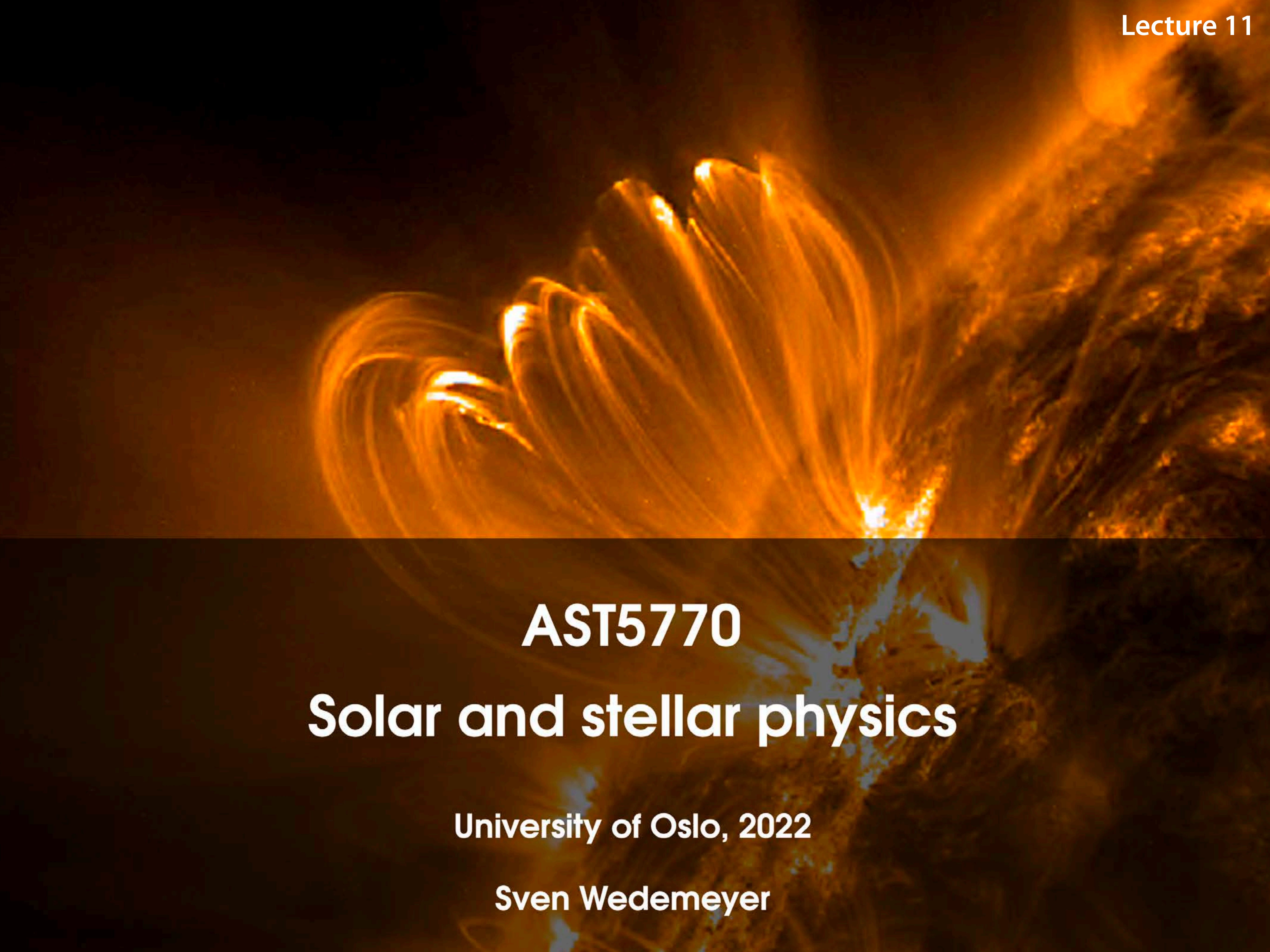


Lecture 11

AST5770

Solar and stellar physics

University of Oslo, 2022

Sven Wedemeyer

Practical information

Updates

- **Deadline for 2nd assignment postponed** by one week to March 11
- Template for 3rd assignment available.

- **Next group sessions — March**
  - 14:15 — 15:00: "interactive writing workshop".
  - The aim is to help you with any problem/challenge you have with writing the introduction . For instance, unsure how to connect two things, how express facts, etc.

  ➡ Please **send** me questions and/or difficult(even unfinished) paragraphs you may have **by the end of March 8** so that we can work on them as examples.
  (I can keep it anonymous.)

Helioseismology continued

Helioseismology

p-modes

- Note the grouping of modes
- Large separations

$$(n, l) \longleftrightarrow (n\text{-}1, l)$$

- Small separations

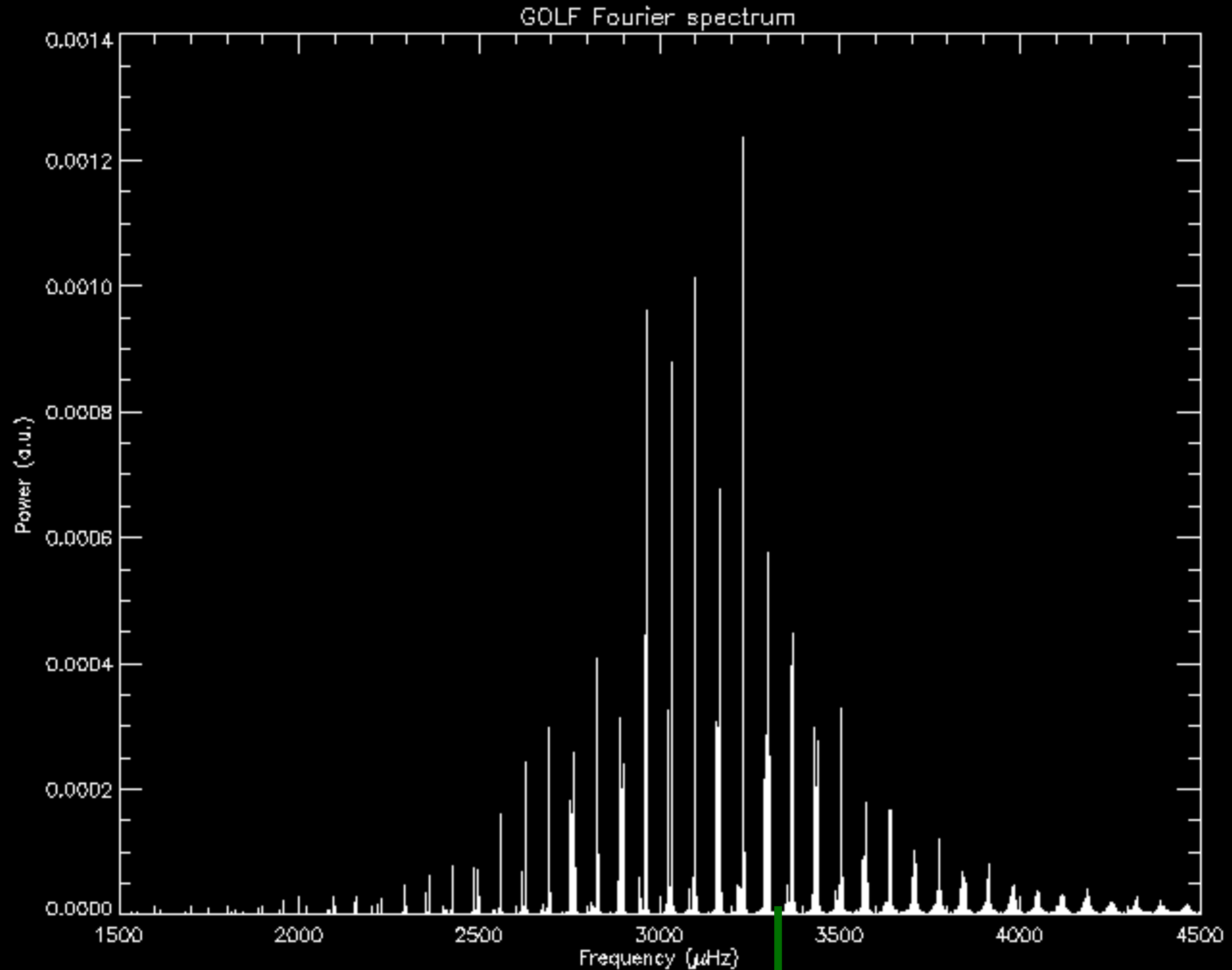
$$(n, l) \longleftrightarrow (n\text{-}1, l{+}2)$$

- n = number or radial nodes
- l = number of nodes on the solar surface

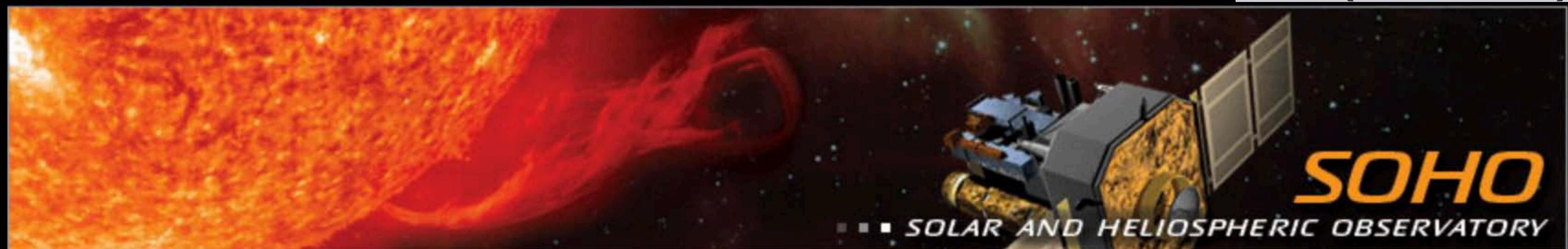
5min = 300 s

ν = 1/300 s = 3.333 mHz

GOLF (Global Oscillations at Low Frequencies)



SOHO (ESA & NASA)



Helioseismology

Types of solar eigenmodes:

p-modes (sound waves) restoring force = pressure	**g-modes** (buoyancy modes) restoring force = gravity
• Excited by **turbulence** associated with convection, mainly by the more vigorous motions at the **surface** (granulation)	• Randomn vertical displacements (**buoyancy**) in a convectively stable medium
• Propagate in the interior but evanescent in the solar atmosphere • So far only p-modes have been detected on the Sun with certainty!	• Propagate in the radiative interior and in the atmosphere but evanescent in the convection zone • No definite observational proof for g-modes on the Sun yet.
• p-modes propagate throughout the solar interior at sound speed c_s ➡ Spend most time where c_s is lowest ➡ Spend most time at the surface (as c_s is lowest there, remember $c_s \sim T^{1/2}$) ➡ Detectable at the surface	• Current upper limit on solar interior g-modes lies below 1 cm/s. • Would probe the centre of the Sun!

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations

- Simplifying assumptions:
    - ***Cowling approximation***
      (neglects any perturbations to the gravitational potential)
    - Radial changes of stratification small over the scales of the considered oscillations
    - Linear perturbation in radial direction, adiabatic
    - Usage of spherical harmonics for any non-radial (i.e. horizontal) component (here primarily set by degree l)

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations

$$\frac{1}{r^2}\frac{d}{dr}(r^2\xi_r) - \frac{\xi_r g}{c^2} + \frac{1}{\rho_0}\left(\frac{1}{c^2} - \frac{l(l+1)}{r^2\omega^2}\right)P_1 = 0$$

$$\frac{1}{\rho_0}\frac{dP_1}{dr} + \frac{g}{\rho_0 c^2}P_1 - (\omega^2 - N^2)\xi_r = 0\ .$$

Unperturbed background

ϱ_0: Density

P_0: Pressure

c: sound speed

Perturbation:

P_1: Perturbed pressure

ξ_r: radial displacement

- Analytical solutions of these equations for an isothermal medium:

$$\xi_r \sim \rho_0^{-1/2}\exp(ik_r r)$$

$$P_1 \sim \rho_0^{1/2}\exp(ik_r r)\ .$$

- Solutions are oscillations but can also be evanescent (complex k_r)

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations at frequency ω

$$\frac{d^2\xi_r}{dr^2} = \frac{\omega^2}{c_s^2}\left(1 - \frac{N^2}{\omega^2}\right)\left(\frac{S_\ell^2}{\omega^2} - 1\right)\xi_r$$

Unperturbed background

c_s: sound speed

Perturbation:

ξ_r: radial displacement

ω: angular frequency

N

Brunt–Väisälä frequency

$$N^2 = g\left(\frac{1}{\Gamma_1 P}\frac{dp}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)$$

S_l

Lamb frequency

$$S_l^2 = \frac{l(l+1)}{r^2}c^2$$

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations at frequency ω

➡ **Dispersion relation**:

$$k_r^2 = \frac{\omega^2 - \omega_\mathrm{A}^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

Unperturbed background

c_s: sound speed

Perturbation:

ξ_r: radial displacement

ω: angular frequency

k_r: radial wavenumber

N

Brunt–Väisälä frequency

$$N^2 = g\left(\frac{1}{\Gamma_1 P}\frac{dp}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)$$

S_l

Lamb frequency

$$S_l^2 = \frac{l(l+1)}{r^2}c^2$$

ω_A

Acoustic cutoff frequency

$$\omega_\mathrm{A} = c/2H$$

- density scale height H: locally approx. constant $H \equiv -\rho_0/(d\rho_0/dr) = \left(\frac{g}{c^2} + \frac{N^2}{g}\right)^{-1}$

Helioseismology

Oscillation equations — k_h-ω-plane

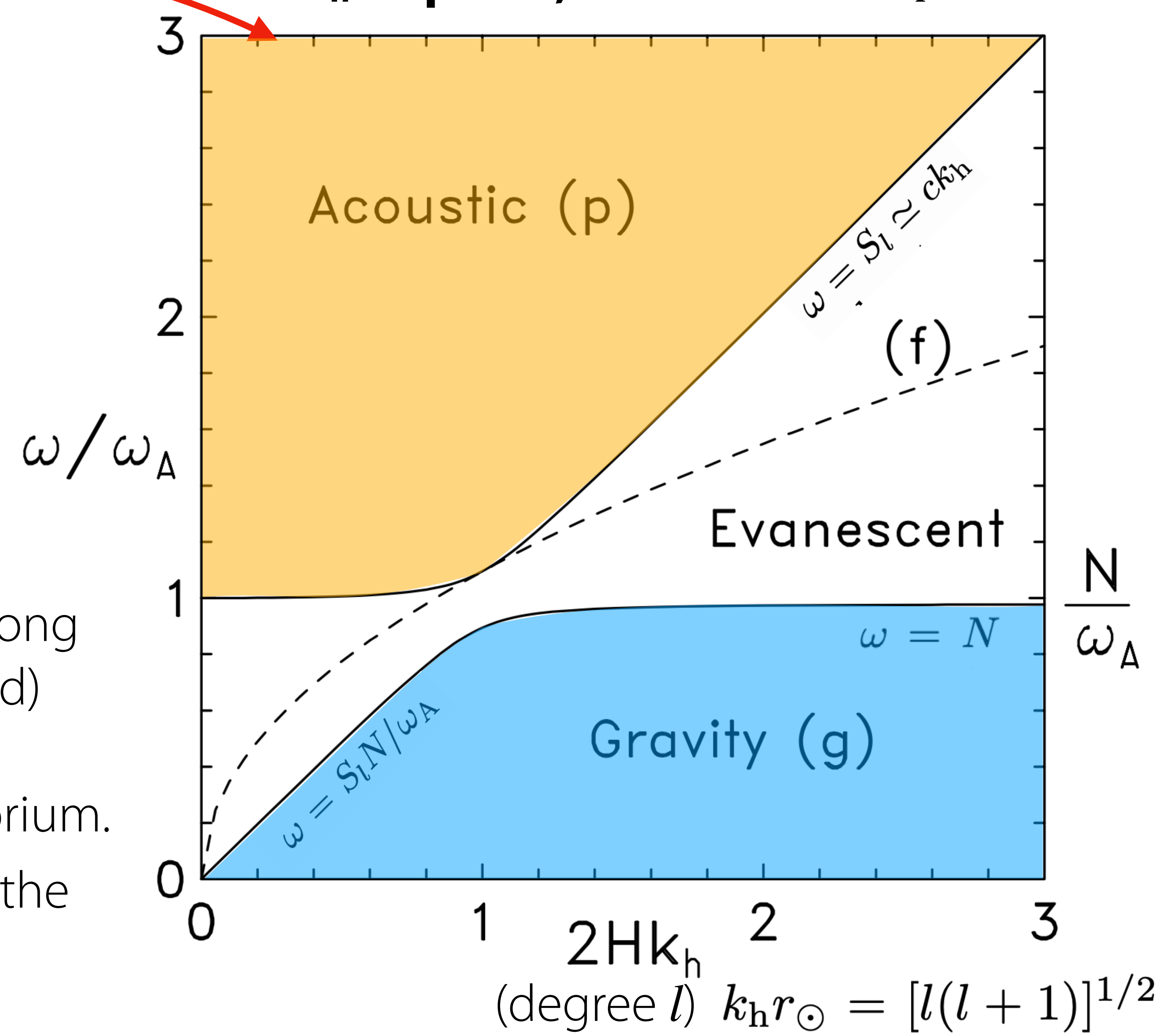
- **Dispersion relation**:

$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

- In the two regimes of acoustic waves and gravity waves:
  $k_r^2 > 0$
- Between: regime of evanescent waves (exponential damping)
  $k_r^2 < 0$
  - Evanescent waves occur if period so long that the whole (exponentially stratified) medium has time to adapt to the perturbation, achieving a new equilibrium.
  - ➡ Wave does not propagate, but rather the medium as a whole oscillates

