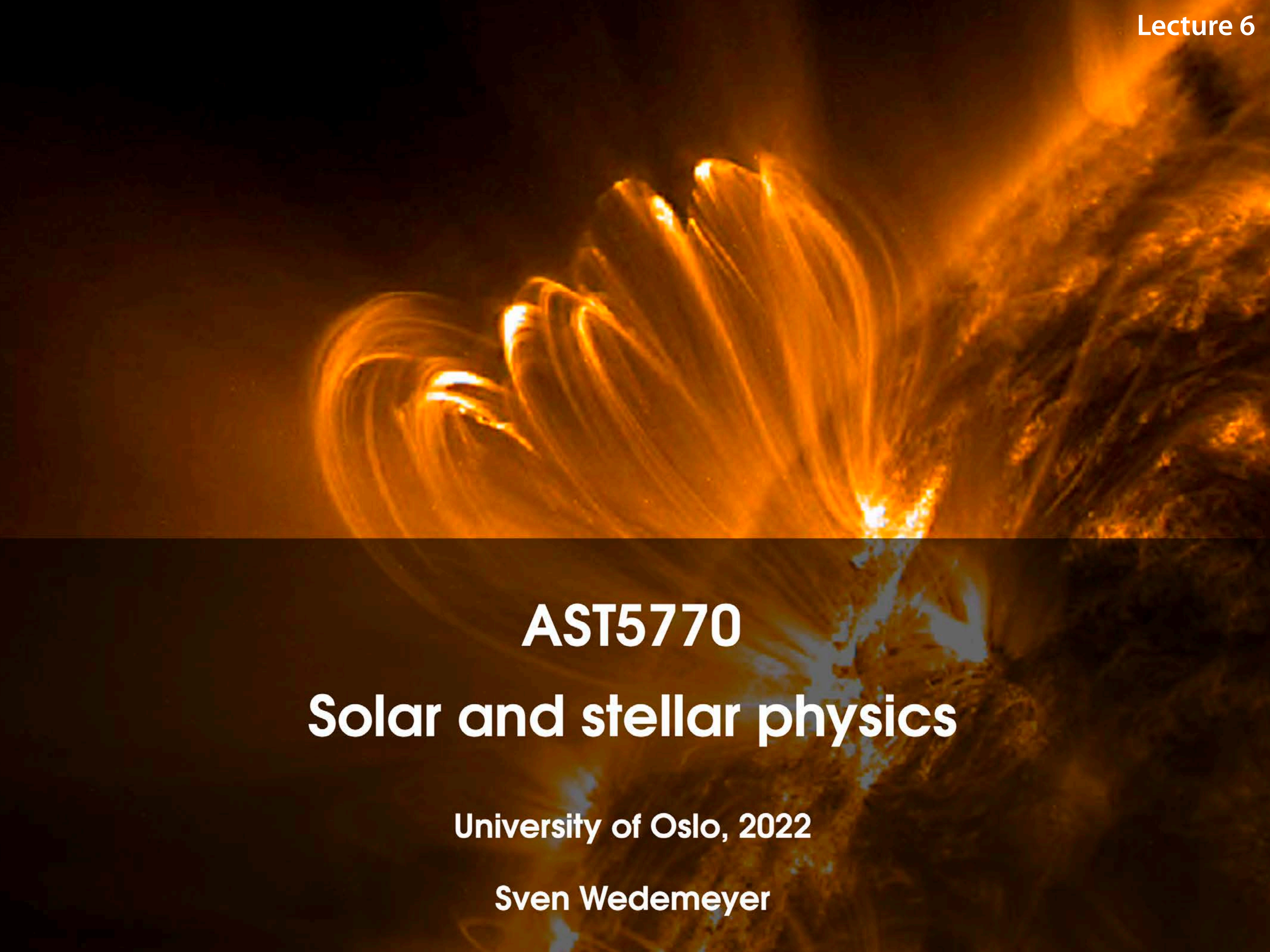


# AST5770

# Solar and stellar physics

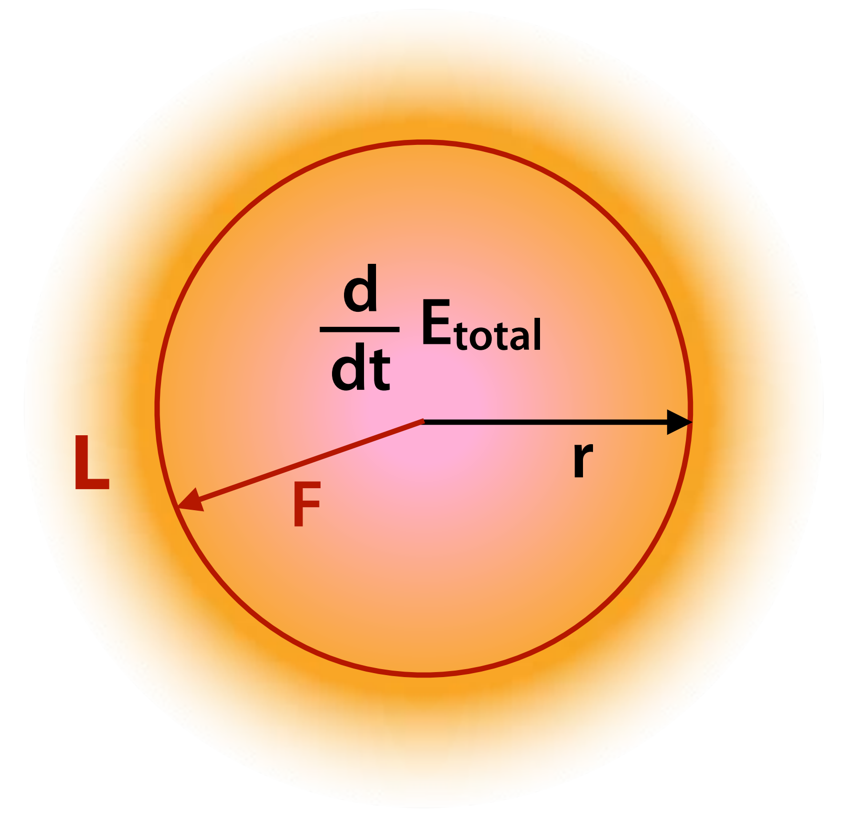
University of Oslo, 2022

Sven Wedemeyer

# Energy transport

## Overview

- **Energy flux** (outward) $F = F_\mathrm{R} + F_\mathrm{C} = L/4\pi r^2$
  - Conduction is negligible in the Sun
  - The contributions of convection and radiation change as function of radius
  - Luminosity: energy flux arriving and being emitted from the surface at radius r
  - Flux driven by a temperature gradient

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- Note: **Neutrinos** produced by fusion carry energy
  - Carry comparatively little energy and thus neglected here (strictly speaking, the total energy production due to all sources balances the luminosity and neutrino flux)
  - Neutrinos escape from normal stars essentially without interaction with matter (but that is no longer true for very dense stars/ stellar remnants)
  - Strictly speaking: Energy production rate $\varepsilon_\nu$ for neutrinos should be taken into account

# Energy transport

## Reminder

- **Diffusion:** A general concept, time-dependent: **Net transport** of particles or energy
  - Driven by a corresponding gradient — towards equilibrium
  - Random microscopic motion

➡ (net) energy flux: $\boldsymbol{F} = -D\,\boldsymbol{\nabla} U,$

U: Energy density
D: Diffusion coefficient

- Gradient in energy density connected to temperature gradient:

$$\nabla U = (\partial U/\partial T)_V \, \nabla T = C_V \, \nabla T$$

➡ Heat conduction: $\boldsymbol{F} = -K\,\nabla T$ with $K = \frac{1}{3}\bar{v}\ell C_V$

$\bar{v}$: avg. velocity
$l$: mean free path
$C_V$: specific heat capacity per constant volume
$K$: conductivity

➡ valid for all particles in LTE, including gas particles but also photons

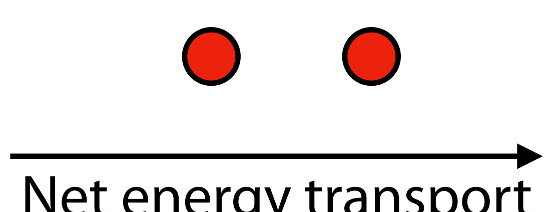
# Energy transport

## Reminder

### Diffusion:

- A general concept, time-dependent: **Net transport** of particles or energy
- Driven by a corresponding gradient — towards equilibrium
- **Random microscopic motion**

- Diffusion of particles: **Conduction** Particles pass on their internal (kinetic/ potential) energy to neighbouring particles without moving over large distances
- **Radiative diffusion** via photons

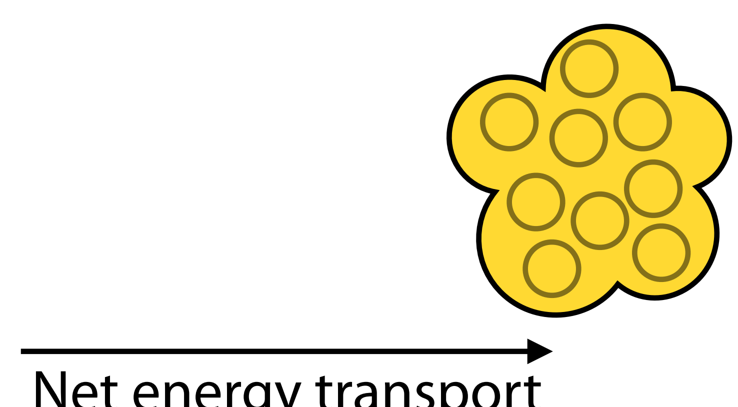



### Advection:

- Particles move over longer distances (and transport heat); e.g. as part of a fluid with macroscopic (large-scale) motion

- **Macroscopic (bulk) motion** (particles/mass)

- Convection with macroscopic motion



# Energy transport

## Diffusive energy transport in stellar interiors

| **Conduction** | **vs** | **Radiation** |
|---|---|---|
| **Gas particles (electrons)** | | **Photons** |
| **Energy** carried by a typical particle: $E = {}^3/_2\, k\, T$ | **Comparable** | **Energy** carried by a typical photon: $E = h\, c\, /\, \lambda$ |
| Number density of particles | **>>** | Number density of photons. |
| Mean free path (between collisions) Typically $10^{-10}$ m | **<<** | Mean free path before being absorbed or scattered Typically $10^{-2}$ m |

- Smaller number of **photons** is far outweighed by their **much larger mean free path**!

