

AST5770
Solar and stellar physics

University of Oslo, 2022

Sven Wedemeyer

Practical information

Updates

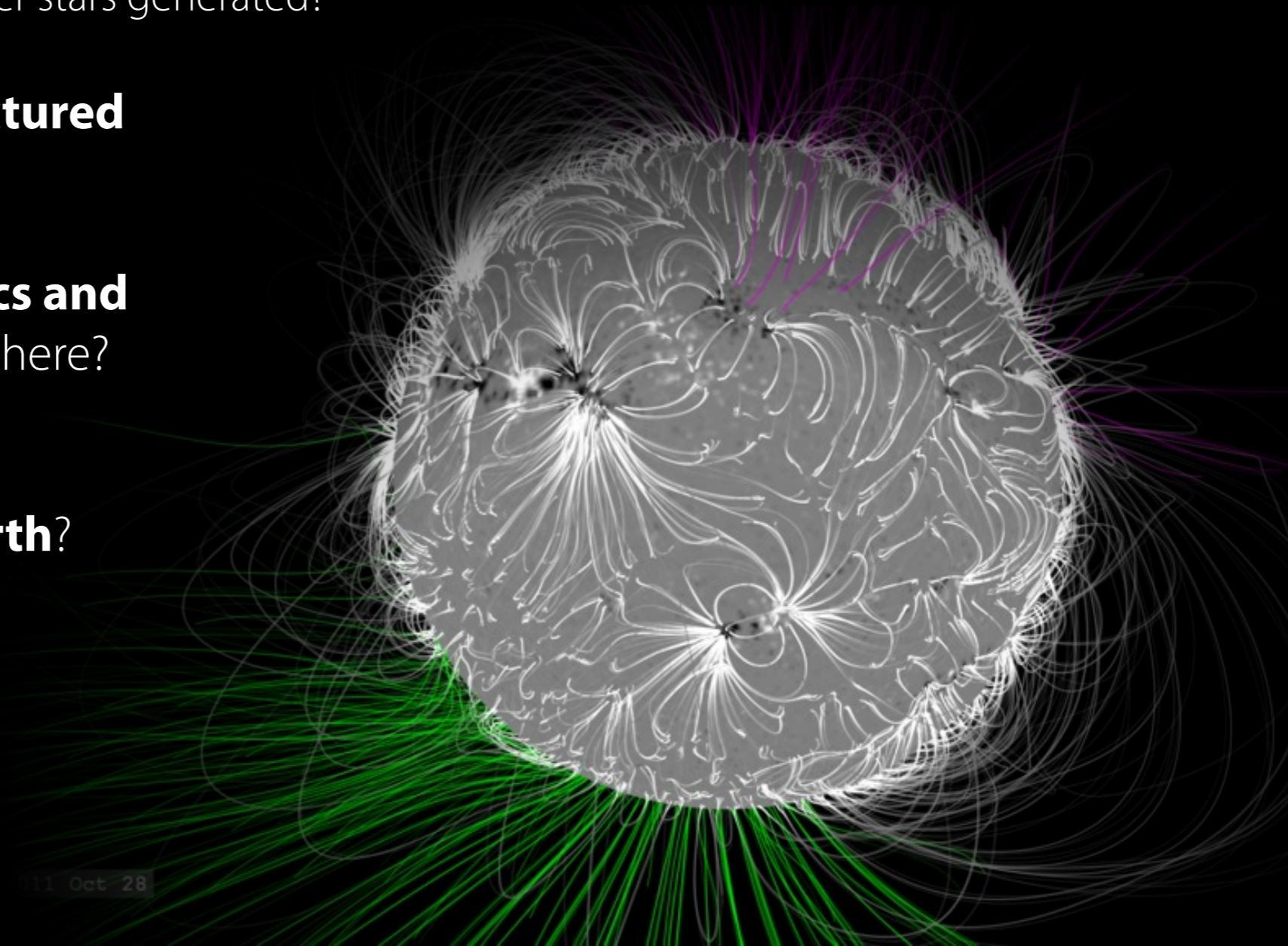
- **Assignment #2:** New minimum word count = 1500
 - Remember: Not the introduction of a real scientific article but starting at a bit lower level and can contain “textbook level” content.
- **New deadline for assignment #4:** April 20 (previously April 18)
- **Group class next week** (March 16):
 - Group teacher Aditi
 - Discussion on Fourier transformations and spatial/temporal power spectra
- **Week 12** (21.3. - 25.3.): No lectures and no group class (due to clash with block course)
- **Last lecture** (most likely): May 12
 - More time to work on the final project assignment (due May 31)

Magnetism

Magnetism

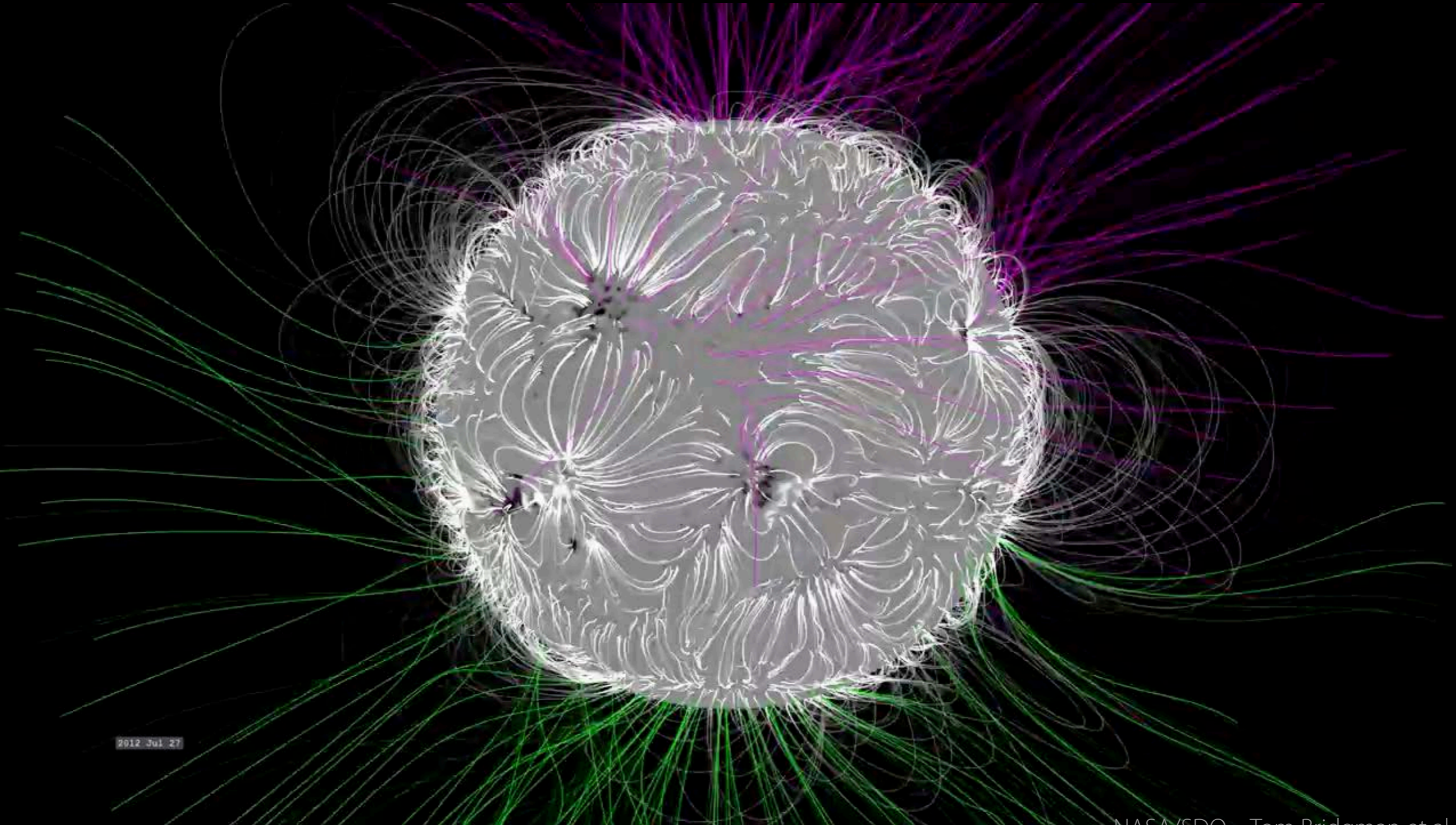
Many important questions ...