- ω_A : Acoustic cutoff frequency
- N : Brunt–Väisälä frequency
- S_l : Lamb frequency

k_h-ω-plane, solid curves: $k_r^2 = 0$



Helioseismology

Oscillation equations — k_h-ω-plane

- **Dispersion relation**:

$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

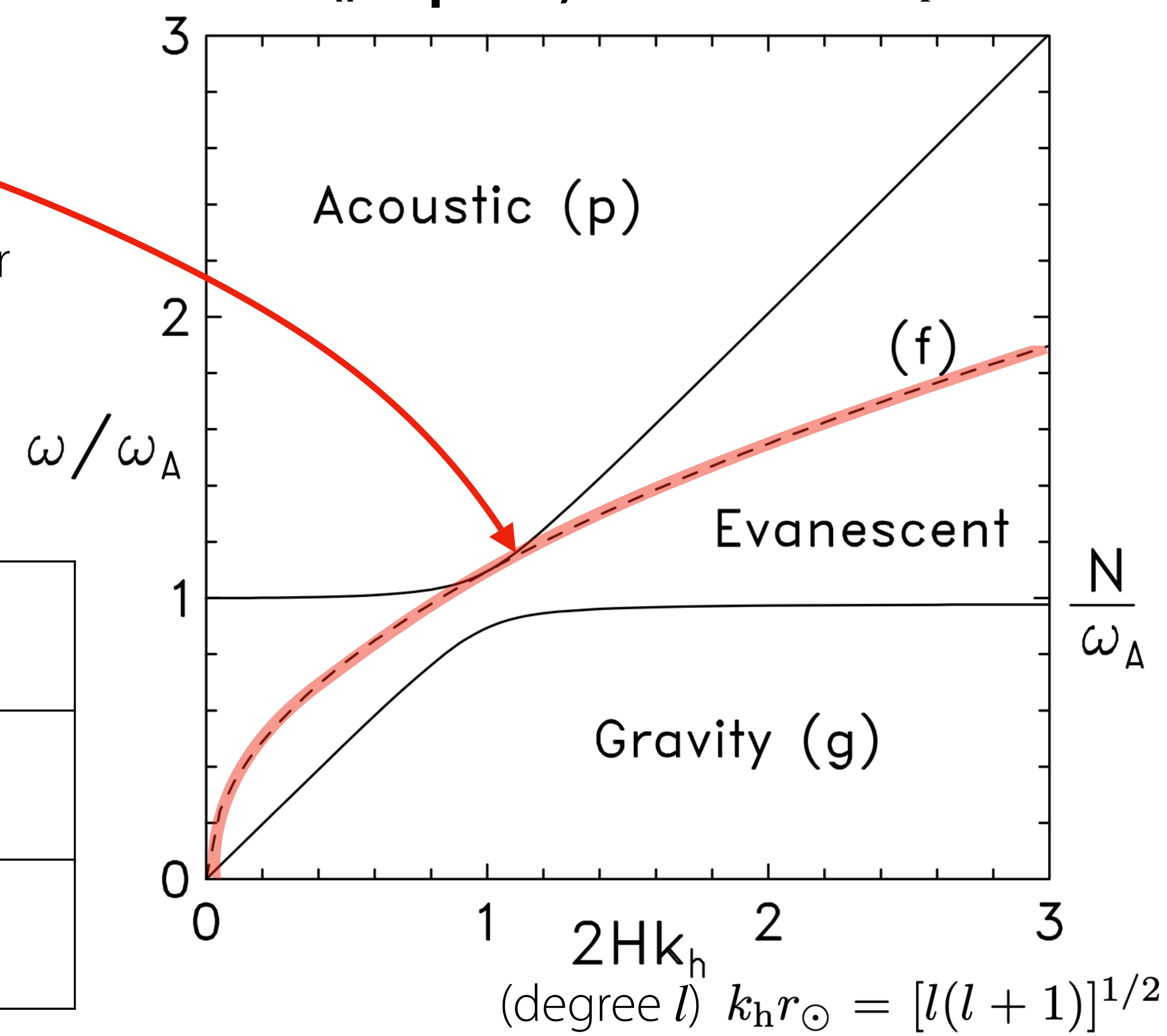
- **Fundamental mode (f-mode):** essentially without compression, resembles a surface wave on deep water

$$\omega = \sqrt{g k_h}$$

Acoustic waves	➡ p-modes
Buoyancy waves	➡ g-modes
Surface gravity waves	➡ f-mode

- ω_A : Acoustic cutoff frequency
- N : Brunt–Väisälä frequency
- S_l : Lamb frequency

k_h-ω-plane, solid curves: $k_r^2 = 0$



Helioseismology

Oscillation equations — k_h-ω-plane

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$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

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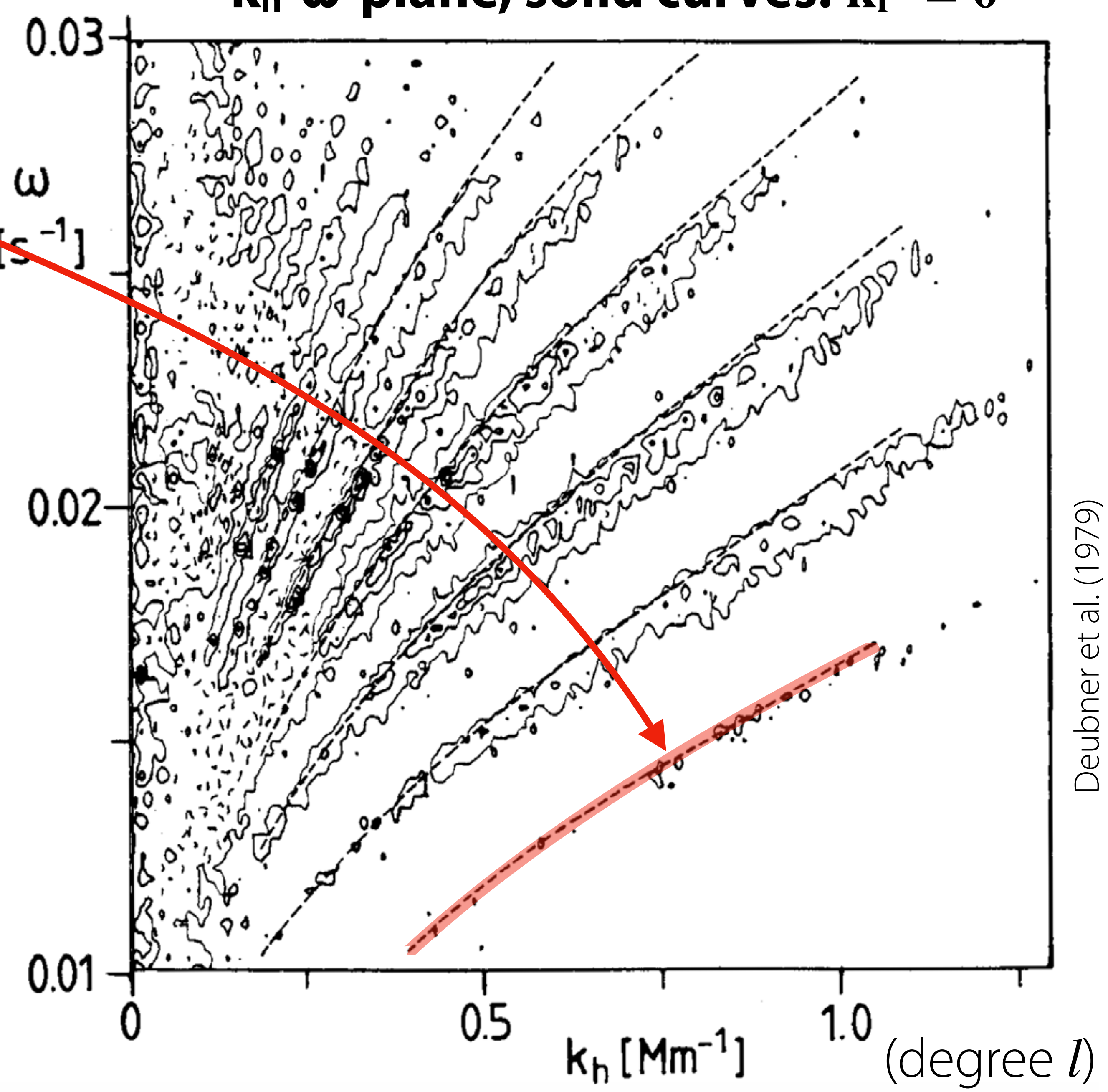
- S_l : Lamb frequency

- **Fundamental mode (f-mode):** essentially without compression, resembles a surface wave on deep water

$$\omega = \sqrt{g k_h}$$

- In diagnostic diagrams: f-mode = lowest ridge

k_h-ω-plane, solid curves: $k_r^2 = 0$



Helioseismology

Critical frequencies in the Sun

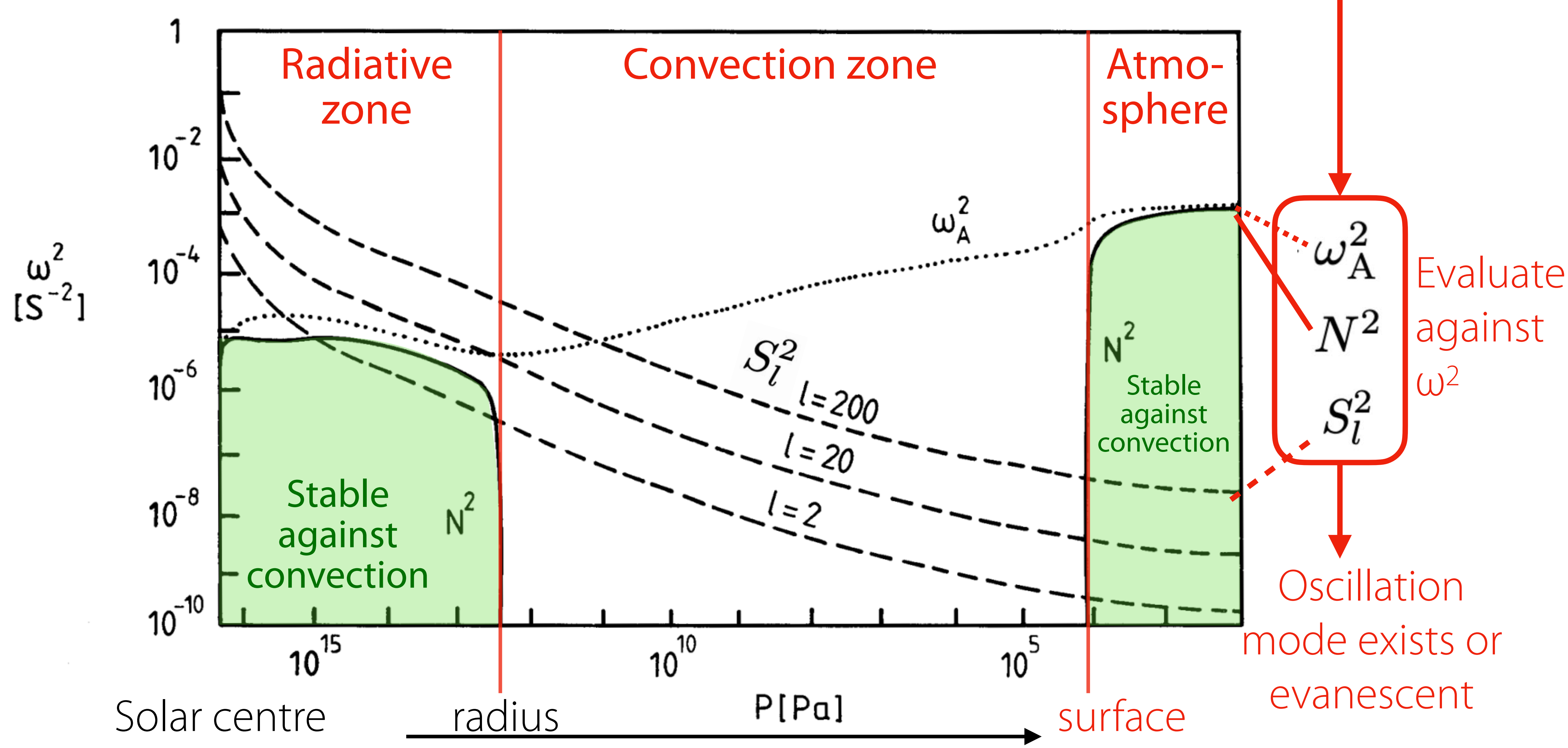
- Mode "trapped" in a layer with oscillatory wave behaviour for this type of mode if the layer is in-between two evanescent layers

➡ Spectrum of oscillation frequencies is <u>discrete</u>.