➡ Photons get easier from location with high temperature to one with lower temperature

➡ Larger transport of energy

➡ Radiation is the dominant energy transport mechanism in most stars.

➡ **Conduction negligible in the interiors** of (nearly all) main sequence stars.

⊙ **Conduction relevant in the solar corona**!

# Energy transport

## Radiative energy transport

- Mean free path of a photon very small in interior of stars

➡ Location where photon is emitted and location where photon absorbed have nearly same temperature

➡ Conditions of local thermodynamic equilibrium fulfilled

➡ Source function = Kirchhoff–Planck function

- **Radiative energy diffusion**

➡ $\boldsymbol{F} = -K \nabla T$ *with* $\boldsymbol{K} = \frac{1}{3}\bar{v}\ell C_V$

- Velocity $\bar{v} = c$
- Energy density $U = aT^4$

➡ $C_V = dU/dT = 4\,a\,T^3$ with the radiation constant $a = \frac{8\pi^5 k^4}{15 h^3 c^3} = 7.56 \times 10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}}.$

➡ How do we derive the free mean free path?

# Energy transport

## Radiative energy transport

- Local luminosity $l\ (r)$ :
  rate at which energy (as heat) flows outward through a sphere of radius r

- In spherical symmetry: $l$ related to radial energy flux $F$

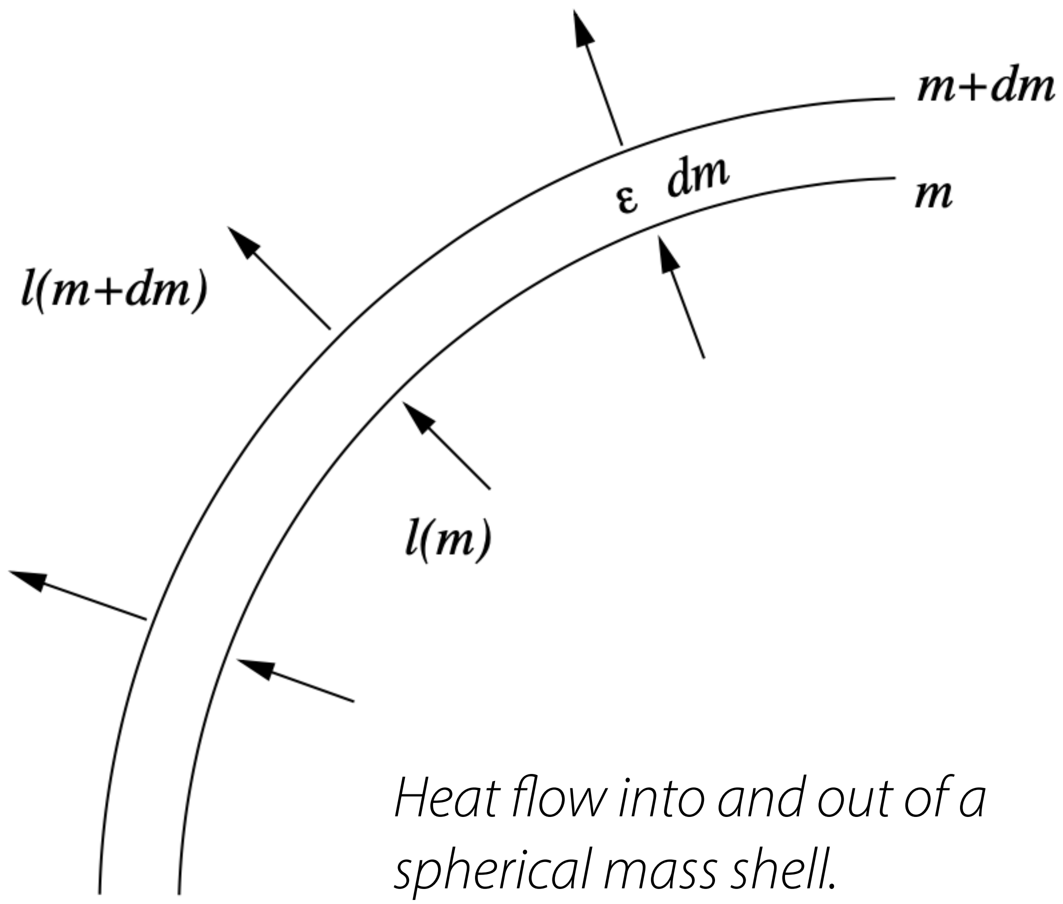
$$l\ (r) = 4\pi r^2 F$$

  - At the surface: $l = L$
  - At the centre: $l = 0.$

- Normally heat flows outwards, in direction of decreasing temperature (gradient!)

  ➡ $l$ is usually positive

- $l$ negative under special circumstances (e.g. neutrino emission cooling the core)




*Heat flow into and out of a spherical mass shell.*

# Energy transport

## Radiative energy transport

- Considered here: Energy transport only by radiation
  - If mean free path of photons short, radiative energy transport as diffusion process
  
  ➡ Radiative transfer handled with **diffusion approximation**

- Radiative transfer equation $$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu \rho\, I_\nu$$

➡ Intensity $I_\nu$ diminished over distance s (in absence of emission)

➡ **mean free path** = distance over which the intensity decreases by a factor of e

$$\ell_{\mathrm{ph}} = \frac{1}{\kappa\rho}.$$

$\kappa$: opacity

$\rho$: mass density

➡ Radiative conductivity $$K_{\mathrm{rad}} = \frac{4}{3}\frac{acT^3}{\kappa\rho}.$$

➡ Radiative energy flux $$\boldsymbol{F}_{\mathrm{rad}} = -K_{\mathrm{rad}}\,\boldsymbol{\nabla} T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\boldsymbol{\nabla} T.$$

# Energy transport

## Radiative energy transport

- Radiative energy flux $\boldsymbol{F}_{\mathrm{rad}} = -K_{\mathrm{rad}} \boldsymbol{\nabla} T = -\frac{4}{3} \frac{acT^3}{\kappa\rho} \boldsymbol{\nabla} T.$
- With $F_{rad} = l / 4\pi r^2$ in spherical geometry (radius r): $\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi acT^3} \frac{l}{r^2}$
- With the equation for mass conservation $\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \Longrightarrow \boxed{\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac} \frac{\kappa l}{r^4 T^3}}$

**➡ Temperature gradient required to carry the entire luminosity *l* by radiation.**

- A region with this gradient = in radiative equilibrium ($\Longrightarrow$ radiative zone).

# Energy transport

## Radiative energy transport

- **Temperature gradient for radiative energy transport** $$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$$

- **Note**:
  - Valid only if conditions for **LTE** are fulfilled (requires <u>short</u> mean free paths, much shorter than the radius, $l_{ph} \ll R$)
  - Not valid if $l_{ph}$ becomes much longer (at the surface, near photosphere where photons escape into space)

    ➡ Diffusion approximation is no longer valid

    ➡ Solution of full equations of radiative transfer necessary!

- **In practice (in simulations):**
  - Stellar interiors can be handled with the diffusion approximation up to some depth below the surface.

    ➡ Computationally much cheaper!
  - Surface-near layers + atmosphere to be treated with full radiative transfer equations!

    ➡ Computationally much more demanding!

