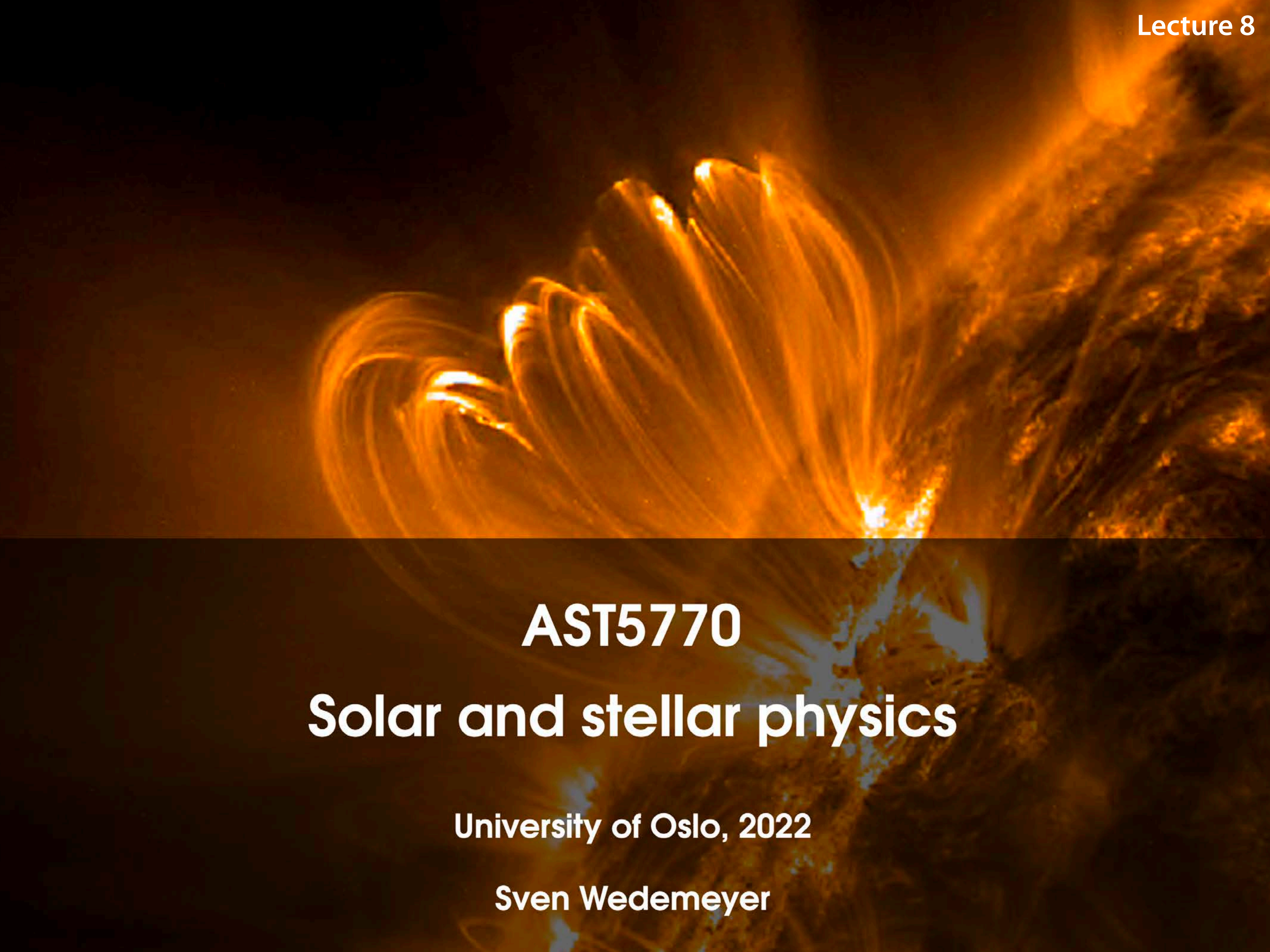


Lecture 8

AST5770

Solar and stellar physics

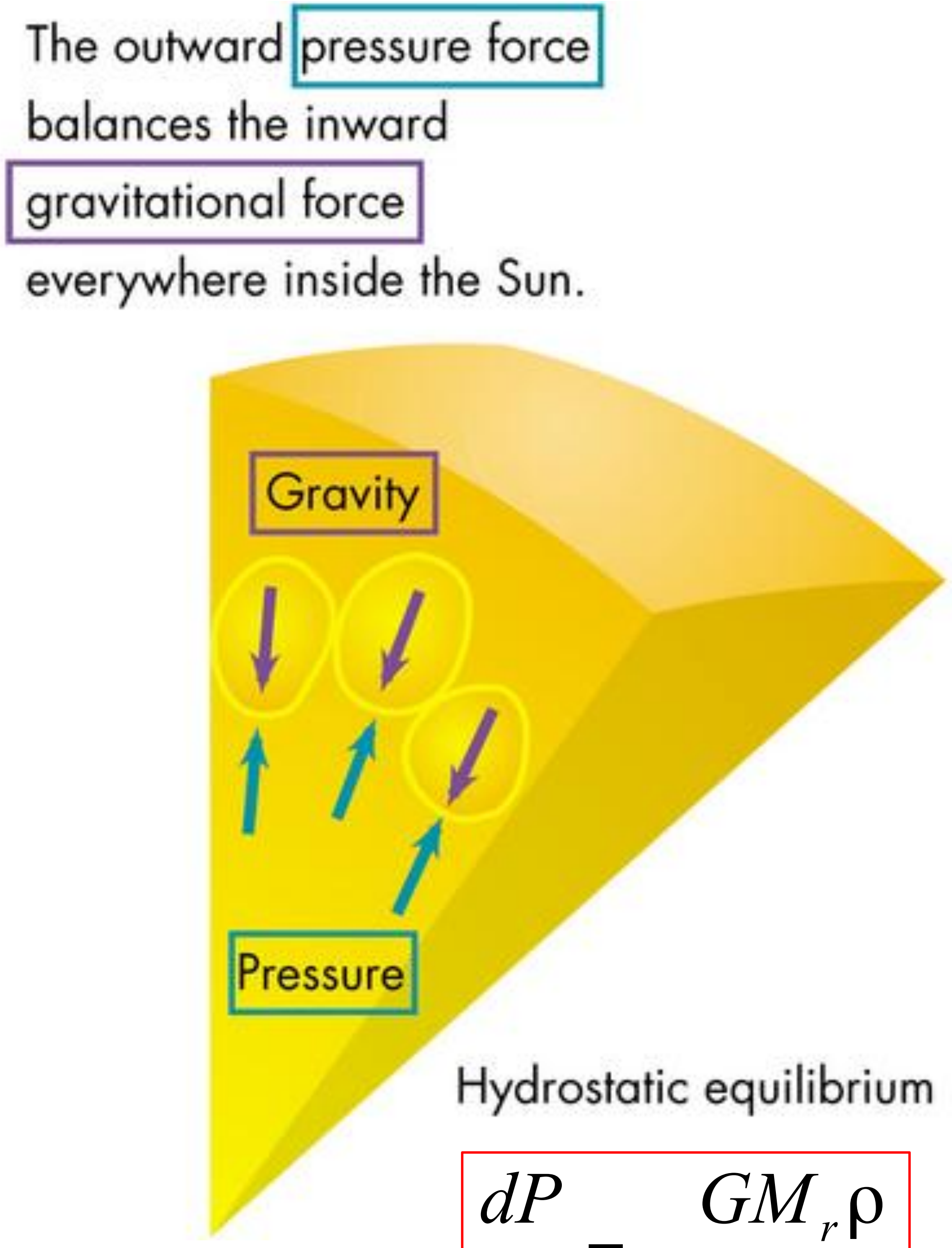
University of Oslo, 2022

Sven Wedemeyer

Equations of stellar structure

Stellar structure

Hydrostatic Equilibrium

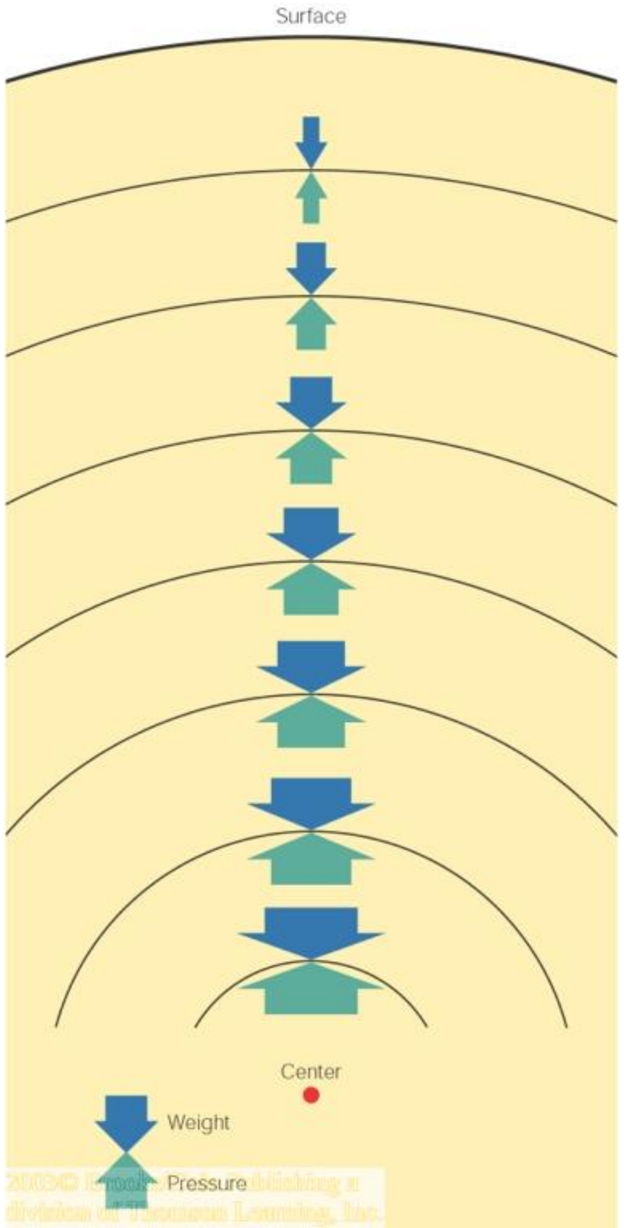


The outward pressure force balances the inward gravitational force everywhere inside the Sun.

Hydrostatic equilibrium

$$\boxed{\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}}$$

- Outward pressure of hot gas in the center balances the inward force due to gravity.
  - ➡ At any given radius balancing the weight of all layers above
  - ➡ Imbalance at some radius will result in corresponding adjustment of the stratification
  - ➡ Determines the interior structure (stratification)
- Main Sequence in the Hertzsprung-Russell diagram is a narrow strip as it requires stability over long enough time



Stellar interior

Equations of stellar structure — Recap

- The interior structure of a star in equilibrium can be described with the following “ingredients””
- Variables and their stratification (function of radius) P, ϱ, T, M_r, L_r
- The following equations :

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$
Mass conservation/continuity	$\frac{dM_r}{dr} = 4\pi r^2\rho$
Equation of state	$P = \frac{\rho kT}{\mu m_H}$
Energy “generation”	$\frac{dL_r}{dr} = 4\pi r^2\rho\varepsilon$
Energy transport	$\frac{dT}{dr} = -\frac{3\kappa\rho}{16\pi ac}\frac{L_r}{r^2T^2}$

A “small” problem:
We cannot observe the interior of stars directly
but rely mostly on few measurable properties (M, L,…), assumptions, (tested) physical laws and material properties
(often with substantial uncertainties)

- And these boundary conditions:
  - In the centre (r=0): $M_r(0) = 0, \quad L_r(0) = 0$
  - At the surface (r=R): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$
- Note that the chemical composition affects the density and thus the stratification via the mean molecular weight μ as function of radius (see equation of state)

Stellar interior

Equations of stellar structure — Recap

- The interior structure of a star in equilibrium can be described with the following “ingredients””
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Solution

Standard Model

assumes
[X:Y:Z]
=
[0.73:0.25:0.015]

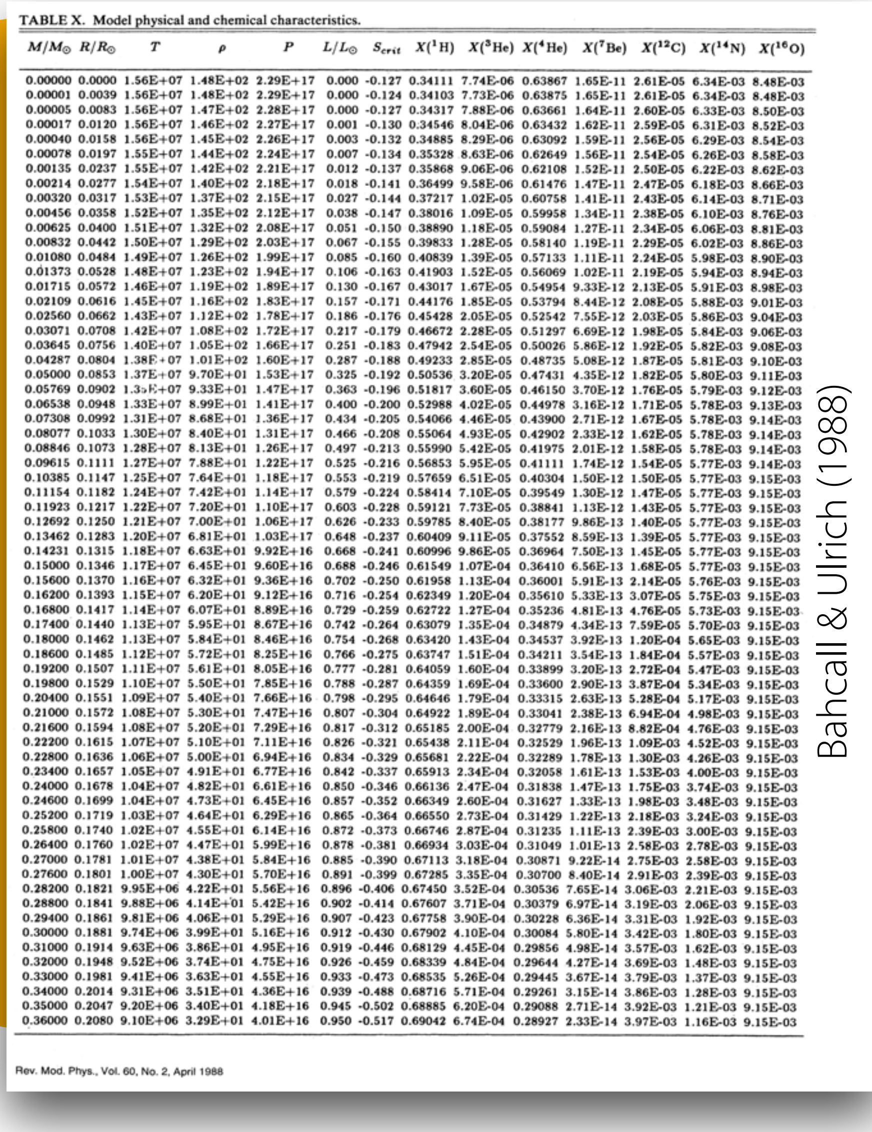

TABLE X. Model physical and chemical characteristics.