$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

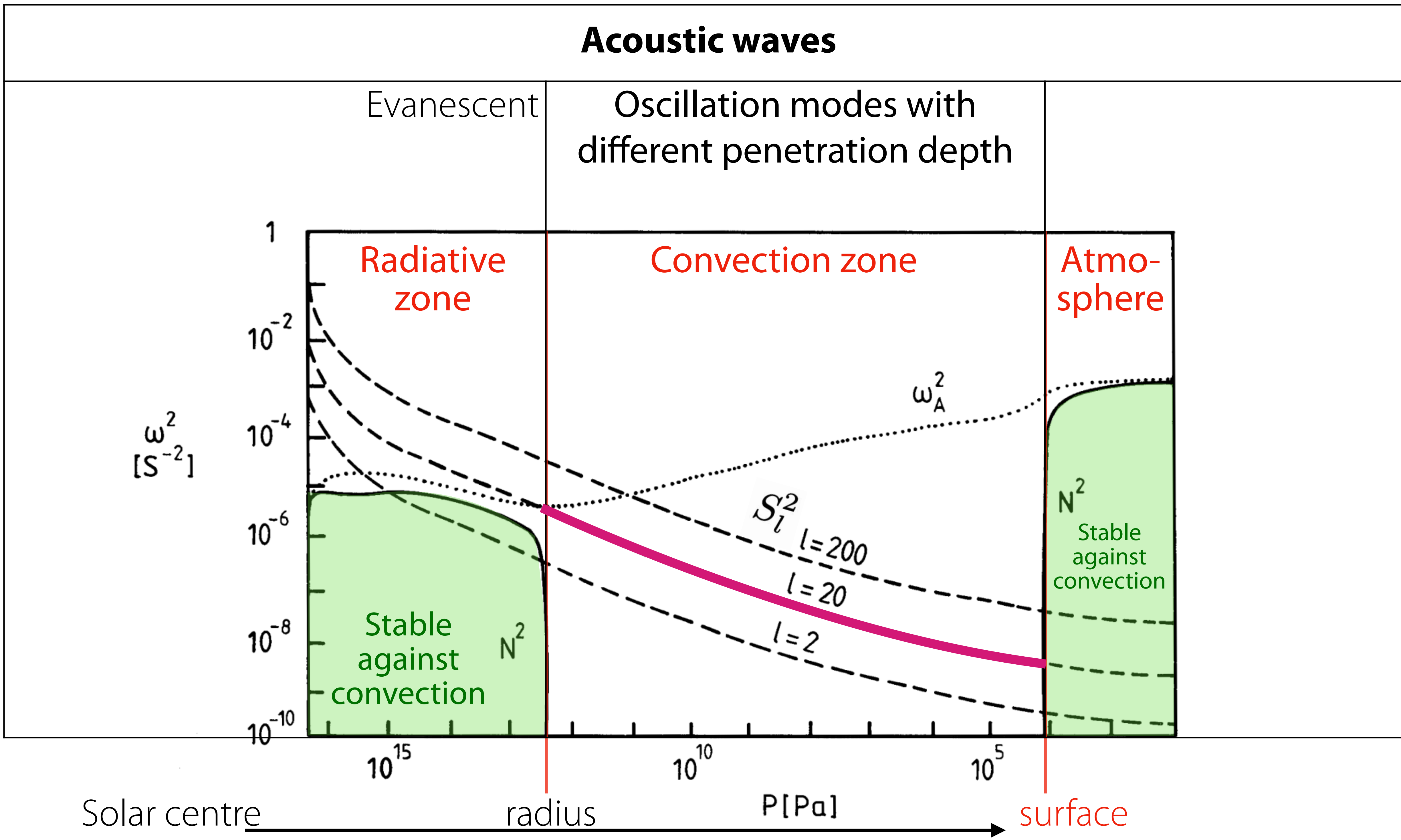
$k_r^2 > 0$: Oscillation mode exists

$k_r^2 < 0$: Evanescent



Helioseismology

Critical frequencies in the Sun

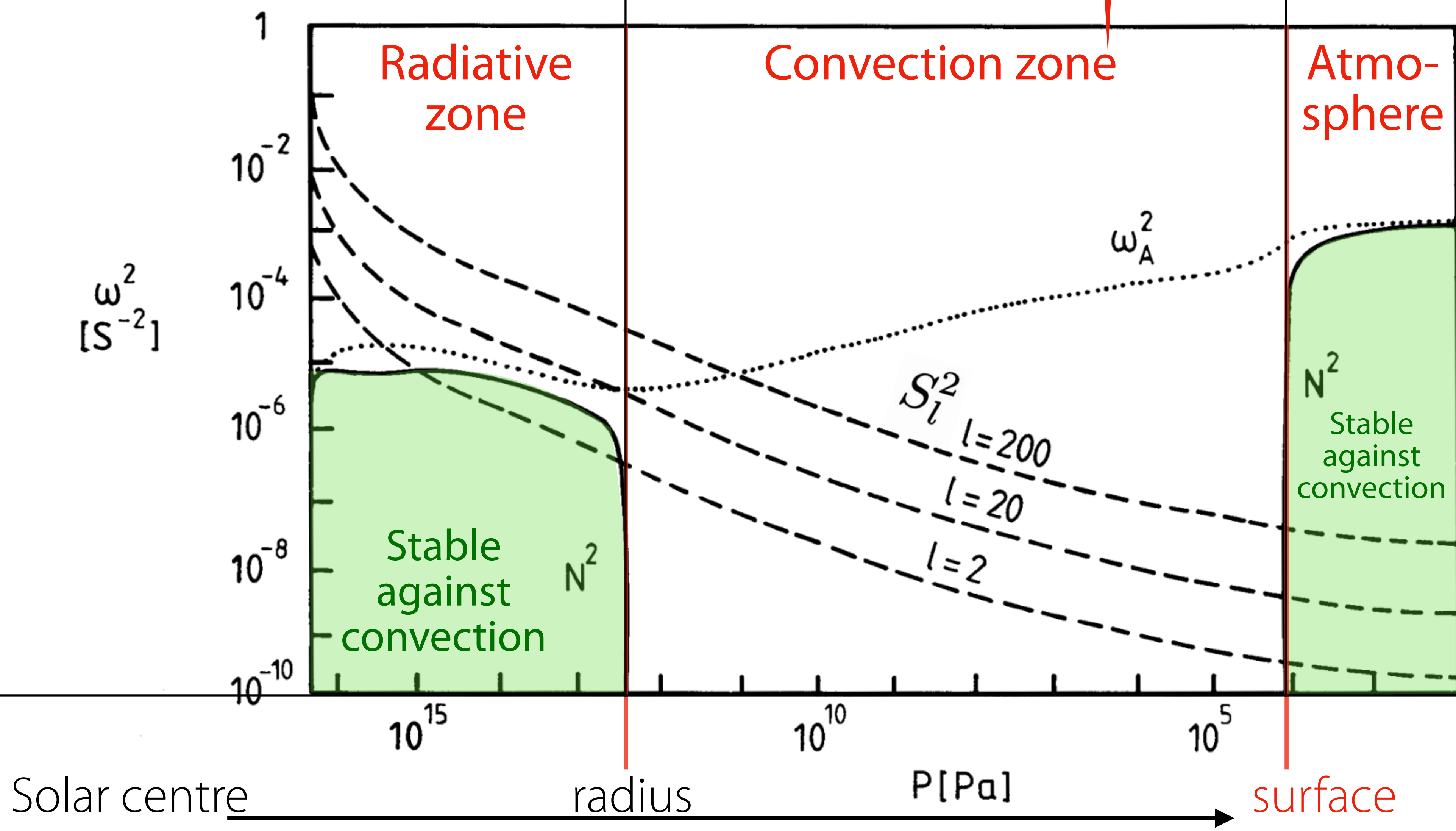


Helioseismology

Critical frequencies in the Sun

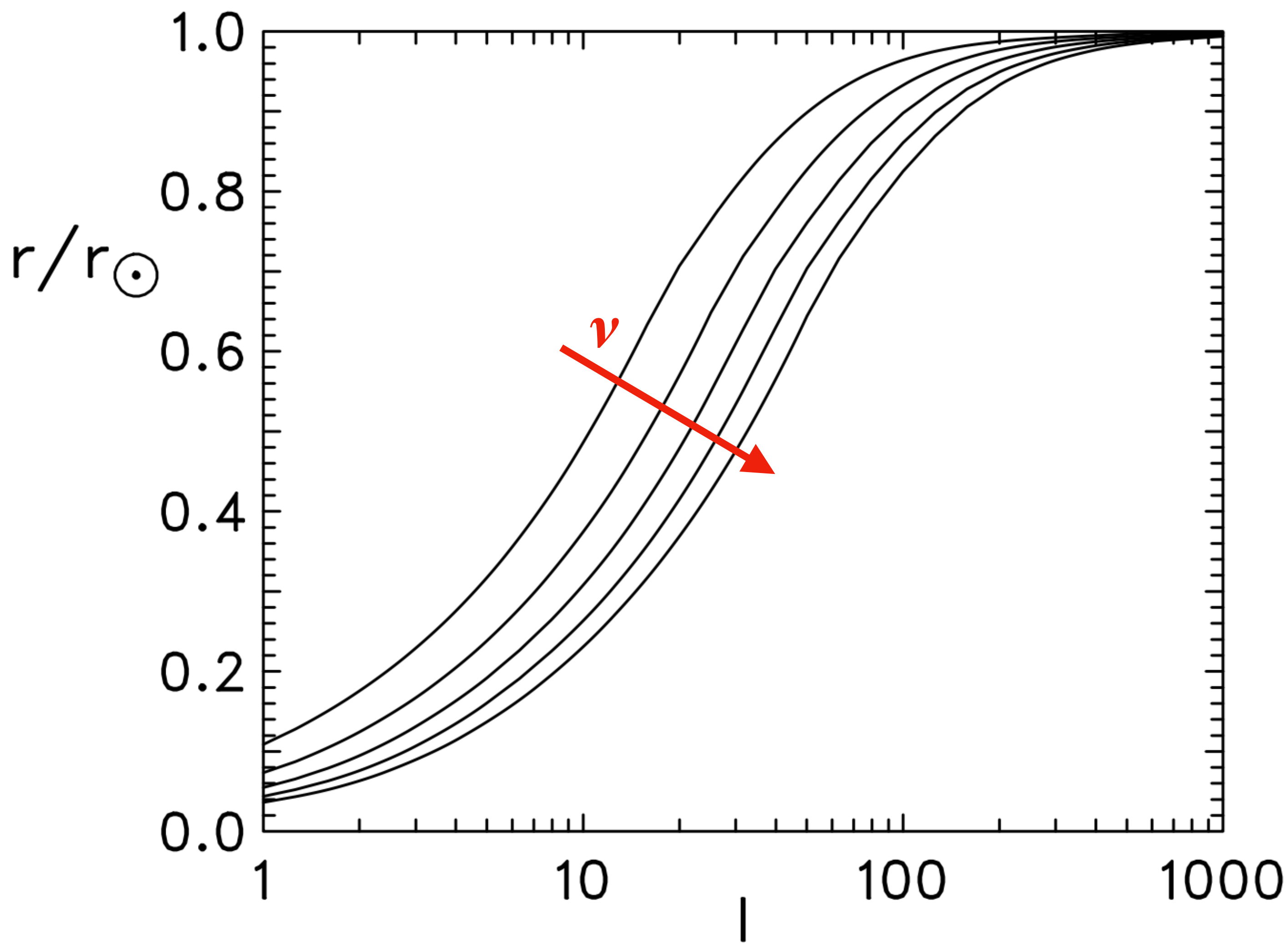
Note: The exact mode spectrum is set by the interior structure and thus expected to differ as function of **spectral type** etc.

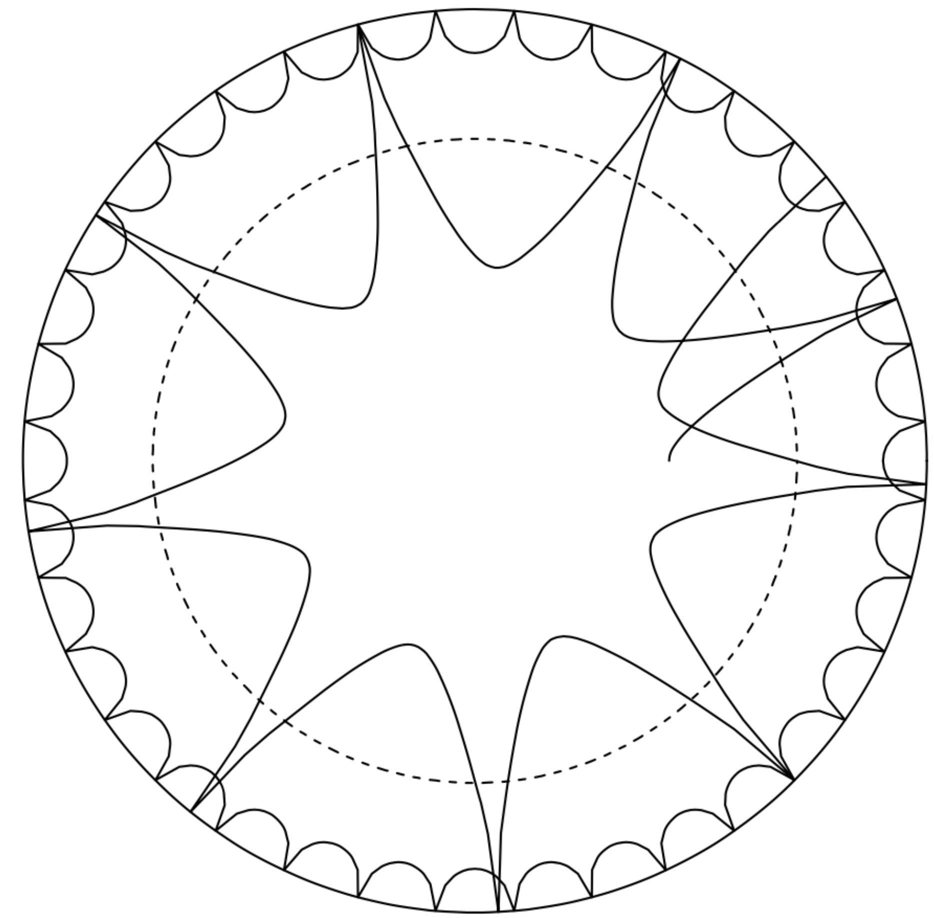
Internal gravity waves		
trapped, form a discrete frequency spectrum of g modes	Evanescent	Should be able to occur but no direct observational proof (yet)



Helioseismology

Reflection of p-modes





Stix

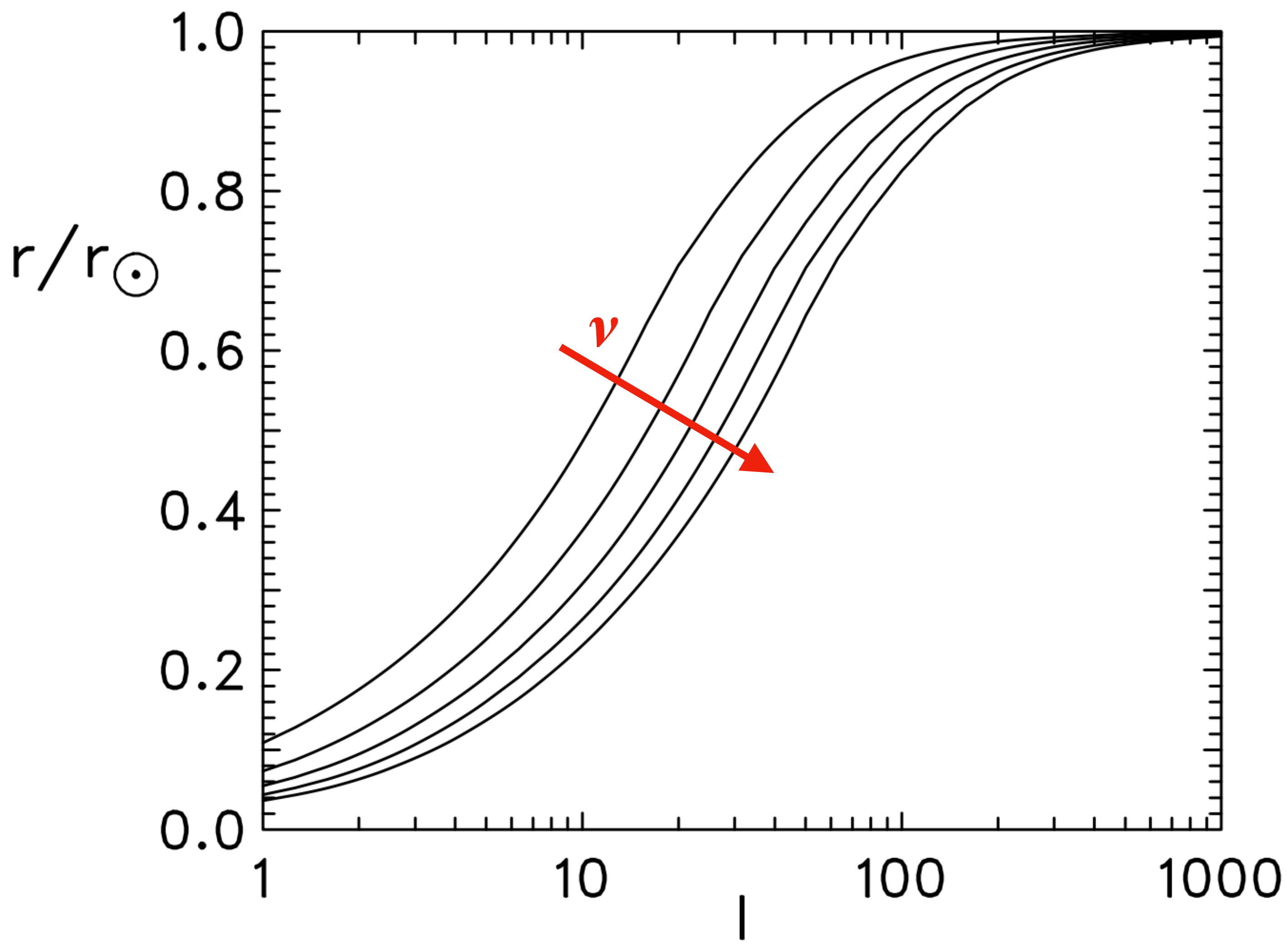
- Lower penetration depth with increasing degree l

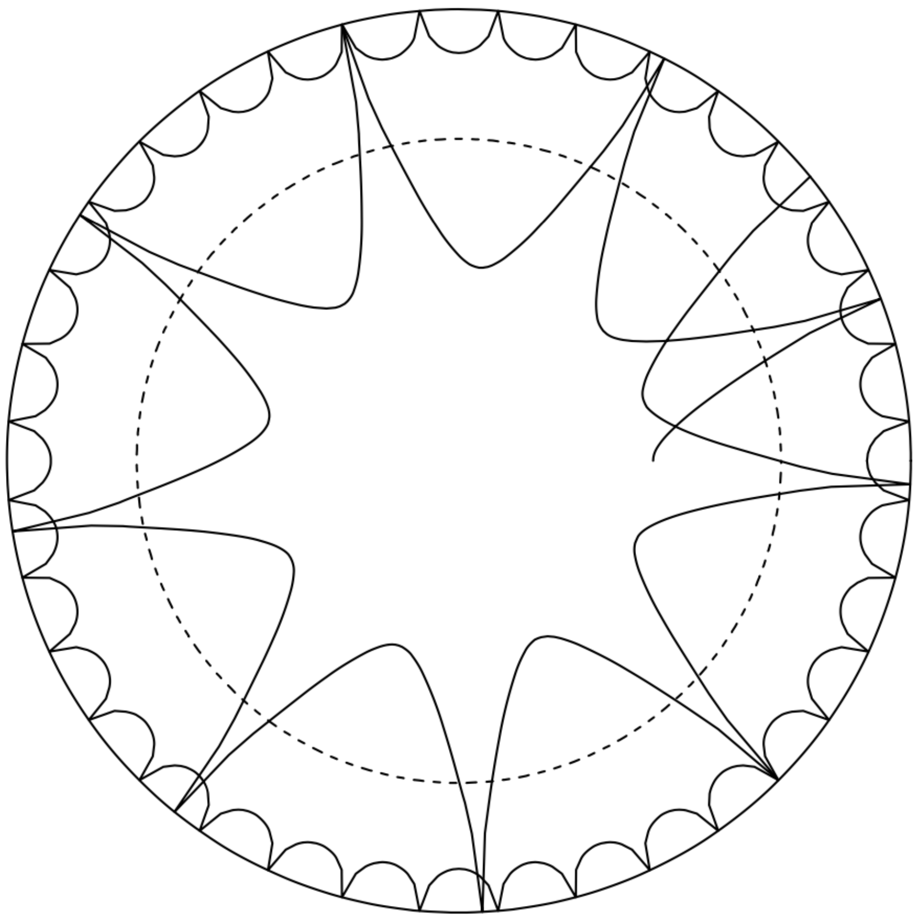
➡ Modes with high l mostly in the top of the convection zone (near surface)

➡ Modes with low l can probe the deep interior

Helioseismology

Reflection of p-modes





Stix

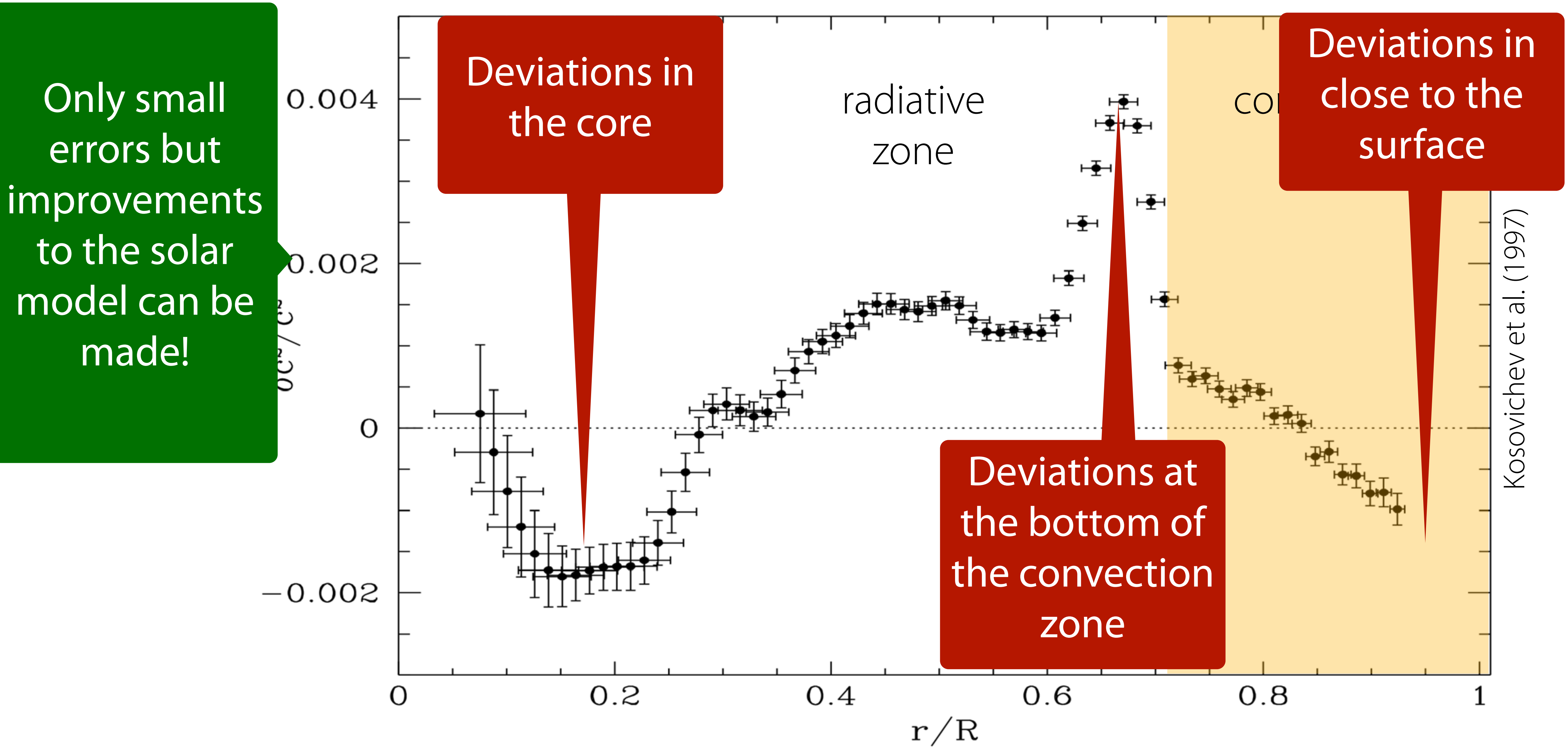
We have now a theoretical framework for oscillation modes in the Sun
and their connection to the physical properties of the solar interior (incl. the stratification),
which is otherwise not directly observable!