# Energy transport

## Radiative energy transport

- In hydrostatic equilibrium: $$\frac{\mathrm{d}P}{\mathrm{d}m} = -\frac{Gm}{4\pi r^4}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$$

$$\frac{\mathrm{d}T}{\mathrm{d}m} = \frac{\mathrm{d}P}{\mathrm{d}m}\cdot\frac{\mathrm{d}T}{\mathrm{d}P} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\cdot\frac{\mathrm{d}\log T}{\mathrm{d}\log P}$$

- **Radiative temperature gradient** $$\nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\log T}{\mathrm{d}\log P}\right)_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa l P}{m T^4}$$
  - Describes logarithmic variation of T with depth for a star in hydrostatic equilibrium and pure radiative energy transport (with pressure as depth coordinate)

# Energy transport

## Radiative energy transport

- **Frequency-dependence:** Radiative flux in frequency interval $[\nu, \nu + d\nu]$: $F_\nu d\nu$

➡ $$\boldsymbol{F}_\nu = -D_\nu \boldsymbol{\nabla} U_\nu = -D_\nu \frac{\partial U_\nu}{\partial T} \boldsymbol{\nabla} T \quad \text{with} \quad D_\nu = \tfrac{1}{3} c \ell_\nu = \frac{c}{3\kappa_\nu \rho}$$

➡ Integral over all frequencies: $$\boldsymbol{F} = -\left[\frac{c}{3\rho} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U_\nu}{\partial T} \mathrm{d}\nu\right] \boldsymbol{\nabla} T.$$

➡ Can be written (as before) as $\boldsymbol{F} = -K_{rad} \nabla T$

**but** radiative conductivity needs to look like this now: $$K_\mathrm{rad} = \frac{c}{3\rho} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U_\nu}{\partial T} \mathrm{d}\nu.$$

➡ Proper average of opacity $\kappa_\nu$ needed! $$\frac{1}{\kappa} = \frac{1}{4aT^3} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial U_\nu}{\partial T} \mathrm{d}\nu.$$

$$4aT^3 = \int_0^\infty (\partial U_\nu / \partial T)\, \mathrm{d}\nu$$

➡ Energy density $U_\nu$ in same frequency interval proportional to **Planck function**! $U_\nu \propto B_\nu$

# Energy transport

## Radiative energy transport

- **Rosseland mean absorption coefficient**

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$

Used for the radiative temperature gradient

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac} \frac{\kappa l}{r^4 T^3}$$

# Energy transport

## Radiative energy transport

- **Rosseland mean absorption coefficient**

$$\frac{1}{\kappa_R} = \frac{\int\limits_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int\limits_0^\infty \frac{dB_\nu}{dT} d\nu}$$

- Weighted with $1/\kappa_\nu$ $\Longrightarrow$ More energy transported at frequencies where the matter is more transparent.
- Weighted with $dB_\nu/dT$ $\Longrightarrow$ More energy transported at frequencies where the radiation field is more temperature-dependent (stronger gradients).
- $\kappa$ essentially as an inverse conduction coefficient!

# Energy transport

## Conductive energy transport

- Collisions between the gas particles (ions and electrons) can also transport heat.
- Energy flux due to heat conduction (equivalently to radiative energy flux)

$$\boldsymbol{F}_{cd} = -K_{cd} \nabla T$$

- Conductive and radiative energy flux can be combined:

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{rad}} + \boldsymbol{F}_{\mathrm{cd}} = -(K_{\mathrm{rad}} + K_{\mathrm{cd}}) \nabla T$$

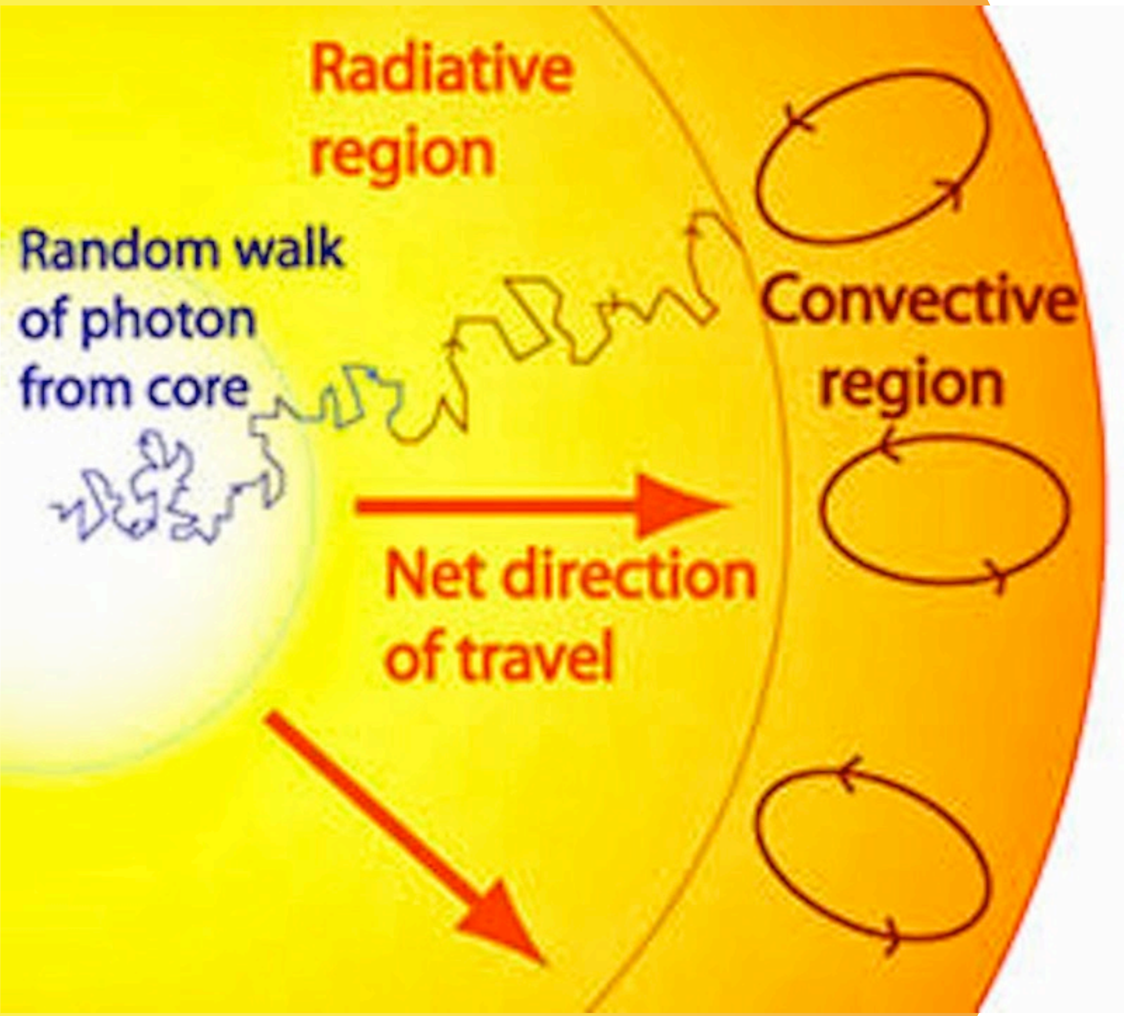
- Define an equivalent conductive opacity $K_{\mathrm{cd}} = \dfrac{4acT^3}{3\kappa_{\mathrm{cd}}\,\rho}$

- Combined energy flux $\boldsymbol{F} = -\dfrac{4acT^3}{3\kappa\rho}\boldsymbol{\nabla}T$ with $\dfrac{1}{\kappa} = \dfrac{1}{\kappa_{\mathrm{rad}}} + \dfrac{1}{\kappa_{\mathrm{cd}}}$

- Transport mechanism with largest flux dominates
(= mechanism for which the plasma is more transparent)

# Energy transport

## Radiative energy transport




- Time for a photon to travel from centre to surface without interaction:
  ~2s
- **But: mean free path of photons very small!**
- In the dense solar interior: Mean free path of a photon only $10^{-2}$ m
  - Random walk,
    photon absorbed and re-emitted
    ~$10^{22}$ times before reaching surface

  ➡ Time ~ thermal timescale of the Sun
  **~ 2 $10^7$ yr**

  ➡ Observed radiation due to fusion reactions (on average) tens of millions of years ago.

  - Net direction towards surface due to gradients (opacity)

# Energy transport

## Radiative energy transport

- **Radiation pressure** $P_{\rm rad} = {}^1/_3\, a\, T^4$
- Outward force that must be smaller than gravitational force in order to maintain hydrostatic equilibrium (HE)!

$$\left|\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}r}\right| < \left|\left(\frac{\mathrm{d}P}{\mathrm{d}r}\right)_{\rm HE}\right| \quad \Rightarrow \quad \frac{\kappa\rho}{4\pi c}\frac{l}{r^2} < \frac{Gm\rho}{r^2}$$

➡ Upper limit to the local luminosity: (local) **Eddington luminosity:** Maximum energy flux that can be carried by radiation

$$l < \frac{4\pi c G m}{\kappa} = l_{\rm Edd}$$

➡ Can get violated by intense nuclear burning

➡ In these situations, radiative energy transport insufficient for maintaining hydrostatic equilibrium

