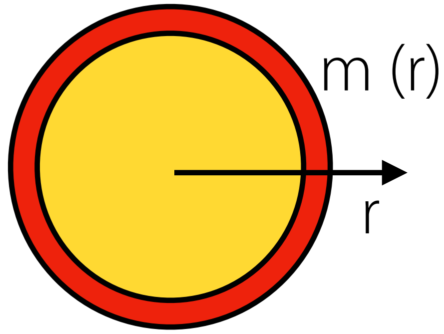
➡ Resulting luminosity of a star = temporal change of the star's energy content:

$$L = -\frac{d}{dt} E_{total}$$

Energy reservoirs

Gravitational potential

- **Gravitational potential energy** for a system of two particles: $U = -\frac{G m_1 m_2}{r}$
- **Contraction**: masses closer together
  ➡ potential energy becomes more negative.



- Instead now integration over thin shells of thickness dr

$$U = -G \int_0^R \frac{m(r) 4\pi r^2 \rho}{r} dr \quad = -\frac{3GM^2}{5R}$$

- Apply **virial theorem:**
  total energy =1/2 (time-averaged) potential energy →

$$E_{total} = \frac{3GM^2}{10R}$$

- **Example: Contraction of the Sun at current luminosity**

$$t \approx \frac{E_{total}}{L_\odot} \approx \frac{1.1\ 10^{41}\ \text{J}}{3.84\ 10^{26}\ \text{W}} \approx 2.9\ 10^{14}\ \text{s} \approx 9\ 10^6\ \text{yr}$$

(Very simplified estimate!)

- Proposed as energy source for the Sun by Kelvin and Helmholtz in late 19th century *(before fusion was known)*
- **Dismissed** due to clear geological and biological indications of much higher age

Energy reservoirs

Chemical energy

- Chemical reactions: based on interactions of orbital electrons in atoms.
  - Typical energy differences between atomic orbitals ΔE ~10 eV.

- **Example:** Assume the Sun is pure hydrogen

➡ Total number of H atoms in the Sun: $n = \frac{M_{Sun}}{m_H} = \frac{1.99 \times 10^{30}}{1.67 \times 10^{-27}} = 1.19 \times 10^{57}$

- Assume all atoms release ΔE ~ 10 eV each due to chemical reactions

$$L = - \frac{d}{dt} E_{total}$$

➡ Total released chemical energy: $\approx 10^{58}\, eV = 10^{39}\, J$

➡ **How long could the Sun radiate at current luminosity?**

$$t \approx \frac{E_{chem}}{L_\odot} \approx \frac{10^{39}\ J}{3.84\ 10^{26}\ W} \approx 10^5\ yr$$

➡ Purely based on chemical energy: only 100,000 years
(~100 times less than for the gravitational potential energy)

➡ Obviously not a viable energy source.

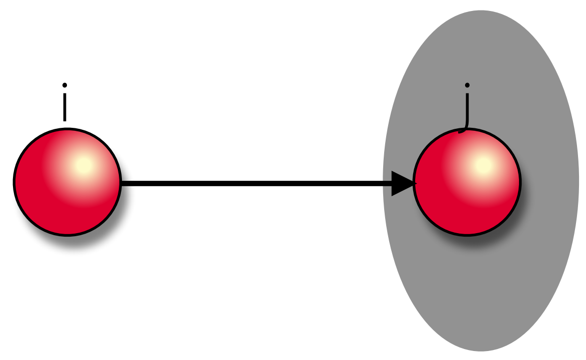
Energy reservoirs

Nuclear energy

- **Wanted: Energy production rate as function of temperature, density, chemical composition**
- Needed:
  1. What is the probability of a certain type of reaction to occur?

     ➡ Cross-section and number densities of reactants!

     ➡ Reaction rates
  2. How much energy is released in each reaction?



- Energy production rate must be related to the rate $\mathbf{r_{i,j}}$ of collisions between two particles (i,j)
  - Amount of energy released per unit mass if each reaction releases an energy $\mathbf{\Lambda}$:

$$\varepsilon_{ij} = \left(\frac{\Lambda}{\rho}\right) r_{ij}$$

ρ: mass density

Energy reservoirs

Nuclear energy — Reaction rates

- Rate proportional to number of i-j pairs in the volume.

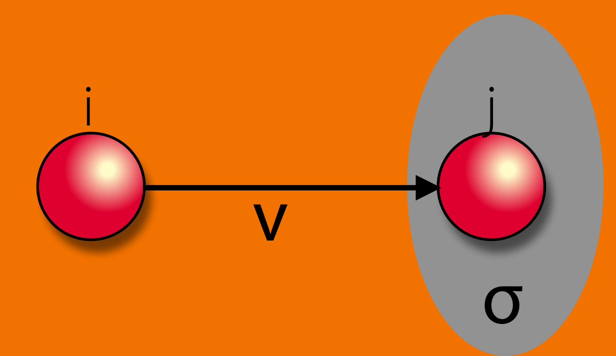
➡ Reaction rate per second and cm³ :

$$r_{ij} = \sigma \, v \, n_i \, n_j$$

v: relative velocity between particles
σ: cross-section

- Interaction between particles of same species: divide rate by 2:

$$r_{ij} = \frac{1}{1+\delta_{ij}} \, n_i \, n_j \, \sigma \, v = \frac{1}{1+\delta_{ij}} \, \frac{X_i \, \rho}{A_i \, m_u} \, \frac{X_j \, \rho}{A_j \, m_u} \, \sigma \, v$$



In this context wanted:
$\langle \sigma \, v \rangle$

- **Number density n_i:** number of particles per cm³

$$n_i = \frac{N_i}{V} = \frac{X_i \, \rho}{m_i}$$

$$\mathbf{m_i} \approx A_i \cdot m_u$$

- Abundance of species I: $Y_i = \frac{X_i}{A_i}$

Particles=nuclei of species i
N_i: Absolute number of particles of species I
V: Volume
ρ : mass density (g/cm3)
m_i: mass of nucleus of species i
X_i : fraction of total mass of gas consisting of particle type i
A_i : atomic weight
m_u: atomic mass unit

Use cgs units…

Energy reservoirs

Nuclear energy — recap: fusion

- **Coulomb barrier** due to electric charges of particles

$$U_C = \frac{1}{4\pi\varepsilon_0}\frac{Z_i Z_j e^2}{d}$$

(Coulomb potential energy)

- Particle energy needed for reaction to become possible:

$$E = 3/2\, kT \geq U_c$$

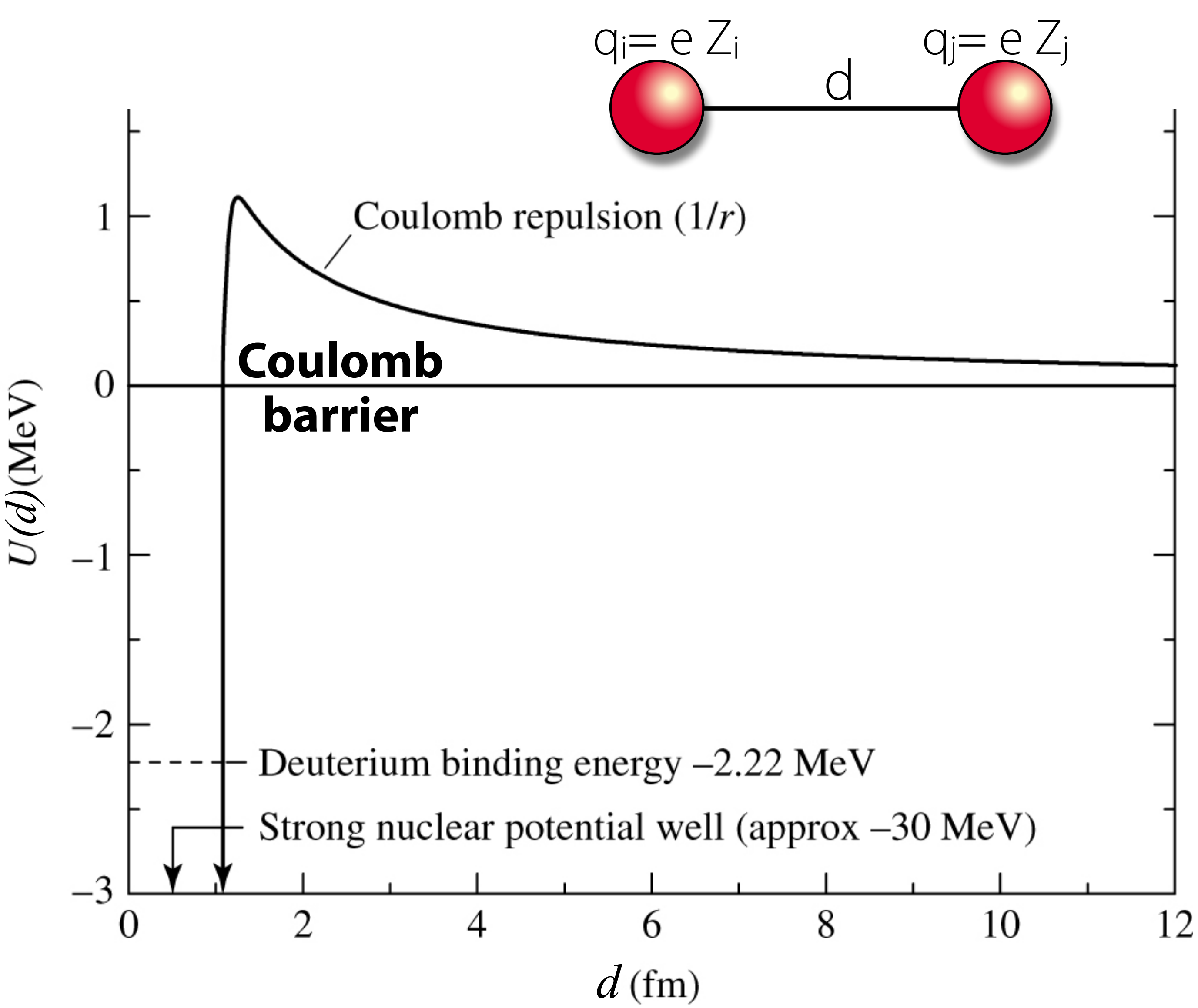
- Equivalently: Required particle velocity :

$$v >> \sqrt{3kT/(2m)}$$

Mean thermal velocity

- Quantummechanical **tunneling** "through" Coulomb barrier!

p-p interaction



Energy reservoirs

Nuclear energy — recap: fusion

- **Coulomb barrier** due to electric charges

$$U_C = \frac{1}{4\pi\varepsilon_0} \frac{Z_i Z_j e^2}{d}$$

(Coulomb potential energy)

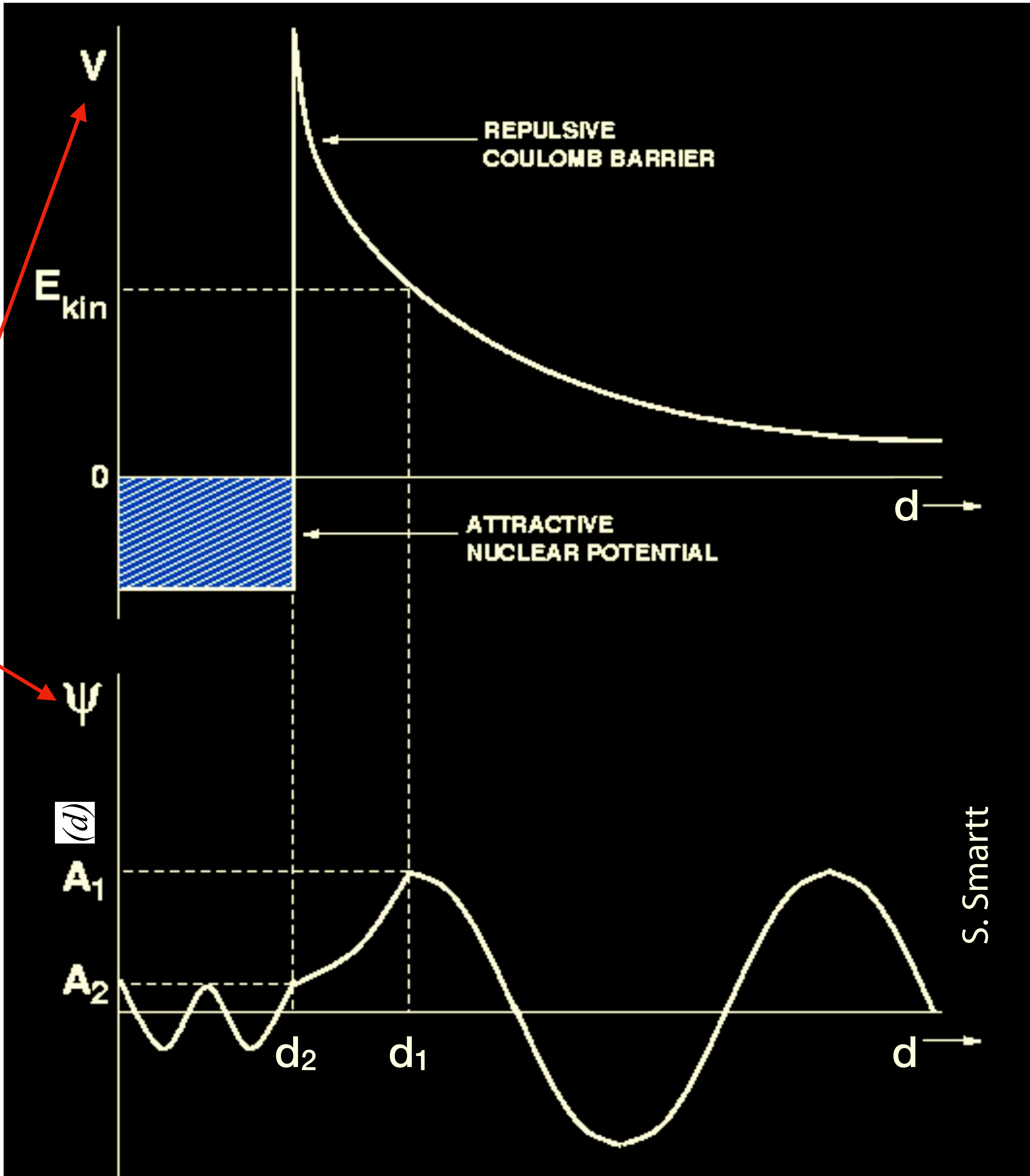
Penetration probability $\propto e^{\frac{-\pi Z_i Z_j e^2}{\varepsilon_0 h v}}$

(Gamow)

- Quantummechanical **tunneling** "through" Coulomb barrier!