➡ Use this knowledge to invert real observations!

Helioseismology

Probing the solar interior — inversion results

- The measured mode spectrum now allows for deriving the stratification of the solar interior
- Derived sound speed **differs from the standard model** of the Sun!

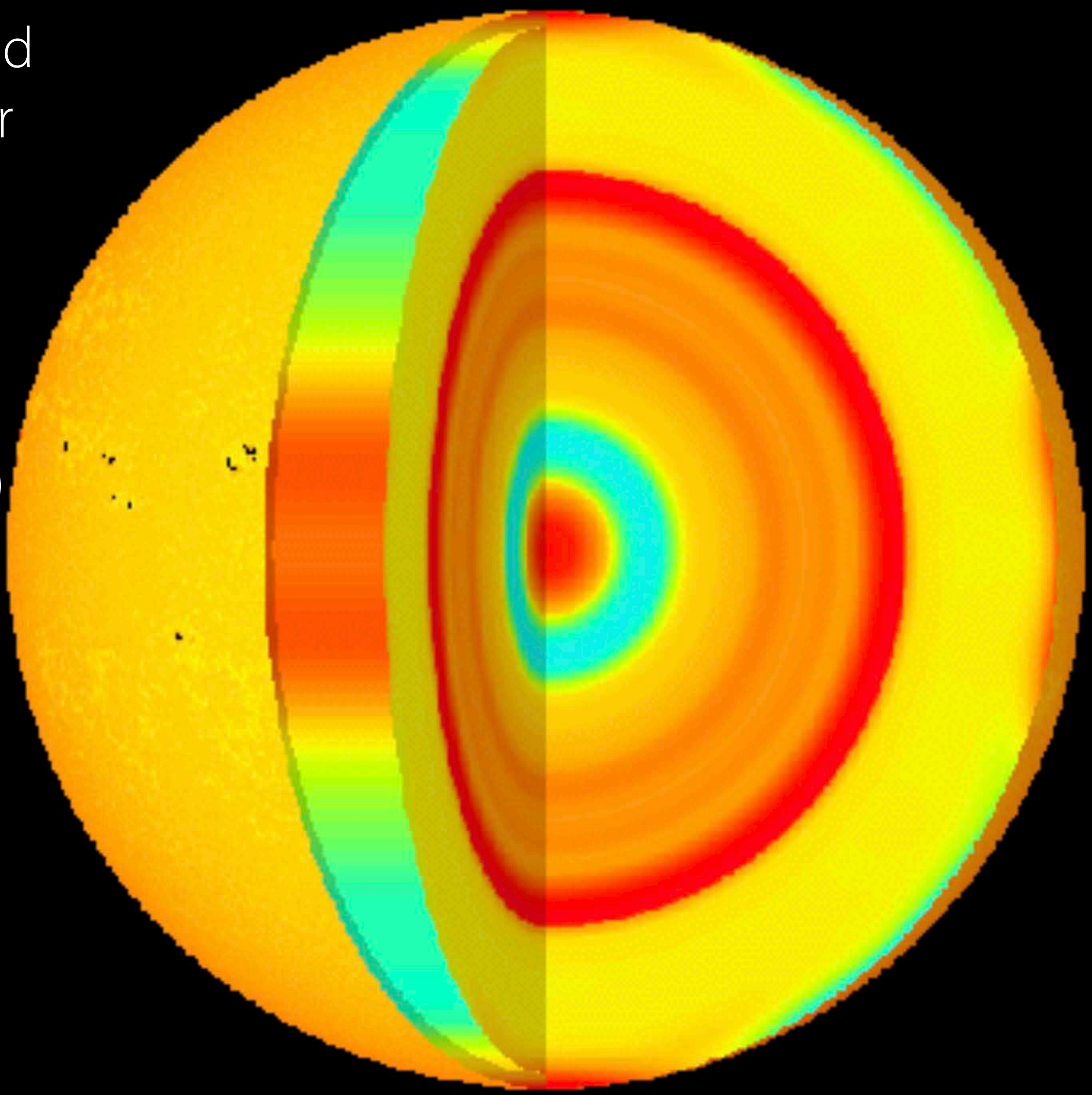


Relative difference between the squared sound speed as inferred from 2 months of MDI data and the standard solar model of Christensen-Dalsgaard et al. (1996)

Helioseismology

Probing the solar interior — inversion results

- Derived sound speed differs from the standard model of the Sun!
- Radial and latitudinal variations of the sound speed in the Sun relative to a standard solar model.
- Temperature does differ correspondingly
- Red = positive variations (`hotter' regions)
- Blue = negative variations (`cooler' regions)



Kosovichev

Helioseismology

Implications

- So far, despite (relatively small deviations): **good agreement** between sound speed predicted by models of the solar interior and helioseismological measurements

- New determination of chemical abundances — updated (suggested) values
  - Among them, C, N, O: significant **opacity sources in the solar interior**

  ➡ Changed abundances result in (slightly) different density and temperature stratification

  ➡ **Disagreement** with standard solar interior models, most notably just below convection zone including predicted depth of convection zone

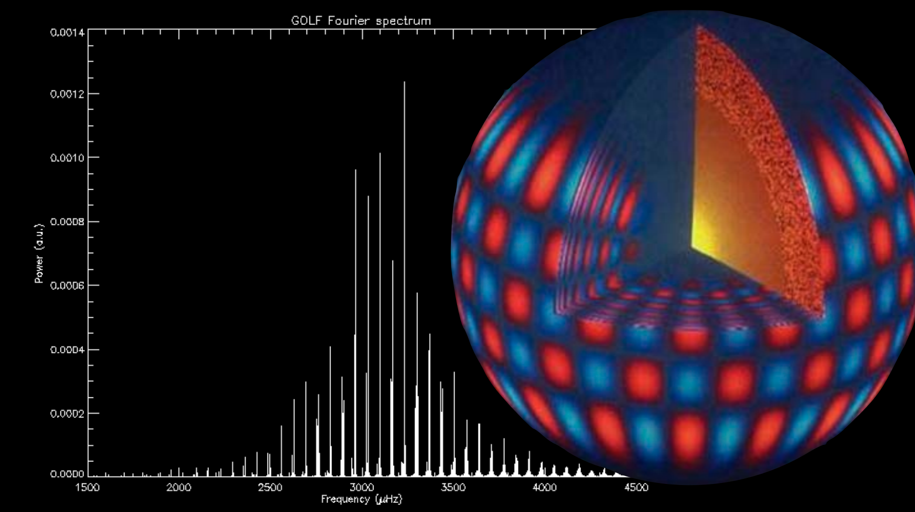
- Different explanations debated
  - Possible: New solar abundances are not precise enough yet as a lot of effects to be taken into account
    - Active field of research, advanced use of 3D spectral line synthesis (based on 3D magnetohydrodynamical models)
    - Relatively new and challenging

- Please also note that the standard solar model assumes perfect spherical symmetry and no rotation!

See spectra line data files provided for the assignments.

Helioseismology

Rotational splitting

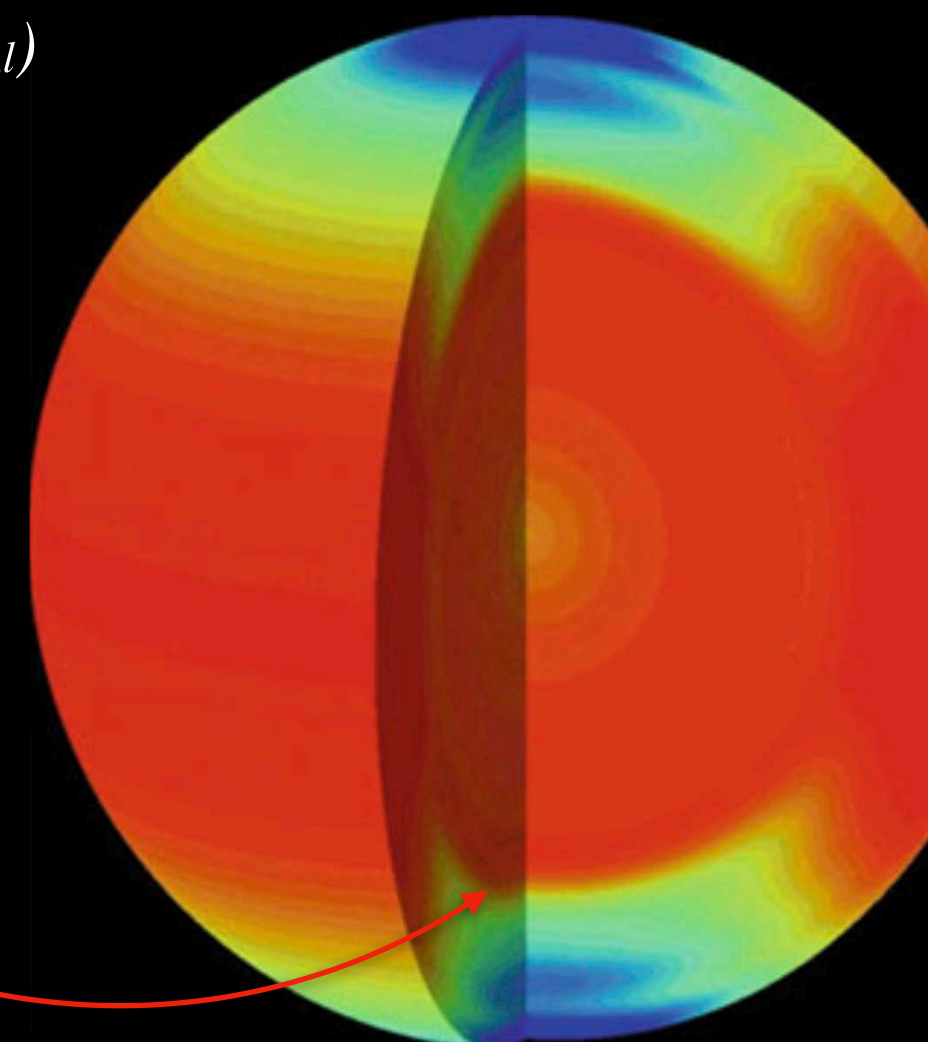


- So far neglected dependence on m
  (azimuthal, number of nodes passing through poles)
- Closer look: Frequency spectrum is (finely) split regarding m due to the Sun's rotation

➡ **Rotational splitting**

  - Waves propagate prograde or retrograde $(\nu_{-m,l} - \nu_{+m,l})$
- Can be exploited to derive the rotation rate throughout the solar interior
- Discovery of a rapid change in rotation rate at the bottom of the convection zone
- That layer is called **tachocline**
  - Depth: $r = 0.712 \pm 0.005\ R_\odot$ with a
  - Thickness $\Delta r = 0.04 \pm 0.01\ R_\odot$

fast rotation rate (P ≈ 25 days)
slow rotation rate (P ≈ 35 days)

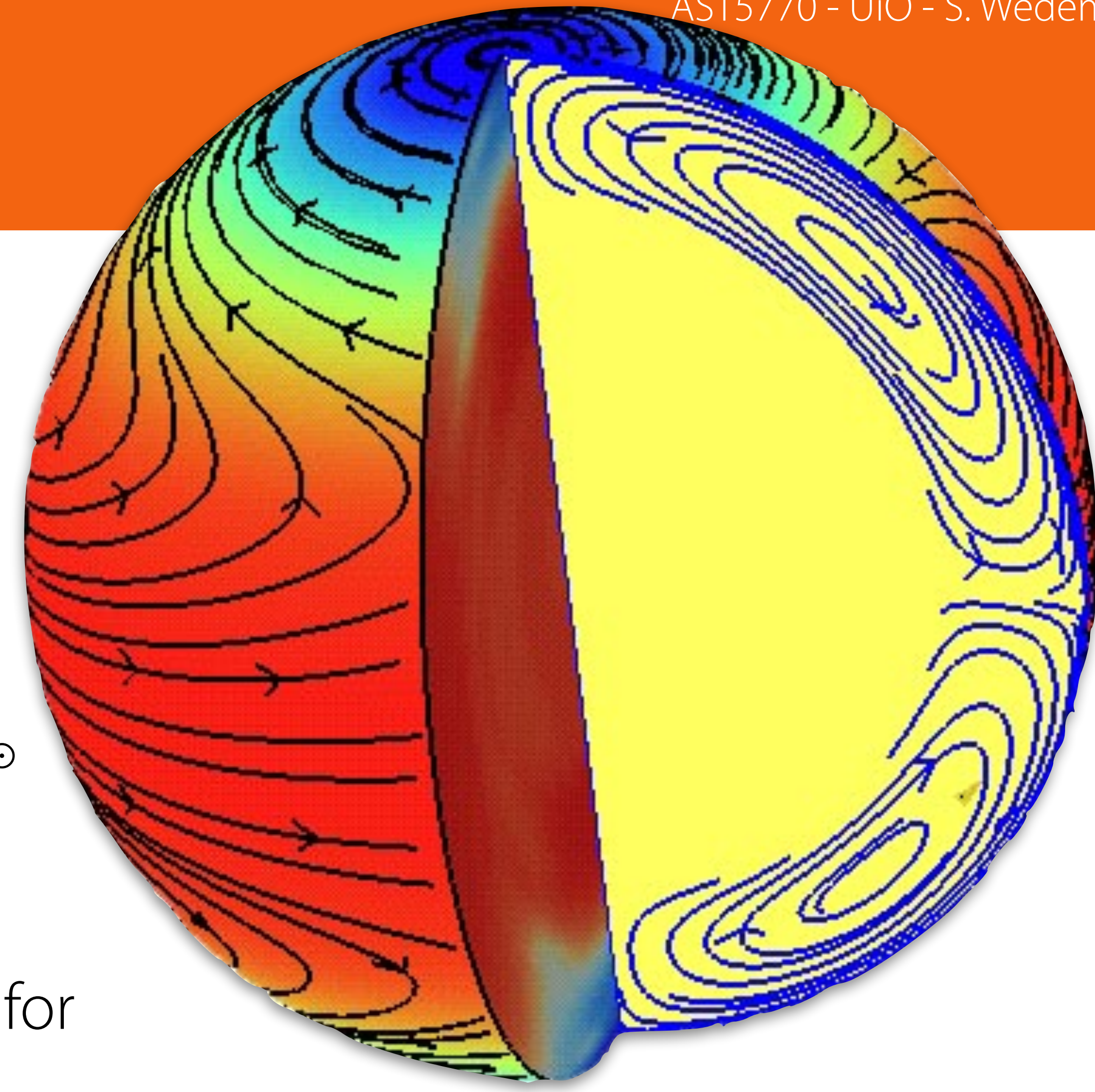


MDI/SOHO Team

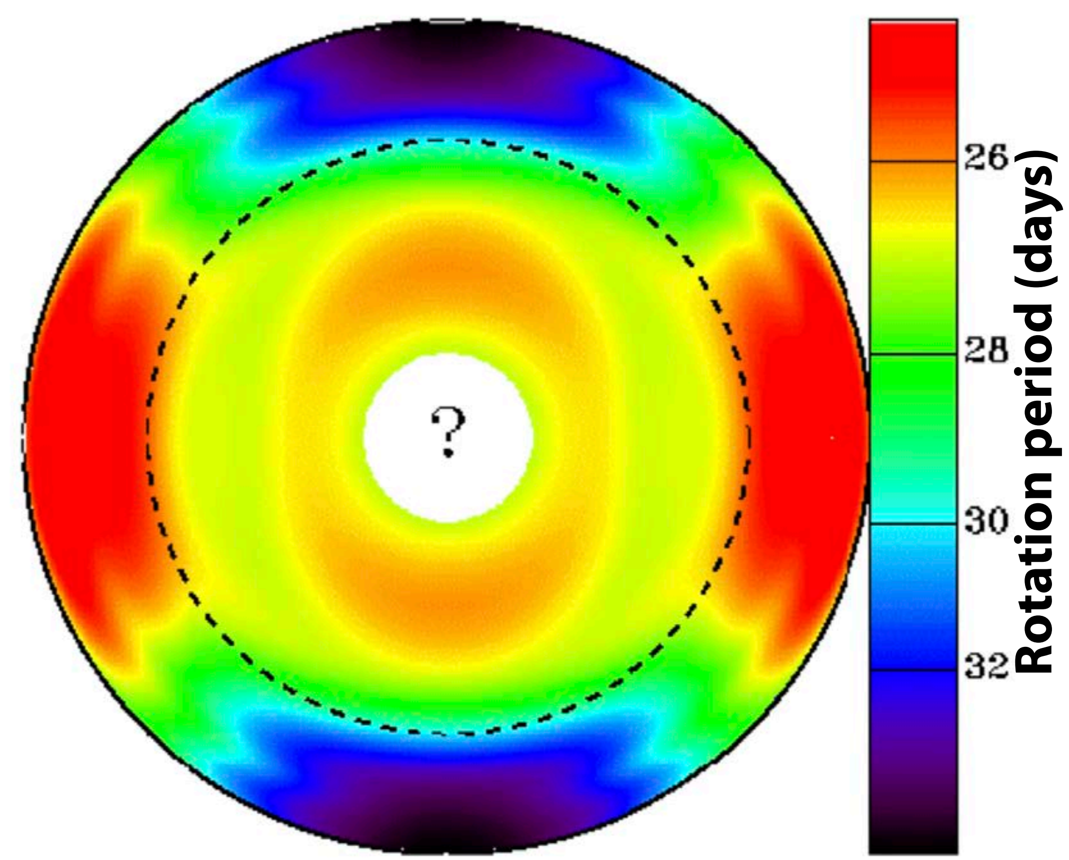
Helioseismology

Rotation and meridional flows

- From helioseismology:
  meridional flow poleward at /close to the surface (down to ~30 Mm), speeds ~ 20 m/s
- "Return flow" towards equator at a depth ~ 0.77 $R_\odot$ (deep convection zone)
- Bottom of convection zone = lower turning point for oscillation modes with $l \approx 20$
- Below convection zone (0.2 – 0.7 $R_\odot$):
  - rotation approx. constant with radius
  - spherically symmetric
  - Well-described by rigid-body rotation
- Not much known about rotation of the innermost core
  (probed only by very low-l degree p-modes,
  internal g-modes not accessible)