- At the surface (m=M): $L < L_{\rm Edd} = \frac{4\pi c G M}{\kappa}$

↑ In photosphere

**Condition violated?**

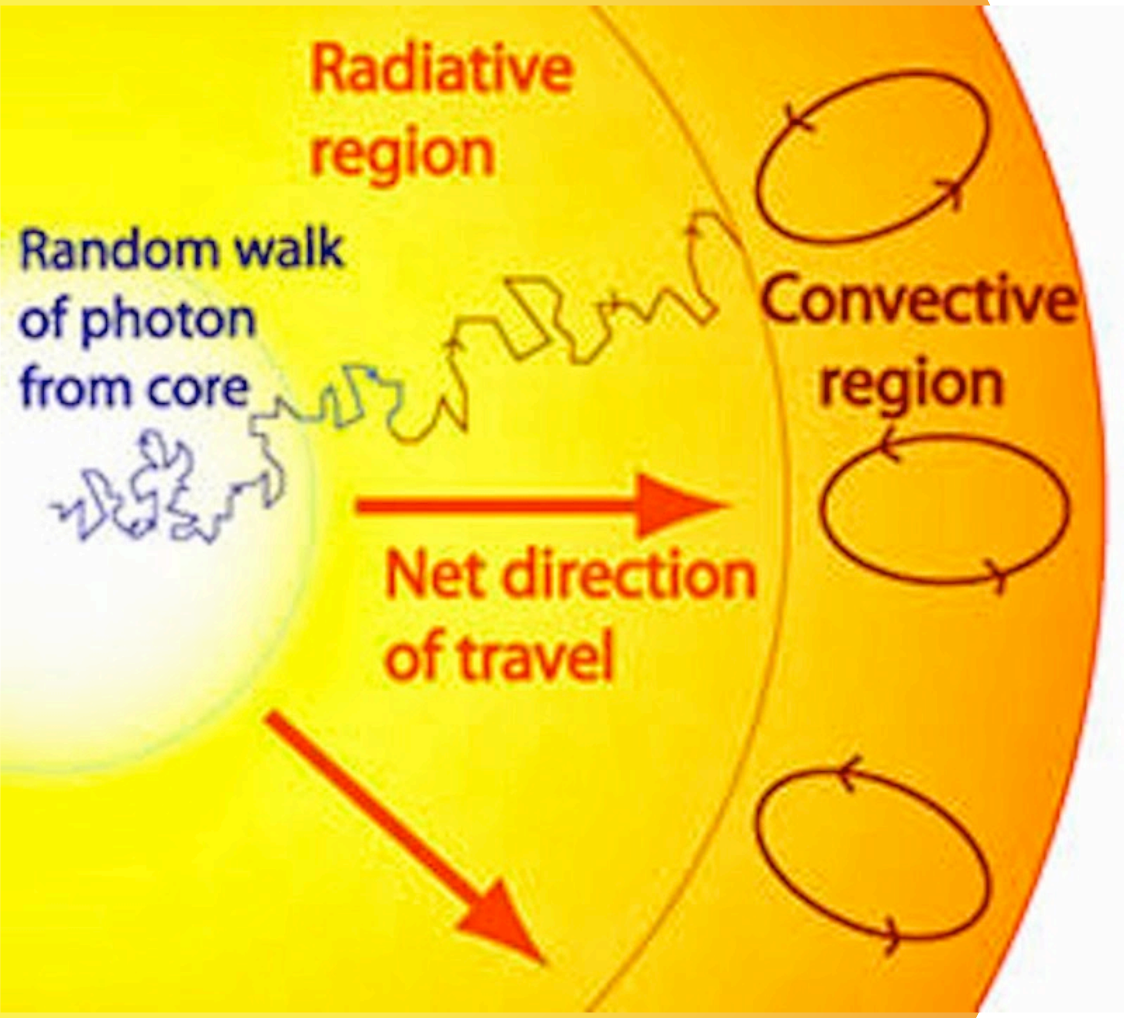
➡ No hydrostatic equilibrium!

➡ Gas accelerated outwards due to radiative pressure.

➡ Mass loss!

# Energy transport

## Radiative energy transport




- Note: At high temperature all atom are completely ionised
  - ➡ Photons move through a plasma consisting of free electrons and atomic nuclei (incl. protons)
- Towards surface (in solar-like stars): Not all atoms are completely ionised anymore
  - ➡ Changes in mean molecular weight and gradients
  - ➡ Convection becomes the dominant mode of energy transport subject to <u>stability criterion</u>
- Note: Radiative energy transport the "default" in convectively stable regions

# Energy transport

## Convection

$$F = F_{\mathrm{rad}} + F_{\mathrm{c}} + F_{\mathrm{cd}} = L / 4\pi r^2$$

$$F_{\mathrm{rad}} \propto \nabla T$$

- Transport for larger energy flux (luminosity) requires larger temperature gradient required.
- But: Upper limit to $\nabla T$ in stellar interior
  - ➡ Stratification becomes unstable if limit exceeded
  - ➡ (Additional) dominating energy transport via convection

# Energy transport

**Literature:** Nordlund, Stein, Asplund
Living Rev. Solar Phys., 6, (2009), 2
http://www.livingreviews.org/lrsp- 2009- 2

## Surface convection — granulation

- Density, temperature decreases with radius
- Eventually plasma transparent enough (longer mean free path)

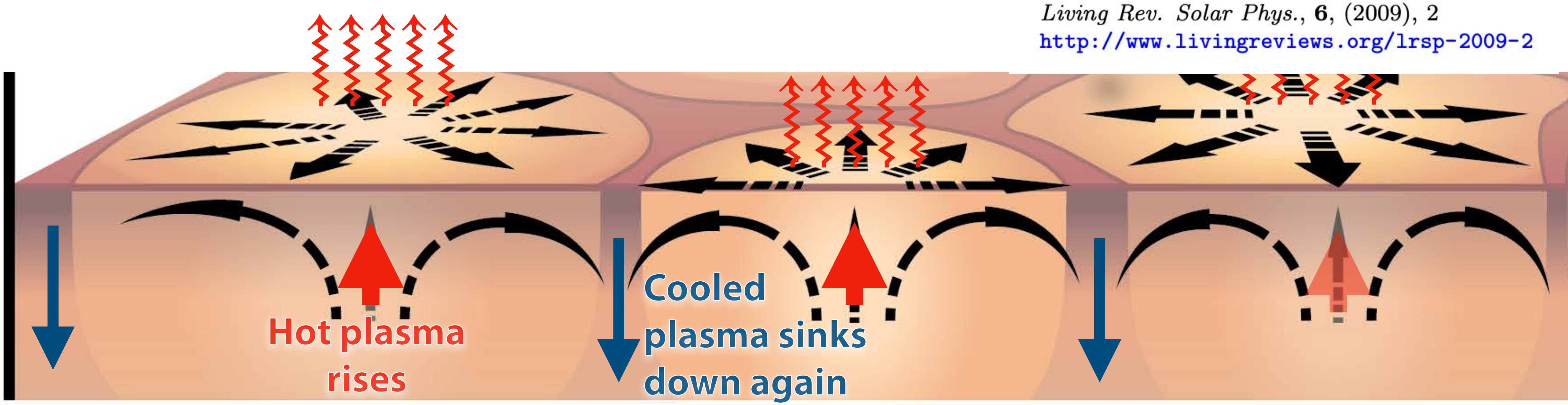
➡ **Radiation effectively removes heat from rising convective cells** at surface

➡ Plasma cools

➡ Advected sideways (pushed away from more upwelling gas below)