$M/M_\odot$	$R/R_\odot$	T	ρ	P	$L/L_\odot$	S_{crit}	$X(^1H)$	$X(^3He)$	$X(^4He)$	$X(^7Be)$	$X(^{12}C)$	$X(^{14}N)$	$X(^{16}O)$
0.00000	0.0000	1.56E+07	1.48E+02	2.29E+17	0.000	-0.127	0.34111	7.74E-06	0.63867	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00001	0.0039	1.56E+07	1.48E+02	2.29E+17	0.000	-0.124	0.34103	7.73E-06	0.63875	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00005	0.0083	1.56E+07	1.47E+02	2.28E+17	0.000	-0.127	0.34317	7.88E-06	0.63661	1.64E-11	2.60E-05	6.33E-03	8.50E-03
0.00017	0.0120	1.56E+07	1.46E+02	2.27E+17	0.001	-0.130	0.34546	8.04E-06	0.63432	1.62E-11	2.59E-05	6.31E-03	8.52E-03
0.00040	0.0158	1.56E+07	1.45E+02	2.26E+17	0.003	-0.132	0.34885	8.29E-06	0.63092	1.59E-11	2.56E-05	6.29E-03	8.54E-03
0.00078	0.0197	1.55E+07	1.44E+02	2.24E+17	0.007	-0.134	0.35328	8.63E-06	0.62649	1.56E-11	2.54E-05	6.26E-03	8.58E-03
0.00135	0.0237	1.55E+07	1.42E+02	2.21E+17	0.012	-0.137	0.35868	9.06E-06	0.62108	1.52E-11	2.50E-05	6.22E-03	8.62E-03
0.00214	0.0277	1.54E+07	1.40E+02	2.18E+17	0.018	-0.141	0.36499	9.58E-06	0.61476	1.47E-11	2.47E-05	6.18E-03	8.66E-03
0.00320	0.0317	1.53E+07	1.37E+02	2.15E+17	0.027	-0.144	0.37217	1.02E-05	0.60758	1.41E-11	2.43E-05	6.14E-03	8.71E-03
0.00456	0.0358	1.52E+07	1.35E+02	2.12E+17	0.038	-0.147	0.38016	1.09E-05	0.59958	1.34E-11	2.38E-05	6.10E-03	8.76E-03
0.00625	0.0400	1.51E+07	1.32E+02	2.08E+17	0.051	-0.150	0.38890	1.18E-05	0.59084	1.27E-11	2.34E-05	6.06E-03	8.81E-03
0.00832	0.0442	1.50E+07	1.29E+02	2.03E+17	0.067	-0.155	0.39833	1.28E-05	0.58140	1.19E-11	2.29E-05	6.02E-03	8.86E-03
0.01080	0.0484	1.49E+07	1.26E+02	1.99E+17	0.085	-0.160	0.40839	1.39E-05	0.57133	1.11E-11	2.24E-05	5.98E-03	8.90E-03
0.01373	0.0528	1.48E+07	1.23E+02	1.94E+17	0.106	-0.163	0.41903	1.52E-05	0.56069	1.02E-11	2.19E-05	5.94E-03	8.94E-03
0.01715	0.0572	1.46E+07	1.19E+02	1.89E+17	0.130	-0.167	0.43017	1.67E-05	0.54954	9.33E-12	2.13E-05	5.91E-03	8.98E-03
0.02109	0.0616	1.45E+07	1.16E+02	1.83E+17	0.157	-0.171	0.44176	1.85E-05	0.53794	8.44E-12	2.08E-05	5.88E-03	9.01E-03
0.02560	0.0662	1.43E+07	1.12E+02	1.78E+17	0.186	-0.176	0.45428	2.05E-05	0.52542	7.55E-12	2.03E-05	5.86E-03	9.04E-03
0.03071	0.0708	1.42E+07	1.08E+02	1.72E+17	0.217	-0.179	0.46672	2.28E-05	0.51297	6.69E-12	1.98E-05	5.84E-03	9.06E-03
0.03645	0.0756	1.40E+07	1.05E+02	1.66E+17	0.251	-0.183	0.47942	2.54E-05	0.50026	5.86E-12	1.92E-05	5.82E-03	9.08E-03
0.04287	0.0804	1.38E+07	1.01E+02	1.60E+17	0.287	-0.188	0.49233	2.85E-05	0.48735	5.08E-12	1.87E-05	5.81E-03	9.10E-03
0.05000	0.0853	1.37E+07	9.70E+01	1.53E+17	0.325	-0.192	0.50536	3.20E-05	0.47431	4.35E-12	1.82E-05	5.80E-03	9.11E-03
0.05769	0.0902	1.35E+07	9.33E+01	1.47E+17	0.363	-0.196	0.51817	3.60E-05	0.46150	3.70E-12	1.76E-05	5.79E-03	9.12E-03
0.06538	0.0948	1.33E+07	8.99E+01	1.41E+17	0.400	-0.200	0.52988	4.02E-05	0.44978	3.16E-12	1.71E-05	5.78E-03	9.13E-03
0.07308	0.0992	1.31E+07	8.68E+01	1.36E+17	0.434	-0.205	0.54066	4.46E-05	0.43900	2.71E-12	1.67E-05	5.78E-03	9.14E-03
0.08077	0.1033	1.30E+07	8.40E+01	1.31E+17	0.466	-0.208	0.55064	4.93E-05	0.42902	2.33E-12	1.62E-05	5.78E-03	9.14E-03
0.08846	0.1073	1.28E+07	8.13E+01	1.26E+17	0.497	-0.213	0.55990	5.42E-05	0.41975	2.01E-12	1.58E-05	5.78E-03	9.14E-03
0.09615	0.1111	1.27E+07	7.88E+01	1.22E+17	0.525	-0.216	0.56853	5.95E-05	0.41111	1.74E-12	1.54E-05	5.77E-03	9.14E-03
0.10385	0.1147	1.25E+07	7.64E+01	1.18E+17	0.553	-0.219	0.57659	6.51E-05	0.40304	1.50E-12	1.50E-05	5.77E-03	9.15E-03
0.11154	0.1182	1.24E+07	7.42E+01	1.14E+17	0.579	-0.224	0.58414	7.10E-05	0.39549	1.30E-12	1.47E-05	5.77E-03	9.15E-03
0.11923	0.1217	1.22E+07	7.20E+01	1.10E+17	0.603	-0.228	0.59121	7.73E-05	0.38841	1.13E-12	1.43E-05	5.77E-03	9.15E-03
0.12692	0.1250	1.21E+07	7.00E+01	1.06E+17	0.626	-0.233	0.59785	8.40E-05	0.38177	9.86E-13	1.40E-05	5.77E-03	9.15E-03
0.13462	0.1283	1.20E+07	6.81E+01	1.03E+17	0.648	-0.237	0.60409	9.11E-05	0.37552	8.59E-13	1.39E-05	5.77E-03	9.15E-03
0.14231	0.1315	1.18E+07	6.63E+01	9.92E+16	0.668	-0.241	0.60996	9.86E-05	0.36964	7.50E-13	1.45E-05	5.77E-03	9.15E-03
0.15000	0.1346	1.17E+07	6.45E+01	9.60E+16	0.688	-0.246	0.61549	1.07E-04	0.36410	6.56E-13	1.68E-05	5.77E-03	9.15E-03
0.15600	0.1370	1.16E+07	6.32E+01	9.36E+16	0.702	-0.250	0.61958	1.13E-04	0.36001	5.91E-13	2.14E-05	5.76E-03	9.15E-03
0.16200	0.1393	1.15E+07	6.20E+01	9.12E+16	0.716	-0.254	0.62349	1.20E-04	0.35610	5.33E-13	3.07E-05	5.75E-03	9.15E-03
0.16800	0.1417	1.14E+07	6.07E+01	8.89E+16	0.729	-0.259	0.62722	1.27E-04	0.35236	4.81E-13	4.76E-05	5.73E-03	9.15E-03
0.17400	0.1440	1.13E+07	5.95E+01	8.67E+16	0.742	-0.264	0.63079	1.35E-04	0.34879	4.34E-13	7.59E-05	5.70E-03	9.15E-03
0.18000	0.1462	1.13E+07	5.84E+01	8.46E+16	0.754	-0.268	0.63420	1.43E-04	0.34537	3.92E-13	1.20E-04	5.65E-03	9.15E-03
0.18600	0.1485	1.12E+07	5.72E+01	8.25E+16	0.766	-0.275	0.63747	1.51E-04	0.34211	3.54E-13	1.84E-04	5.57E-03	9.15E-03
0.19200	0.1507	1.11E+07	5.61E+01	8.05E+16	0.777	-0.281	0.64059	1.60E-04	0.33899	3.20E-13	2.72E-04	5.47E-03	9.15E-03
0.19800	0.1529	1.10E+07	5.50E+01	7.85E+16	0.788	-0.287	0.64359	1.69E-04	0.33600	2.90E-13	3.87E-04	5.34E-03	9.15E-03
0.20400	0.1551	1.09E+07	5.40E+01	7.66E+16	0.798	-0.295	0.64646	1.79E-04	0.33315	2.63E-13	5.28E-04	5.17E-03	9.15E-03
0.21000	0.1572	1.08E+07	5.30E+01	7.47E+16	0.807	-0.304	0.64922	1.89E-04	0.33041	2.38E-13	6.94E-04	4.98E-03	9.15E-03
0.21600	0.1594	1.08E+07	5.20E+01	7.29E+16	0.817	-0.312	0.65185	2.00E-04	0.32779	2.16E-13	8.82E-04	4.76E-03	9.15E-03
0.22200	0.1615	1.07E+07	5.10E+01	7.11E+16	0.826	-0.321	0.65438	2.11E-04	0.32529	1.96E-13	1.09E-03	4.52E-03	9.15E-03
0.22800	0.1636	1.06E+07	5.00E+01	6.94E+16	0.834	-0.329	0.65681	2.22E-04	0.32289	1.78E-13	1.30E-03	4.26E-03	9.15E-03
0.23400	0.1657	1.05E+07	4.91E+01	6.77E+16	0.842	-0.337	0.65913	2.34E-04	0.32058	1.61E-13	1.53E-03	4.00E-03	9.15E-03
0.24000	0.1678	1.04E+07	4.82E+01	6.61E+16	0.850	-0.346	0.66136	2.47E-04	0.31838	1.47E-13	1.75E-03	3.74E-03	9.15E-03
0.24600	0.1699	1.04E+07	4.73E+01	6.45E+16	0.857	-0.352	0.66349	2.60E-04	0.31627	1.33E-13	1.98E-03	3.48E-03	9.15E-03
0.25200	0.1719	1.03E+07	4.64E+01	6.29E+16	0.865	-0.364	0.66550	2.73E-04	0.31429	1.22E-13	2.18E-03	3.24E-03	9.15E-03
0.25800	0.1740	1.02E+07	4.55E+01	6.14E+16	0.872	-0.373	0.66746	2.87E-04	0.31235	1.11E-13	2.39E-03	3.00E-03	9.15E-03
0.26400	0.1760	1.02E+07	4.47E+01	5.99E+16	0.878	-0.381	0.66934	3.03E-04	0.31049	1.01E-13	2.58E-03	2.78E-03	9.15E-03
0.27000	0.1781	1.01E+07	4.38E+01	5.84E+16	0.885	-0.390	0.67113	3.18E-04	0.30871	9.22E-14	2.75E-03	2.58E-03	9.15E-03
0.27600	0.1801	1.00E+07	4.30E+01	5.70E+16	0.891	-0.399	0.67285	3.35E-04	0.30700	8.40E-14	2.91E-03	2.39E-03	9.15E-03
0.28200	0.1821	9.95E+06	4.22E+01	5.56E+16	0.896	-0.406	0.67450	3.52E-04	0.30536	7.65E-14	3.06E-03	2.21E-03	9.15E-03
0.28800	0.1841	9.88E+06	4.14E+01	5.42E+16	0.902	-0.414	0.67607	3.71E-04	0.30379	6.97E-14	3.19E-03	2.06E-03	9.15E-03
0.29400	0.1861	9.81E+06	4.06E+01	5.29E+16	0.907	-0.423	0.67758	3.90E-04	0.30228	6.36E-14	3.31E-03	1.92E-03	9.15E-03
0.30000	0.1881	9.74E+06	3.99E+01	5.16E+16	0.912	-0.430	0.67902	4.10E-04	0.30084	5.80E-14	3.42E-03	1.80E-03	9.15E-03
0.31000	0.1914	9.63E+06	3.86E+01	4.95E+16	0.919	-0.446	0.68129	4.45E-04	0.29856	4.98E-14	3.57E-03	1.62E-03	9.15E-03
0.32000	0.1948	9.52E+06	3.74E+01	4.75E+16	0.926	-0.459	0.68339	4.84E-04	0.29644	4.27E-14	3.69E-03	1.48E-03	9.15E-03
0.33000	0.1981	9.41E+06	3.63E+01	4.55E+16	0.933	-0.473	0.68535	5.26E-04	0.29445	3.67E-14	3.79E-03	1.37E-03	9.15E-03
0.34000	0.2014	9.31E+06	3.51E+01	4.36E+16	0.939	-0.488	0.68716	5.71E-04	0.29261	3.15E-14	3.86E-03	1.28E-03	9.15E-03
0.35000	0.2047	9.20E+06	3.40E+01	4.18E+16	0.945	-0.502	0.68885	6.20E-04	0.29088	2.71E-14	3.92E-03	1.21E-03	9.15E-03
0.36000	0.2080	9.10E+06	3.29E+01	4.01E+16	0.950	-0.517	0.69042	6.74E-04	0.28927	2.33E-14	3.97E-03	1.16E-03	9.15E-03

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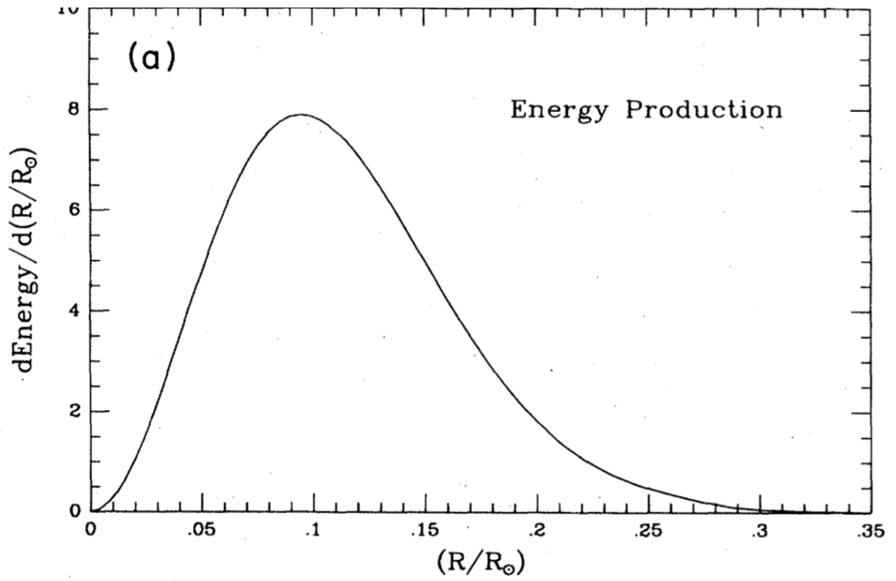
Bahcall & Ulrich (1988)

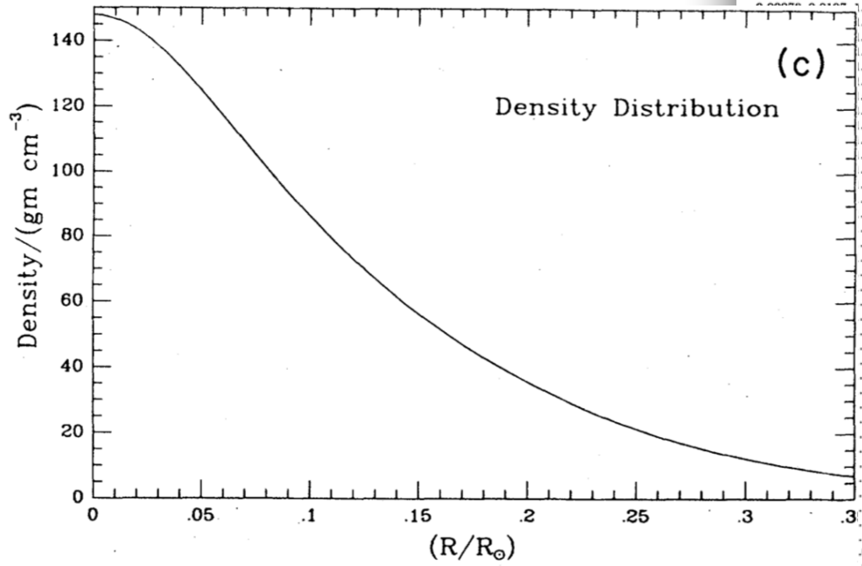
- And these boundary conditions:
  - In the centre (r=0): $M_r(0) = 0, \quad L_r(0) = 0$
  - At the surface (r=R): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{eff}^4$
- Note that the **chemical composition** affects the density and thus the stratification via the mean molecular weight μ as function of radius (see equation of state)

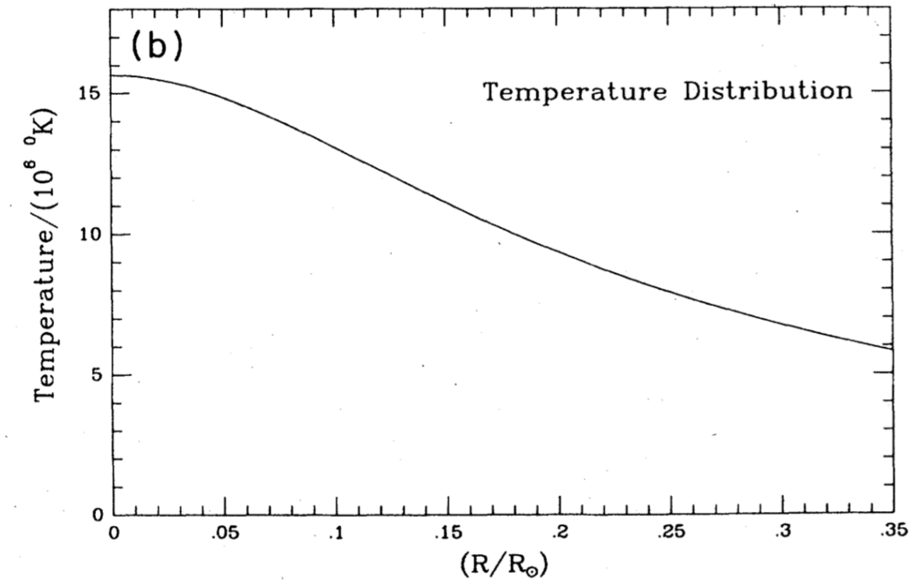
Stellar interior

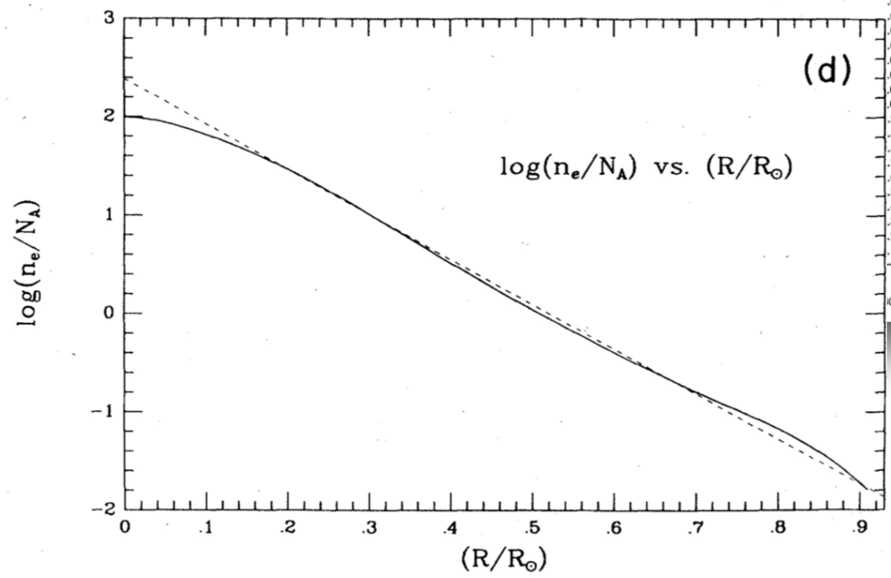
Equations of stellar structure — Recap

- The interior structure of a star in equilibrium can be described with the following “ingredients”"
- Variables and their stratification (function of radius) P, ϱ, T, M_r, L_r
- **Solution: Standard Model** (for the Sun)
  assumes chemical composition [X:Y:Z] =[0.73:0.25:0.015]