Savage & Steigerwald, Stanford
ESA/NASA Solar and Heliospheric Observatory (SOHO)

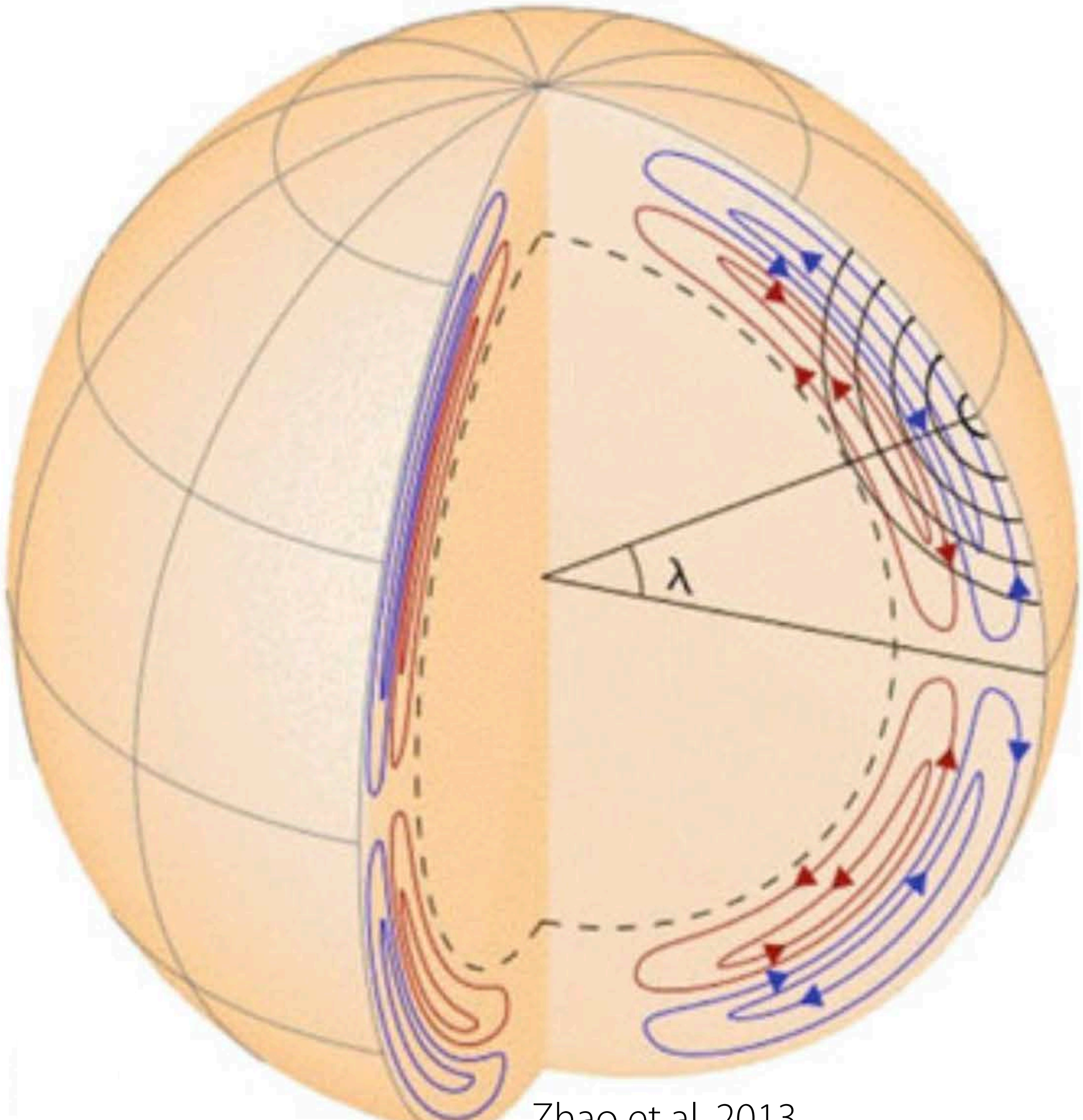


Helioseismology

Rotation and meridional flows

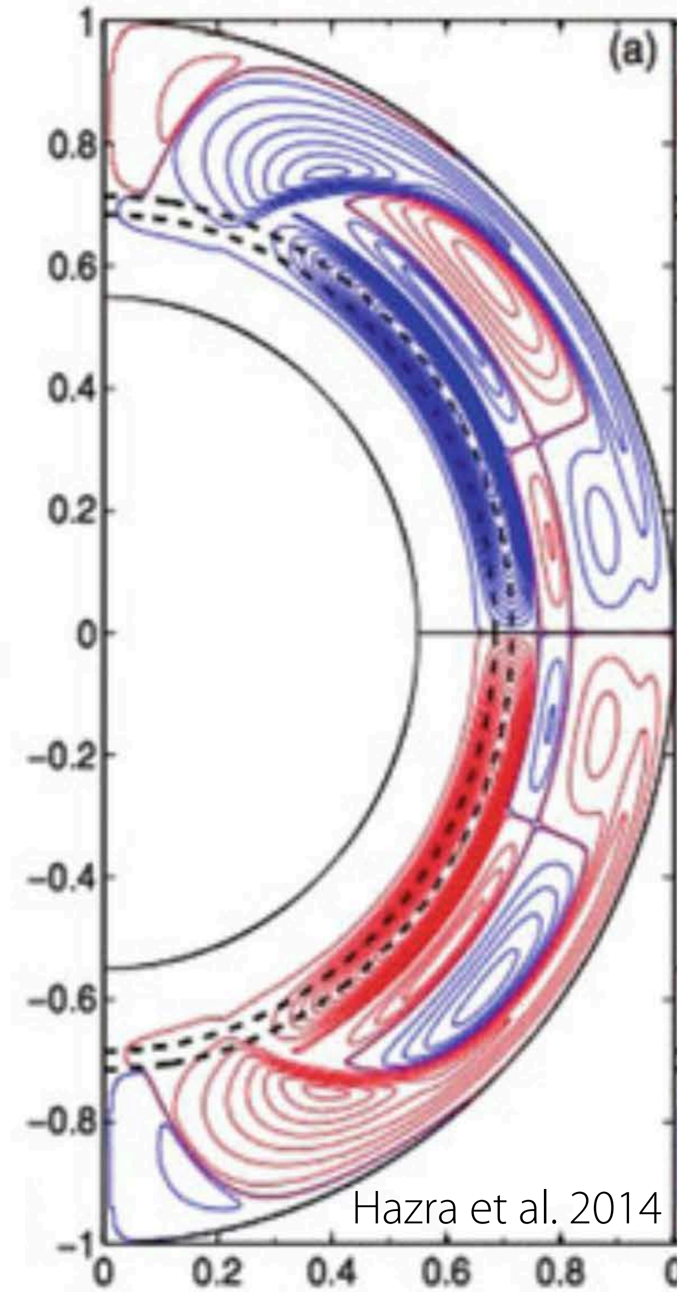
- Further observations + simulations imply several layers of flow cells in radial direction

Based on observations



Zhao et al. 2013

Simulation



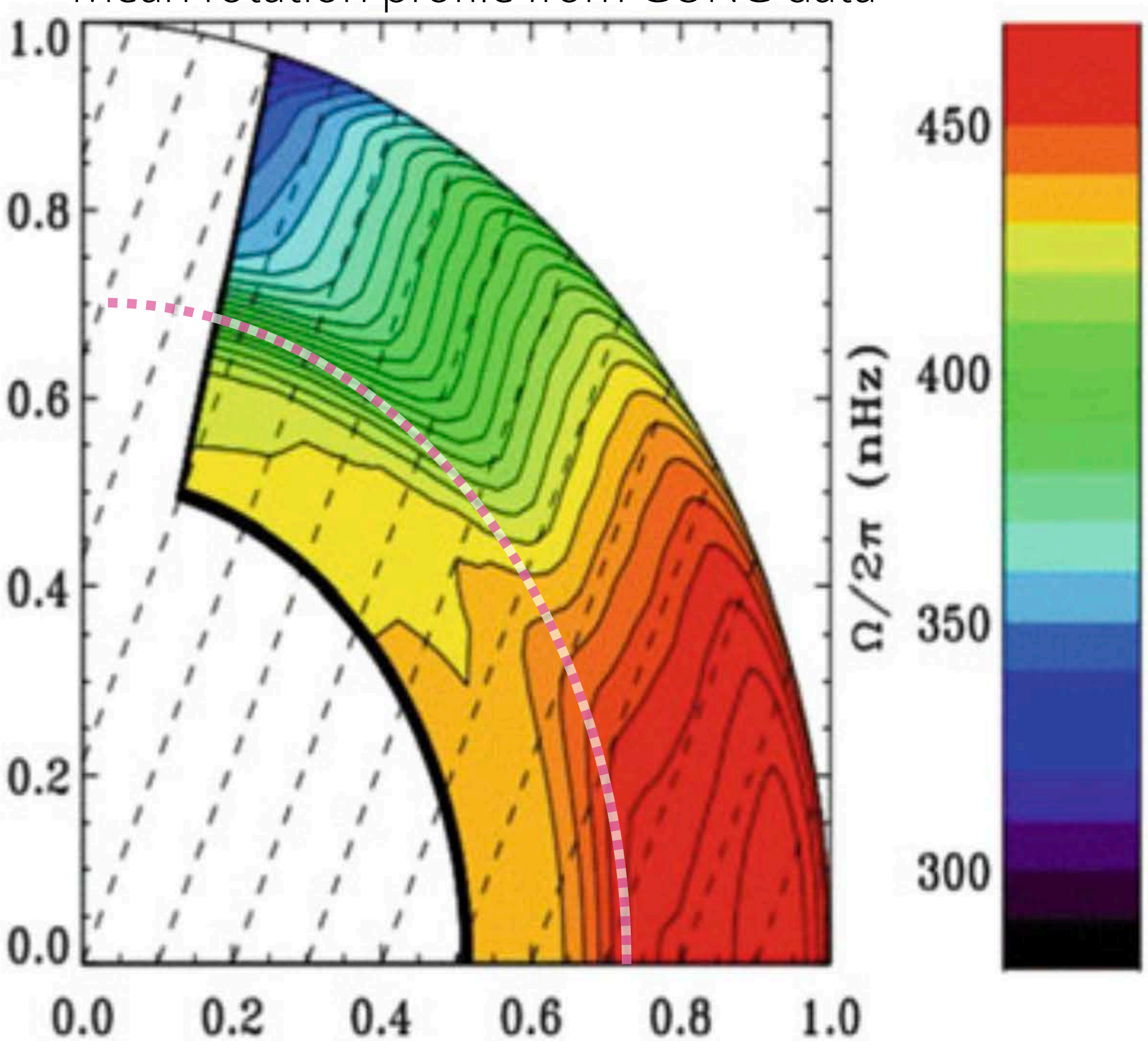
Hazra et al. 2014

Helioseismology

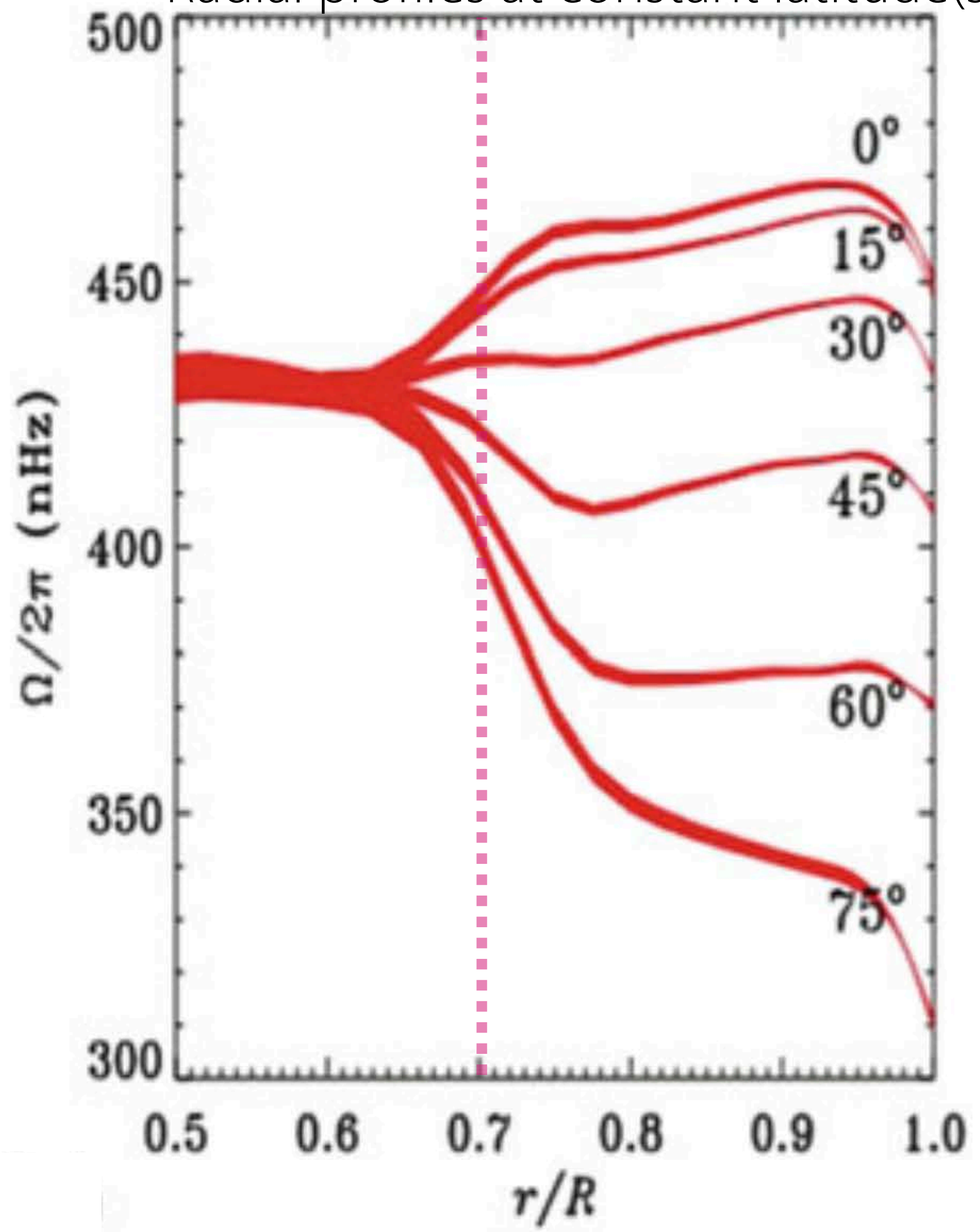
Rotation and meridional flows

- Rotation speed in the convection zone (and at the surface) varies with latitude and depth!
- Differential rotation: The Sun's equator rotates faster than the poles.