➡ Cooled and dense **plasma sinks down again**

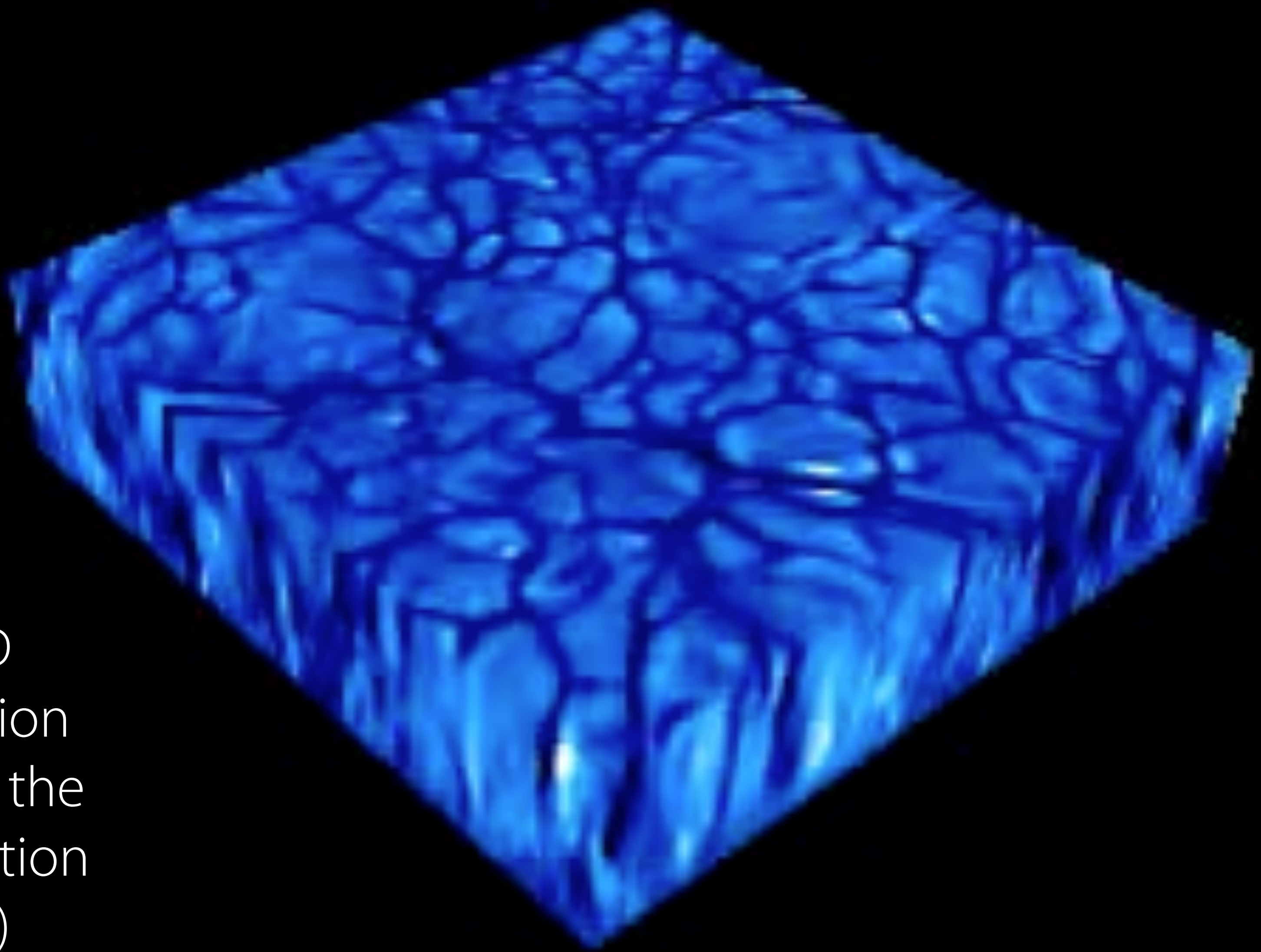

*Living Rev. Solar Phys.*, **6**, (2009), 2
http://www.livingreviews.org/lrsp-2009-2




**Top of convection zone** — energy transport via convection (bulk motion)

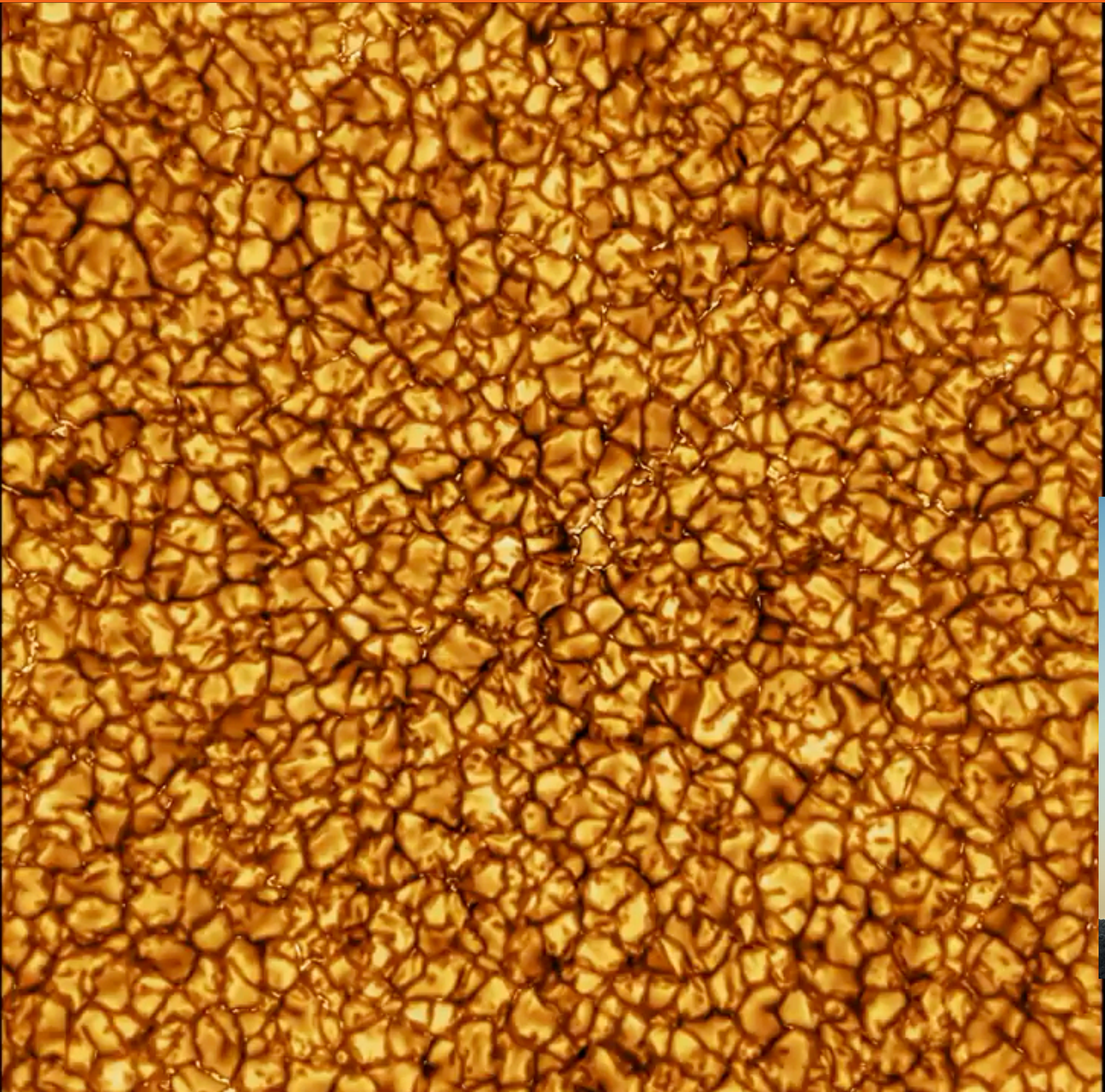
# Energy transport

## Convection — Solar surface convection



Time-dependent 3D hydrodynamic simulation showing convection at the top of the Sun's convection zone (at the surface)

# Energy transport



Highest-resolution observations of the Sun's granulation ever taken.
DKIST (4m)
(NSO/AURA/NSF)


10 Dec. 2019 19:24:31 UT

# Energy transport

## Surface convection — solar granulation

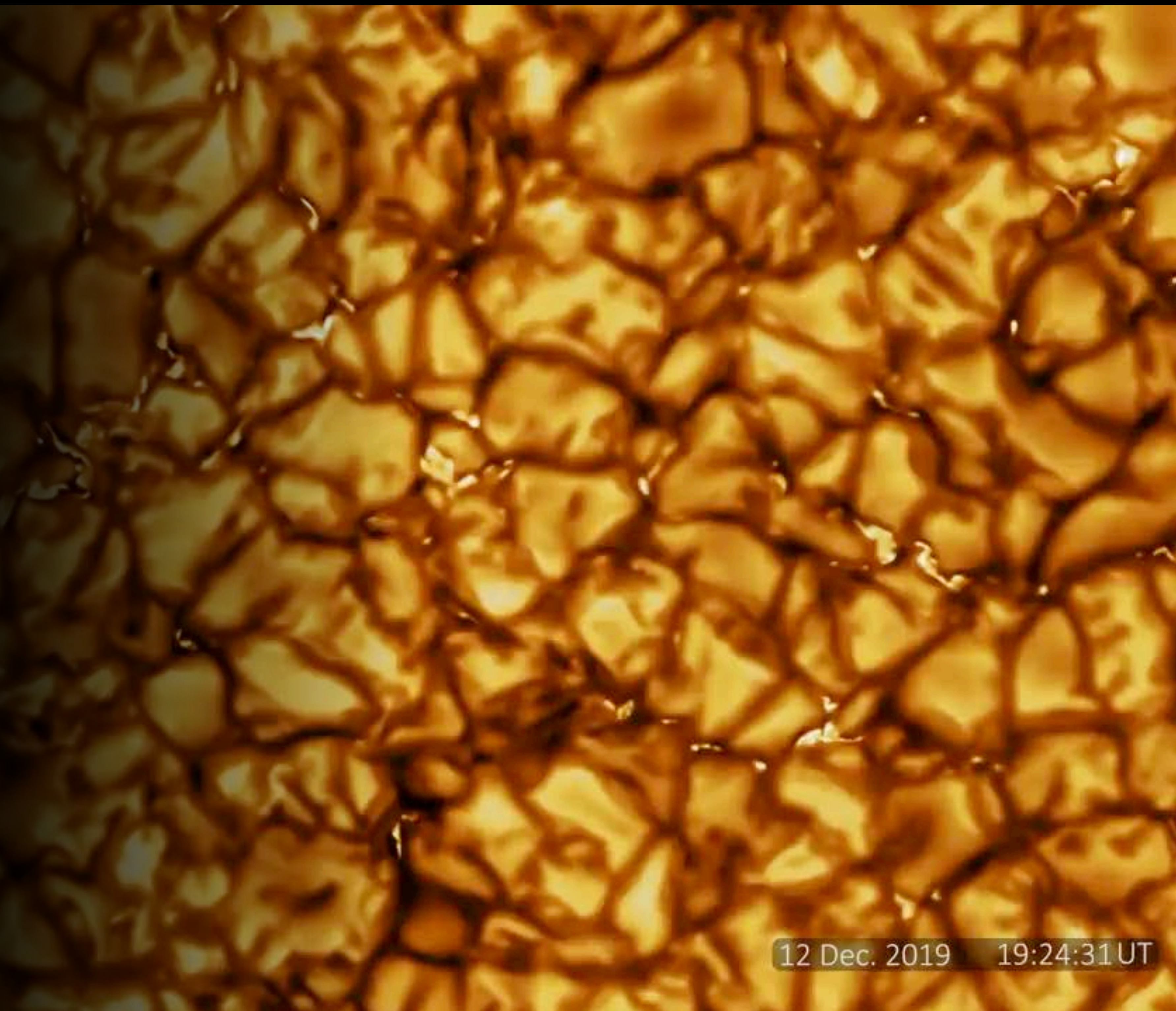
Highest-resolution observations of the Sun's granulation ever taken (DKIST, NSO/AURA/NSF)