- **How is the magnetic field of the Sun generated?**
 - Is the Sun generating “new” magnetic field? Is there a dynamo process at work?
 - Or are those the remainders of a “primordial” magnetic field of the material from which the solar system and the Sun formed?
 - How is the magnetic field of other stars generated?
- How is the magnetic field **structured** in the atmosphere of the Sun?
- How does it affect the **dynamics and energy balance** of the atmosphere?
- And how does it affect the **interplanetary space and Earth?**



Magnetism

Magnetic fields on the Sun — Introduction

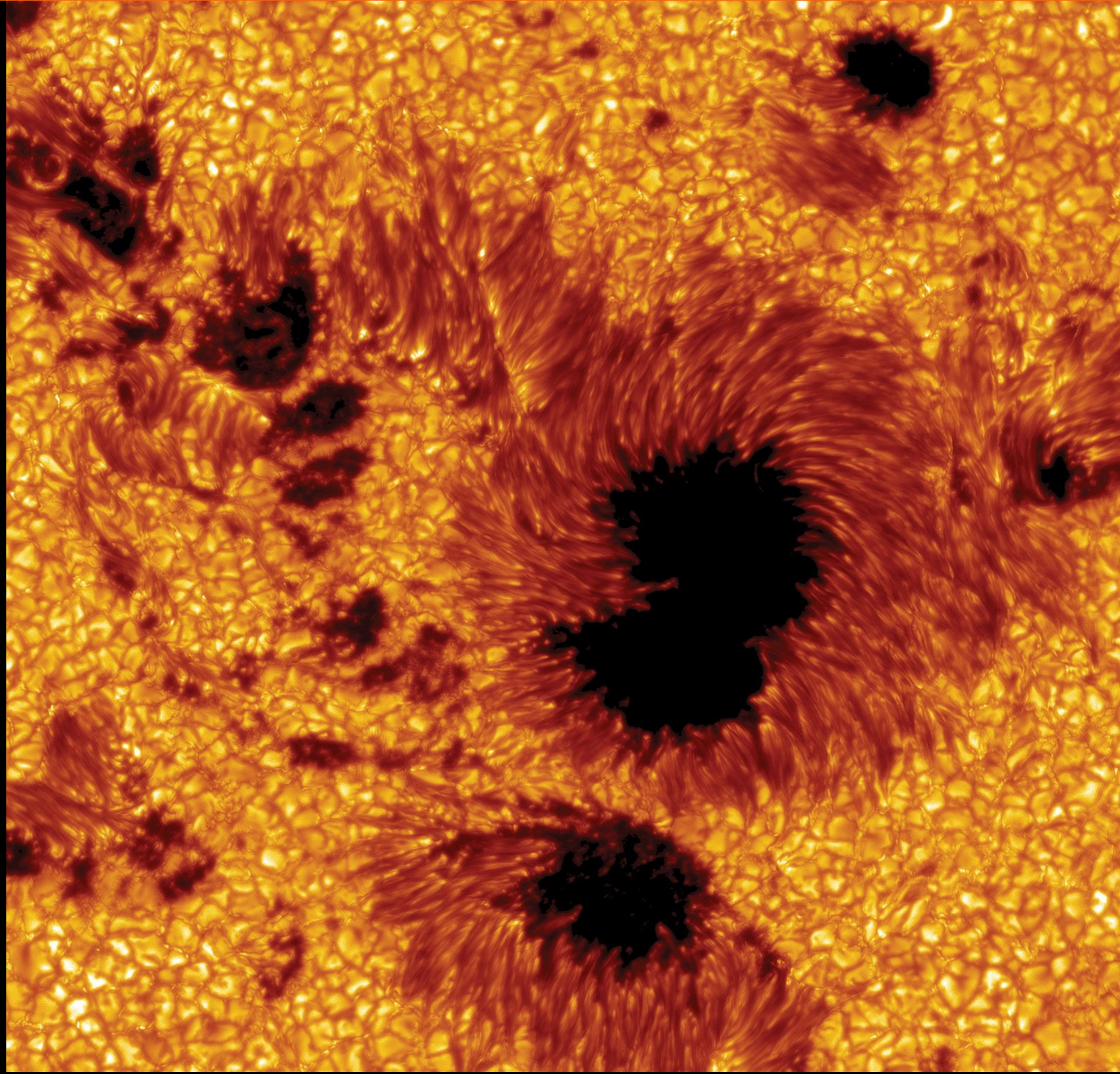


2012 Jul 27

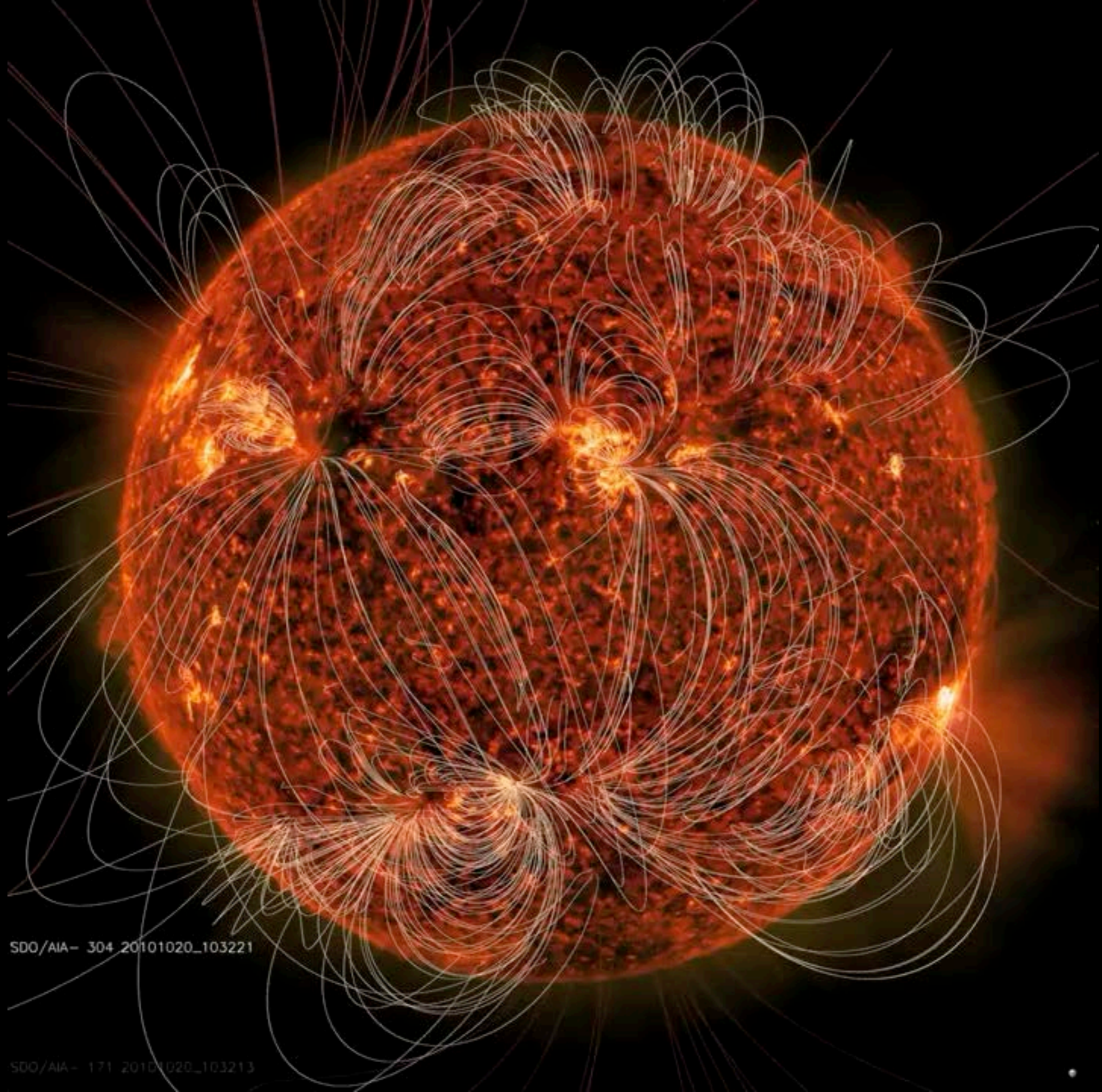
Magnetism

On the Sun

- Sunspots are clear imprints at the surface and in the atmosphere above
- Magnetism on the Sun occurs on all scales.
- Smallest features at or below spatial resolution limit of current telescopes
- Largest features on global scales
- Temporal scales between many years (activity cycle) to extremely short scales ($<1s$).