Mean rotation profile from GONG data



Radial profiles at constant latitude(s)



Howe et al. 2000, 2005

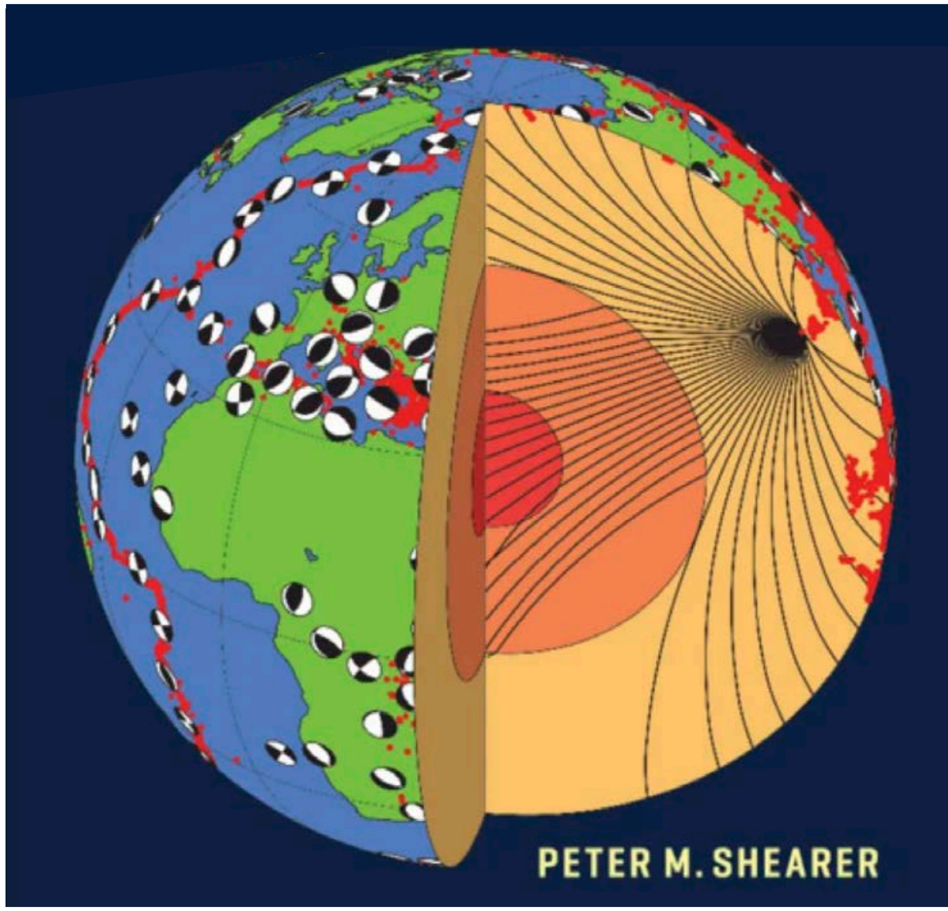
Helioseismology

Time-Distance Methods

- **Seismology on Earth:**
  - Measure arrival time of the initial onset of the disturbance at different locations on the surface
  - If we know the variation of seismic velocity with depth within the earth, then we can calculate the travel time of rays between an earthquake and a receiver.

  ➡ Locate the epicentre of any earthquake

  ➡ Also: Learn about variations of the velocity due to differences in structure/density inside Earth
  (the Earth is not a perfect homogeneous sphere!)



Helioseismology

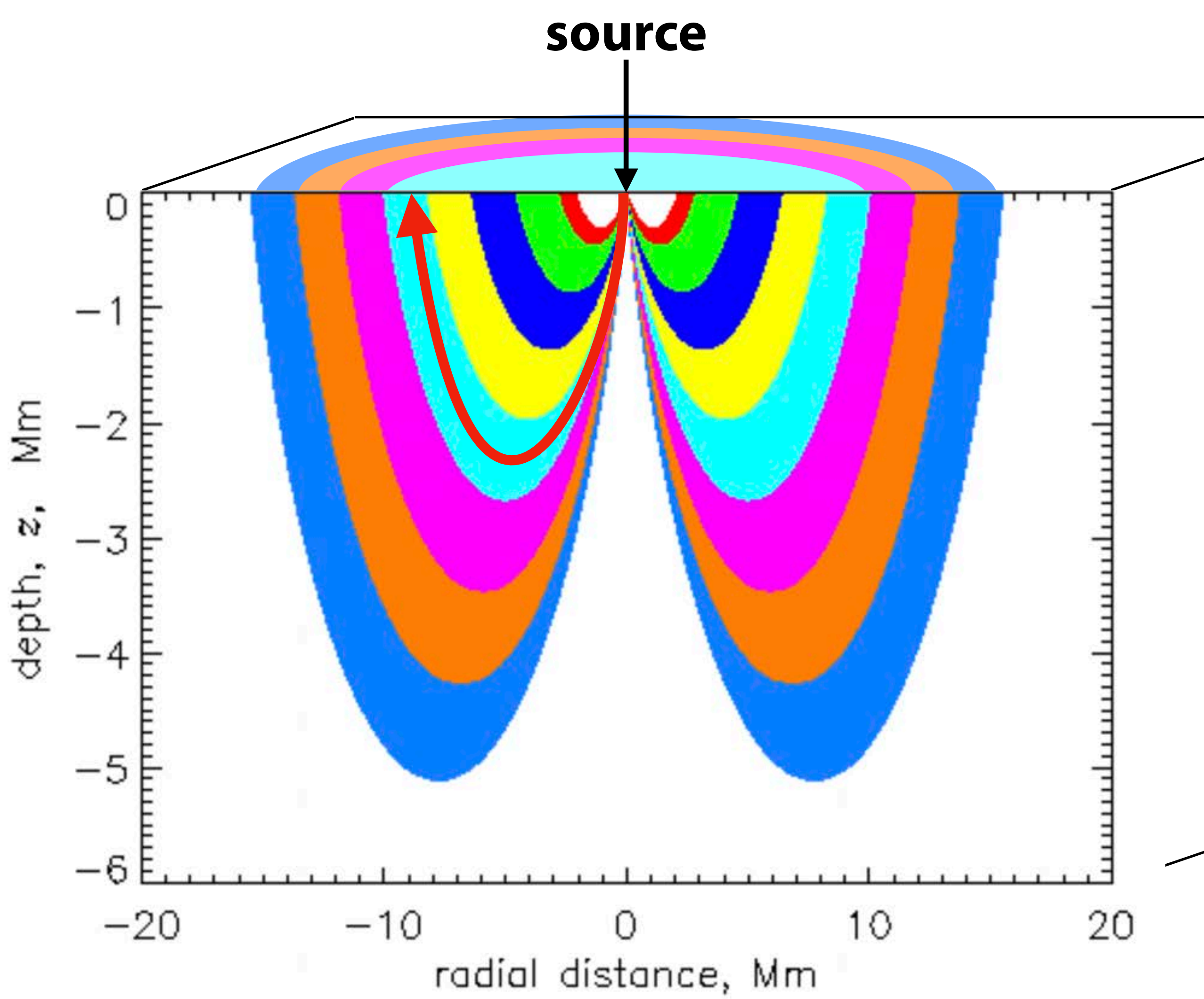
Time-Distance Methods — local helioseismology

- **Time-distance helioseismology**
  — similar idea for the Sun:
  - Select point on surface as the "source"
  - Assume an annulus at some great circle around that point as a destination
  - Calculate correlations between all points in the annulus
  - Any pulse in the correlation function reveals time and distance

  ➡ Probes the sub-surface medium through which the sound waves have traveled

  - Deviations in the arrival times due to inhomogeneities under the surface
  - Remember: Probed depth depends on wavelength of observed p-mode

  ➡ Mapping of sub-surface structure



Helioseismology

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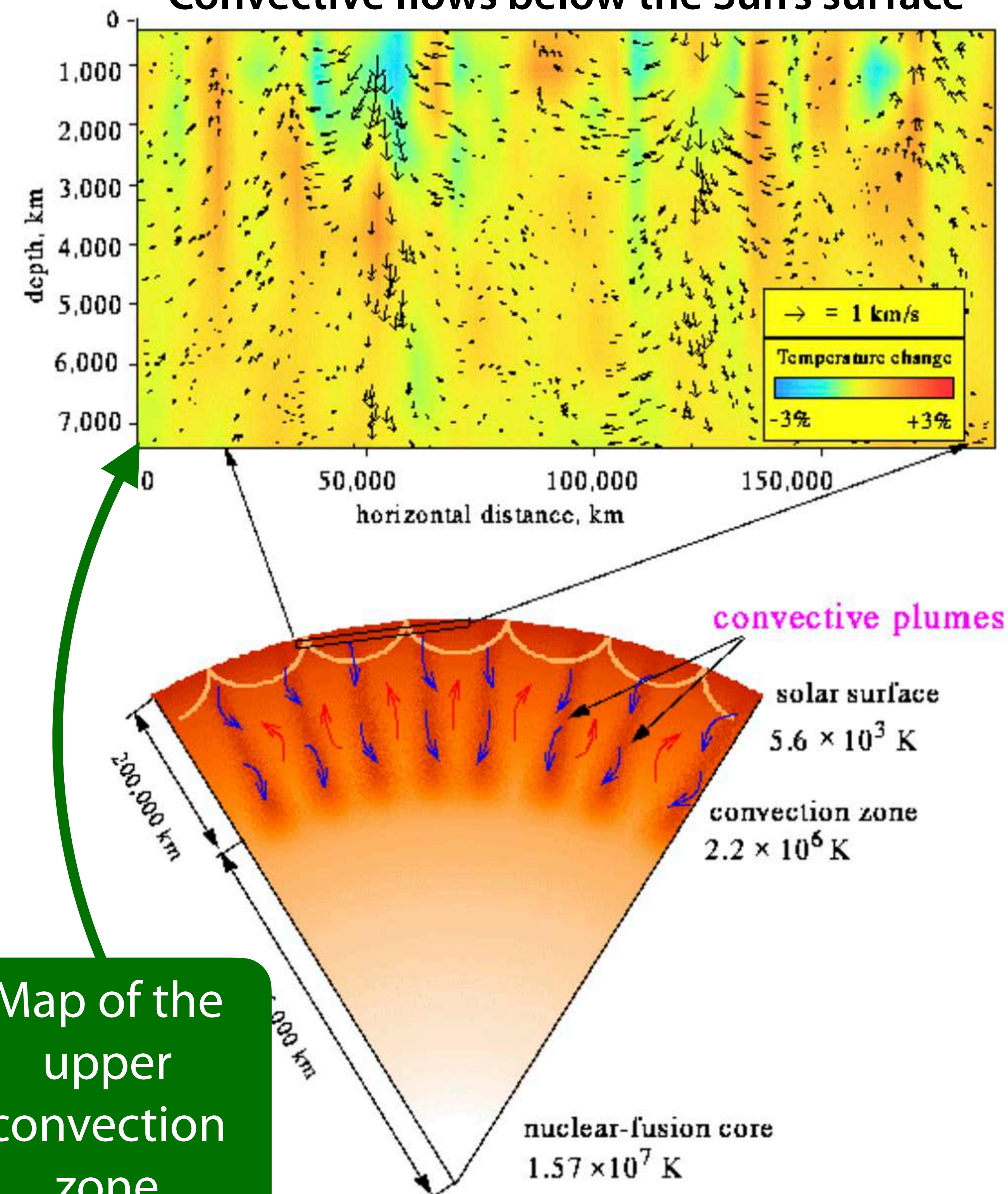
  ➡ Mapping of sub-surface structure

Convective flows below the Sun’s surface

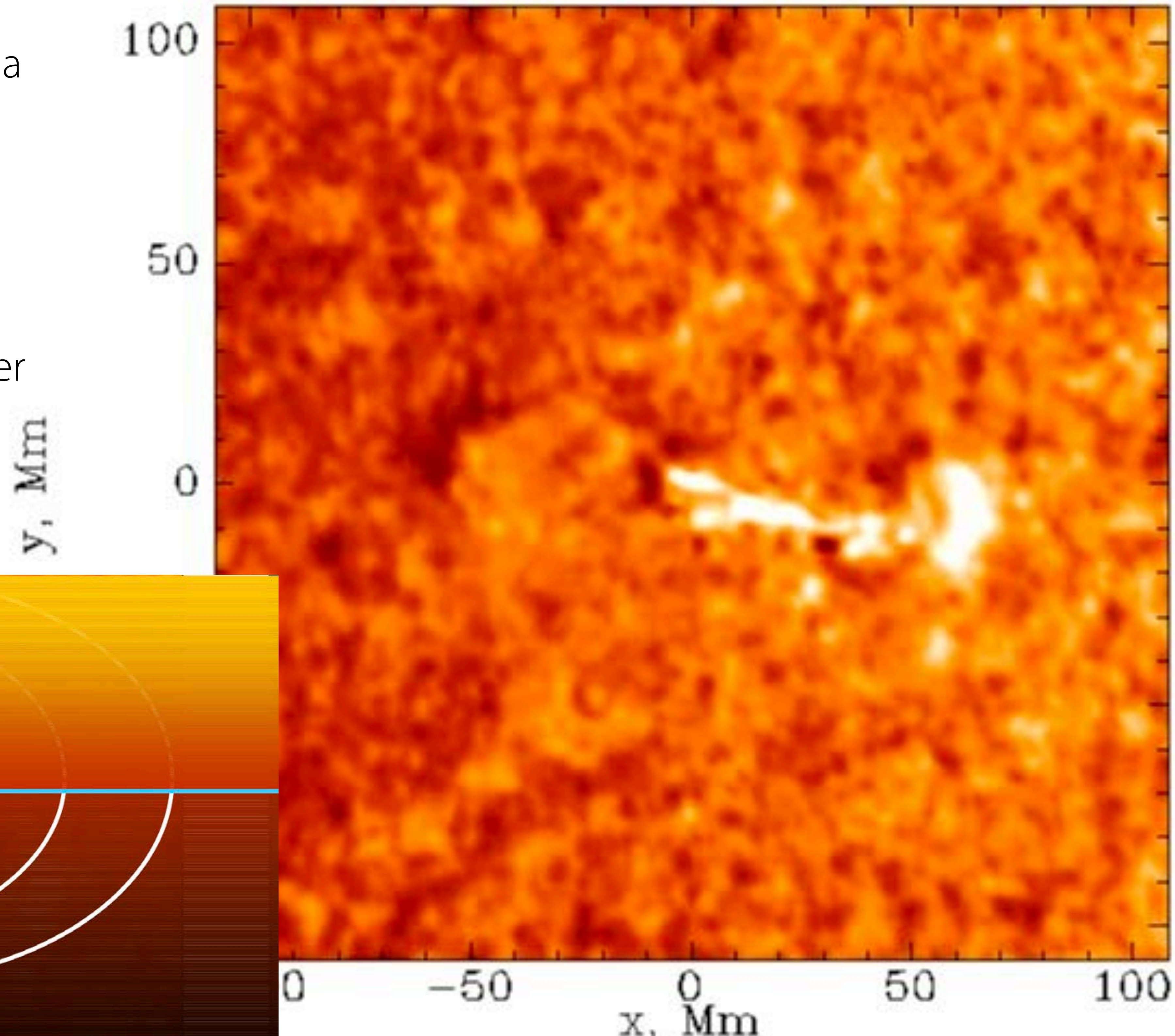


Map of the upper convection zone

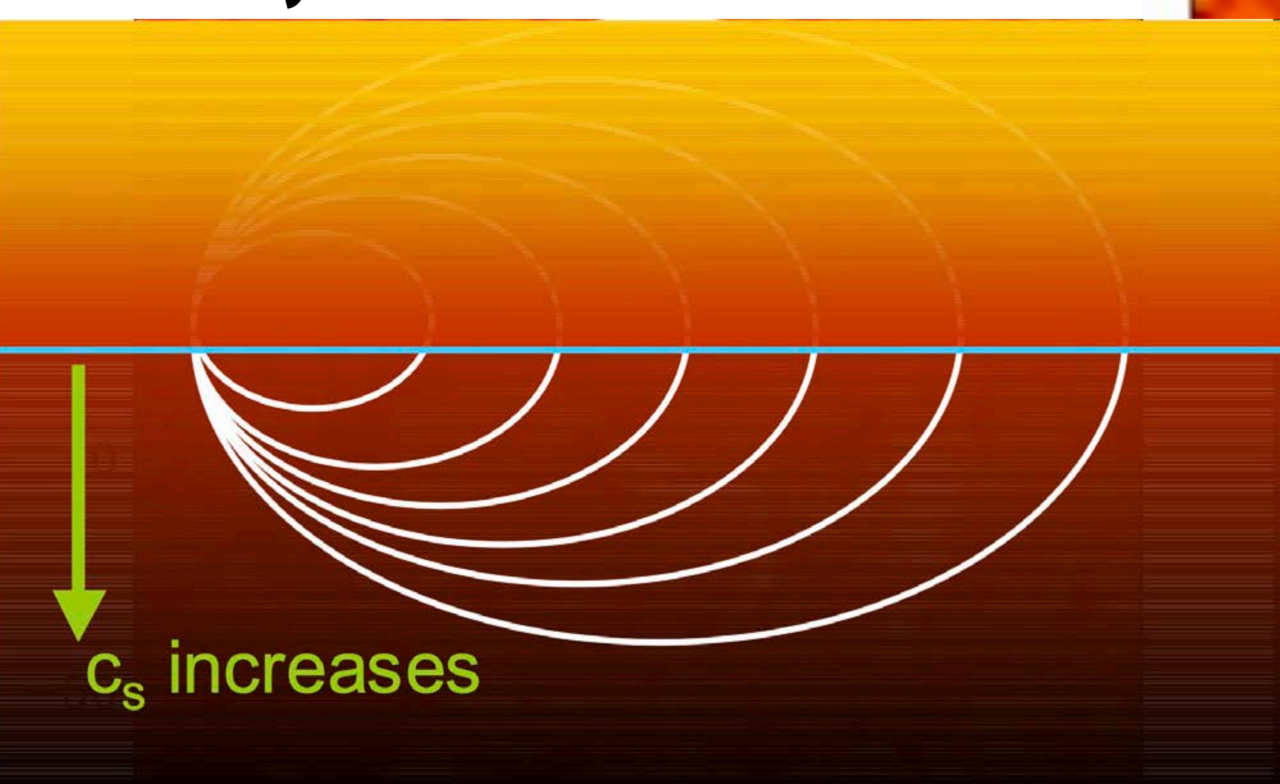
Helioseismology

Sunquake

- Disturbance triggered by a solar flare on 9 July 1996
- Seismic wave ripples outward
- Wave travels not surface, but reaches surface further out at later times.
- Wave seems to speed up! **Why?**



Kosovichev et al (1998)