**Granules**

- Diameters ~1000-2000km
- Lifetime ~8min

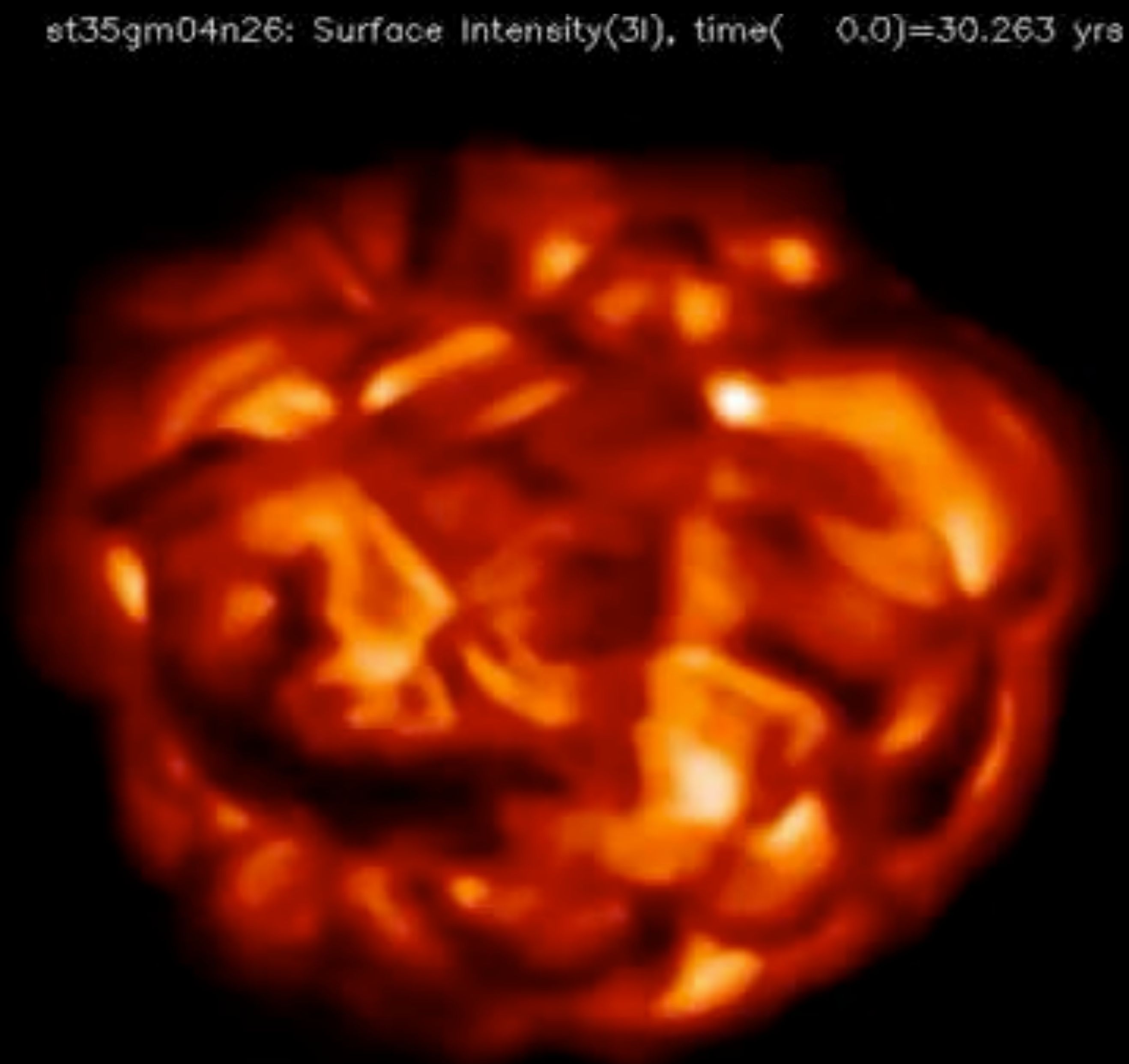
**Bright features in inter-granular lanes**

- (Mostly) due to magnetic field concentrations



# Energy transport

## Convection




Time-dependent 3D hydrodynamic simulation of **Betelgeuse**, here **intensity**

- Large convection cells
- Slower temporal evolution (Note time at top right)

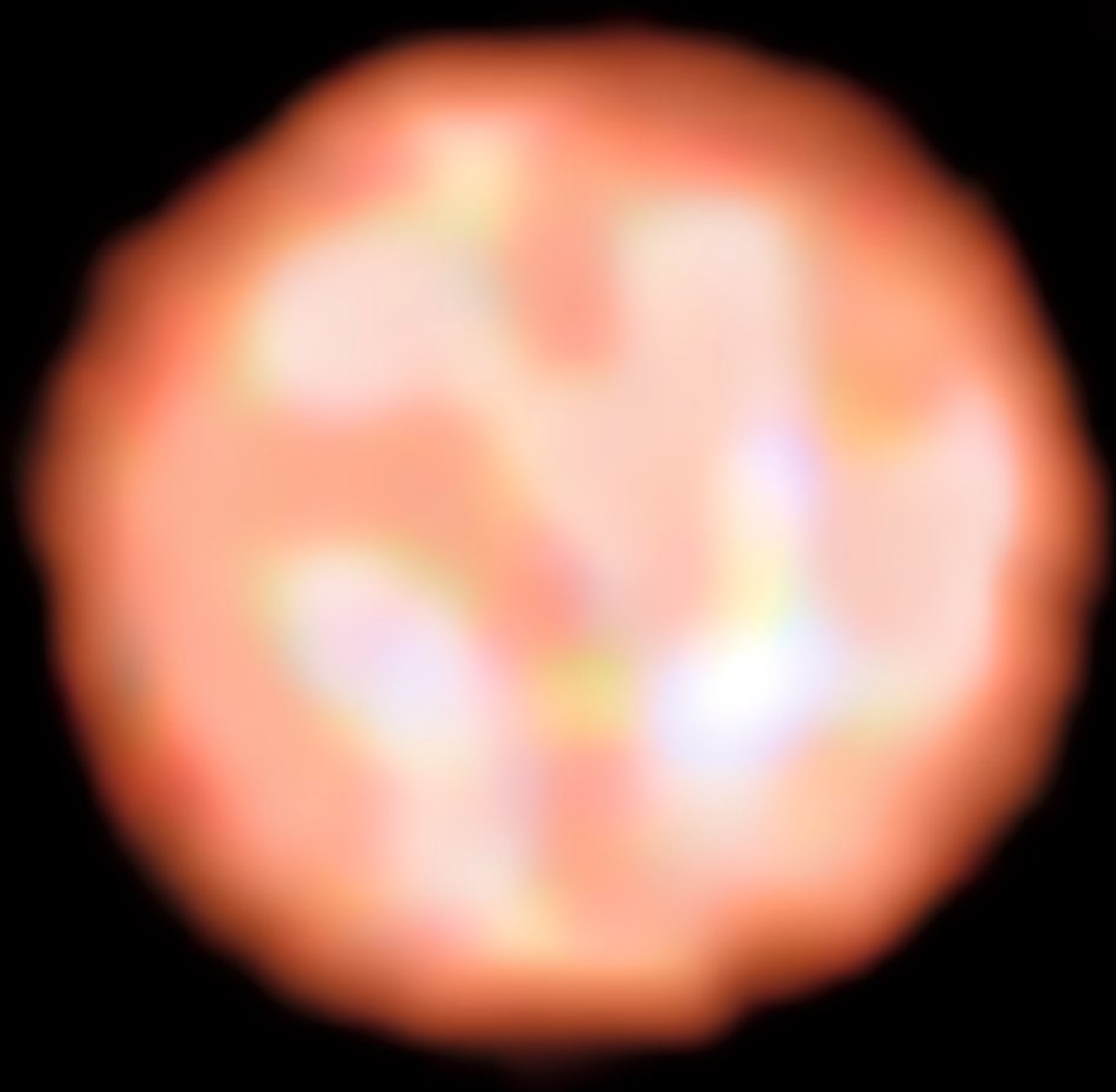
*Simulation produced with the same code as the models for the project assignment (Freytag et al. 2012)*

*Freytag*

# Energy transport

## Surface convection on other stars

- Observations of the cool red giant π[1] Gruis (with PIONIER/VLT)
- Spectral type M1I
- 1.5 solar masses
- 350 times the diameter of the Sun
- Few giant convection cells