- 
- **Elongated “fibrilar” features**
➔ **Outlining magnetic field lines?**



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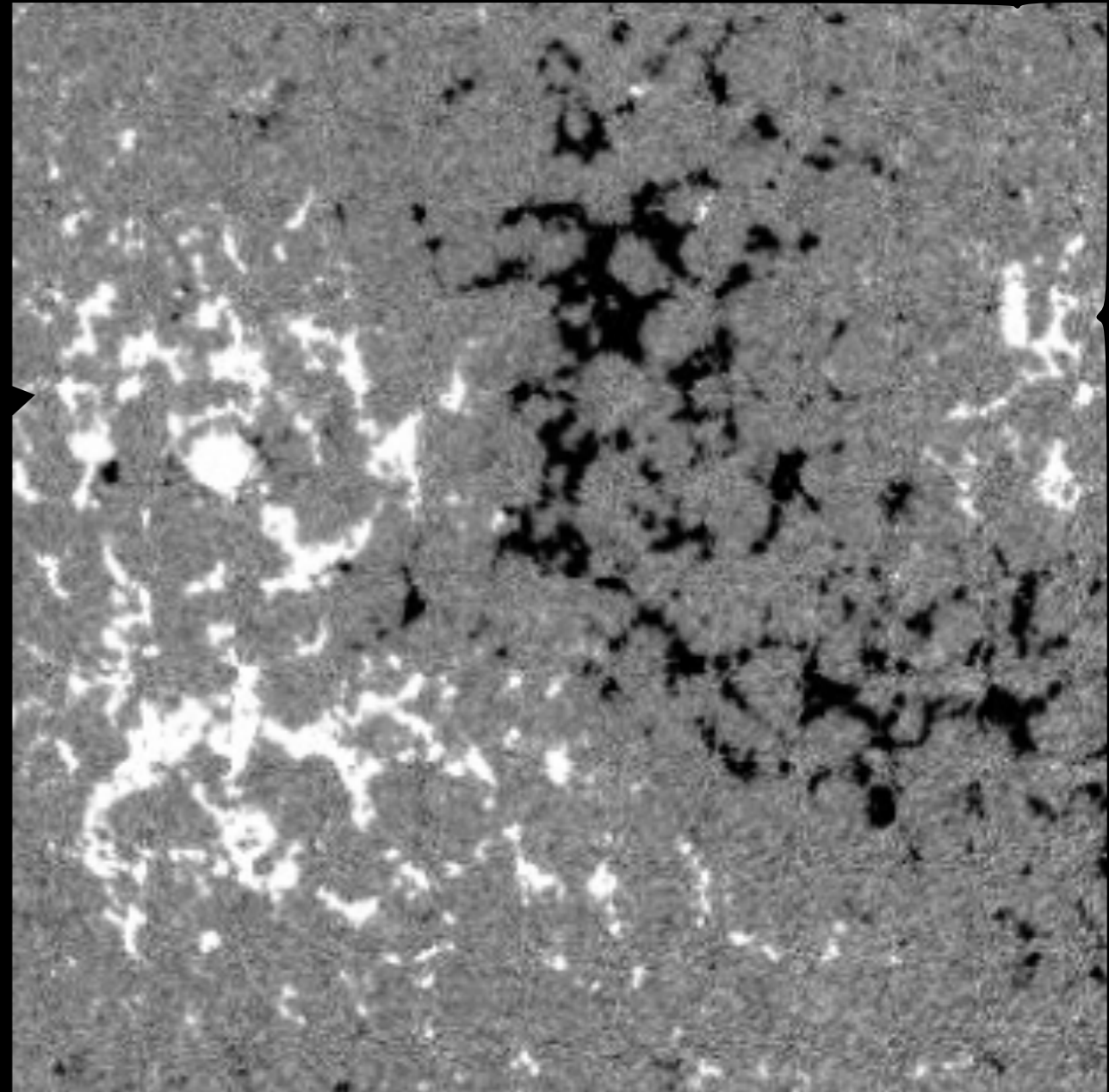
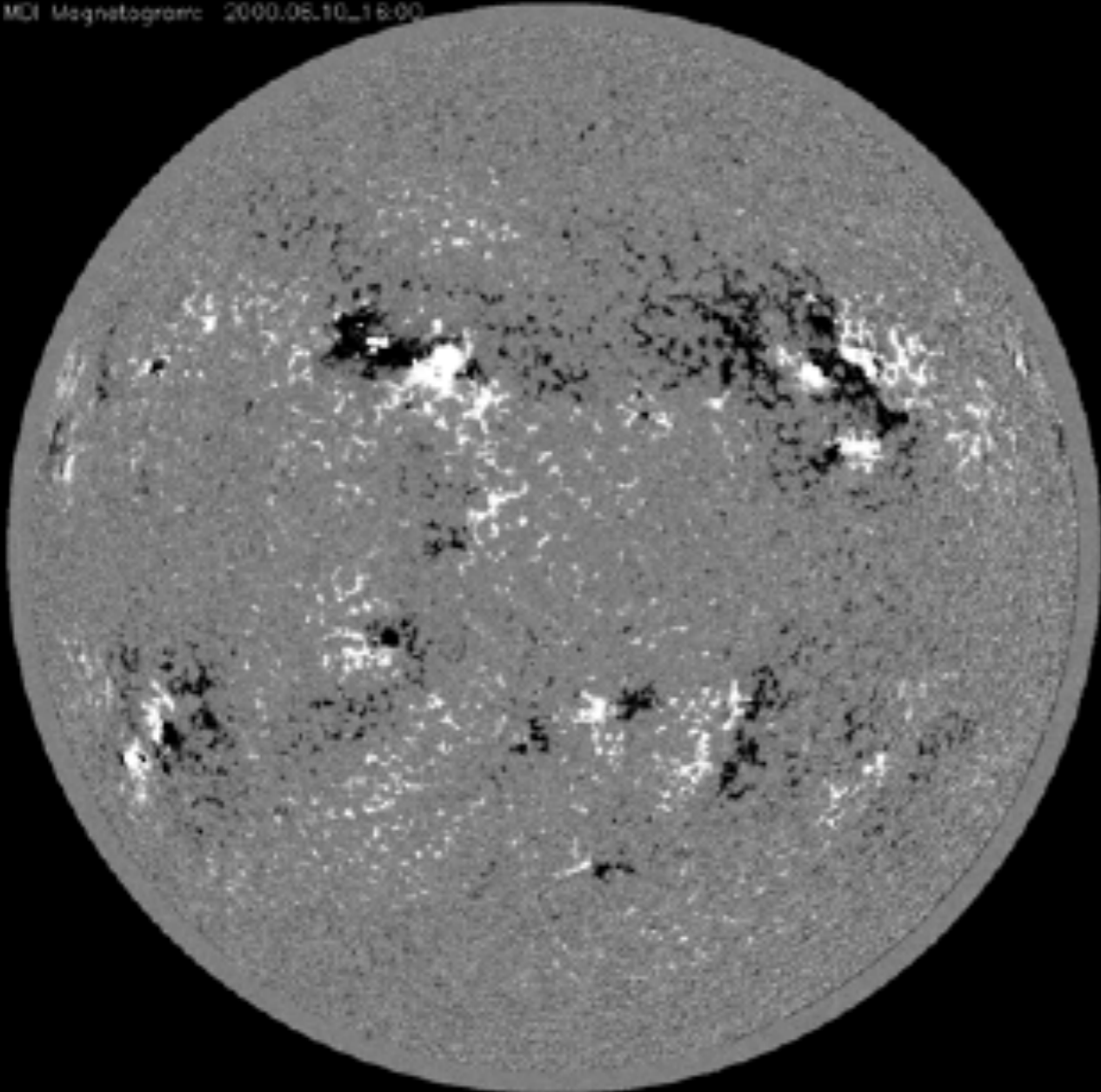
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Magnetism

Photospheric magnetograms

- Active regions, bi-polarity systematic east-west orientation opposite in the south