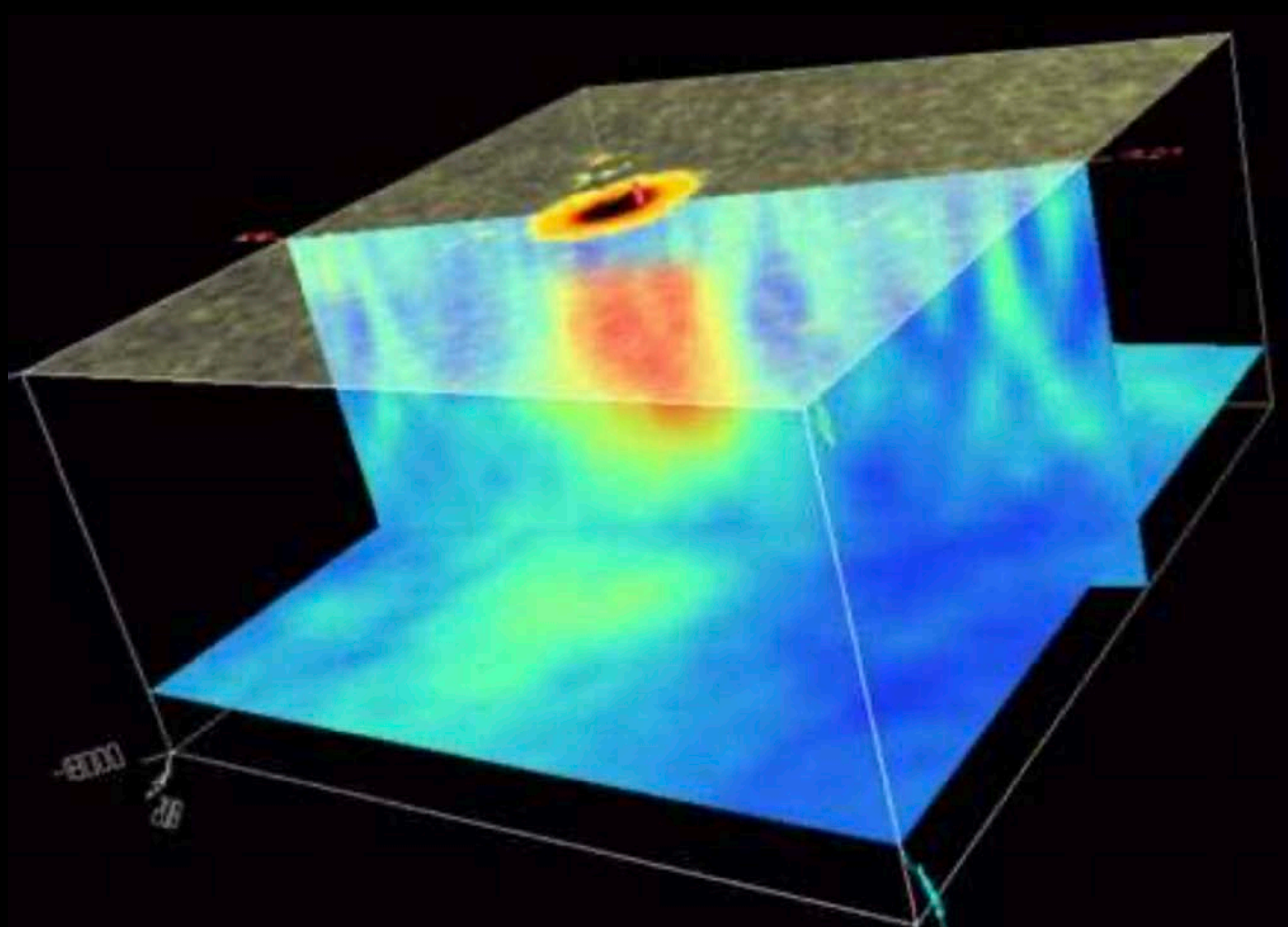
Helioseismology

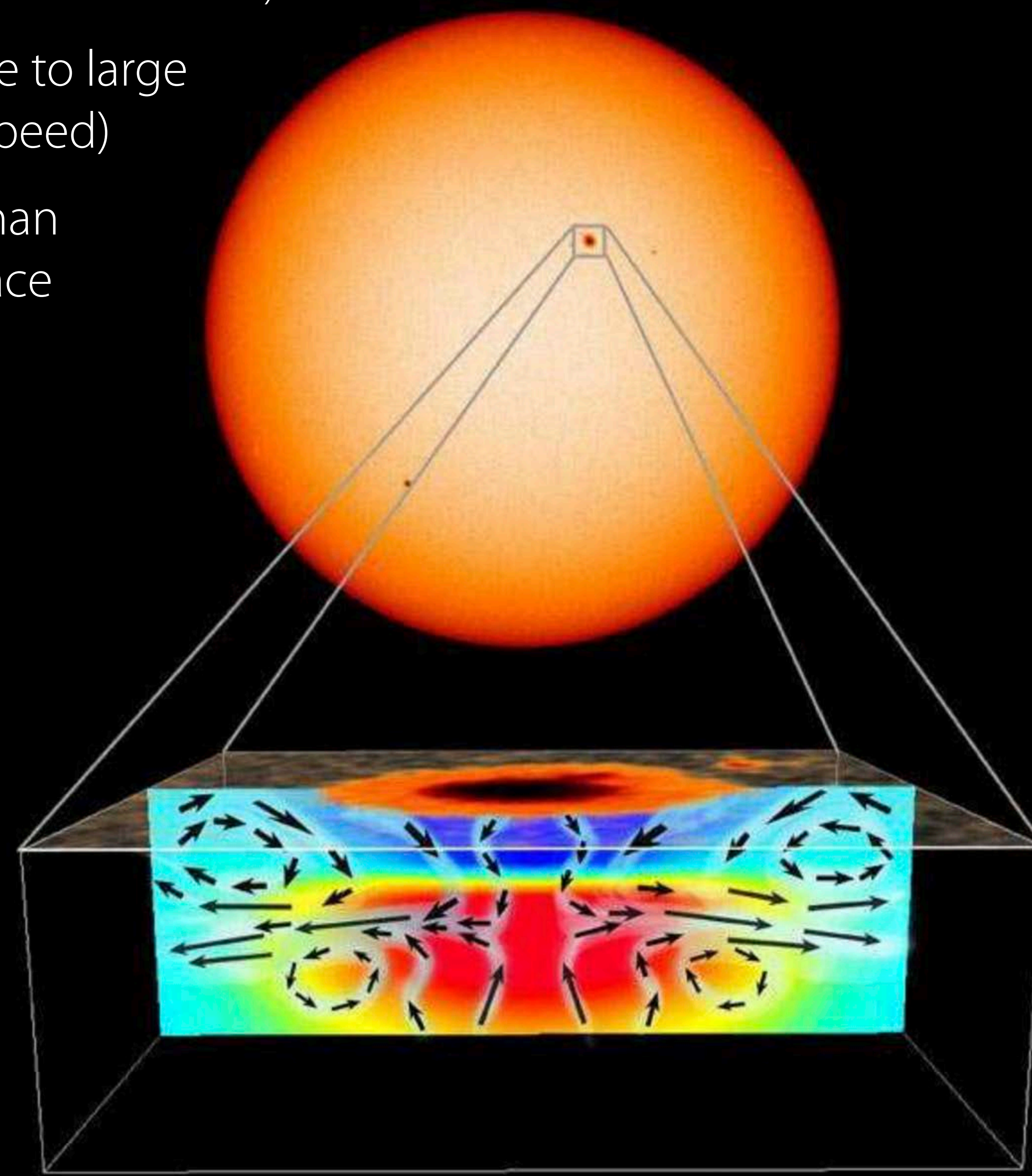
Time-Distance Methods — local helioseismology

- **Subsurface structure of sunspots** (Kosovichev et al. 2000)
- Sunspots good targets for this technique due to large change in temperature (and thus in sound speed)

➡ **Sunspots surprisingly shallow**: warmer than surroundings already ~ 4000 km below surface

- Remaining uncertainty: unknown influence of magnetic field on the wave propagation

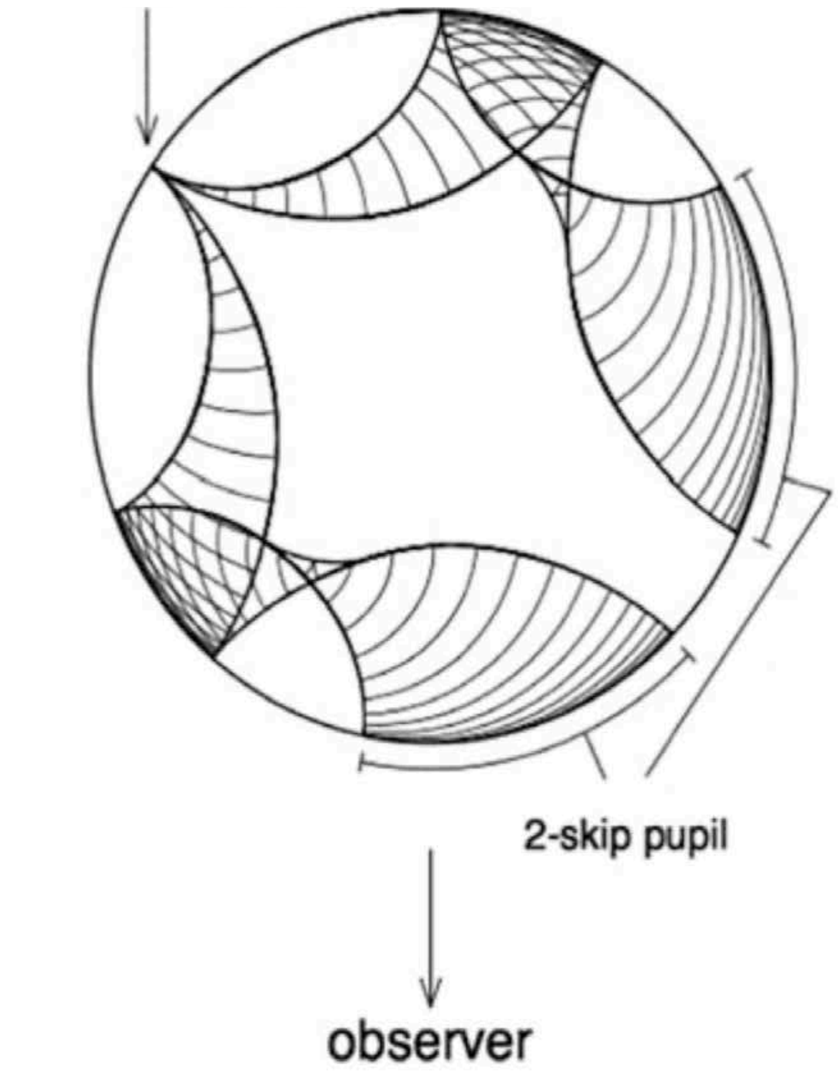




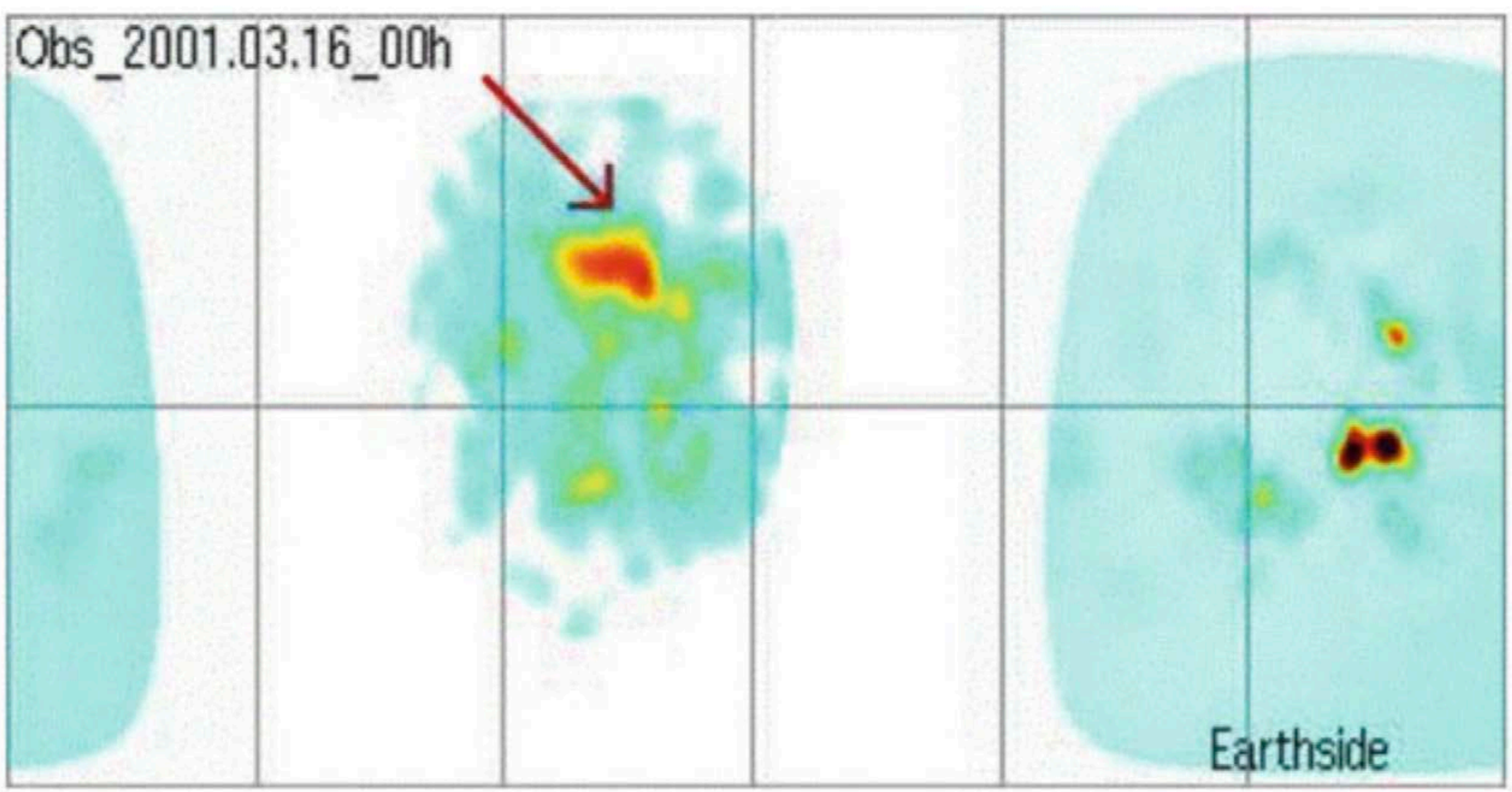
Helioseismology

Mapping the far side

- Two-skip far-side seismic holography
- Difference (delay) in travel time of sound waves from active region on the far-side of the Sun as compared to Quiet Sun ~10 s (on a total travel time of ~6 hr)
- Confirmed by other observations (STEREO satellites)
- Very helpful for space weather forecast — knowing about Active Regions before they rotate on the visible (Earth) side



Braun and Lindsey 2001



Braun and Lindsey 2001; Gizon and Birch 2005)