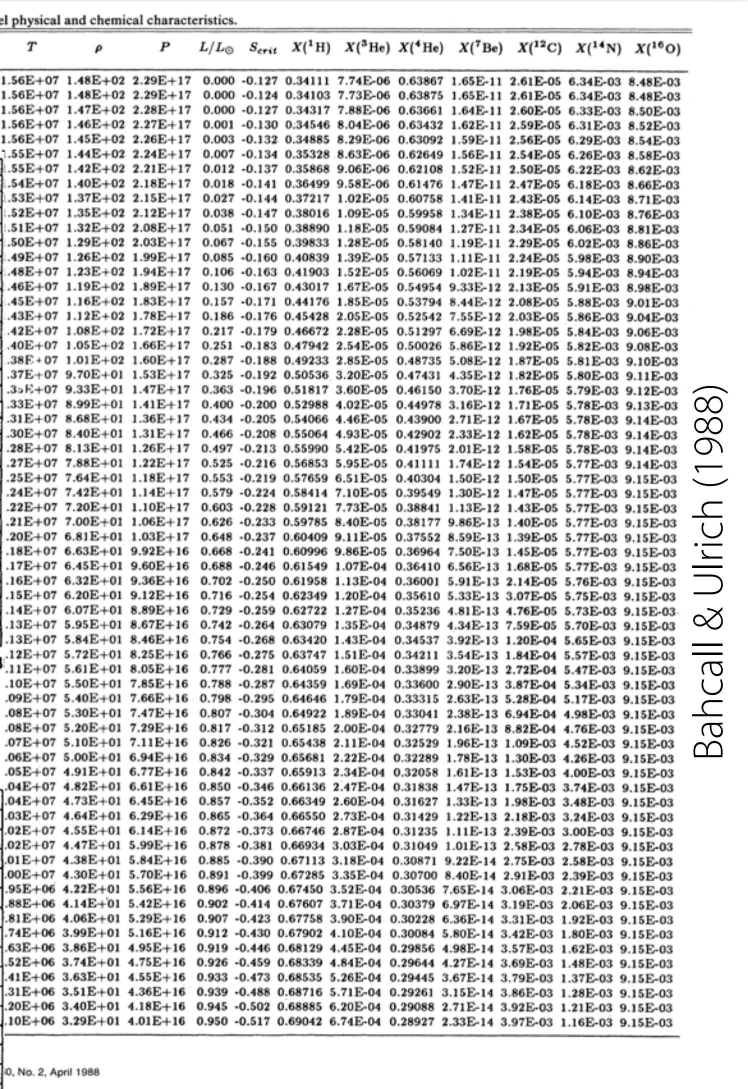


TABLE X. Model physical and chemical characteristics.

$M/M_\odot$	$R/R_\odot$	T	ρ	P	$L/L_\odot$	S_{crit}	$X(^1\mathrm{H})$	$X(^3\mathrm{He})$	$X(^4\mathrm{He})$	$X(^7\mathrm{Be})$	$X(^{12}\mathrm{C})$	$X(^{14}\mathrm{N})$	$X(^{16}\mathrm{O})$
0.00000	0.0000	1.56E+07	1.48E+02	2.29E+17	0.000	-0.127	0.34111	7.74E-06	0.63867	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00001	0.0039	1.56E+07	1.48E+02	2.29E+17	0.000	-0.124	0.34103	7.73E-06	0.63875	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00005	0.0083	1.56E+07	1.47E+02	2.28E+17	0.000	-0.127	0.34317	7.88E-06	0.63661	1.64E-11	2.60E-05	6.33E-03	8.50E-03
0.00017	0.0120	1.56E+07	1.46E+02	2.27E+17	0.001	-0.130	0.34546	8.04E-06	0.63432	1.62E-11	2.59E-05	6.31E-03	8.52E-03
0.00040	0.0158	1.56E+07	1.45E+02	2.26E+17	0.003	-0.132	0.34885	8.29E-06	0.63092	1.59E-11	2.56E-05	6.29E-03	8.54E-03
		.55E+07	1.44E+02	2.24E+17	0.007	-0.134	0.35328	8.63E-06	0.62649	1.56E-11	2.54E-05	6.26E-03	8.58E-03
		.55E+07	1.42E+02	2.21E+17	0.012	-0.137	0.35868	9.06E-06	0.62108	1.52E-11	2.50E-05	6.22E-03	8.62E-03
		.54E+07	1.40E+02	2.18E+17	0.018	-0.141	0.36499	9.58E-06	0.61476	1.47E-11	2.47E-05	6.18E-03	8.66E-03
		.53E+07	1.37E+02	2.15E+17	0.027	-0.144	0.37217	1.02E-05	0.60758	1.41E-11	2.43E-05	6.14E-03	8.71E-03
		.52E+07	1.35E+02	2.12E+17	0.038	-0.147	0.38016	1.09E-05	0.59958	1.34E-11	2.38E-05	6.10E-03	8.76E-03
		.51E+07	1.32E+02	2.08E+17	0.051	-0.150	0.38890	1.18E-05	0.59084	1.27E-11	2.34E-05	6.06E-03	8.81E-03
		.50E+07	1.29E+02	2.03E+17	0.067	-0.155	0.39833	1.28E-05	0.58140	1.19E-11	2.29E-05	6.02E-03	8.86E-03
		.49E+07	1.26E+02	1.99E+17	0.085	-0.160	0.40839	1.39E-05	0.57133	1.11E-11	2.24E-05	5.98E-03	8.90E-03
		.48E+07	1.23E+02	1.94E+17	0.106	-0.163	0.41903	1.52E-05	0.56069	1.02E-11	2.19E-05	5.94E-03	8.94E-03
		.46E+07	1.19E+02	1.89E+17	0.130	-0.167	0.43017	1.67E-05	0.54954	9.33E-12	2.13E-05	5.91E-03	8.98E-03
		.45E+07	1.16E+02	1.83E+17	0.157	-0.171	0.44176	1.85E-05	0.53794	8.44E-12	2.08E-05	5.88E-03	9.01E-03
		.43E+07	1.12E+02	1.78E+17	0.186	-0.176	0.45428	2.05E-05	0.52542	7.55E-12	2.03E-05	5.86E-03	9.04E-03
		.42E+07	1.08E+02	1.72E+17	0.217	-0.179	0.46672	2.28E-05	0.51297	6.69E-12	1.98E-05	5.84E-03	9.06E-03
		.40E+07	1.05E+02	1.66E+17	0.251	-0.183	0.47942	2.54E-05	0.50026	5.86E-12	1.92E-05	5.82E-03	9.08E-03
		.38E+07	1.01E+02	1.60E+17	0.287	-0.188	0.49233	2.85E-05	0.48735	5.08E-12	1.87E-05	5.81E-03	9.10E-03
		.37E+07	9.70E+01	1.53E+17	0.325	-0.192	0.50536	3.20E-05	0.47431	4.35E-12	1.82E-05	5.80E-03	9.11E-03
		.35E+07	9.33E+01	1.47E+17	0.363	-0.196	0.51817	3.60E-05	0.46150	3.70E-12	1.76E-05	5.79E-03	9.12E-03
		.33E+07	8.99E+01	1.41E+17	0.400	-0.200	0.52988	4.02E-05	0.44978	3.16E-12	1.71E-05	5.78E-03	9.13E-03
		.31E+07	8.68E+01	1.36E+17	0.434	-0.205	0.54066	4.46E-05	0.43900	2.71E-12	1.67E-05	5.78E-03	9.14E-03
		.30E+07	8.40E+01	1.31E+17	0.466	-0.208	0.55064	4.93E-05	0.42902	2.33E-12	1.62E-05	5.78E-03	9.14E-03
		.28E+07	8.13E+01	1.26E+17	0.497	-0.213	0.55990	5.42E-05	0.41975	2.01E-12	1.58E-05	5.78E-03	9.14E-03
		.27E+07	7.88E+01	1.22E+17	0.525	-0.216	0.56853	5.95E-05	0.41111	1.74E-12	1.54E-05	5.77E-03	9.14E-03
		.25E+07	7.64E+01	1.18E+17	0.553	-0.219	0.57659	6.51E-05	0.40304	1.50E-12	1.50E-05	5.77E-03	9.15E-03
		.24E+07	7.42E+01	1.14E+17	0.579	-0.224	0.58414	7.10E-05	0.39549	1.30E-12	1.47E-05	5.77E-03	9.15E-03
		.22E+07	7.20E+01	1.10E+17	0.603	-0.228	0.59121	7.73E-05	0.38841	1.13E-12	1.43E-05	5.77E-03	9.15E-03
		.21E+07	7.00E+01	1.06E+17	0.626	-0.233	0.59785	8.40E-05	0.38177	9.86E-13	1.40E-05	5.77E-03	9.15E-03
		.20E+07	6.81E+01	1.03E+17	0.648	-0.237	0.60409	9.11E-05	0.37552	8.59E-13	1.39E-05	5.77E-03	9.15E-03
		.18E+07	6.63E+01	9.92E+16	0.668	-0.241	0.60996	9.86E-05	0.36964	7.50E-13	1.45E-05	5.77E-03	9.15E-03
		.17E+07	6.45E+01	9.60E+16	0.688	-0.246	0.61549	1.07E-04	0.36410	6.56E-13	1.68E-05	5.77E-03	9.15E-03
		.16E+07	6.32E+01	9.36E+16	0.702	-0.250	0.61958	1.13E-04	0.36001	5.91E-13	2.14E-05	5.76E-03	9.15E-03
		.15E+07	6.20E+01	9.12E+16	0.716	-0.254	0.62349	1.20E-04	0.35610	5.33E-13	3.07E-05	5.75E-03	9.15E-03
		.14E+07	6.07E+01	8.89E+16	0.729	-0.259	0.62722	1.27E-04	0.35236	4.81E-13	4.76E-05	5.73E-03	9.15E-03
		.13E+07	5.95E+01	8.67E+16	0.742	-0.264	0.63079	1.35E-04	0.34879	4.34E-13	7.59E-05	5.70E-03	9.15E-03
		.13E+07	5.84E+01	8.46E+16	0.754	-0.268	0.63420	1.43E-04	0.34537	3.92E-13	1.20E-04	5.65E-03	9.15E-03
		.12E+07	5.72E+01	8.25E+16	0.766	-0.275	0.63747	1.51E-04	0.34211	3.54E-13	1.84E-04	5.57E-03	9.15E-03
		.11E+07	5.61E+01	8.05E+16	0.777	-0.281	0.64059	1.60E-04	0.33899	3.20E-13	2.72E-04	5.47E-03	9.15E-03
		.10E+07	5.50E+01	7.85E+16	0.788	-0.287	0.64359	1.69E-04	0.33600	2.90E-13	3.87E-04	5.34E-03	9.15E-03
		.09E+07	5.40E+01	7.66E+16	0.798	-0.295	0.64646	1.79E-04	0.33315	2.63E-13	5.28E-04	5.17E-03	9.15E-03
		.08E+07	5.30E+01	7.47E+16	0.807	-0.304	0.64922	1.89E-04	0.33041	2.38E-13	6.94E-04	4.98E-03	9.15E-03
		.08E+07	5.20E+01	7.29E+16	0.817	-0.312	0.65185	2.00E-04	0.32779	2.16E-13	8.82E-04	4.76E-03	9.15E-03
		.07E+07	5.10E+01	7.11E+16	0.826	-0.321	0.65438	2.11E-04	0.32529	1.96E-13	1.09E-03	4.52E-03	9.15E-03
		.06E+07	5.00E+01	6.94E+16	0.834	-0.329	0.65681	2.22E-04	0.32289	1.78E-13	1.30E-03	4.26E-03	9.15E-03
		.05E+07	4.91E+01	6.77E+16	0.842	-0.337	0.65913	2.34E-04	0.32058	1.61E-13	1.53E-03	4.00E-03	9.15E-03
		.04E+07	4.82E+01	6.61E+16	0.850	-0.346	0.66136	2.47E-04	0.31838	1.47E-13	1.75E-03	3.74E-03	9.15E-03
		.04E+07	4.73E+01	6.45E+16	0.857	-0.352	0.66349	2.60E-04	0.31627	1.33E-13	1.98E-03	3.48E-03	9.15E-03
		.03E+07	4.64E+01	6.29E+16	0.865	-0.364	0.66550	2.73E-04	0.31429	1.22E-13	2.18E-03	3.24E-03	9.15E-03
		.02E+07	4.55E+01	6.14E+16	0.872	-0.373	0.66746	2.87E-04	0.31235	1.11E-13	2.39E-03	3.00E-03	9.15E-03
		.02E+07	4.47E+01	5.99E+16	0.878	-0.381	0.66934	3.03E-04	0.31049	1.01E-13	2.58E-03	2.78E-03	9.15E-03
		.01E+07	4.38E+01	5.84E+16	0.885	-0.390	0.67113	3.18E-04	0.30871	9.22E-14	2.75E-03	2.58E-03	9.15E-03
		.00E+07	4.30E+01	5.70E+16	0.891	-0.399	0.67285	3.35E-04	0.30700	8.40E-14	2.91E-03	2.39E-03	9.15E-03
		.95E+06	4.22E+01	5.56E+16	0.896	-0.406	0.67450	3.52E-04	0.30536	7.65E-14	3.06E-03	2.21E-03	9.15E-03
		.88E+06	4.14E+01	5.42E+16	0.902	-0.414	0.67607	3.71E-04	0.30379	6.97E-14	3.19E-03	2.06E-03	9.15E-03
		.81E+06	4.06E+01	5.29E+16	0.907	-0.423	0.67758	3.90E-04	0.30228	6.36E-14	3.31E-03	1.92E-03	9.15E-03
		.74E+06	3.99E+01	5.16E+16	0.912	-0.430	0.67902	4.10E-04	0.30084	5.80E-14	3.42E-03	1.80E-03	9.15E-03
		.63E+06	3.86E+01	4.95E+16	0.919	-0.446	0.68129	4.45E-04	0.29856	4.98E-14	3.57E-03	1.62E-03	9.15E-03
		.52E+06	3.74E+01	4.75E+16	0.926	-0.459	0.68339	4.84E-04	0.29644	4.27E-14	3.69E-03	1.48E-03	9.15E-03
		.41E+06	3.63E+01	4.55E+16	0.933	-0.473	0.68535	5.26E-04	0.29445	3.67E-14	3.79E-03	1.37E-03	9.15E-03
		.31E+06	3.51E+01	4.36E+16	0.939	-0.488	0.68716	5.71E-04	0.29261	3.15E-14	3.86E-03	1.28E-03	9.15E-03
		.20E+06	3.40E+01	4.18E+16	0.945	-0.502	0.68885	6.20E-04	0.29088	2.71E-14	3.92E-03	1.21E-03	9.15E-03
		.10E+06	3.29E+01	4.01E+16	0.950	-0.517	0.69042	6.74E-04	0.28927	2.33E-14	3.97E-03	1.16E-03	9.15E-03

0, No. 2, April 1988

Bahcall & Ulrich (1988)

Stellar interior

Equations of stellar structure — Recap

- The interior structure of a star in equilibrium can be described with the following “ingredients””
- Variables and their stratification (function of radius) P, ϱ, T, M_r, L_r
- The following equations :

Hydro**dynamic** equilibrium	$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} - \rho\frac{d^2r}{dt^2}$
Mass conservation/continuity	$\frac{dM_r}{dr} = 4\pi r^2\rho$
Equation of state	$P = \frac{\rho kT}{\mu m_H}$
Energy “generation”	$\frac{dL_r}{dr} = 4\pi r^2\rho\left[\varepsilon - T\frac{dS}{dt}\right]$
Energy transport	$\frac{dT}{dr} = -\frac{3\kappa\rho}{16\pi ac}\frac{L_r}{r^2T^2}$