MCI Magnetogram 2000.06.10_18:00

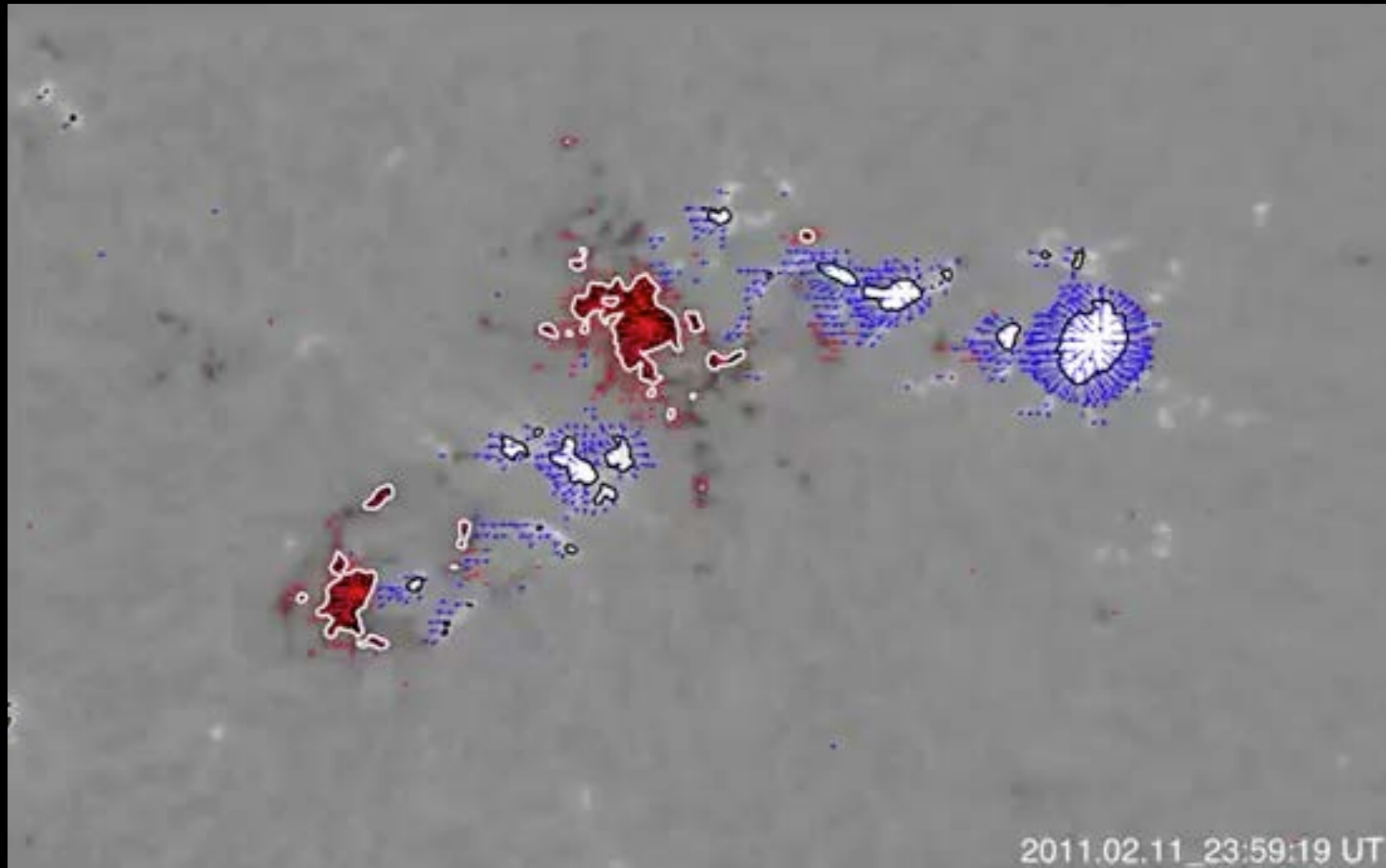


Magnetism

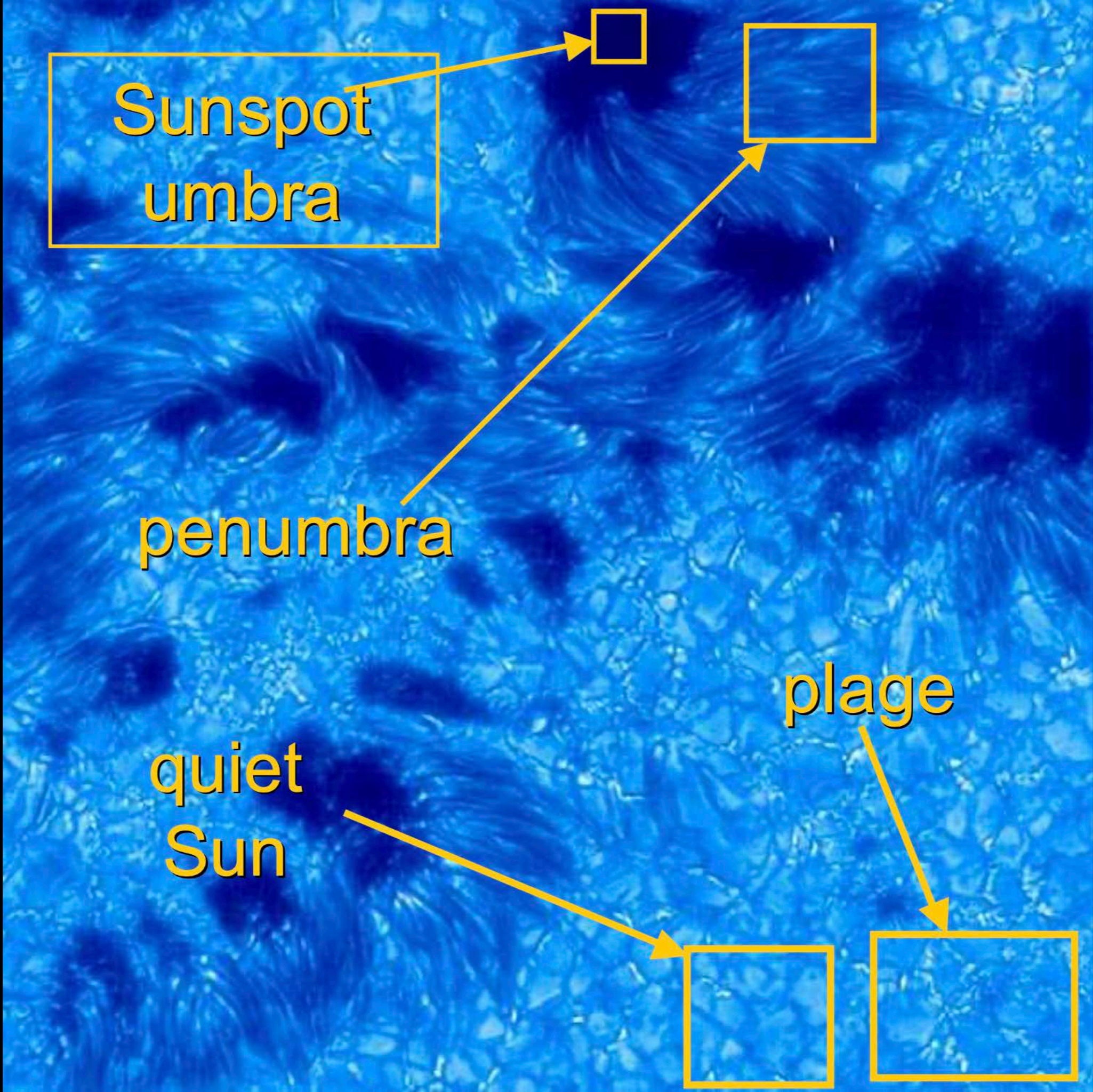
Photospheric magnetograms

- Observation of the Sun with SDO/HMI, 2/2011
- Evolution of magnetic field in an Active Region

Blue arrows: magnetic field lines "point up"
Red arrows: magnetic field lines "point down"



Observation follows target region,
removing effect of solar rotation



Sunspot
umbra

penumbra

quiet
Sun

plage

Magnetism

So far — Radiative-Hydrodynamic Equations

- Hydrodynamic equations:

- Conservation of mass (density):
(*mass continuity equation*)

$$\partial_t \rho = -\nabla \cdot (\rho \mathbf{v}),$$

- Conservation of momentum:

$$\partial_t \rho \mathbf{v} = -\nabla \cdot (\rho \mathbf{v} \mathbf{v} + \underline{\tau}) - \nabla p + \rho \mathbf{g},$$