Helioseismology

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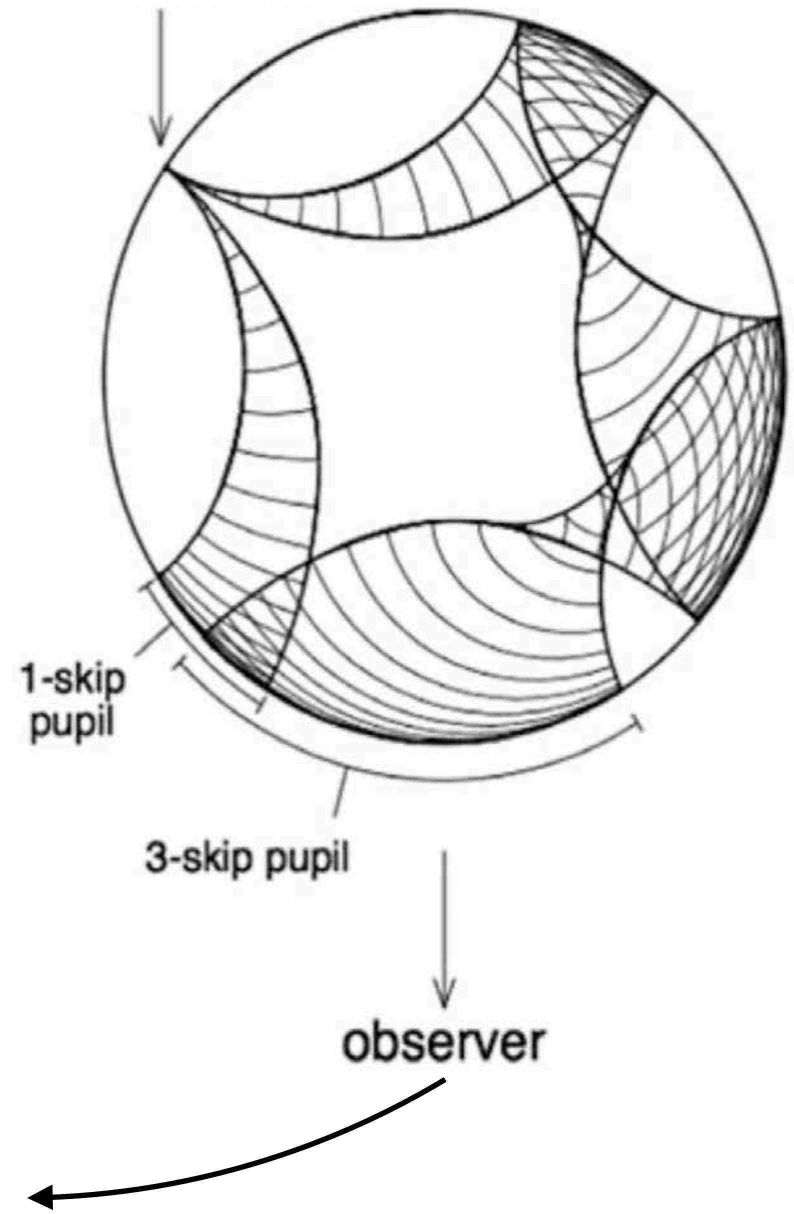
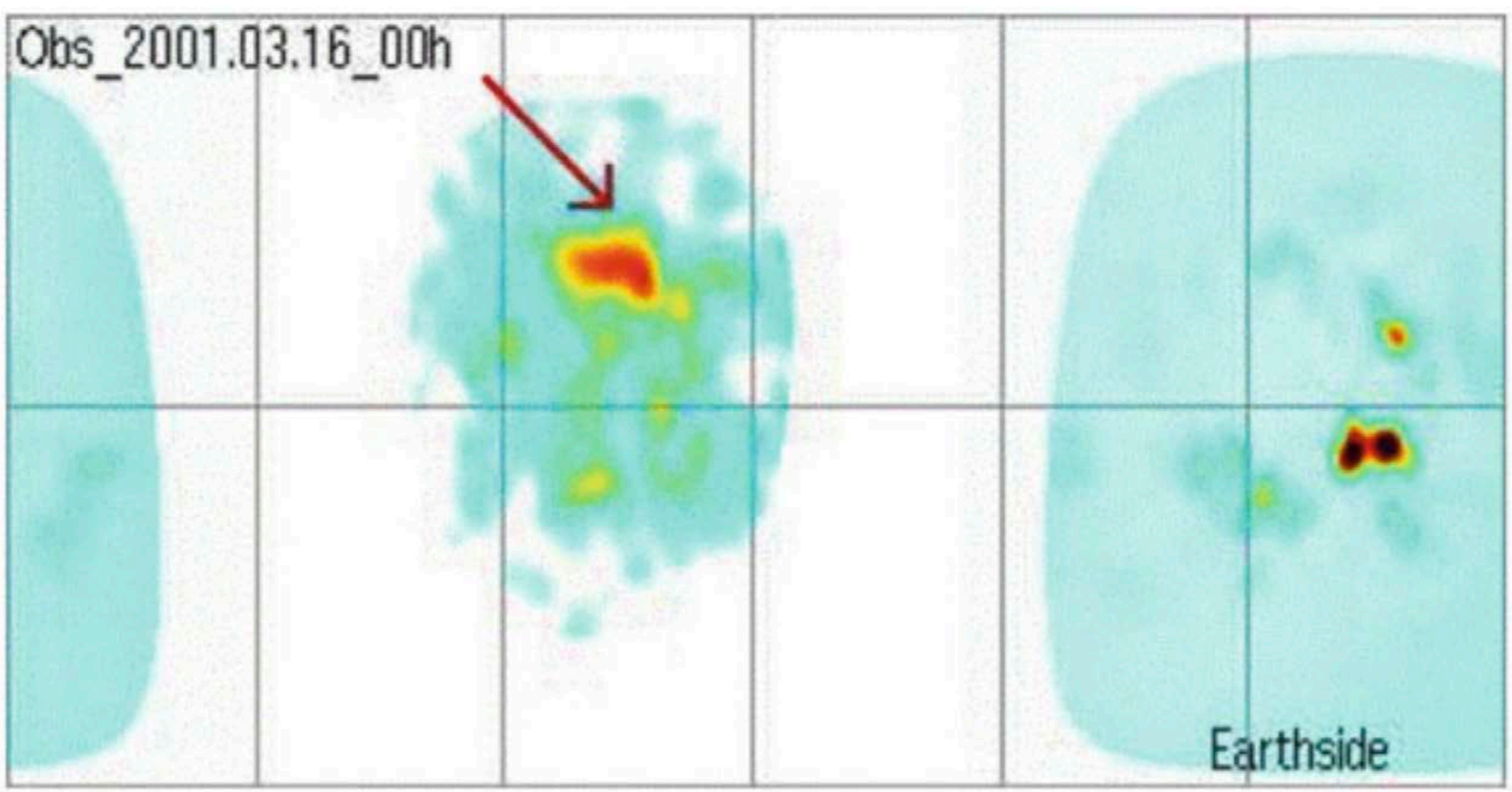
Focal point (location on the other side that is mapped)



Braun and Lindsey 2001



Braun and Lindsey 2001; Gizon and Birch 2005)

Helioseismology

Sun as a star — implications for asteroseismology

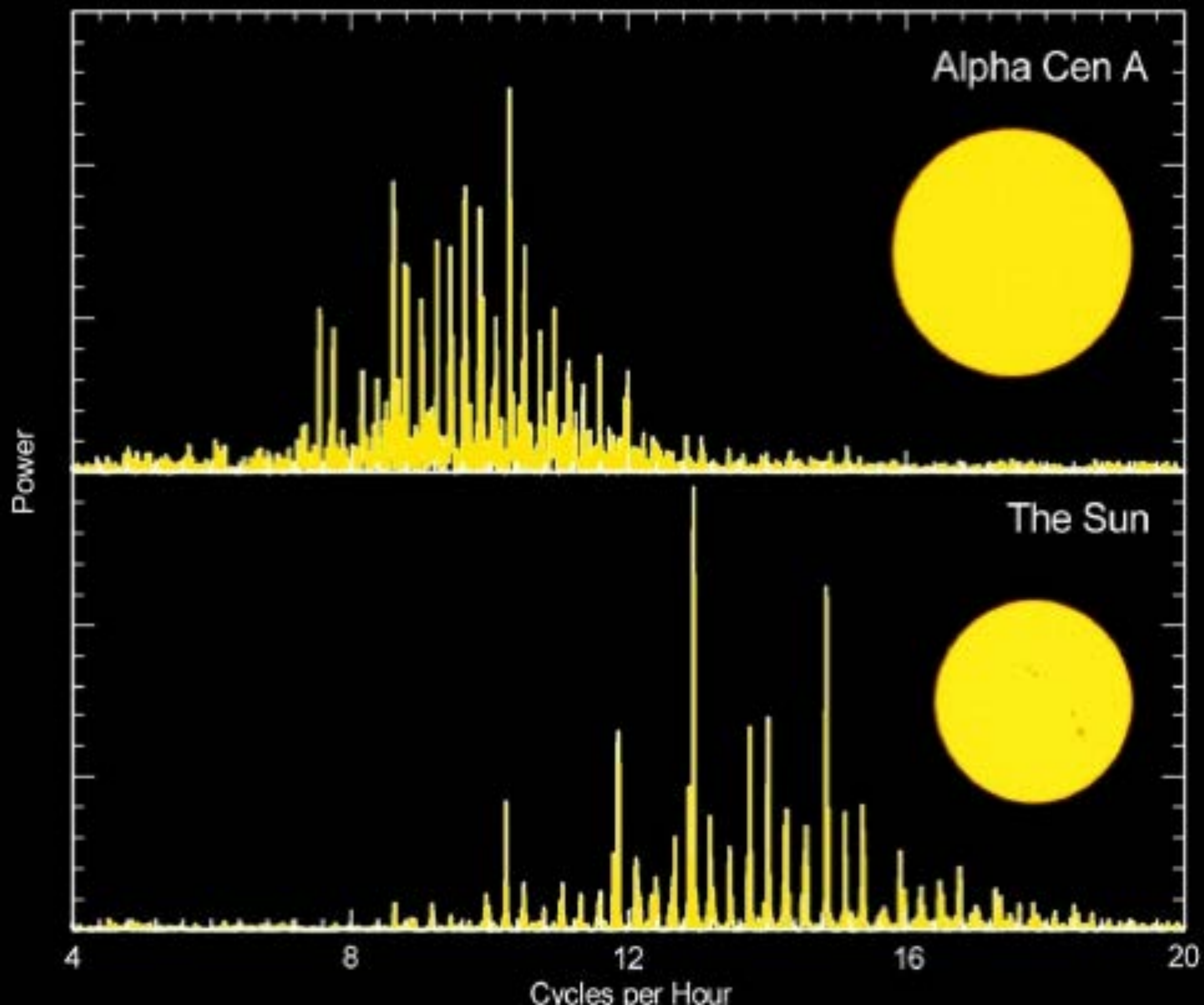
- **“Our usual problem”:** Stars (except for the Sun) observed **not (well) spatially resolved**
- Stellar observations return simpler power spectrum with **only modes with low l**
  - Up to l =3 for intensity variations
  - Up to l =4 for radial velocity variations
  - Imprints of higher modes (with more nodes on stellar surface) cancel out when not spatially resolved, much information on non-radial oscillations lost

- **Low-l modes**:
  - Probe the deep interior (of the Sun)

    ➡ Sometimes called “global” modes.
  - Different peaks in power spectrum for given l correspond to different values of n (radial nodes, n =15...25 are typical)

Asteroseismology

- First reliable detection of oscillations on α Centauri (our nearest solar analogue)
- Power spectrum shifted towards lower frequencies for α Centauri compared to the Sun



α Cen A vs. Sun

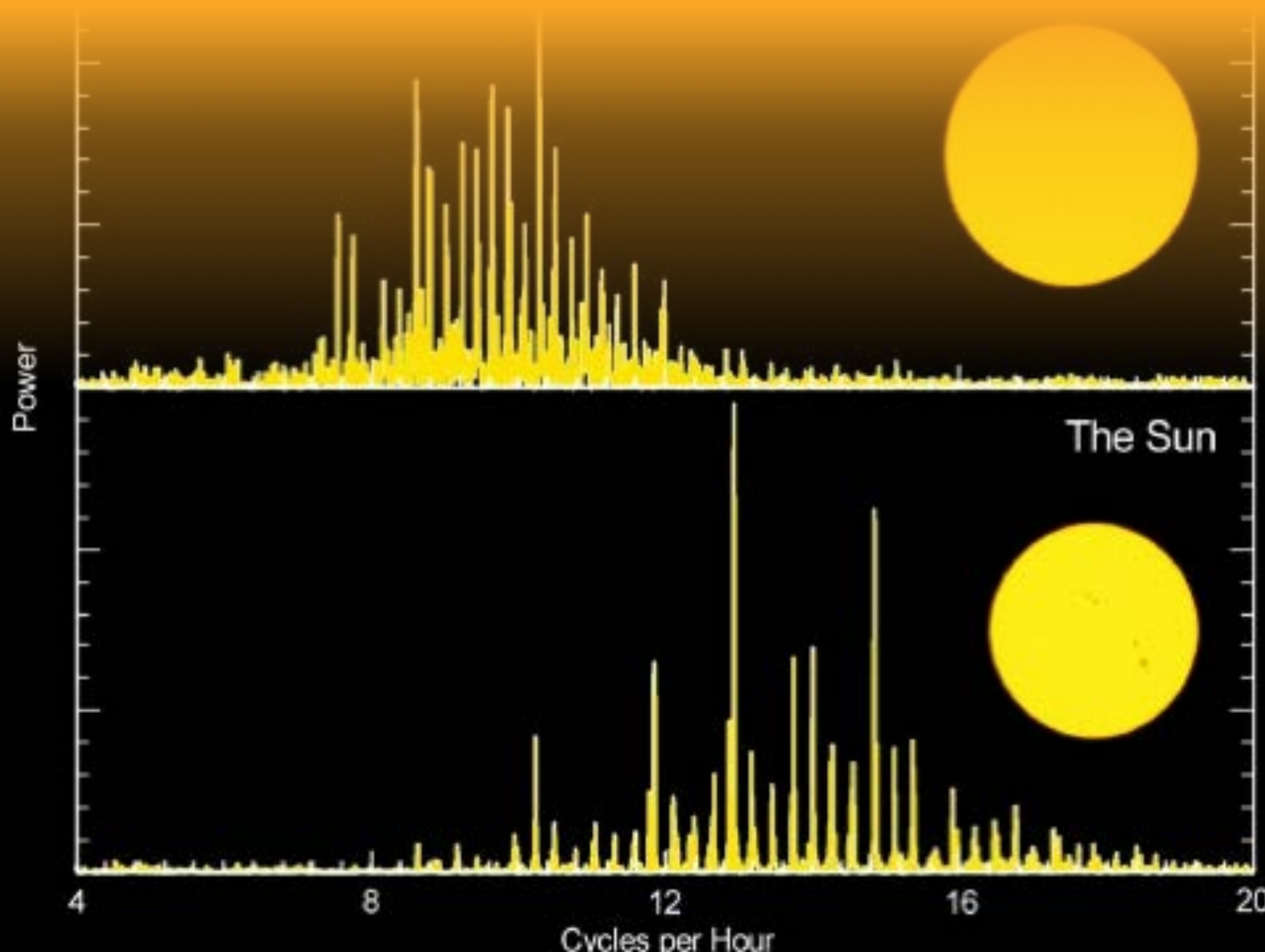
- Age: slightly older
- Mass: a bit more
- Radius: a bit more
- log g: a bit less
- Rotation: a bit faster
- Metallicity: a bit higher

Asteroseismology

- Frequency of the fundamental radial mode $l = 0$, is proportional to a star's dynamical time scale (t_{dyn})

➡ Large frequency spacing $\Delta\nu$ (between adjacent radial modes in oscillation spectrum) proportional to the dynamical time scale

$$\Delta\nu \propto \frac{1}{t_{dyn}} \propto \sqrt{\rho} \propto \left(\frac{M}{R^3}\right)^{\frac{1}{2}}$$



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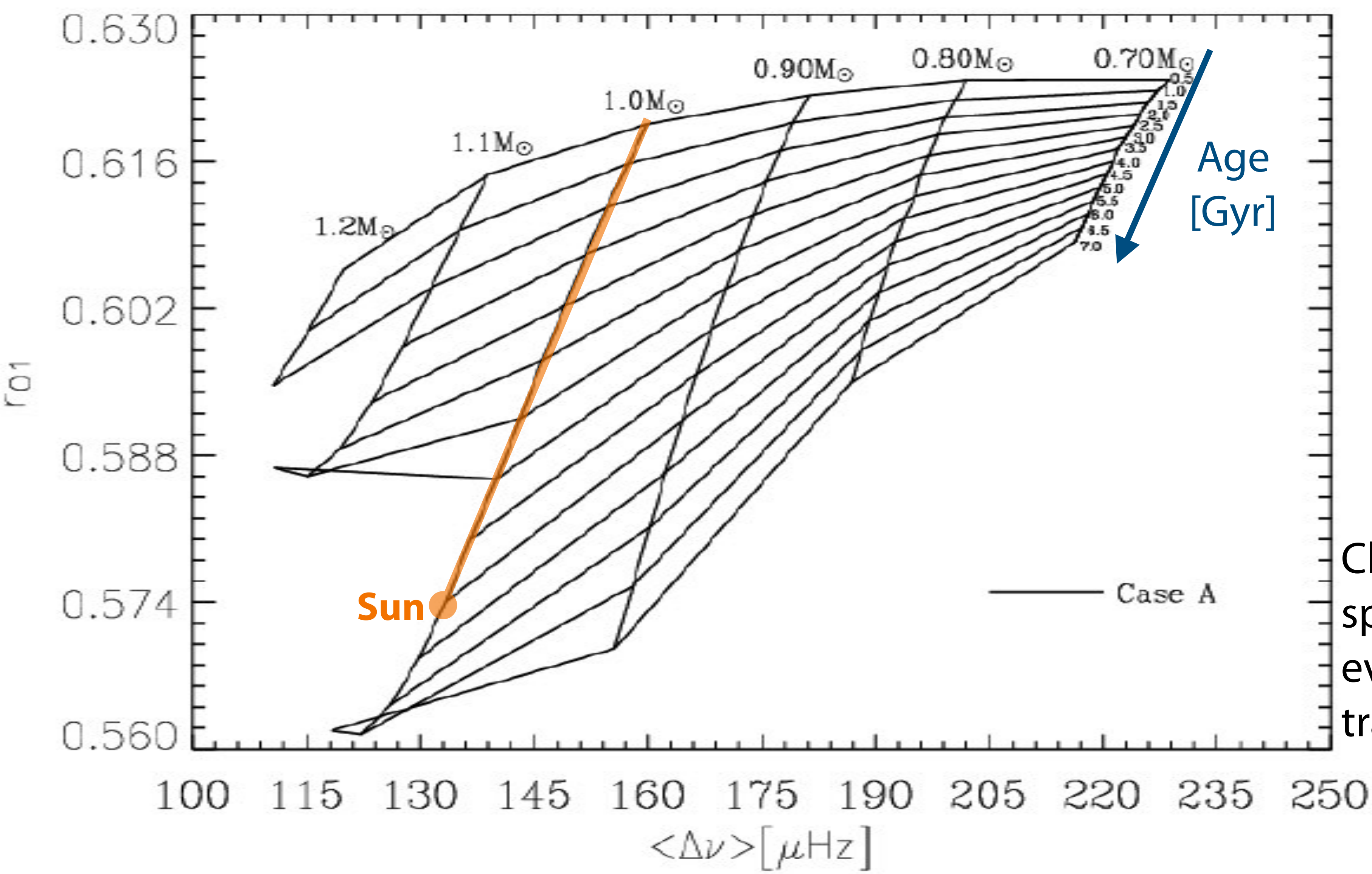
➡ **Freq. spacing depends on stellar properties, changes as star evolves.**



For the Sun (today)
$\Delta\nu \approx 136\ \mu\text{Hz}$

Change of freq. spacing along evolutionary track

Asteroseismology

Summary

- Asteroseismology provides additional constraints on the determinations of stellar parameters (e.g., masses, radii, mean densities, ages)
  - Important tests for stellar structure and evolution models
  - Important constraints for stellar interiors and thus generations of magnetic fields (dynamo) and stellar activity
- Synergies with exoplanet missions that look for small variations in the host star
  - Missions like Kepler, CoRoT, TESS, …
  - Mostly on radial pulsations as limited to low -l modes
- Stellar pulsations to be discussed as part of late evolution stages