More general, **time-dependent form**

Stellar structure is time-dependent and will adjust

➡ S: entropy

➡ T (dS/dt) is the energy of collapse expressed in terms of the entropy change

- And these boundary conditions:
  - In the centre (r=0): $M_r(0) = 0$, $L_r(0) = 0$
  - At the surface (r=R): $M_r(R) = M$, $L_r(R) = L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$

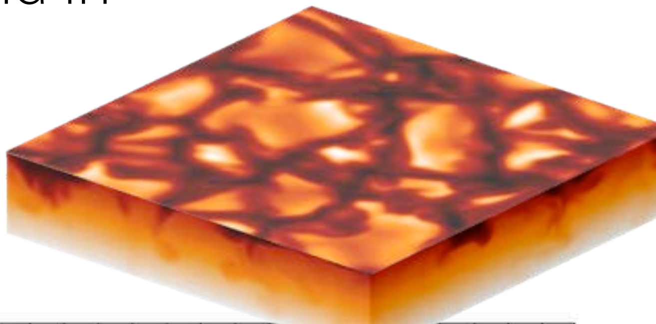
- Note that the mean molecular weight μ and the opacity $\varkappa$ are now functions of radius and time (due to fusion in the core)

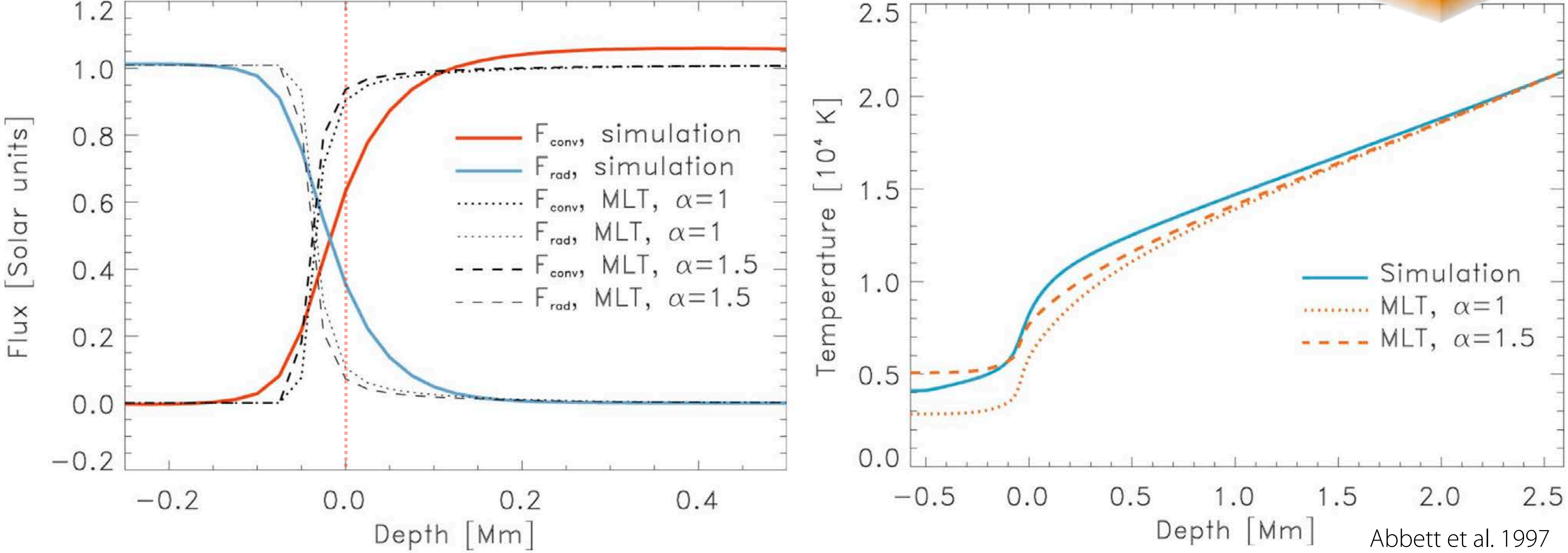
Energy transport

Recap

- Radiative energy transport = default (in a stable layer)
- Convective energy transport very efficient, occurs if a layer is unstable against convection
- For stellar interior structure/evolution simplified mixing length theory valid in good approximation ($l_m = \alpha H_P$)
- Calibration of α from comparison of detailed 3D numerical models

$$\boxed{\nabla_{\text{rad}} < \nabla_{\text{ad}} - \frac{\chi_\mu}{\chi_T} \nabla_\mu}$$





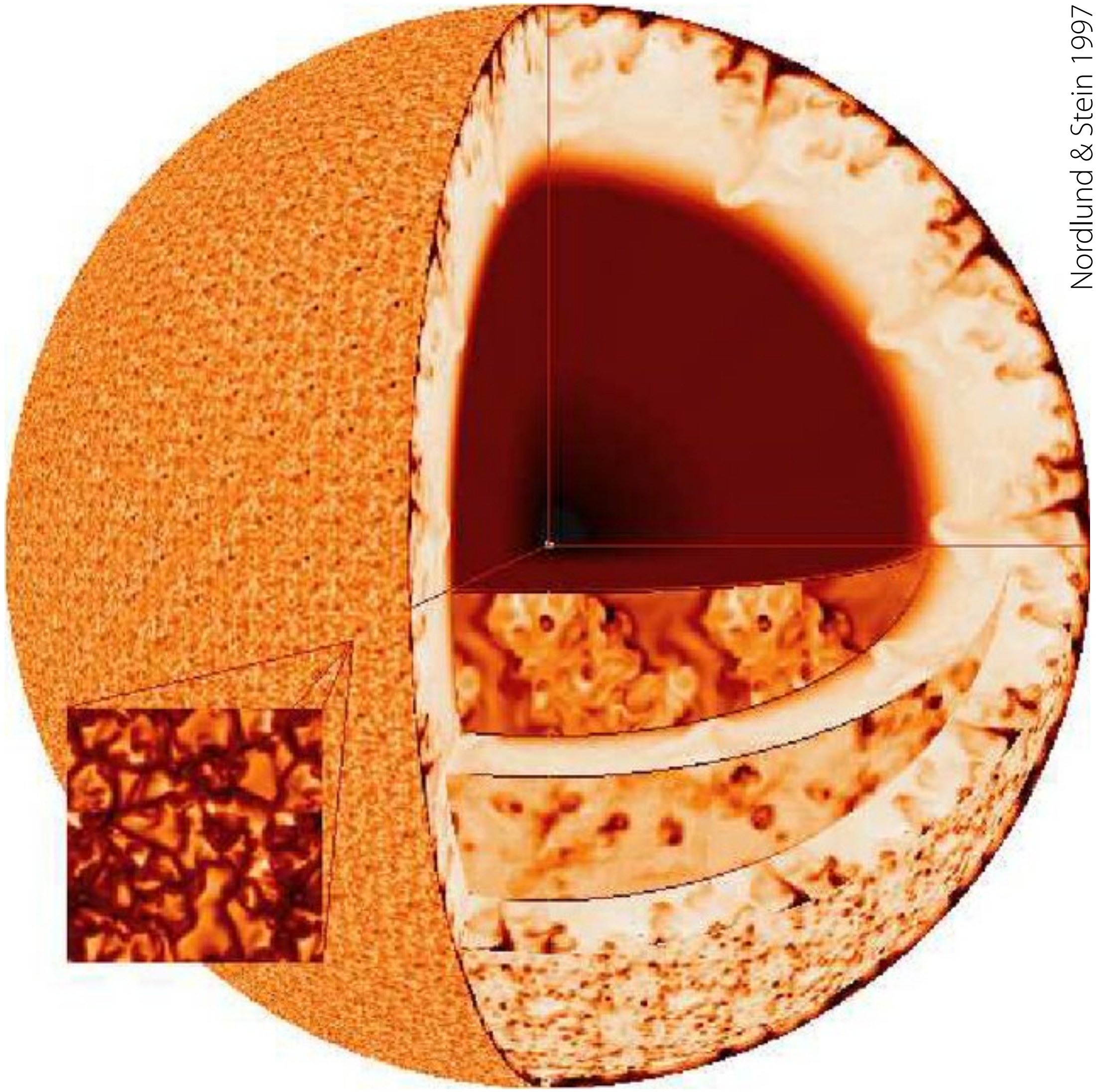
Abbett et al. 1997

Convection

Recap

- Convective energy transport very efficient
- Gradients decisive for convective (in)stability
  - Outer convection zone in the Sun at $r > 0.7\ R_\odot$
- Properties of convection set by local conditions
  - Change with radius/depth due to the larger decrease of temperature, pressure, density
  - Local pressure scale height important
  - Convection cell diameter ~2 Mm near solar surface but much larger deeper in the convection zone
  - Turnover time scale in the Sun between 200s at surface and 25 days at bottom of convection zone
  - Detailed treatment of convection complex and numerically challenging; mixing length theory sufficient in the context of stellar evolution

$$H_P = P/(\varrho g)$$



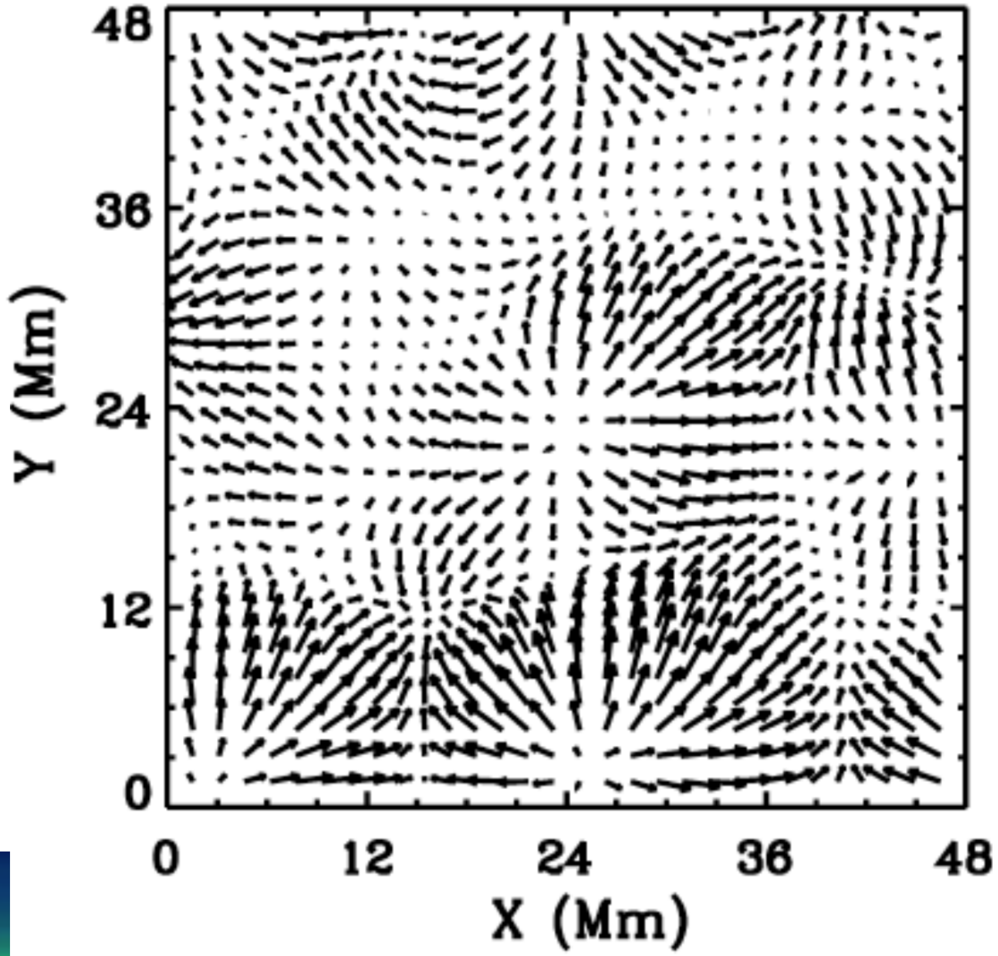
Nordlund & Stein 1997

Convection

Flow field

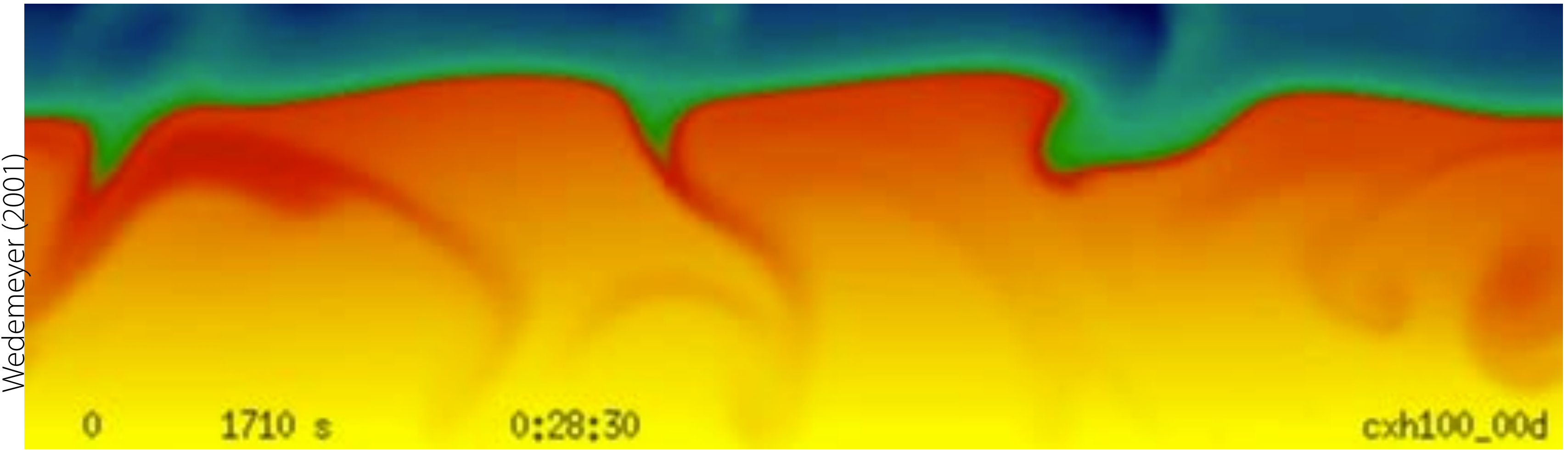
- Hot gas elements rise upwards and diverge, typically within 1-2 pressure scale heights
- Downflowing material denser, can form turbulent plumes

- Velocity field in a horizontal cross-section through a 3D simulation
- Note the flow divergence!



Nordlund et al (2009

- Vertical cross-section at the top of the convection zone — “surface” (photosphere)



Wedemeyer (2001)

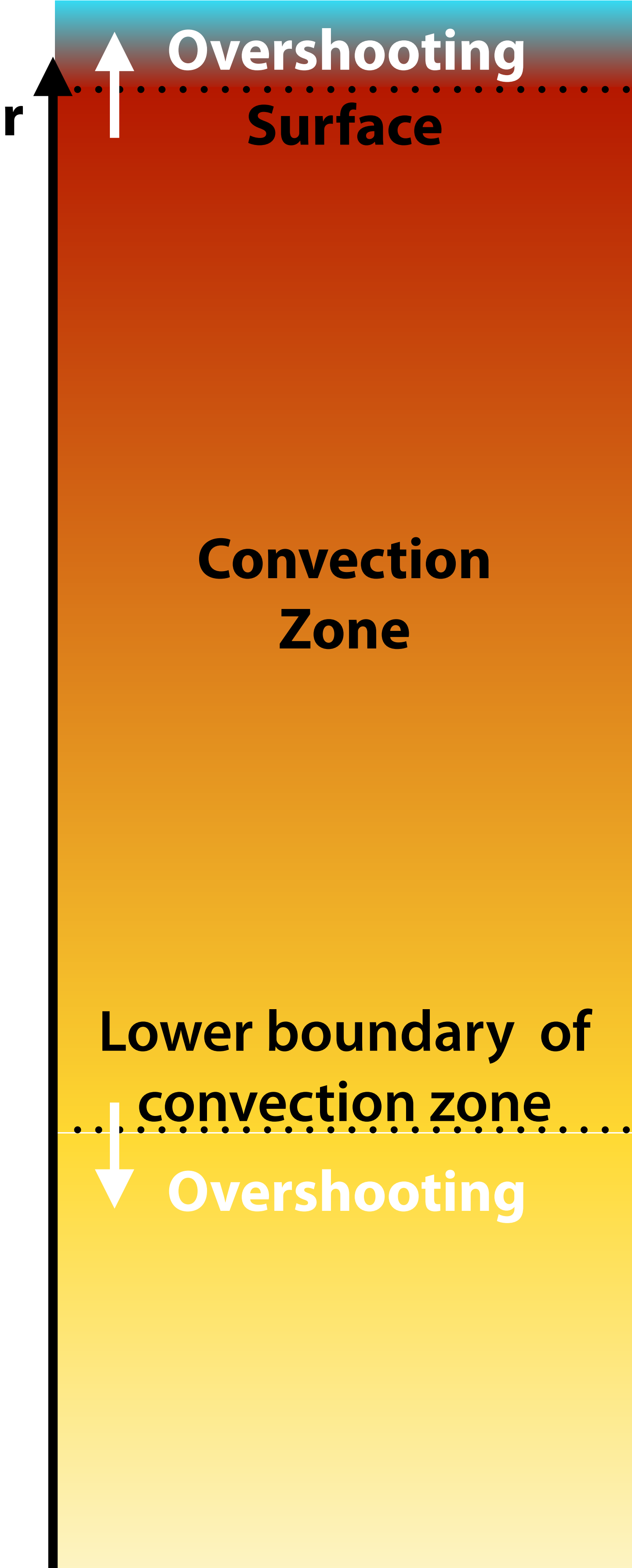
Convection

Convective overshooting

- Chemically homogeneous layer
  (Schwarzschild criterion: $\nabla_{rad} < \nabla_{ad}$):
  - Convection zone stops where $\nabla_{rad} = \nabla_{ad}$
  - There, acceleration due to buoyancy force goes to zero:
    $a \approx g\,(\nabla - \nabla_{ad}) \longrightarrow a=0$
  - Beyond at point, any gas element braked, resulting into convectively stable layer
  - BUT: Gas elements reaching that location have on average still **inertia** and will (on average) **overshoot the boundary by some distance**
  - Overshooting by less than a pressure scale height

➡ Schwarzschild criterion determines boundary of convection zone quite accurately

◉ **IMPORTANT:** No net mass transport but convection **mixes** gas in neighbouring layers!