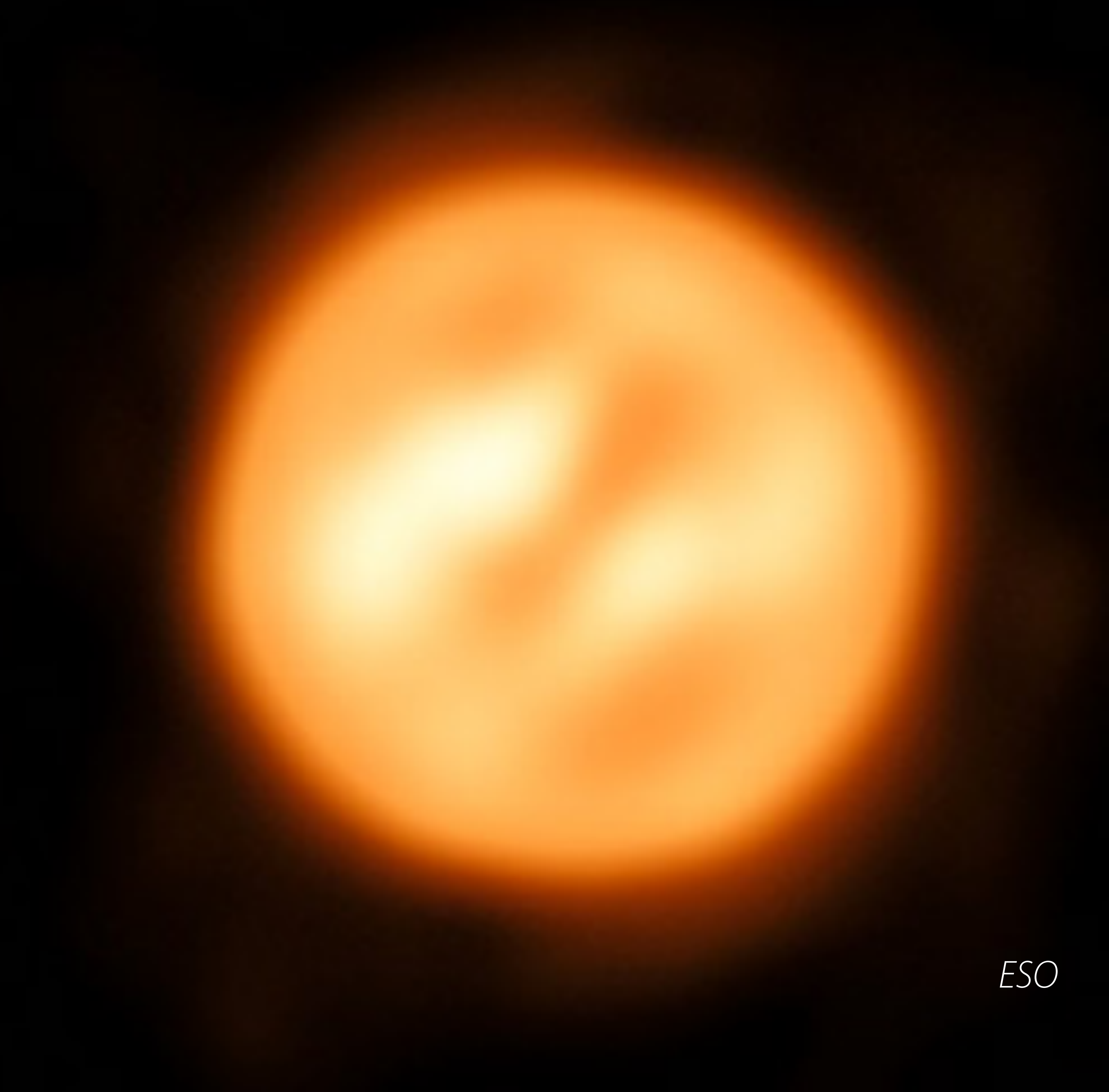


*ESO*

# Energy transport

## Surface convection on other stars

- Observations of the cool red supergiant Antares (with AMBER/VLTI)
- Spectral type M1.5I
- ~11-14 solar masses
- ~700 times the diameter of the Sun



*ESO*

# Energy transport

## Convection — Stability criterion

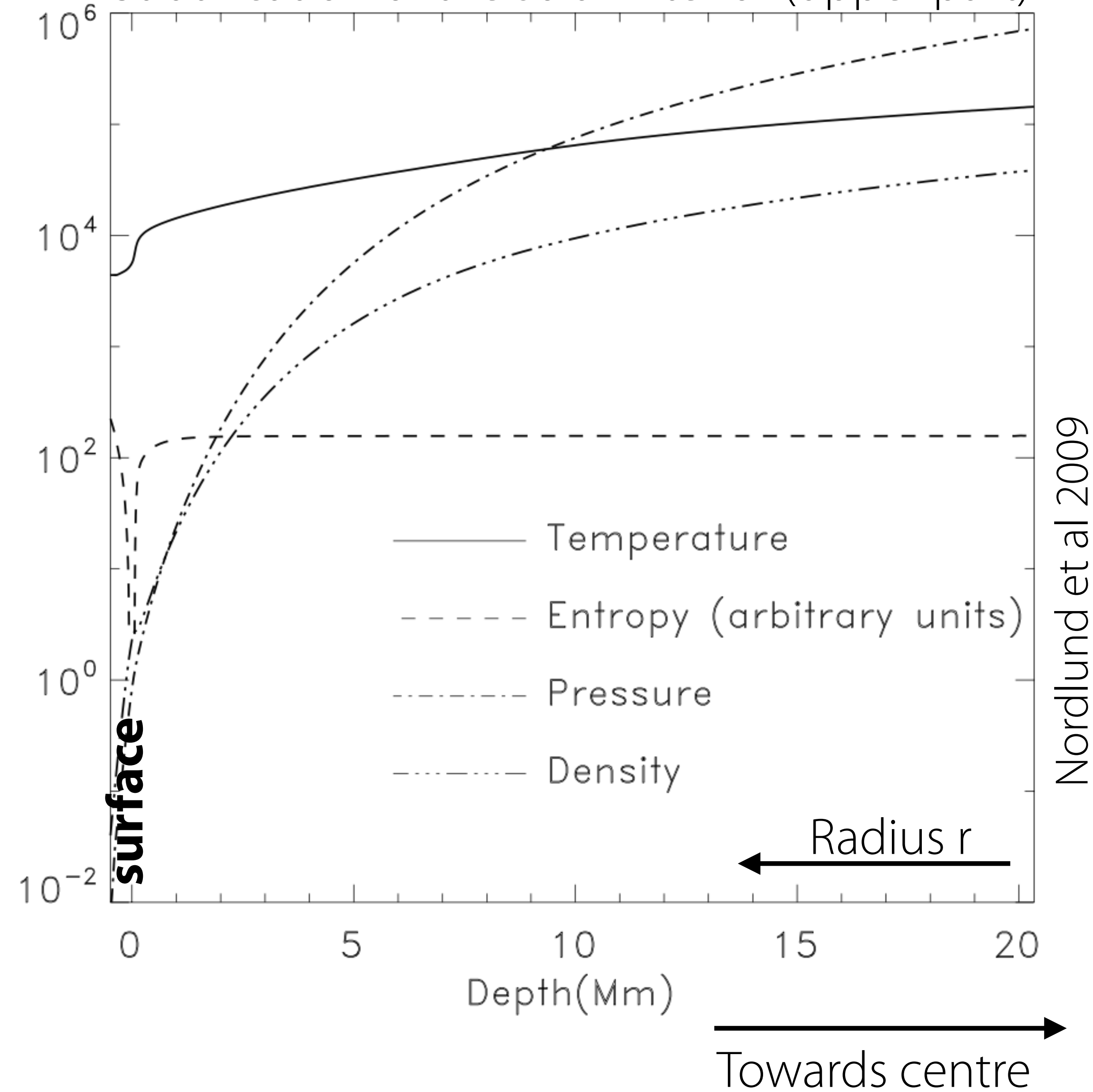
- **Plasma/gas inside a star not a perfectly stratified but small perturbations occur**

➡ Is a layer stable against small perturbations?

➡ Or can initially small perturbations grow and result in significant deviations?