***V* (potential energy)**

Wave function



Energy reservoirs

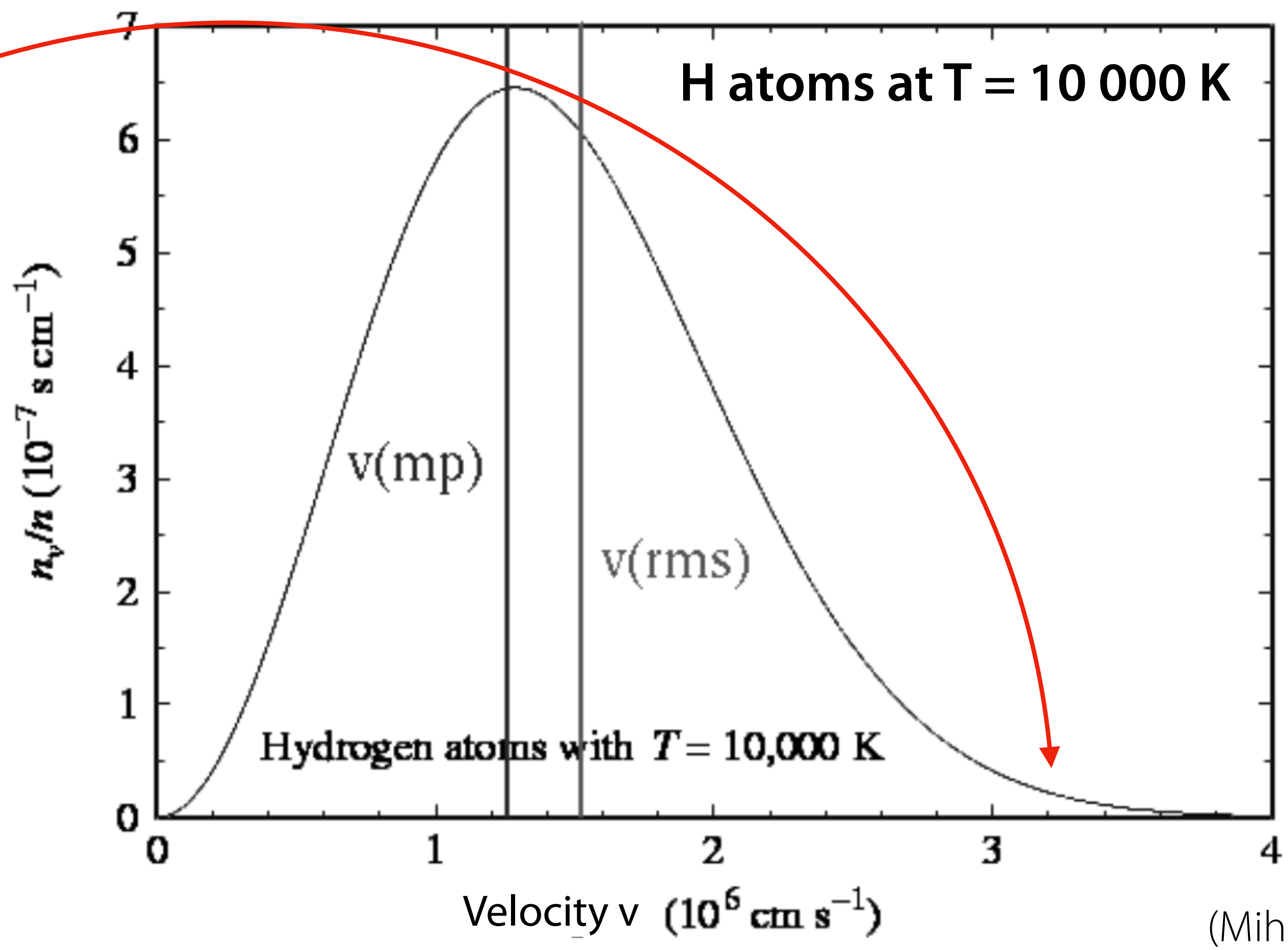
Nuclear energy — recap: fusion

- Distribution of particle velocities: **Maxwellian** velocity distribution

$$P(v)\,dv = 4\pi\left[\frac{\mu}{2\pi kT}\right]^{3/2} e^{-m\,v^2/(2kT)}\,v^2\,dv$$

reduced mass $\mu = m_1 m_2/(m_1 + m_2)$

- **Note the high-velocity tail!**



(Mihos)

Energy reservoirs

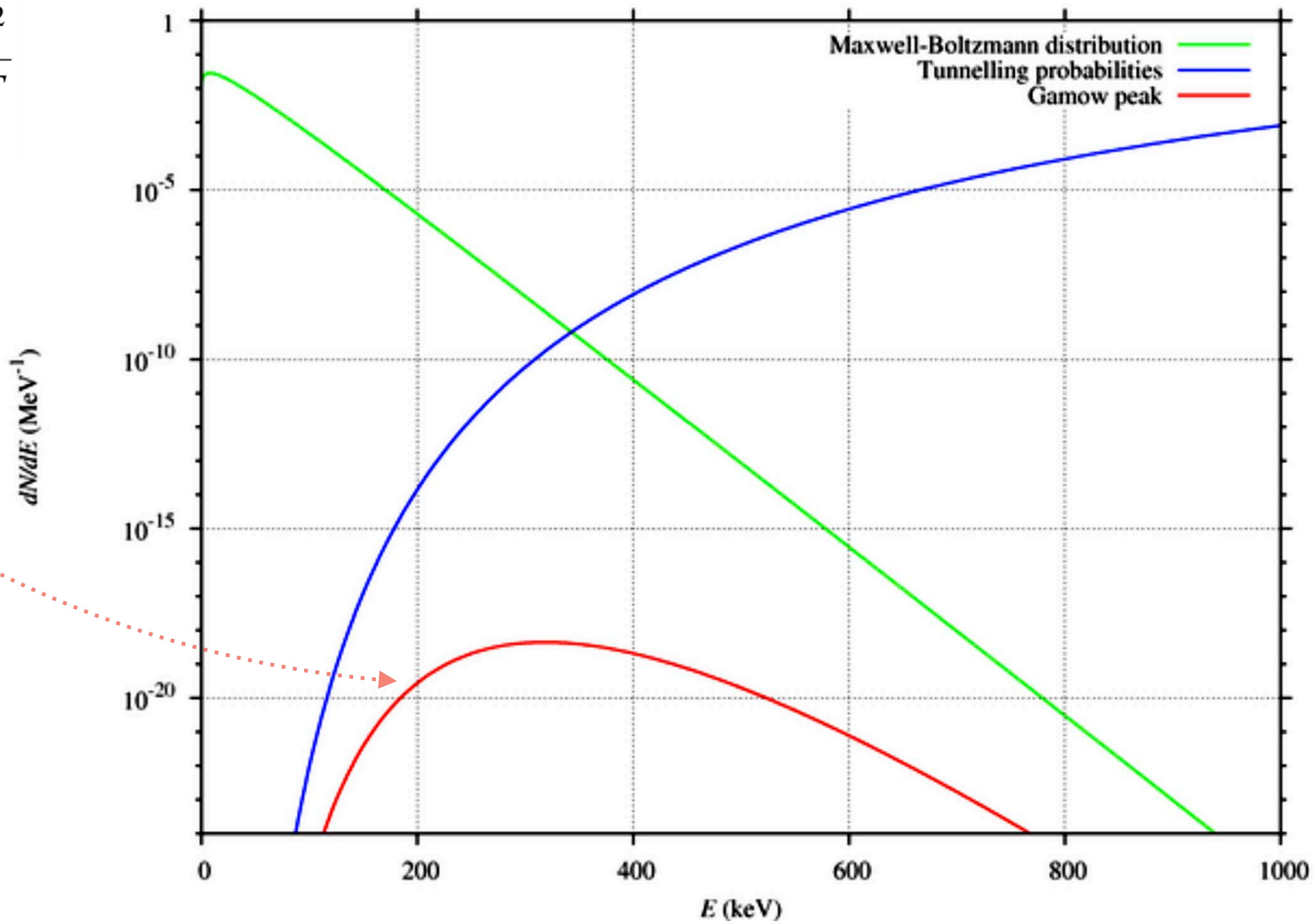
Nuclear energy — recap: fusion

- Combining tail of velocity distribution and tunnelling probability

➡ Probability that this reaction occurs

$$\propto e^{\frac{-\pi Z_i Z_j e^2}{\varepsilon_0 h v}} e^{-\frac{mv^2}{2kT}}$$

➡ Gamow peak



Energy reservoirs

Nuclear energy — Reaction rates

- **Conclusion**: Reaction rates depend on
  - Number of reactants (expressed as **chemical composition and mass density**)
  - **Temperature** (via cross-section, reaction probability)

- For simplicity: Approximate reaction rates as **power laws**

$$r_{ij} \approx r_0 X_i X_j \rho^{1+\alpha} T^{\beta}$$

$$\Rightarrow \varepsilon_{ij} = \left(\frac{\Lambda}{\rho}\right) r_{ij} \approx \varepsilon_0 X_i X_j \rho^{\alpha} T^{\beta}$$

- Parameter $\boldsymbol{\alpha}$: Depends on details of reaction (how many reactants are involved)
  - For two-body interactions (i.e. p+p collisions), $\alpha \sim 1$
- Parameter $\boldsymbol{\beta}$: Temperature dependence