Convection

Convective overshooting

- **Chemically inhomogeneous layer** with $\nabla_{ad} \neq 0$ (Ledoux criterion: $\nabla_{\rm rad} < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T} \nabla_\mu$)
  - Convective mixing decreases $|\nabla - \nabla_{\rm ad}|$ and ∇_μ.

  ➡ Effective buoyancy force increases

  ➡ Positive feedback loop can develop with highly non-linear increase

  ➡ Overshooting gas elements penetrate further and further.
  - Resulting overshoot distance is very uncertain and could be substantial.

- **Mixing due to convective overshooting important for stellar evolution.**
- In the core:
  - Convectively mixed core has “access” to a larger fuel supply for nuclear burning

  ➡ Affects hydrogen-burning lifetime

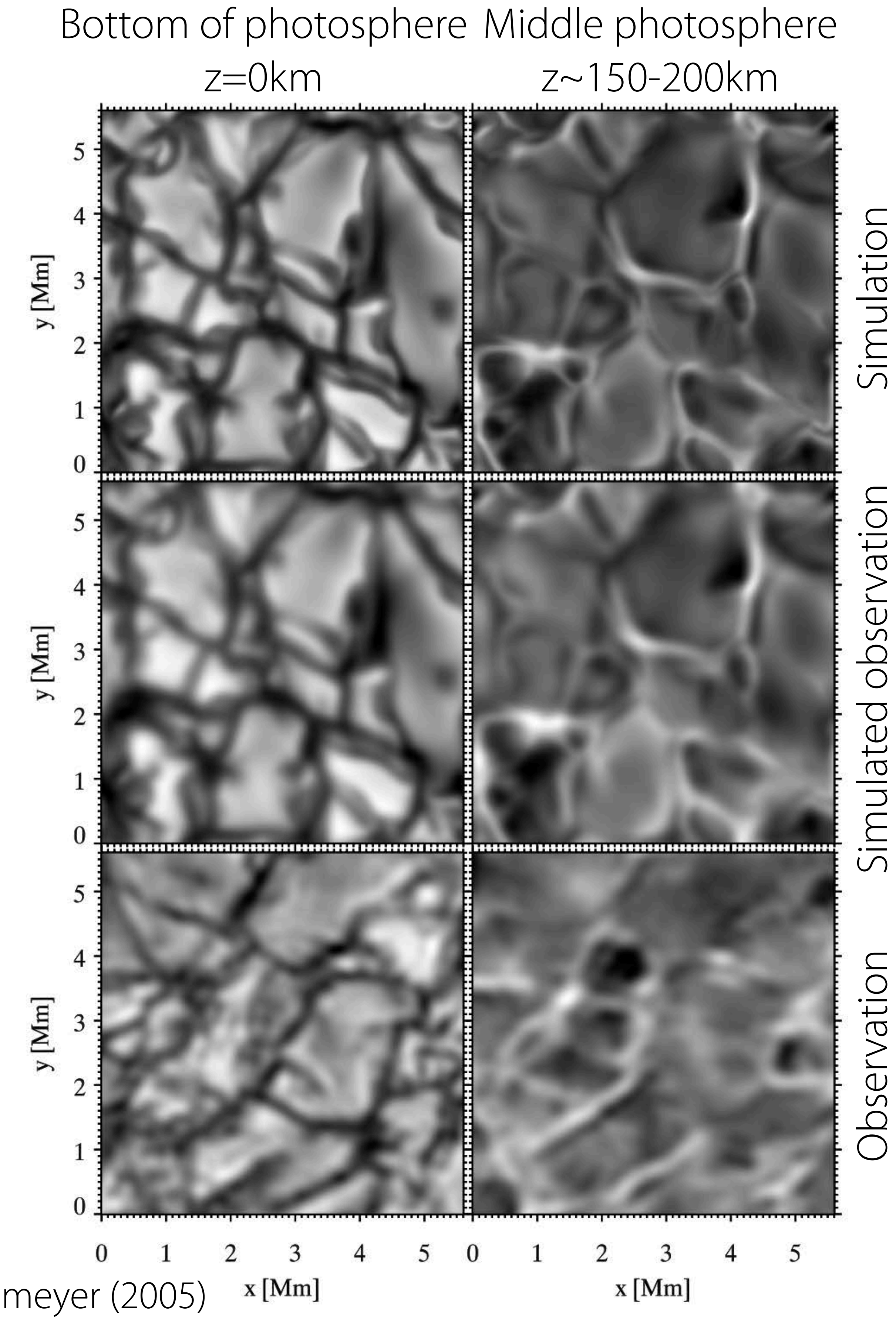
  ➡ Affects further evolution of that star

- Overshooting only taken into account in parameterised form, needs calibration for more detailed modelling

Convection

Convective overshooting

- In the Sun: Convective overshooting at the surface
- Overshooting into the stably (subadiabatically) stratified photosphere.
- Radiative losses make the overshooting material relatively cold and dense.
- Cooling by adiabatic expansion version radiative heating (towards radiative equilibrium from the surrounding)

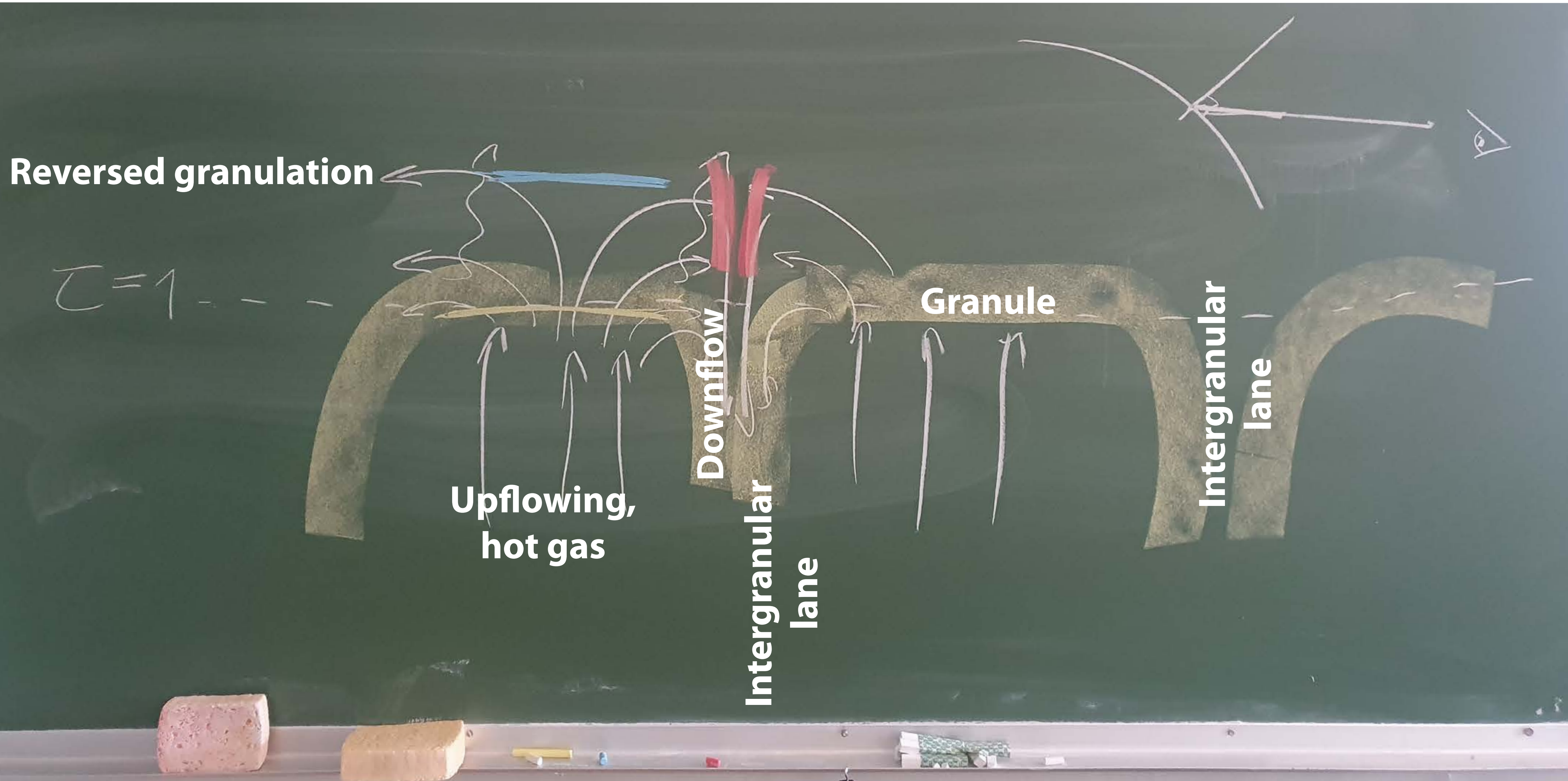


Leenaarts & Wedemeyer (2005)

Convection

Convective overshooting

- Granulation pattern in the low photosphere (where optical depth τ ~ 1)
- Reversed granulation pattern in the middle photosphere



Convection

Convective overshooting

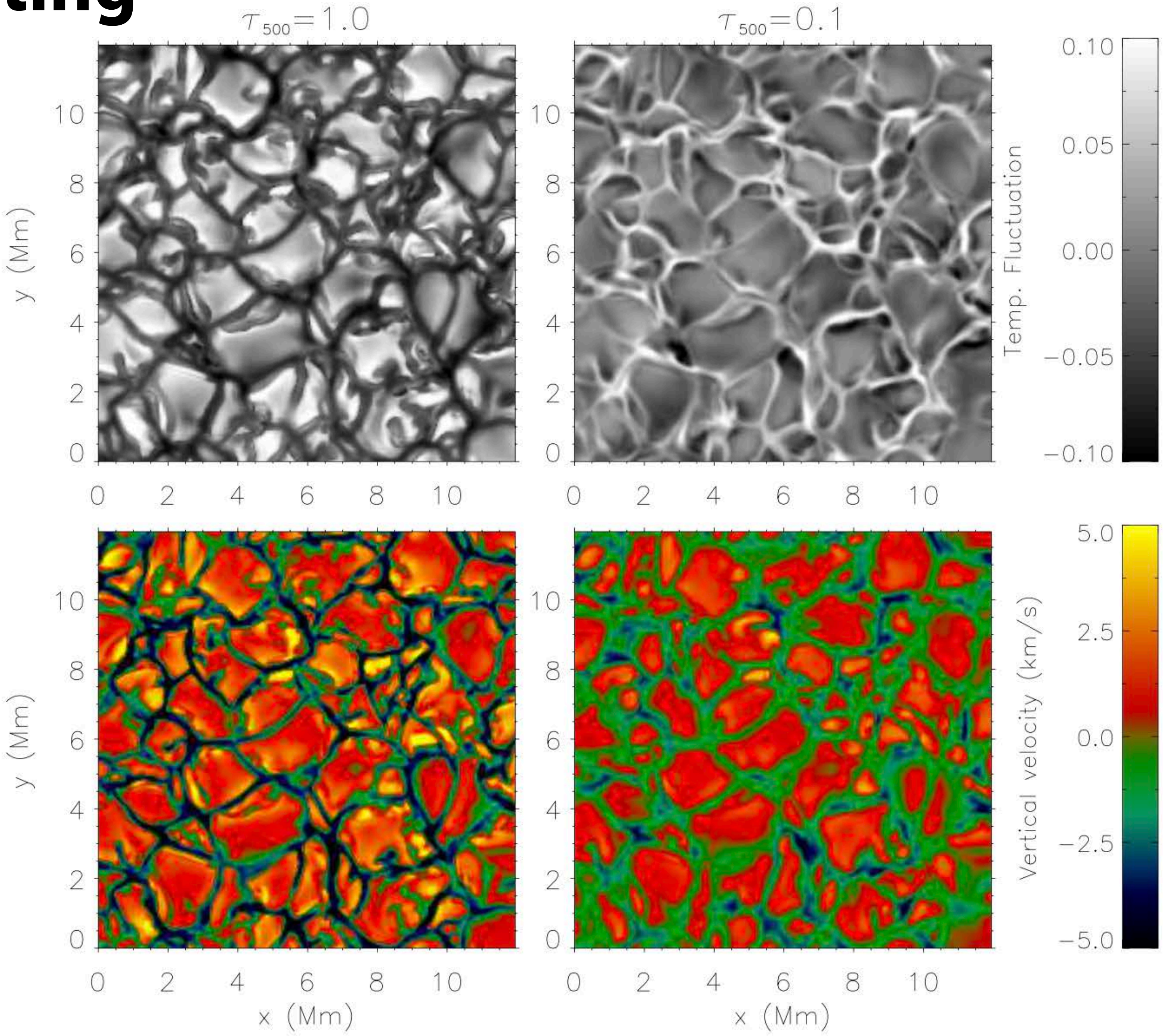


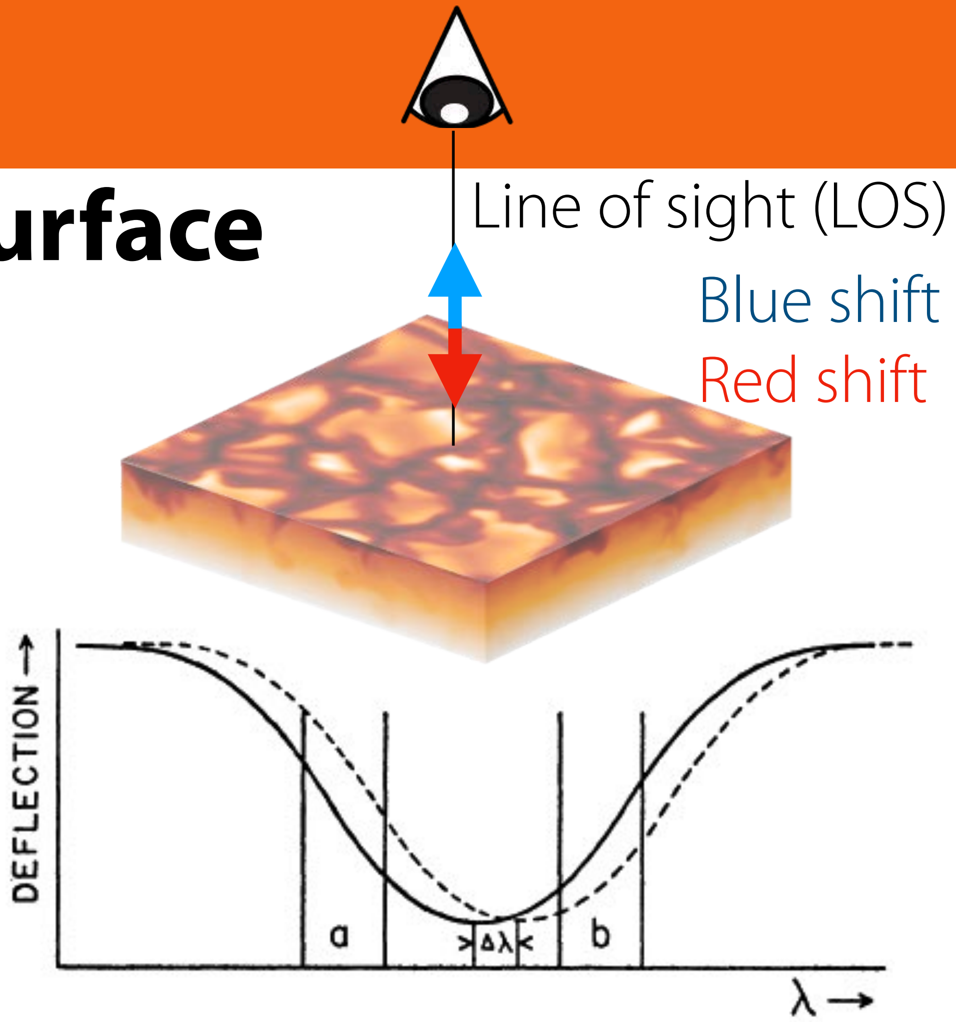
Figure 3: Temperature fluctuations ($\Delta T/\bar{T}$, upper row) and vertical velocity (lower row) at two surfaces of constant optical depth. The left column shows the patterns at $\tau_{500} = 1$, the right $\tau_{500} = 0.1$. The reversed granulation corresponds to a reversal of the temperature fluctuations while the velocity pattern remains qualitatively unchanged.