- **In the stratified stellar interior:**
  Pressure and density decreases with radius r towards surface

Stratification of the solar interior (upper part)



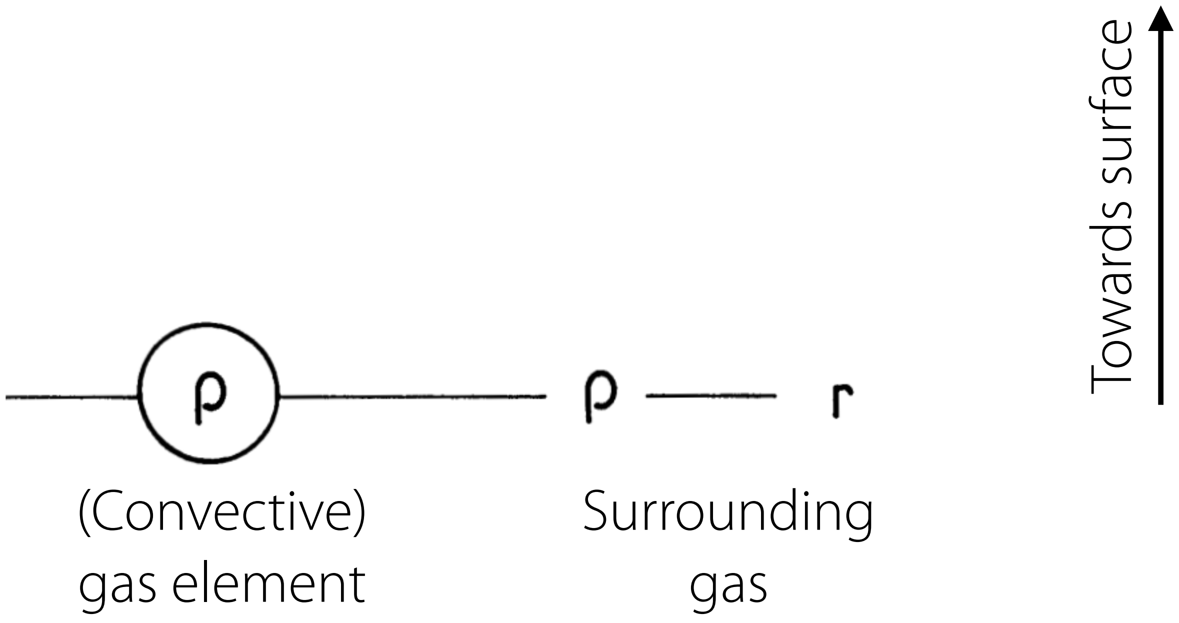

Nordlund et al 2009

# Energy transport

## Convection — Stability criterion

- **Gas element** at distance r from the centre of the star
- Initially: Element in **equilibrium** with its surroundings at r:

➡ Pressure P and density ρ are the same as in its surroundings.

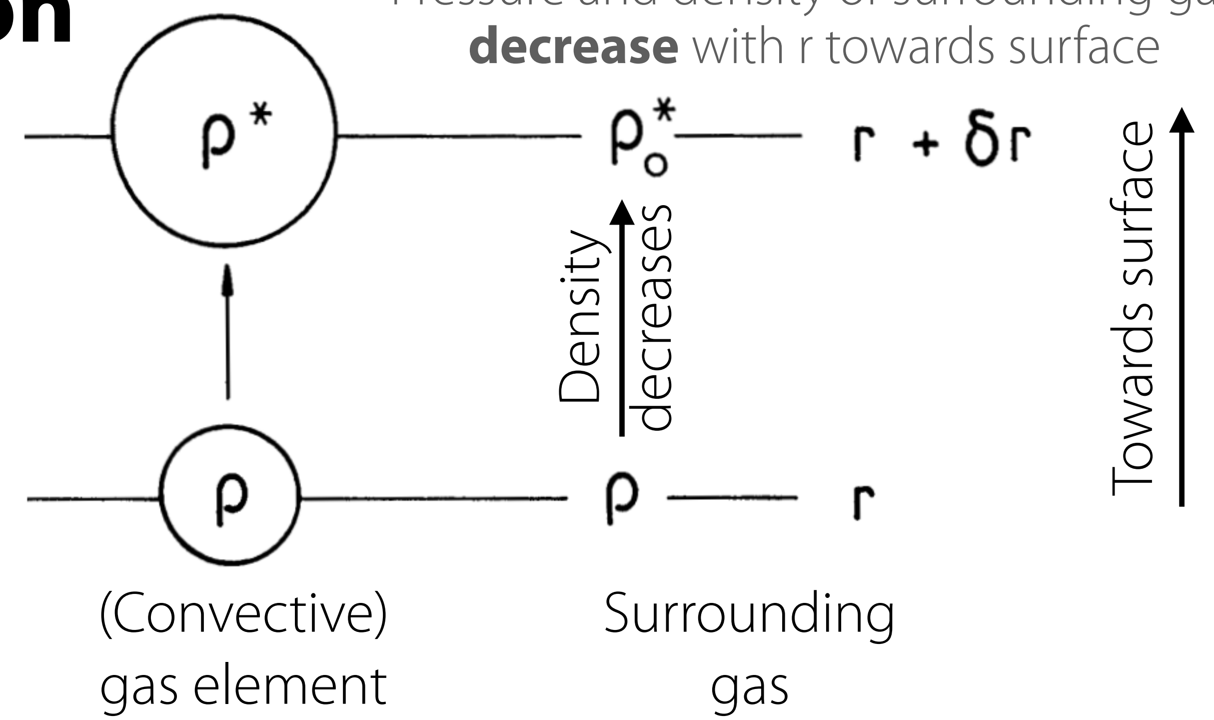


# Energy transport

## Convection — Stability criterion

- Gas element at distance r from the centre of the star
- Initially: Element in equilibrium with its surroundings at r:

➡ Pressure P and density $\rho$ are the same as in its surroundings.




- Now — perturbation: **element** displaced (**rises**) a vertical distance $\delta r$ **adiabatically** (no heat exchanged with environment) but slow enough that pressure is adjusted to new balance with outside pressure
  - Occurs when <u>time scale</u> of heat exchange is long compared to time scale of expansion of the element (the latter = local dynamical time scale, set by local sound speed); happens in the optically thick solar interior

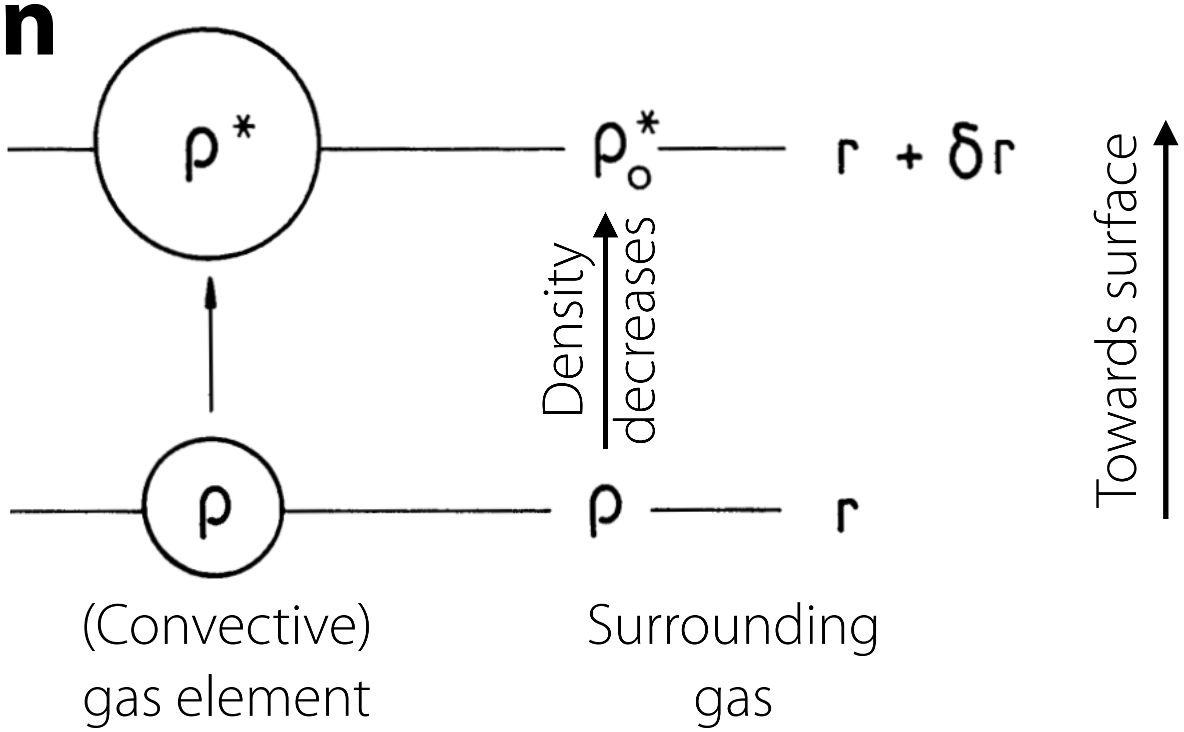
➡ Element expands to restore pressure equilibrium with surrounding

➡ Pressure in the element reduce as it rises

➡ Now at r+δr: Compare density of the element $\rho^*$ with density of (new) surrounding $\rho_0^*$

# Energy transport

## Convection — Stability criterion




- Gas element at distance r from the centre of the star rises …
- Initially in equilibrium with its surroundings at r

- $\varrho^* > \varrho_0^*$: Element will fall back to initial height — stratification is **convectively stable**
- $\varrho^* < \varrho_0^*$: Element will keep rising up (net buoyancy!) — stratification is **convectively unstable**

- At $r+\delta r$: Density difference $\varrho^*$ - $\varrho_0^*$ — gradients!
- For adiabatic expansion: $$\frac{\delta P^*}{P^*} = \gamma_{\rm ad} \frac{\delta \varrho^*}{\varrho^*}$$

➡ For the situation above, we derive the following criterion for the gas remaining convectively stable:

$$\frac{d \log \rho}{d \log P} > \frac{1}{\gamma_{\rm ad}}$$