➡ Fusion reactions known to very sensitively dependent on temperature

➡ β can have a wide range of values

Energy reservoirs

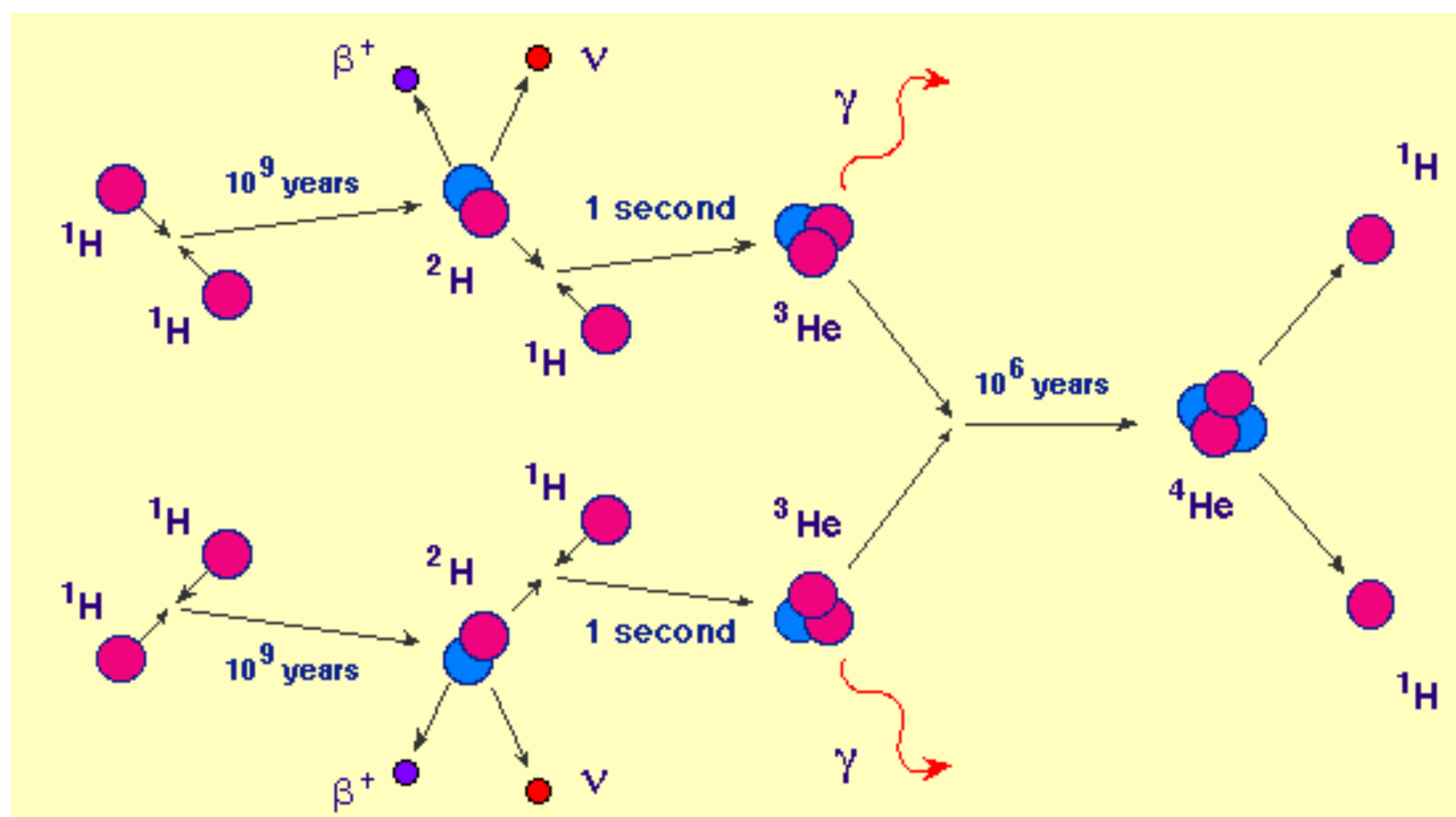
Nuclear energy — Hydrogen burning

- Different fusion reactions with different chains, branches, steps
- Individual reactions reaction rates (and a reaction being relevant) depends on the local plasma properties such as temperature, density, and abundance of the required reactants
  - Will change as function of radius from centre to surface

- **Example:** Hydrogen burning in the Sun (and solar-like stars):
  - Mostly pp chain with three different branches with different relative occurrence

- **PP1 in the Sun:**

Step 1	${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \upsilon_e$	10^9 yr
Step 2	${}^2_1H + {}^1_1H \rightarrow {}^3_2He + \gamma$	~1s
Step 3	${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2\,{}^1_1H$	10^6 yr



Energy reservoirs

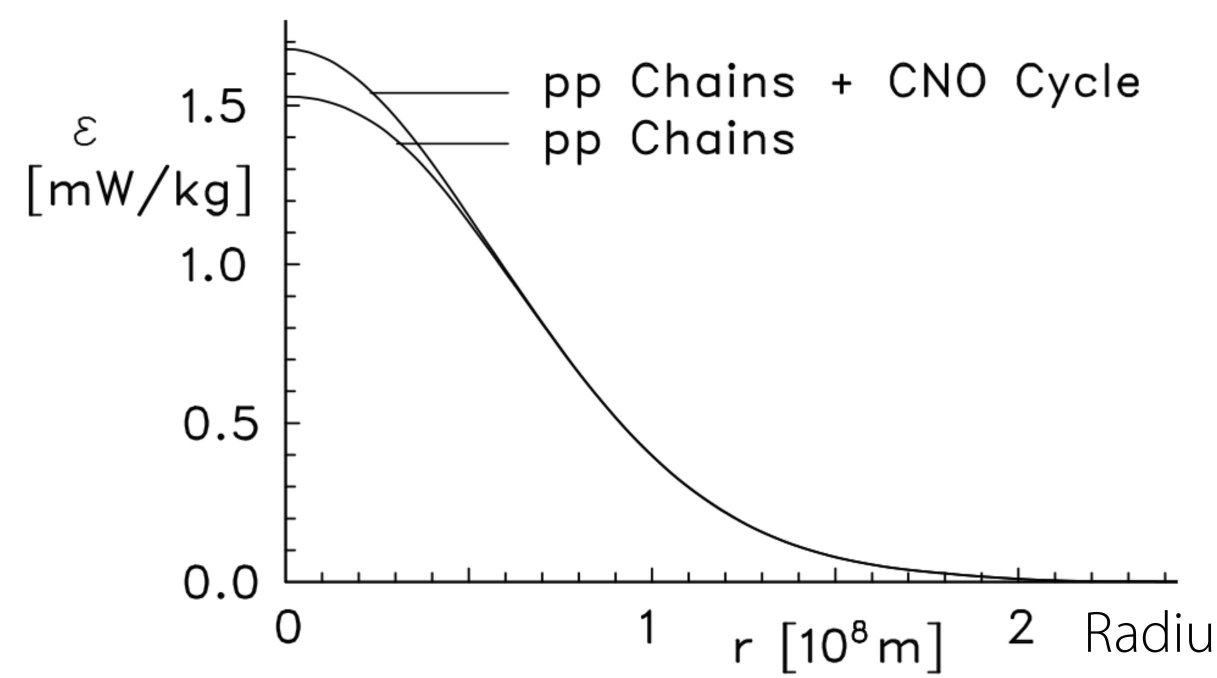
Nuclear energy — Hydrogen burning

PP1	PP2	PP3	CNO
${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \upsilon_e$ ${}^2_1H + {}^1_1H \rightarrow {}^3_2He + \gamma$ ${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1H$	${}^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$ ${}^7_4Be + e^- \rightarrow {}^7_3Li + \upsilon_e$ ${}^7_3Li + {}^1_1H \rightarrow 2{}^4_2He$	${}^7_4Be + {}^1_1H \rightarrow {}^8_5B + \gamma$ ${}^8_5B \rightarrow {}^8_4Be + e^+ + \upsilon_e$ ${}^8_4Be \rightarrow 2{}^4_2He$	${}^{12}_6C + {}^1_1H \rightarrow {}^{13}_7N + \gamma$ ${}^{13}_7N \rightarrow {}^{13}_6C + e^+ + \nu_e$ ${}^{13}_6C + {}^1_1H \rightarrow {}^{14}_7N + \gamma$ ${}^{14}_7N + {}^1_1H \rightarrow {}^{15}_8O + \gamma$ ${}^{15}_8O \rightarrow {}^{15}_7N + e^+ + \upsilon_e$ ${}^{15}_7N + {}^1_1H \rightarrow {}^{12}_6C + {}^4_2He$
69 %	31 %	<0.3%	
T < 1.4 10^7 K	T > 1.4 10^7 K	T > 3 10^7 K	

Relative occurrence in the Sun. **PP1 dominates in the current Sun.**

Temperature at which this branch dominates

- Overall nuclear energy production rate ε, either in total or by fusion chain derived as sum over all involved reactions