Cheung et al. (2006)

Convection

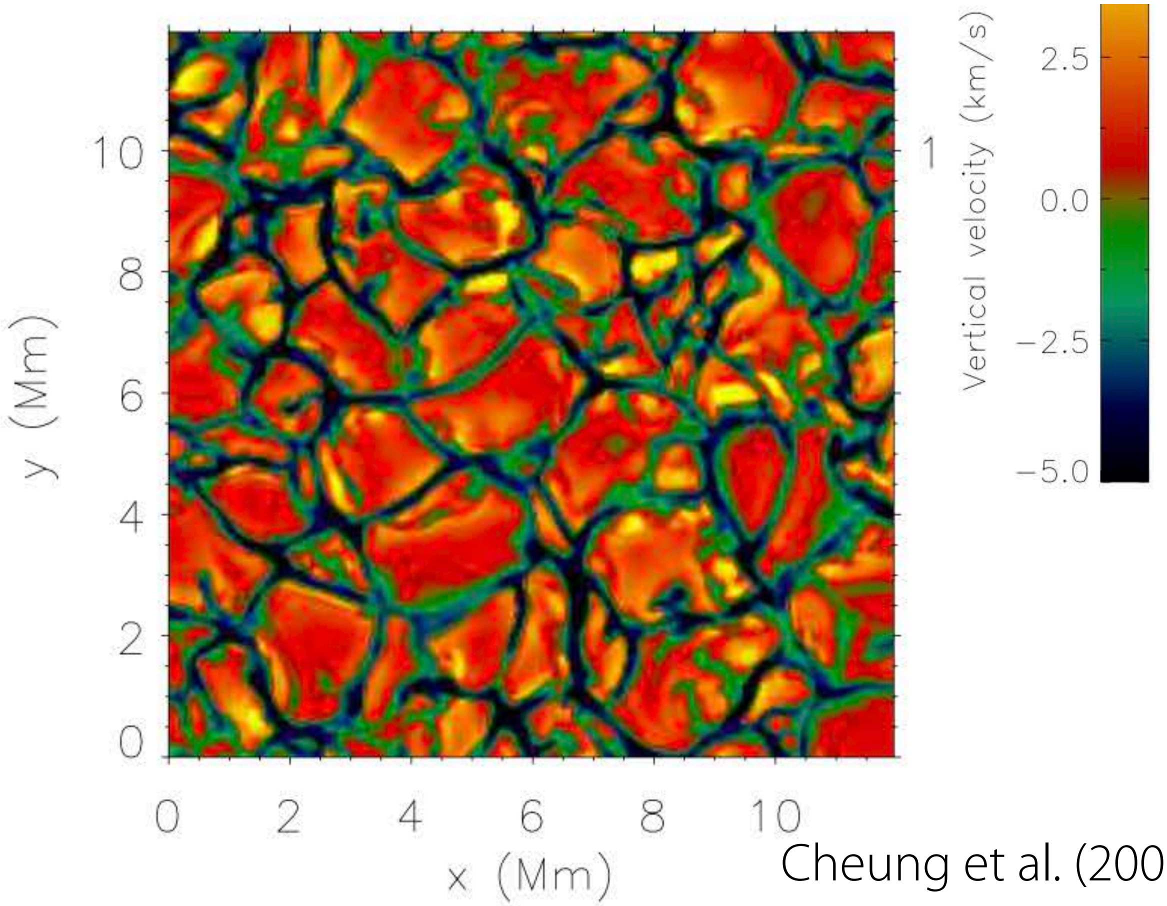
Velocity field — observable at the surface

- Granules visible in intensity but also in vertical velocity
- Vertical velocity (at solar disk-centre, observing from above) can be determined from **Doppler shifts** of spectral line cores that are formed deep in the photosphere
- Horizontal velocities can be determined by tracing the horizontal motion of granules (or even smaller feature): **local correlation tracking**
- Be aware of **projection effects** when not looking at the centre of the disk


Line of sight (LOS)

Blue shift

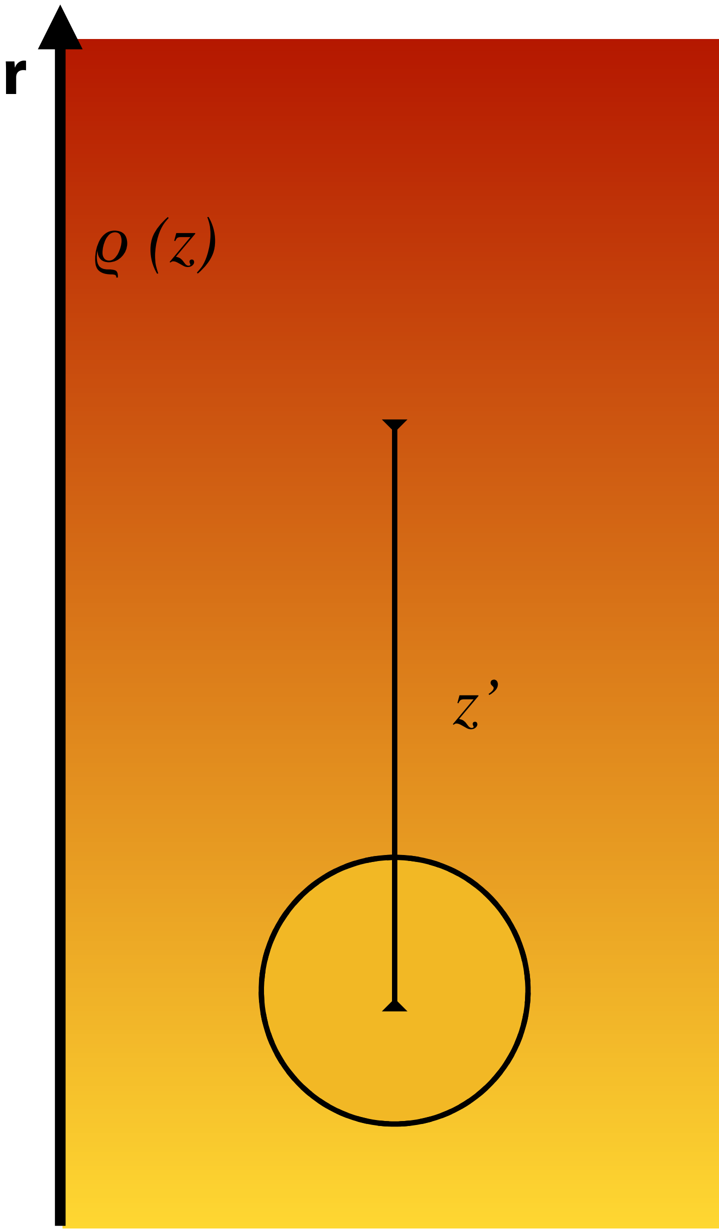
Red shift



Cheung et al. (2006)

Convection

Stability against convection



- Stability of a vertically displaced gas element against convection can be evaluated in terms of the **Brunt–Väisälä frequency** (buoyancy frequency)
  - Frequency at which a vertically displaced gas element oscillates in a convectively stable layer
- Gravity as restoring force (working against upwards displacement)
  - gravitational acceleration:

$$g = -\rho_0^{-1} dP_0/dr.$$

- Now: vertical displacement by z'

➡ $$\rho_0 \frac{\partial^2 z'}{\partial t^2} = -g\left[\rho(z) - \rho(z+z')\right]$$

➡ $$\frac{\partial^2 z'}{\partial t^2} = \frac{g}{\rho_0} \frac{\partial \rho(z)}{\partial z} z'$$

- Solutions of the form $z' = z'_0 e^{i\sqrt{N^2}t}$ with **Brunt–Väisälä frequency** N

➡Oscillations may occur depending on N!

Convection

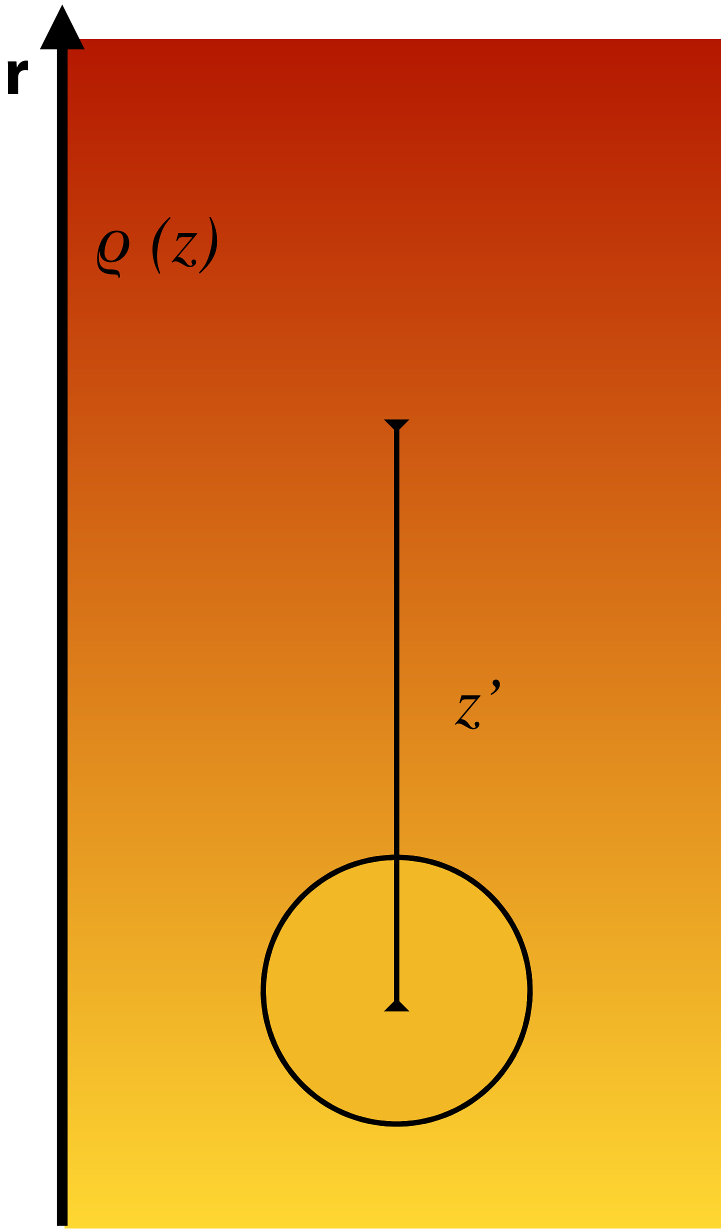
Stability against convection

- **Brunt–Väisälä frequency** N: $$N^2 = g\left(\frac{1}{\Gamma_1 P_0}\frac{dP_0}{dz} - \frac{1}{\rho_0}\frac{d\rho_0}{dz}\right)$$

- $N^2 > 0$: Oscillation around the height where density of surrounding matches density of gas element
- $N^2 = 0$: Gas element in rest after displacement
- $N^2 < 0$: (N imaginary) perturbation leads to instability, run away growth

- Buoyancy vs gravity as restoring force
- Gravity waves — g-modes *(not to be confused with gravitational waves)*

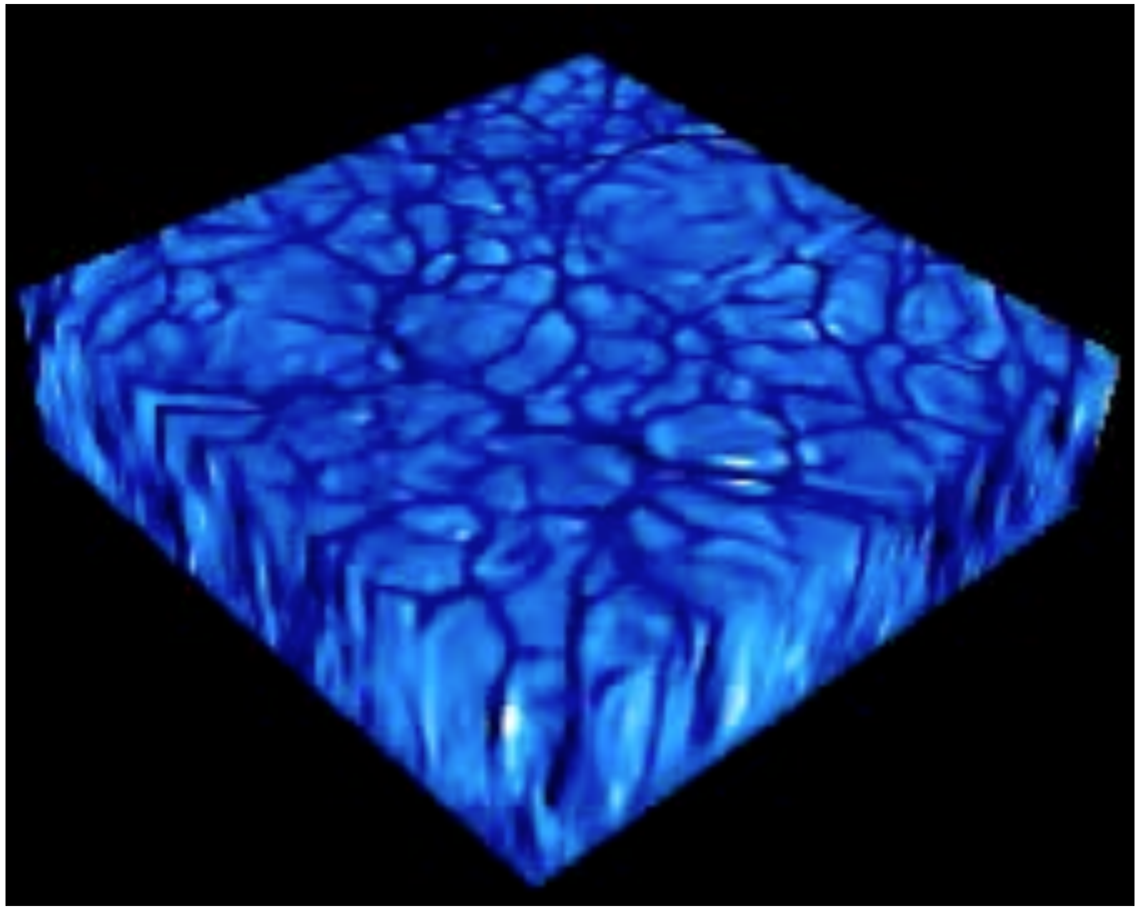
- Condition $N^2 > 0$ is equivalent to Schwarzschild / Ledoux criteria for stability against convection



Stellar interior

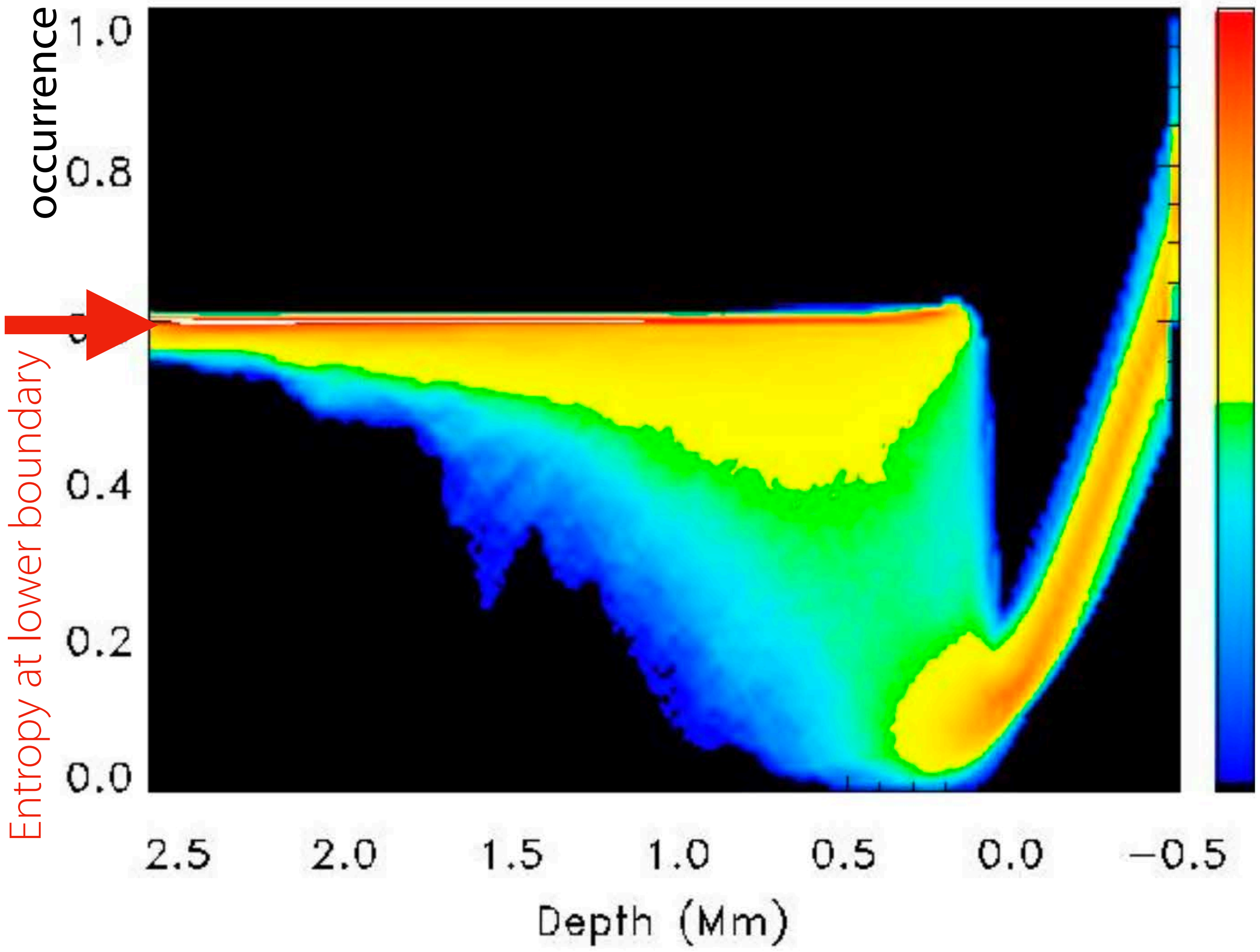
Entropy

- Entropy quite constant in the upper solar convection zone (in contrast to stratification in temperature, pressure etc.)
- Exploited by some simulation codes for the lower boundary (less sensitive to the exact depth, prescribed entropy of gas inflowing through the lower boundary)