- Conservation of energy:

$$\partial_t e = -\nabla \cdot (e \mathbf{v}) - p \nabla \cdot \mathbf{v} + q_{\text{rad}} + q_{\text{visc}},$$

- Coupling with the radiation field is give by the radiative cooling and heating term (as derived from the radiative transfer equation)

$$q_{\text{rad}} = 4\pi\rho \int_{\lambda} \kappa_{\lambda} (J_{\lambda} - S_{\lambda}) d\lambda,$$

- Equation of state

Magnetism

Magnetohydrodynamics (MHD)

- MHD equations describe how a magnetic field interacts with a continuous plasma (ionized gas).
 - “Like hydrodynamics”, but with extra equations (Lorentz-force etc.)
- **Ideal MHD**
 - Simplifying assumption: infinite conductivity (no resistance), perfectly conducting (and thus fully ionised) plasma, no dissipation of electro-magnetic energy
 - Currents are present, but no charge densities
 - Applicable in the solar interior and in good approximation in the solar atmosphere (note that there will be deviations)
- **Non-ideal MHD conditions:**
 - Ions and neutrals slip past each other (ambipolar diffusion)
 - Magnetic reconnection (localized events, flares)
 - Turbulence induced reconnection

Magnetism

Magnetohydrodynamics (MHD)

- MHD equations describe how a magnetic field interacts with a continuous plasma (ionized gas).
- primary variables v , B , p , ρ and T

Conservation of mass (density) (mass continuity equation)	$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0$
Conservation of momentum — extra term	$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F}_g + \mathbf{F}_v$
Conservation of energy	$\frac{\rho^\gamma}{\gamma - 1} \frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = -\nabla \cdot \mathbf{q} - L_r + \frac{j^2}{\sigma} + F_H$
Equation of state	$p = \frac{k_B}{m} \rho T \quad \left(= \frac{\tilde{R}}{\tilde{\mu}} \rho T \right)$

$\mathbf{j} \times \mathbf{B}$: Lorentz force per unit volume — describes the interaction between the magnetic field and the plasma

j^2/σ : Ohmic dissipation

\mathbf{q} : heat flux due to particle conduction

L_r : net radiation

Magnetism

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Equation of state	$p = \frac{k_B}{m} \rho T \quad \left(= \frac{\tilde{R}}{\tilde{\mu}} \rho T \right)$
Induction equation	$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$
Ohm's Law	$\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})$

- Further reading: *Solar Magnetohydrodynamics*, E. Priest (2014), downloadable via UiO bib.

Magnetism

Magnetohydrodynamics (MHD)

- MHD equations describe how a magnetic field interacts with a continuous plasma (ionized gas).
- primary variables: v , B , p , ρ , T

- Numerical simulations need to account for a lot physics — computationally very challenging
- Certain terms can be neglected depending on the simulated scenario in order to make problem computationally feasible
 - Validity of resulting model then limited by these simplifying assumptions
- Numerical simulations have historically developed from simplified cases to increasingly more complex and realistic cases

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F}_g + \mathbf{F}_v$$

$$\frac{\rho^\gamma}{\gamma - 1} \frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = -\nabla \cdot \mathbf{q} - L_r + j^2 / \sigma + F_H$$

$$p = \frac{k_B}{m} \rho T \quad \left(= \frac{\tilde{R}}{\tilde{\mu}} \rho T \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

$$\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

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Magnetism

Magnetohydrodynamics (MHD)

- Additional equation can be derived from the Maxwell Equations and the Lorentz force

Maxwell's Equations	Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho_q}{\epsilon_0}$
	Gauss's law of magnetism	$\nabla \cdot \mathbf{B} = 0$
	Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$
	Ampère's circuital law (with Maxwell's addition)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$
Lorentz force (electromagnetic force) <i>Charged particle moving in electric and magnetic fields.</i>		$\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$

- Further reading: *Solar Magnetohydrodynamics*, E. Priest (2014), downloadable via UiO bib.

Magnetism

Fundamental plasma properties

- **Electrical conductivity** σ

- For a fully ionised, collision-dominated plasma: $\sigma = n_e e^2 \tau_{ei} / m_e$

n_e : number density of electrons

e : electric charge

m_e : electron mass

τ_{ei} : electron-ion collision time

- **Magnetic diffusivity** $\eta = 1/(\mu\sigma)$

μ : magnetic permeability;
magnetic field production due to moving electric charge (current)

- **Magnetic Reynolds number**

$$R_m = \frac{\text{induction or advection of a magnetic field due to the motion of a conducting medium}}{\text{magnetic diffusion}}$$

- Small R_m : Advection is unimportant, magnetic field is diffusive.
- Large R_m : Diffusion unimportant. Magnetic field advected with the fluid flow.
 - In the Sun: $R_m \sim 10^6$ (very large) — Diffusion and related dissipation of magnetic field unimportant.

Magnetism

Charge Conservation

- Charge is also a conserved quantity, the MHD charge continuity equation:

$$\frac{\partial \rho_q}{\partial t} + \nabla \cdot (\rho_q \mathbf{v}) = 0$$

$$\rho_q = q n$$

$$\rho_q \mathbf{v} = \mathbf{j}$$

$$\Rightarrow \frac{\partial \rho_q}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

\mathbf{v} : Velocity

ρ_q : Charge density

q : Charge per particle

n : Number density

\mathbf{j} : Current

Magnetism

Ohm's law

- Lorentz force combined for large number of charged particles (making up a plasma)

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic force}} \qquad Q_q \mathbf{v} = \mathbf{j}$$

- For a plasma moving at a non-relativistic velocity v in the presence of a magnetic field
 - ➔ Electric field ($v \times B$) in addition to the electric field (E) which would act on material at rest.
 - ➔ Current density

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- **More general:** For a plasma that consists of a mix of electrons, protons and neutral atoms:
 - ➔ Generalised Ohm's law

Magnetism

Generalized Ohms Law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}$$

Electric field
in a moving
plasma

Resistive
term (cf
 $V=IR$; $\eta =$
resistivity)

- **Ideal MHD:** $\eta = 0 \implies \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$
 - A very good approximation for many applications (incl. solar interior)
 - 'frozen-in flux' approximation
 - Simplifies computations!

Magnetism

Generalized Ohms Law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} + \left(\frac{1}{ne} \right) (\mathbf{j} \times \mathbf{B}) - \left(\frac{1}{ne} \right) \nabla \cdot \mathbf{P}_e + \left(\frac{m_e}{ne^2} \right) \frac{\partial \mathbf{j}}{\partial t}$$

Electric field
in a moving
plasma

Resistive
term (cf
 $V=IR$; $\eta =$
resistivity)

Hall
term

Electron
anisotropy

Electron
inertia

These terms are small on long length and time scales
and/or in regions of weak currents \rightarrow often ignored

- **Ideal MHD:** $\eta = 0 \implies \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$
 - A very good approximation for many applications (incl. solar interior)
 - 'frozen-in flux' approximation
 - Simplifies computations!