Stix

Energy reservoirs

Nuclear energy

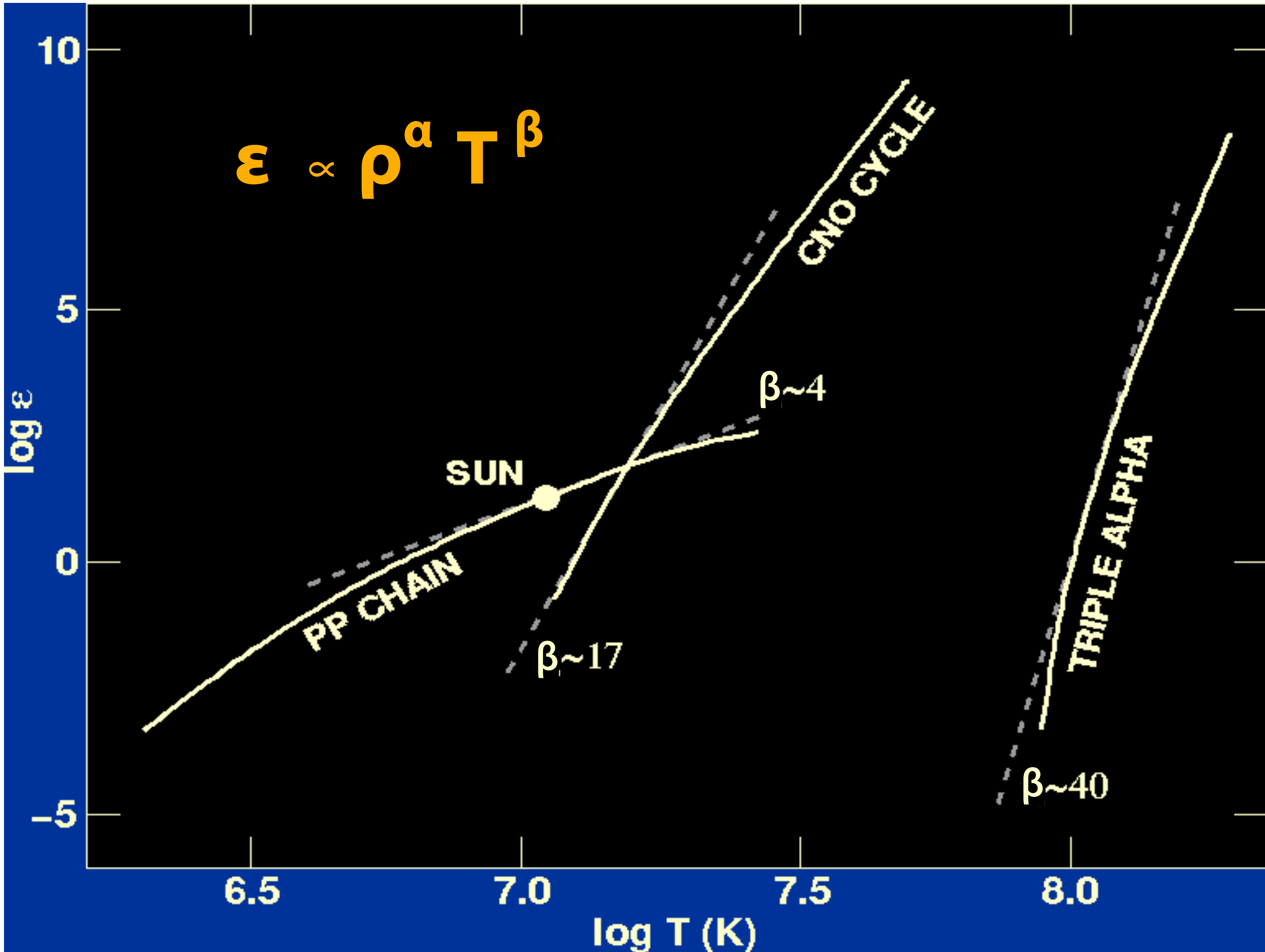
	H-burning		He-burning
	pp chain	**CNO cycle**	**Triple-α**
Energy released per complete reaction chain	26.0 MeV plus ~0.73 MeV as neutrinos	25 MeV	7.3 MeV
Overall contribution in the current Sun	99 %	1 %	~0
Energy production rate	$\varepsilon \propto \rho T^{4}$	$\varepsilon \propto \rho T^{17}$	$\varepsilon \propto \rho T^{40}$

$$^{4}He + ^{4}He \rightarrow ^{8}Be$$
$$^{8}Be + ^{4}He \rightarrow ^{12}C + \gamma$$

Energy reservoirs

Nuclear energy production rates

- **Nuclear energy production rate depends sensitively on temperature (increasing β)**
- Fusion reactions involve successively heavier elements
  - in ascending order: the PP chain, the CNO cycle and the triple-alpha reaction)
- Higher fusion reactions become more temperature dependent and require **higher temperatures** to operate *(larger Coulomb barrier to overcome for heavier, more positively charged nuclei)*



- Dependence on density
  - linear ($\alpha=1$) for two-particle reactions (pp chain, CNO cycle …)
  - quadratic ($\alpha=2$) for three-particle reactions (e.g., triple-alpha process)

Energy reservoirs

Higher burning stages

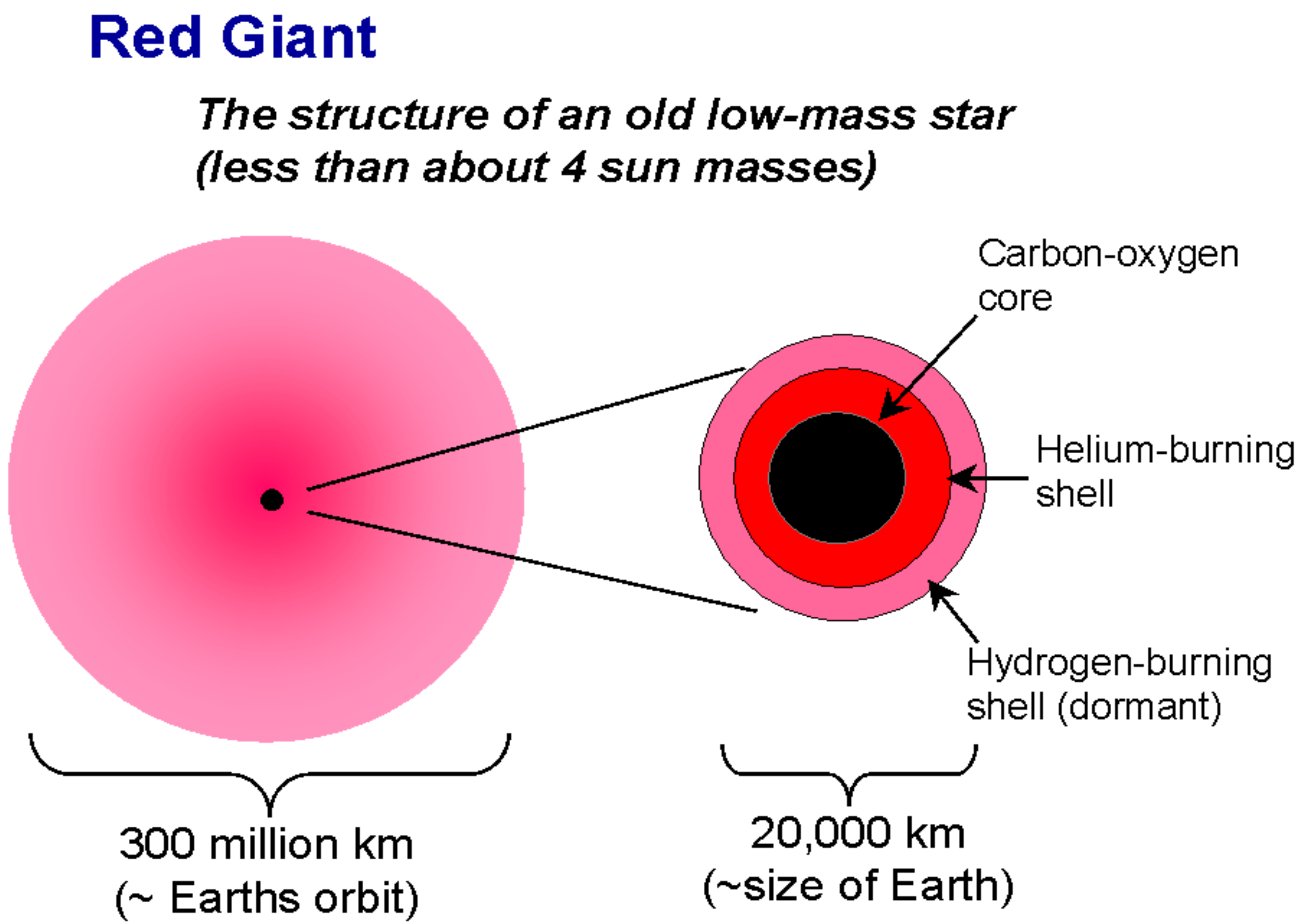
- Further burning stages require higher central temperature
- Achieved at progressively larger stellar masses (e.g. Carbon burning needs M > 4MS)
- Examples of higher burning stages:

At $T \sim 6\ 10^8$ K	$O_8^{16} + He_2^4 \rightarrow Ne_{10}^{20}$ $Ne_{10}^{20} + He_2^4 \rightarrow Mg24$ $Mg^{24} + He_2^4 \rightarrow Si_{14}^{28}$	4.7 MeV 9.3 MeV 10.0 MeV
At $T \sim 10^9$ K	$C_{12} + C_{12} \rightarrow Mg^{24}$ $O_{16} + O_{16} \rightarrow S_{32}$ $Mg^{24} + S_{32} \rightarrow Fe_{56}$	14 MeV 16 MeV END OF FUSION

Energy reservoirs

Higher burning stages

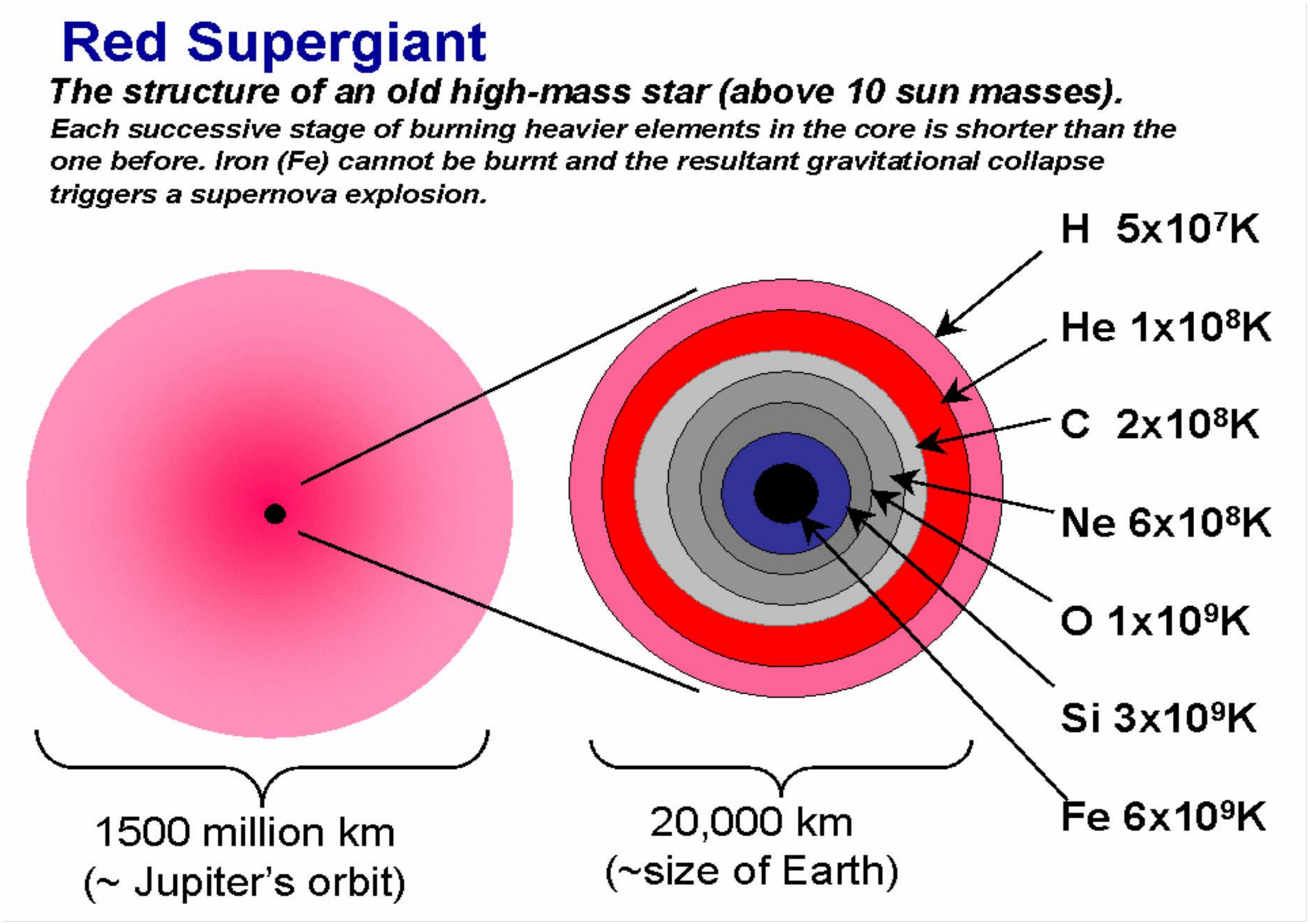
- Shell burning



Energy reservoirs

Higher burning stages

- Shell burning



Energy transport

"Follow the energy"

- **Energy is conserved but will be converted between different forms.**
  - **Source:** nuclear fusion as the by far dominant source (for the Sun: in the core)
  - **Transport** outwards
  - **Leaves** the star at the "surface", mostly in form of radiation

- **Energy transport mechanisms:**
  - **Radiation**: Photons carry energy as propagate through the star (emission/absorption).
  - **Convection**: Net rise of buoyant (hot) gas towards surface.
  - **Conduction**: Transfer of kinetic energy between gas particles during collisions

  - The efficiency / contribution of the different mechanisms depends on the local plasma conditions (such as density, opacity)

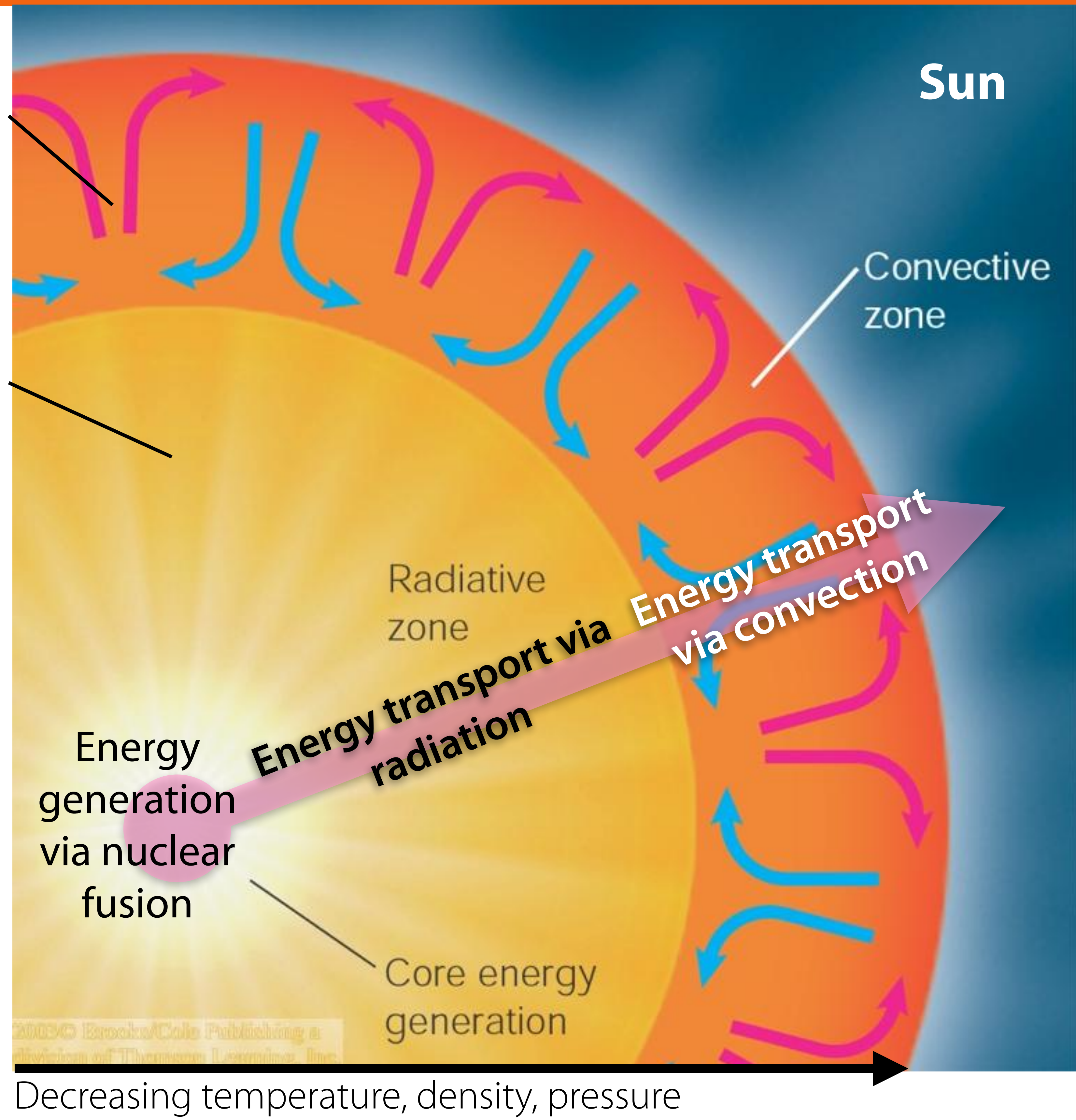
  - Conduction not important in the solar interior but in the corona!