Histogram of the entropy

(<u>logarithmic</u> color scale with arbitrary units) as a function of depth

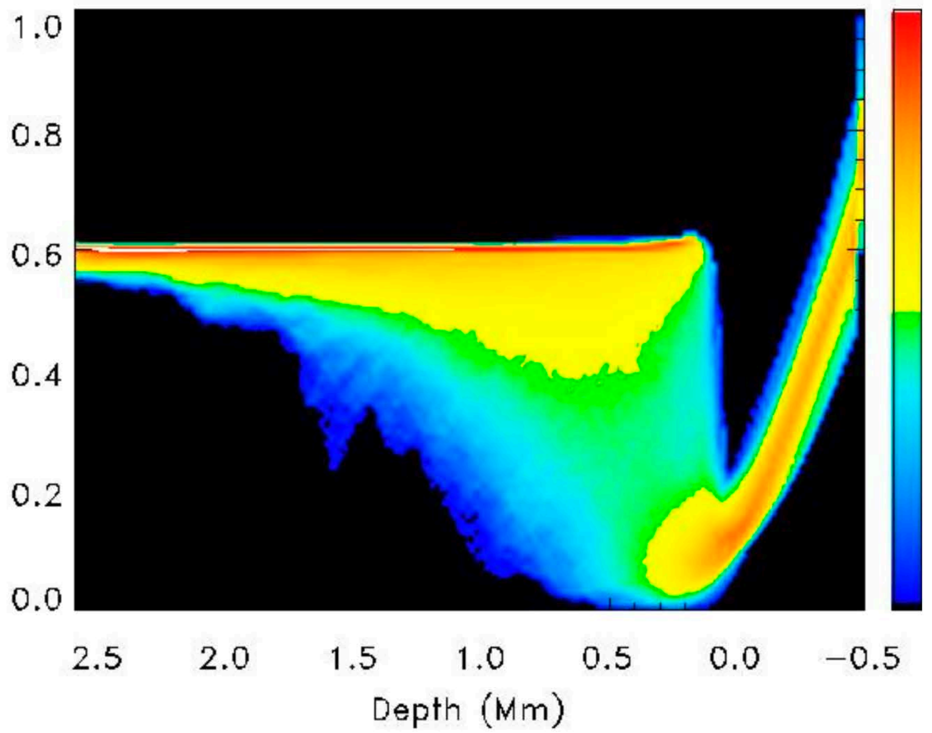


Stellar interior

Entropy

- At the surface: Plasma becomes (more) transparent (longer mean free path of photons)
  ➡ Radiative energy transport becomes efficient
  ➡ Plasma at surface looses thermal energy via radiative cooling
  ➡ Hydrogen ions **recombine** with free electrons to form neutral hydrogen atoms
  ➡ Large amount of ionisation energy set free, also radiated away
  ➡ Escaping photons remove energy and also entropy
  ➡ Resulting overdense fluid sinks back into convection zone (due to gravity)

- Rising plasma in the granules: hot, underdense, high entropy
- Sinking plasma in the intergranular lanes: cool, overdense, low entropy
- Strong impact on entropy at the surface (**entropy jump** + fluctuations) but not so much deeper in the convection zone as diverging upflowing plasma all has almost the same entropy



Stellar interior

Entropy

**difference between actual and adiabatic temperature gradient*

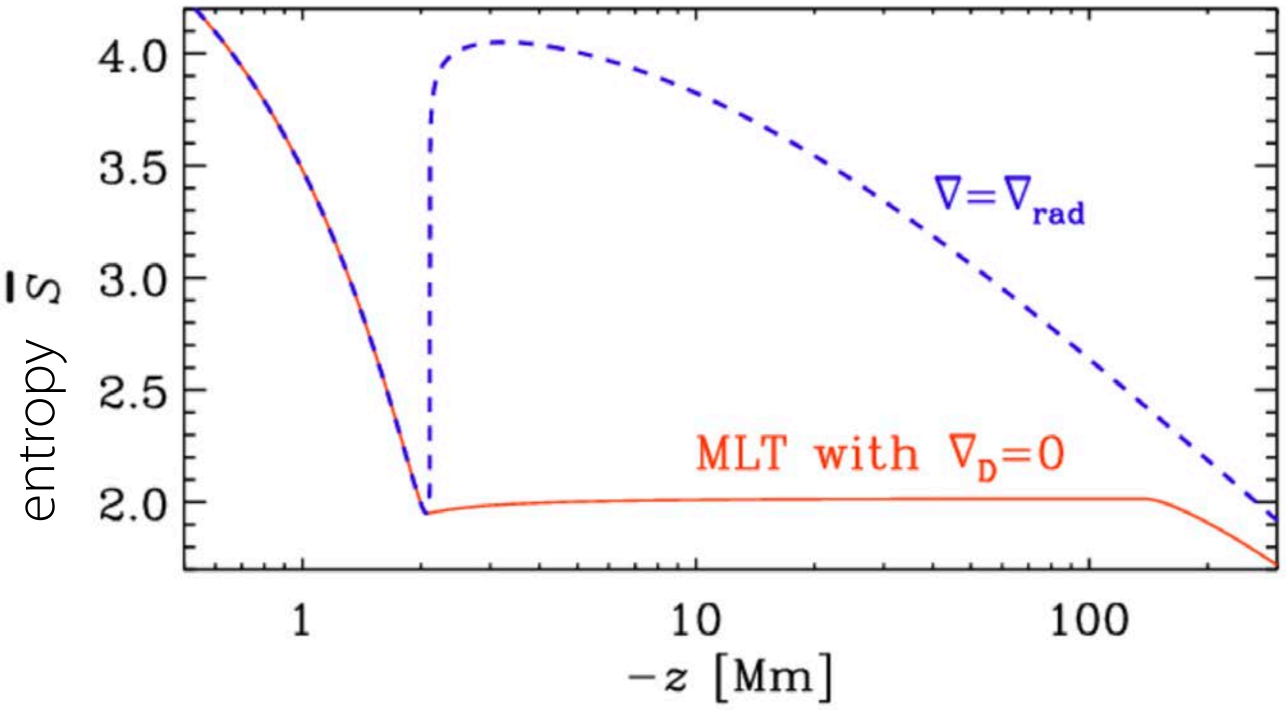
- Convection needs to transport the flux $F = L_\odot / 4 \pi r^2$
- Smaller **superadiabaticity*** $\nabla - \nabla_{ad}$ ⇔ More efficient transport of energy flux by convection
- Related to the excess of specific entropy over the entropy of the marginal state ($\nabla = \nabla_{ad}$)

$$\Delta S = \int c_{\mathrm{P}} (\nabla - \nabla_{\mathrm{a}}) \, d \ln P$$

- ΔS: entropy "jump" across the outermost layers of the convection zone (only there $\nabla = \nabla_{ad}$, significantly larger than zero)

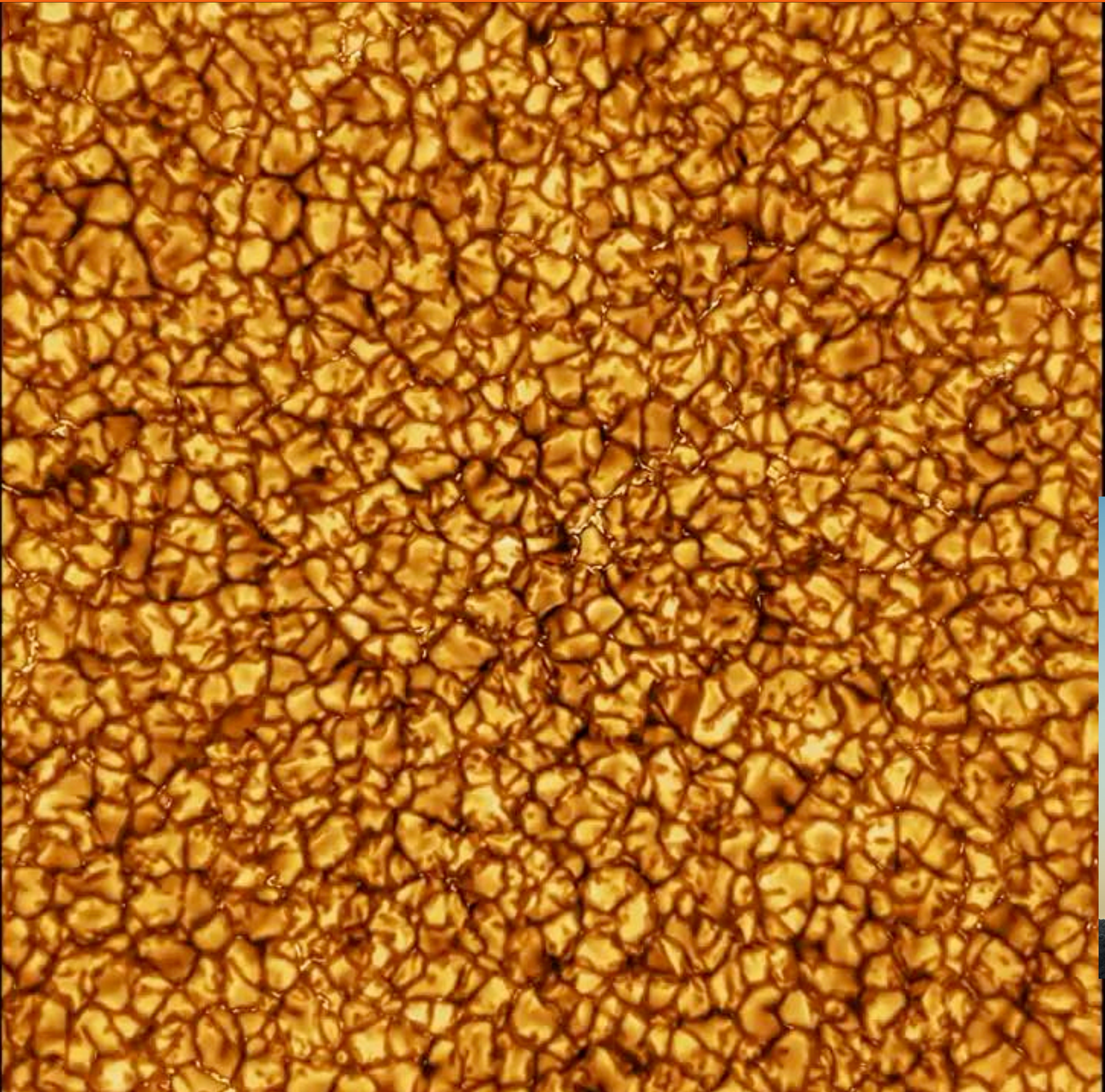
- Note: Efficiency of convection connected to mixing length parameter α
- Detailed properties of convection (including impact of downdraft on entropy) affect the mixing length parameter

Standard convective solution ($\nabla_D = 0$) vs.
nonconvective radiative solution ($\nabla = \nabla_{rad}$)



Brandenburg 2015

Granulation



Highest-resolution observations of the Sun's granulation ever taken. DKIST (4m) (NSO/AURA/NSF)

- Prominent scale: Granulation with 1-2Mm cell diameters

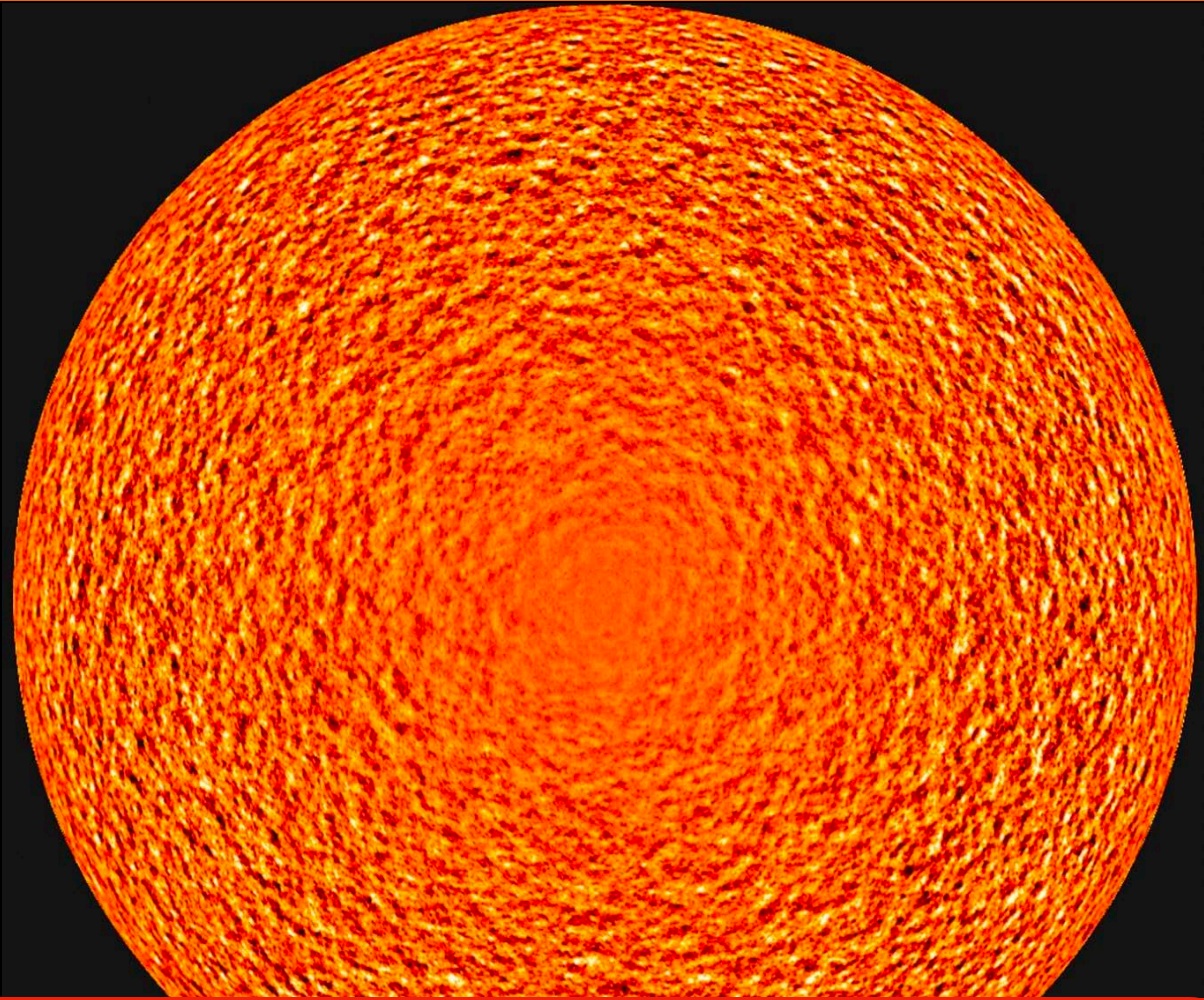
KRajala

10 Dec. 2019 19:24:31 UT

Granulation

Supergranulation

- Dopplergram revealing the supergranulation pattern (credits SOHO/MDI/ESA).
- Prominent scale here: Supergranulation with typical cell diameters of ~30-40 Mm
(wider range 10-70Mm)
- Cell lifetime ~40h
- Typical horizontal velocities ~0.2 km/s
- Convection on larger scales
- But: may only extend 5Mm into convection zone (tbc)



- Supergranulation can be subtle, depending on the way the Sun is observed (wavelength etc.)
- Can be revealed by tracing the horizontal motion of granules / feature (local correlation tracking)
- Observation over such long uninterrupted periods are only possible from space