Magnetism

Momentum Equation

- Conservation of momentum — the MHD equation of motion

$$\underbrace{\rho \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v}}_{\text{convective derivative of the momentum}} = \underbrace{-\nabla \cdot \underline{\underline{\mathbf{P}}} + \rho_q \mathbf{E} + \mathbf{j} \times \mathbf{B}}_{\text{sources and sinks of momentum (forces)}} + \text{any other forces acting on the plasma (e.g. gravity)}$$

- $\nabla \cdot \underline{\underline{\mathbf{P}}}$: plasma pressure gradient
- $\rho_q \mathbf{E}$: electric field force (can be neglected if no net charge density in plasma)
- $\mathbf{j} \times \mathbf{B}$: Lorentz force

Magnetism

Magnetic pressure

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F}_g + \mathbf{F}_v$$

- Assume a static case, no net charge, ignore gravity and viscosity $\implies \mathbf{j} \times \mathbf{B} = \nabla p$
- Magnetic field exerts a pressure force, for short: **magnetic pressure**.

- In SI units: $P_{\text{mag}} = \frac{B^2}{2\mu_0}$

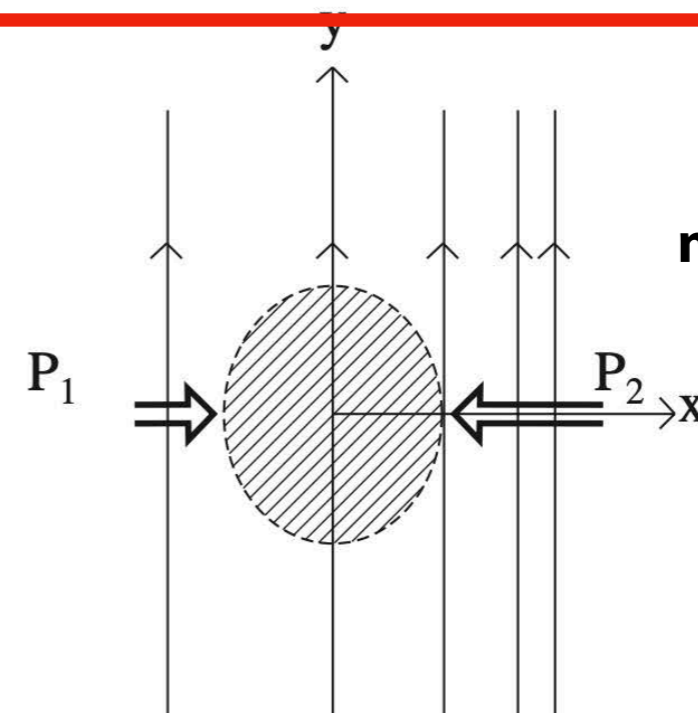
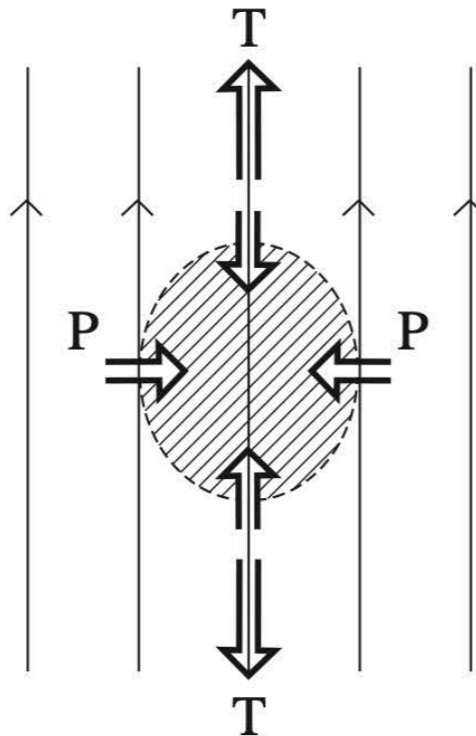
SI units: P in Pa, B in T, μ_0 in H/m

- In cgs units: $P_{\text{mag}} = \frac{B^2}{8\pi}$

cgs units: P in dyn/cm², B in G

μ_0 : magnetic permeability in vacuum

uniform field:
magnetic pressures (P)
and tensions (T)
are in balance



non-uniform field:

$dB/dx > 0$
Imbalance:
 $P_2 > P_1$

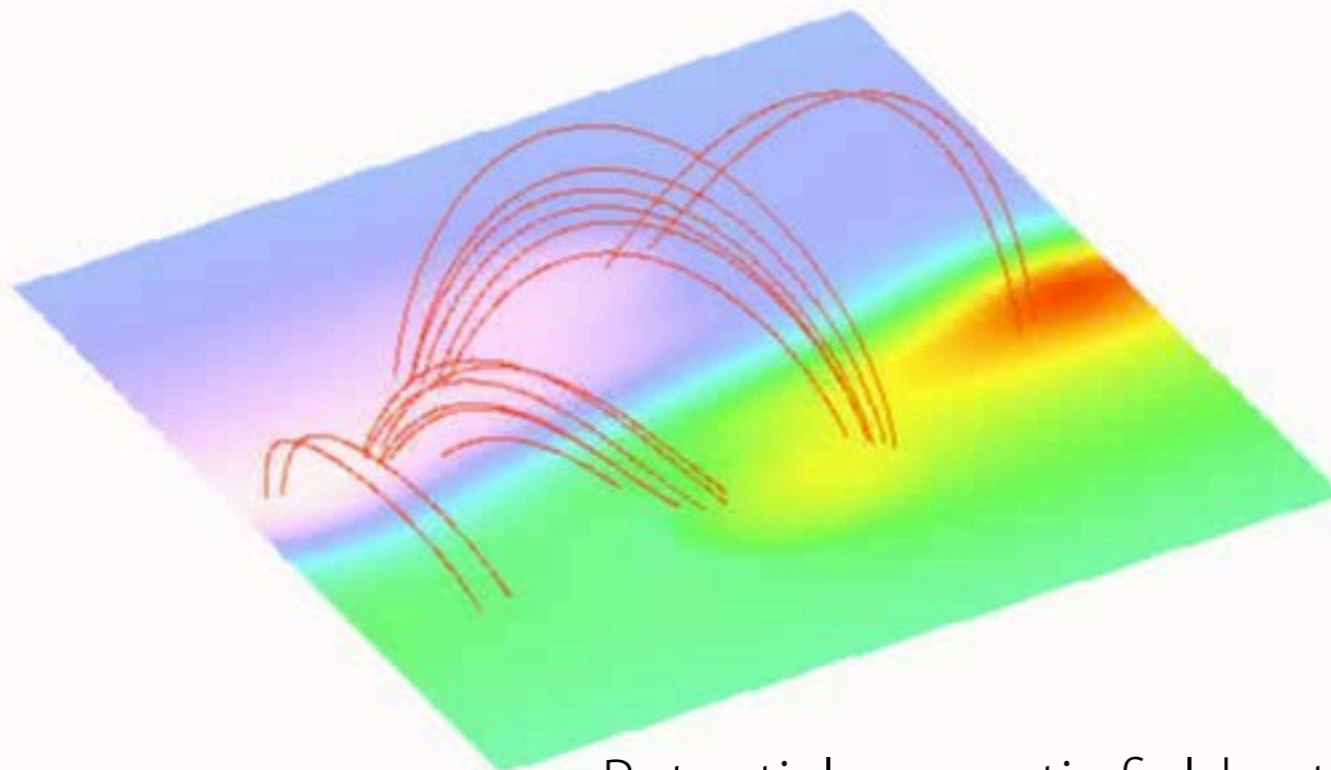
Magnetism

Force-free fields

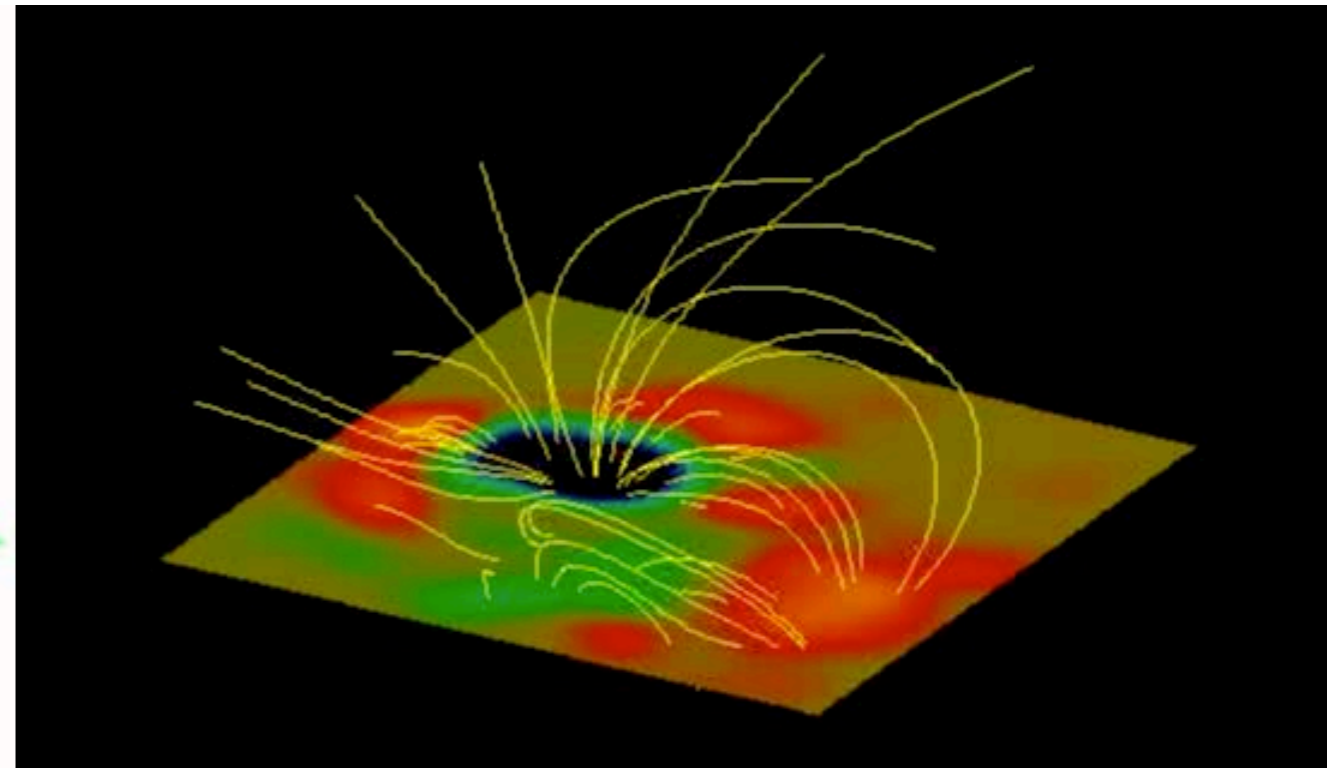
- Magnetic field configuration for which Lorentz force is zero ($\mathbf{j} \times \mathbf{B} = 0$) — called **force-free**