Energy transport

In the Sun and solar-like stars

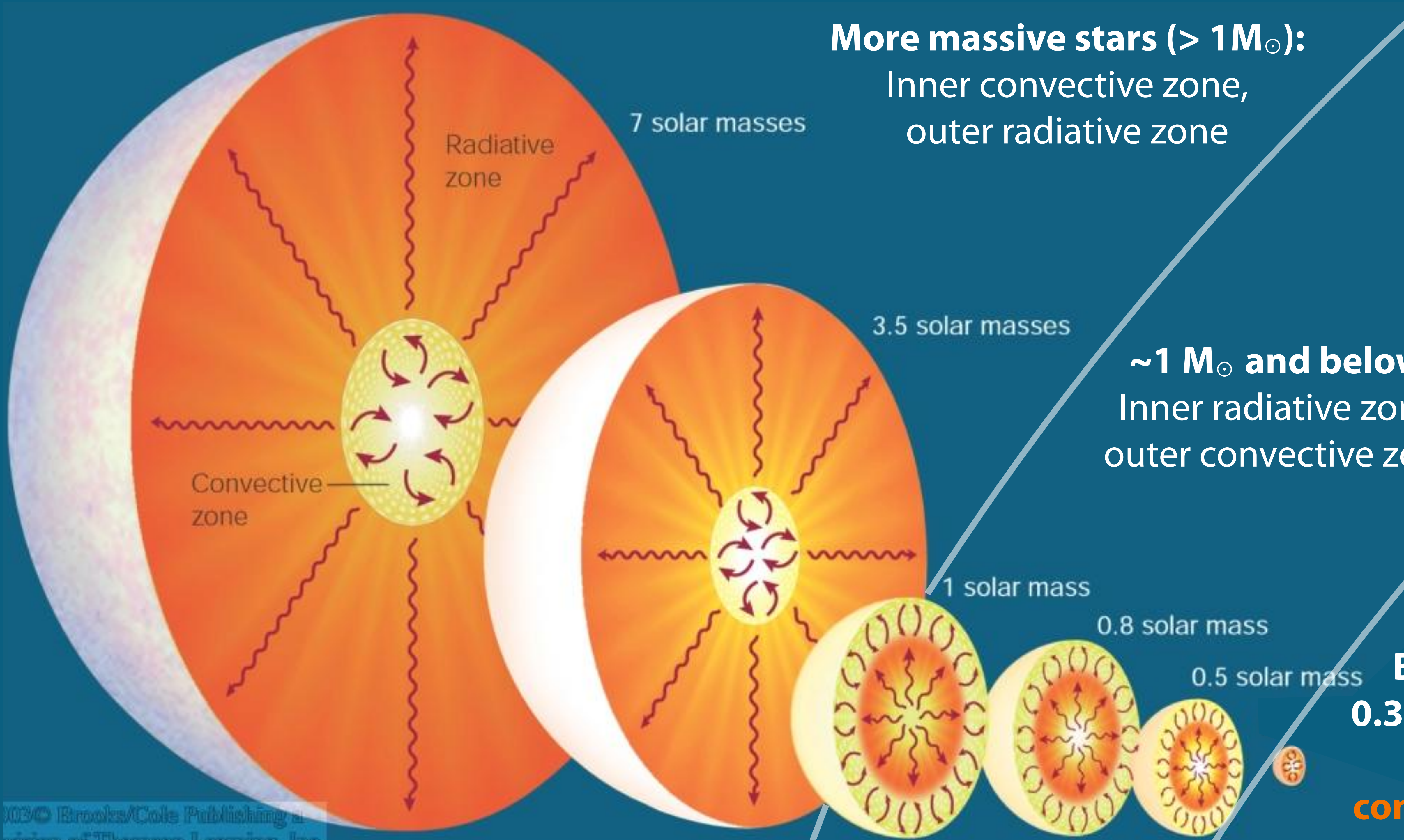
- At any depth in the Sun:
  - Energy flux **F** defined as the luminosity per unit area.
  - Energy transport by radiation (F_R) and by convection (F_C)
  - Conduction insignificant in solar interior

$$F = F_{\mathrm{R}} + F_{\mathrm{C}} = L/4\pi r^2$$



Energy transport

Along the main sequence



More massive stars (> 1M$_\odot$):
Inner convective zone,
outer radiative zone

~1 M$_\odot$ and below:
Inner radiative zone,
outer convective zone

Below 0.3-0.4 M$_\odot$:
Fully convective

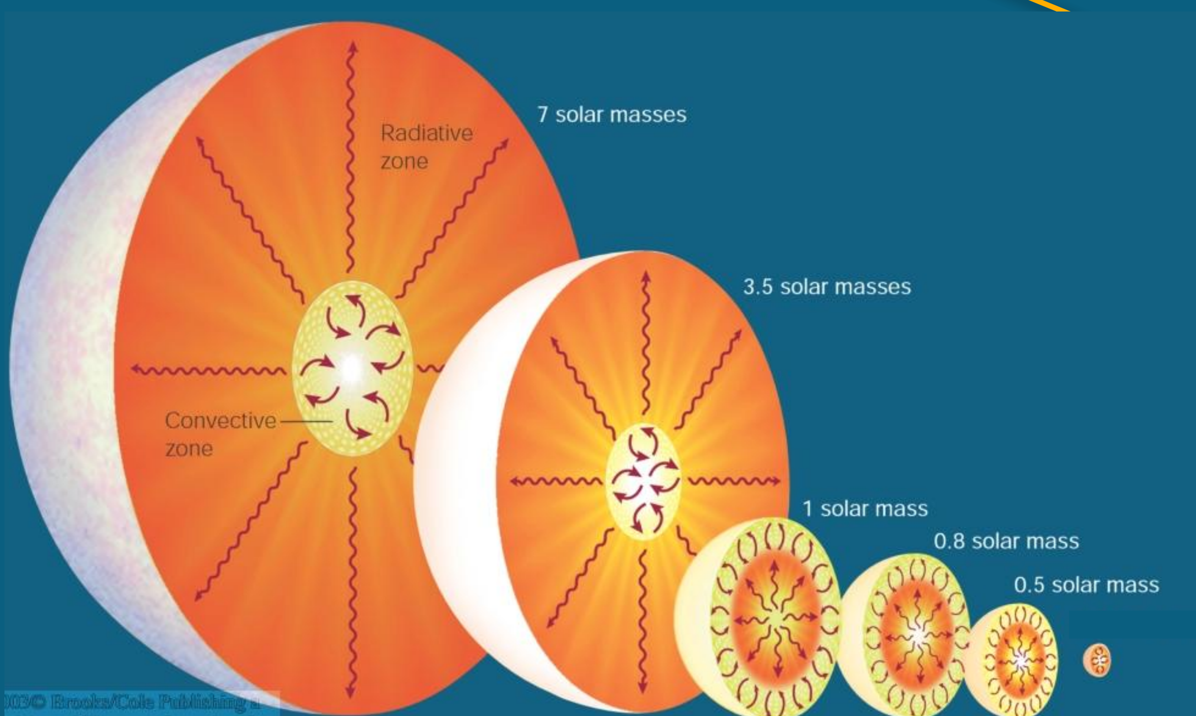
Dominant fusion reaction in stellar core	CNO cycle	pp chain