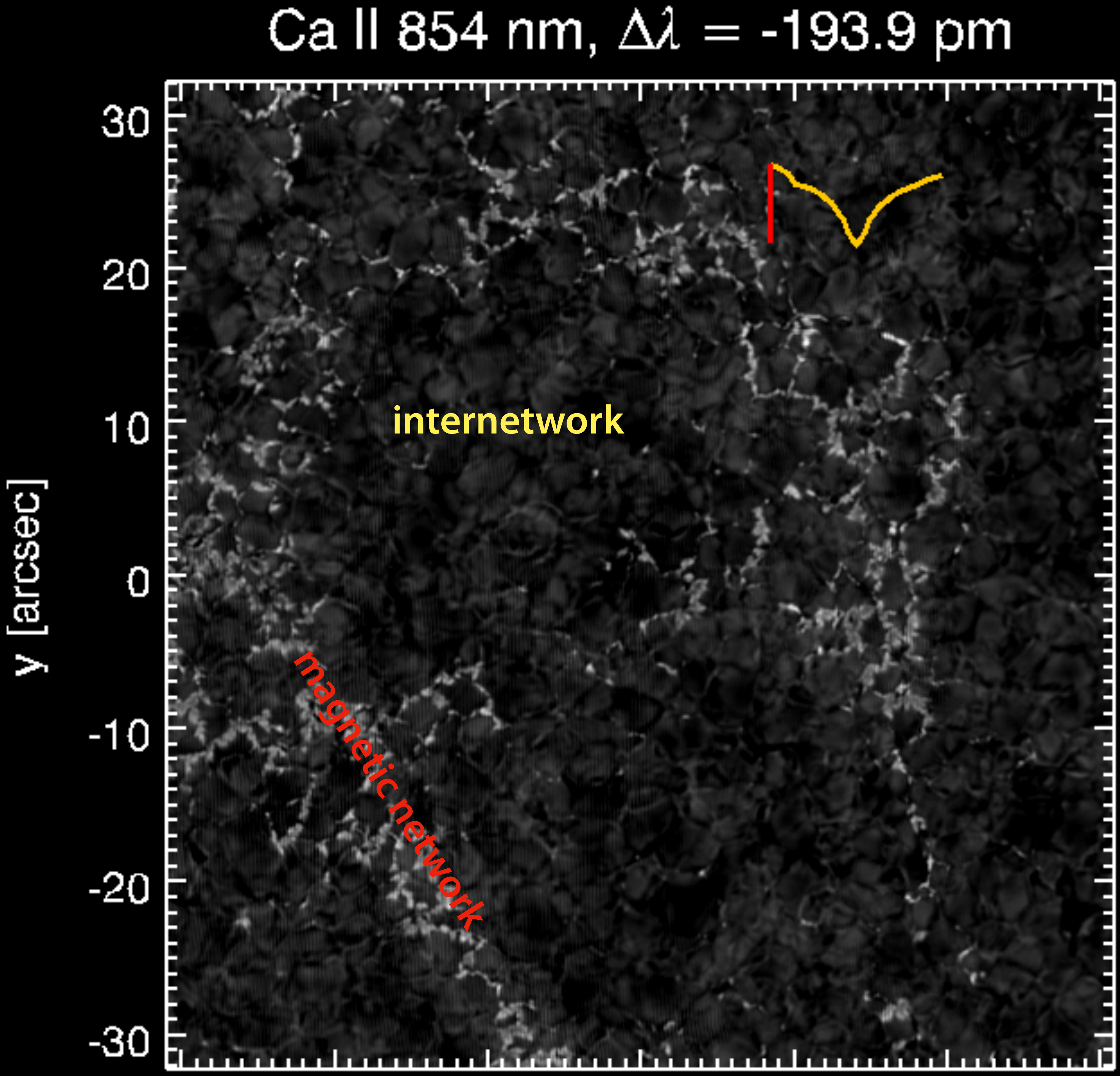
Granulation

Supergranulation

- A bit higher in the atmosphere: map in the Ca II K line core at (393.37 nm), showing the chromospheric network (from Meudon Observatory)
- Also outlines supergranulation scales

Granulation

- Looking a bit higher in the atmosphere
- Spatial scales corresponding to granulation visible
- Prominent scale with super granulation, here with cell sizes of ~30Mm



CRISP@SST, June 2008 (Oslo group)

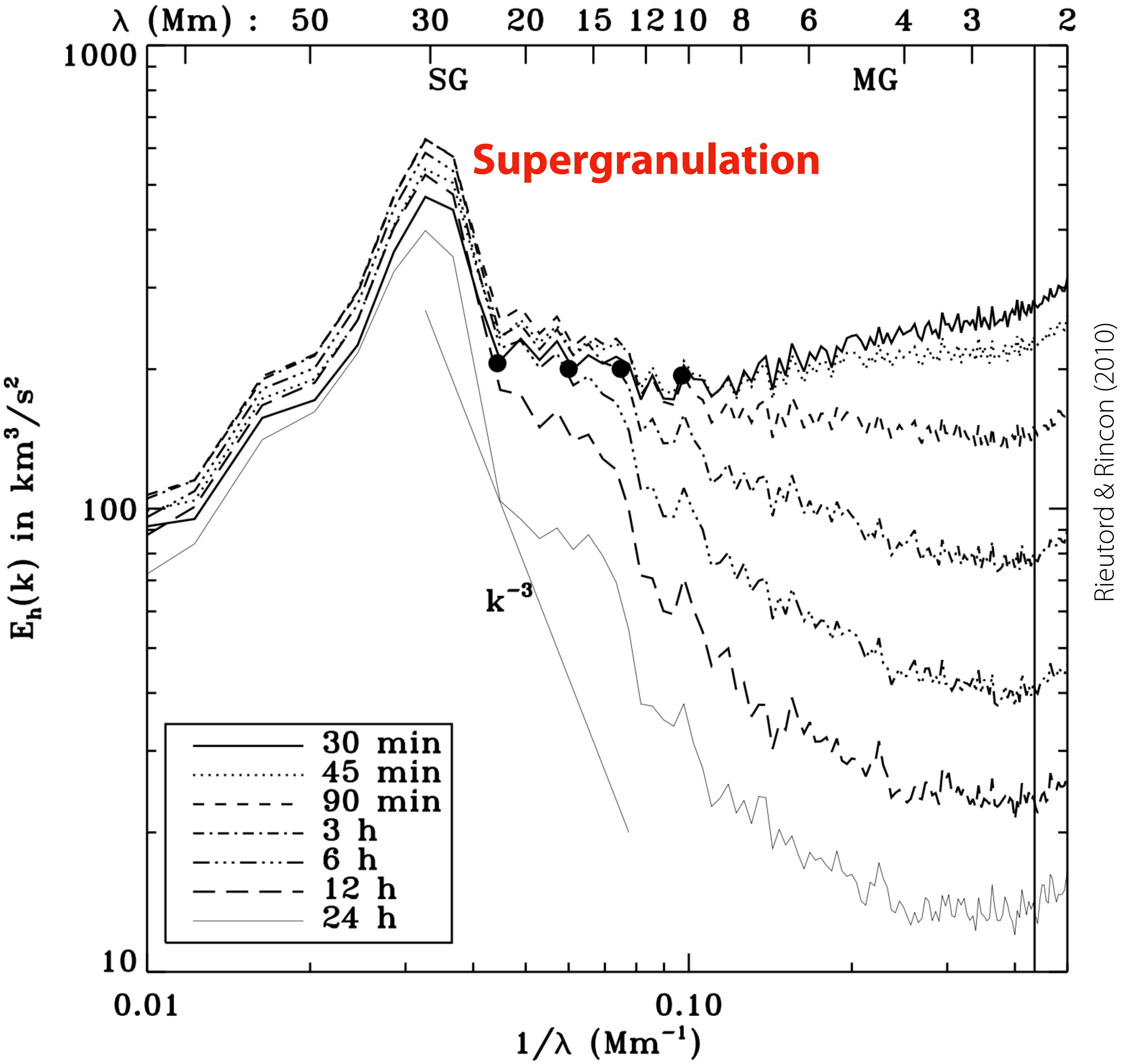
Granulation

Spatial scales

- Spectral density of horizontal kinetic energy $E_\mathrm{h}(k)$
  - Describes the relation between the spatial scale and amplitude of the flow

$$\frac{1}{2}\langle v_h^2\rangle = \int_0^\infty E_h(k)dk$$

  - v_h: horizontal velocity



Rieutord & Rincon (2010)

$$\lambda = 2\pi/k$$

Granulation

Spatial scales

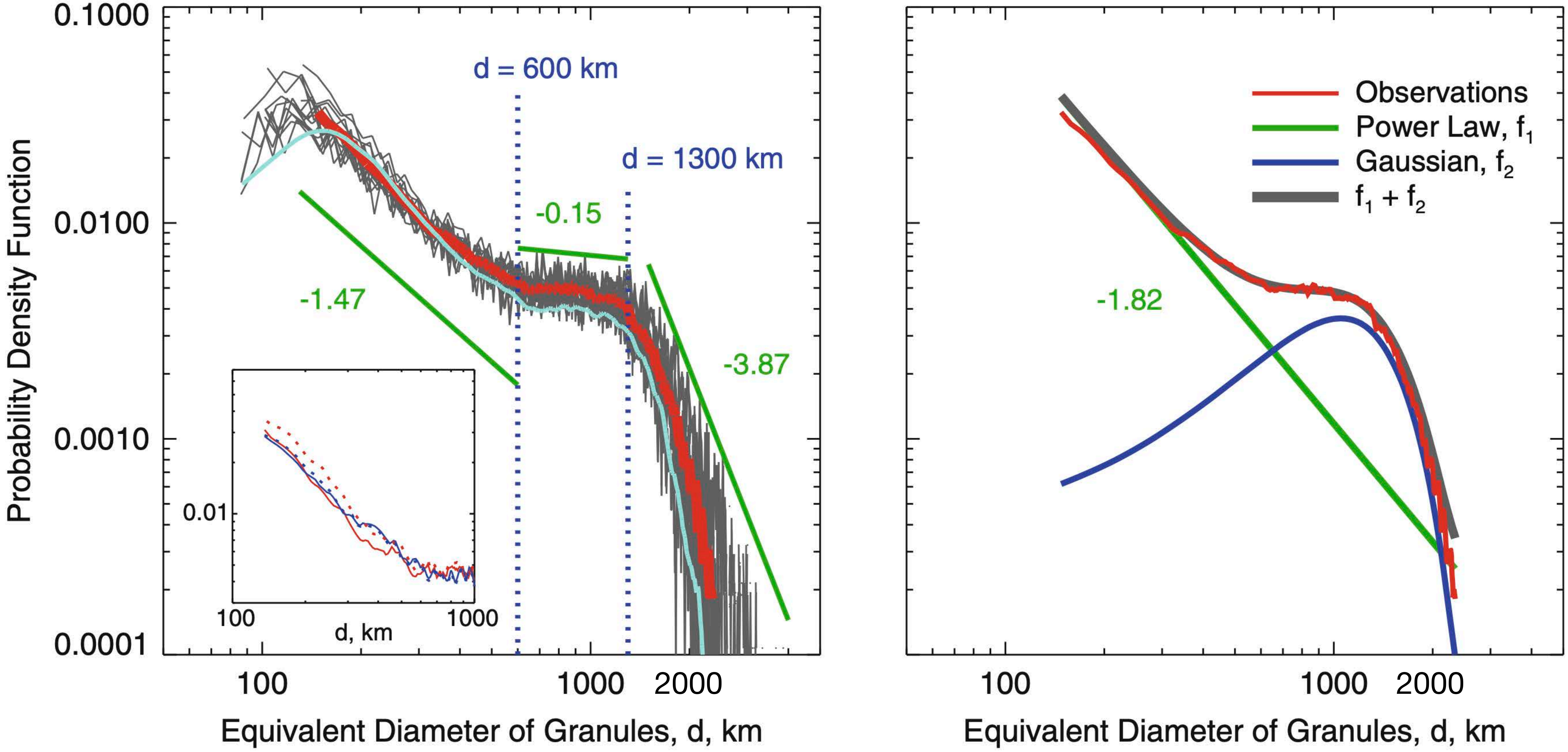


Fig. 4.11 The probability density function of the equivalent diameter of granules (in units of km) is shown, observed in Quiet Sun regions with the *New Solar Telescope (NST)*. The regular granules have a size of $w \approx$ 500–2000 km, while the range of $w \approx$ 100–500 km exhibits the new phenomenon of “mini-granules” (Abramenko et al. 2012)

Granulation

Spatial scales

Giant cells

Supergranulation

Granulation

- 'Velocity spectrum'
- Fourier transformation of velocity

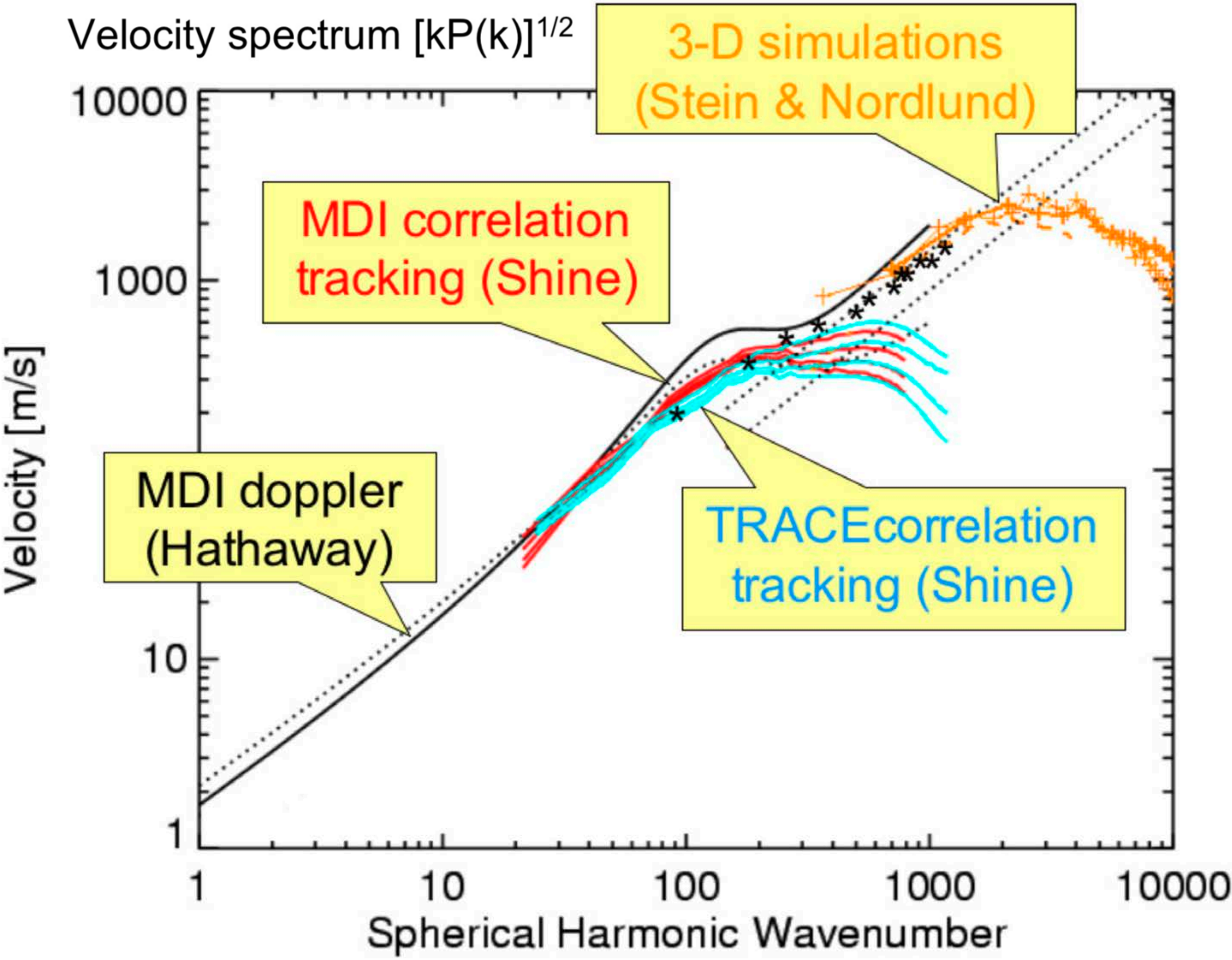
➡ velocity amplitudes over a large range of spatial scales

$$V(k) = \sqrt{kP(k)}$$

\- $P(k)$: power spectrum (velocity power per unit linear wave number)

\- k: wavenumber

($k = 2\pi/\Delta x$)



Nordlund et al. (2009)

Granulation

Spatial scales of solar granulation

- **Mesogranulation:**
    - between granulation and supergranulation, first observational indication in 1980
    - Still debated if a true scale exhibited by convection or not

- **Giant cells:**
    - Larger than supergranulation on scales > 100 Mm
    - Observations need to cover large areas on the Sun over very long time; possible with space-borne telescopes since ~2000 (e.g., SOHO/MDI)

- **Summary:**
    - Continuous distribution of spatial scales and corresponding timescales (lifetimes) for surface convection cells
    - Granulation and supergranulation clearly present
    - Indications for giant cells, while existence of mesogranulation still debated