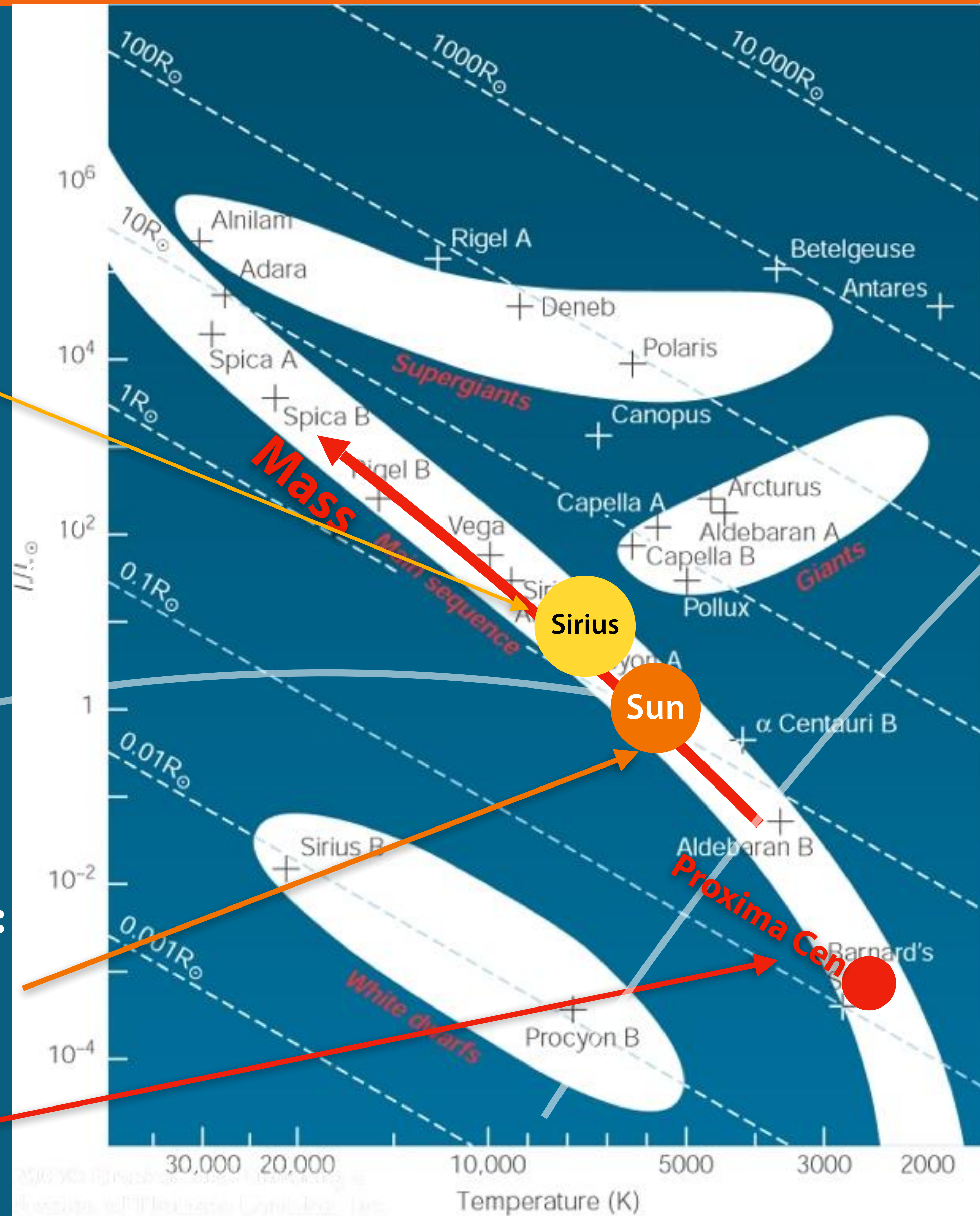
Energy transport

Along the main sequence

Early-type main sequence (> 1M$_\odot$)

Inner convective zone,
outer radiative zone





Late-type main sequence(~1M$_\odot$ — 0.4 M$_\odot$):

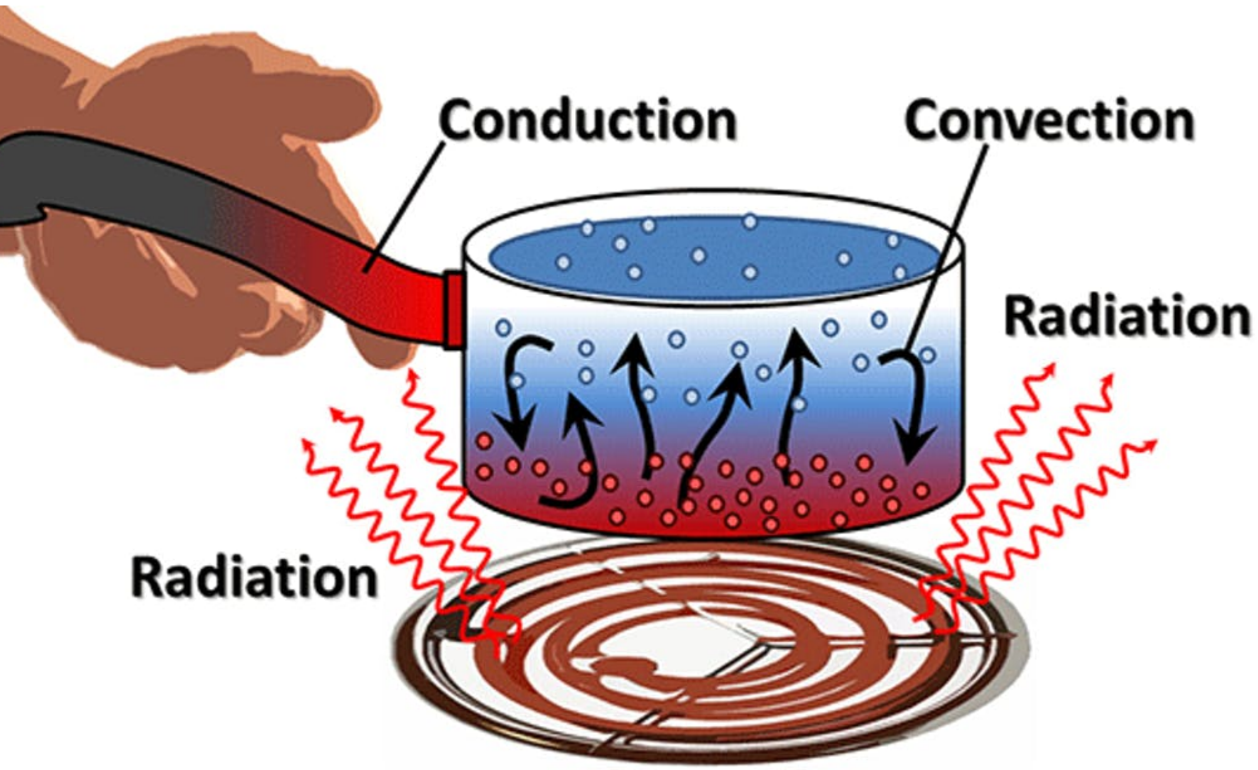
Inner radiative zone, outer convective zone

(Very) low-mass stars (M<0.3-0.4 M$_\odot$:

Fully convective interior

Energy transport

Overview



- **Radiation**: Photons carry energy, travels a distance between emission and being absorbed again
- **Conduction**: Particles carry energy, travels a distance between collisions with other particles (during which energy is exchanged)
- **Convection**: mass motion of elements of gas

- **Conditions** for the occurrence of the different modes of energy transport:
  - Conduction and radiation: whenever a temperature gradient is maintained.
  - Convection: only if the temperature gradient exceeds a critical value.