- **Example:** potential magnetic field configuration with no electric current at all.

$$\mathbf{B} = \nabla \Psi$$

Ψ : magnetic (scalar) potential

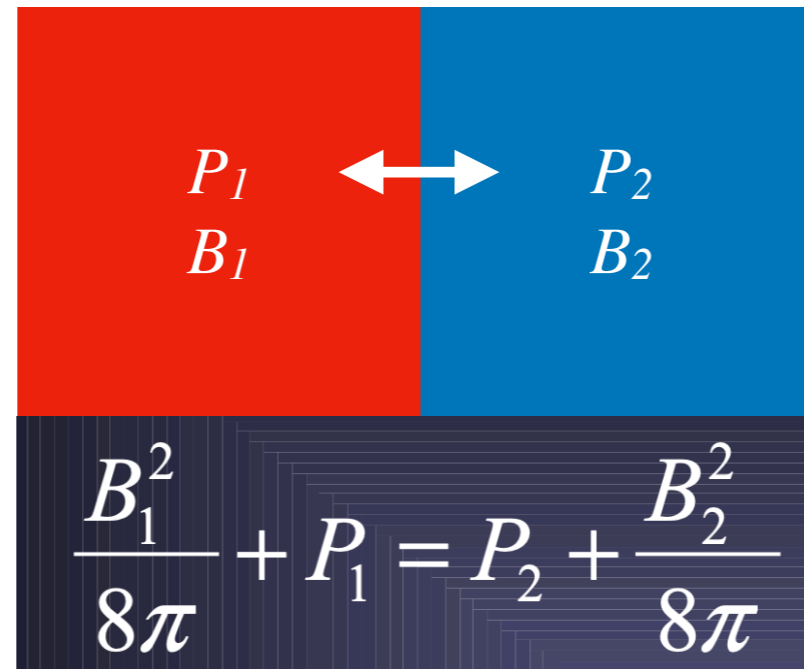


Potential magnetic field extrapolation



Magnetism

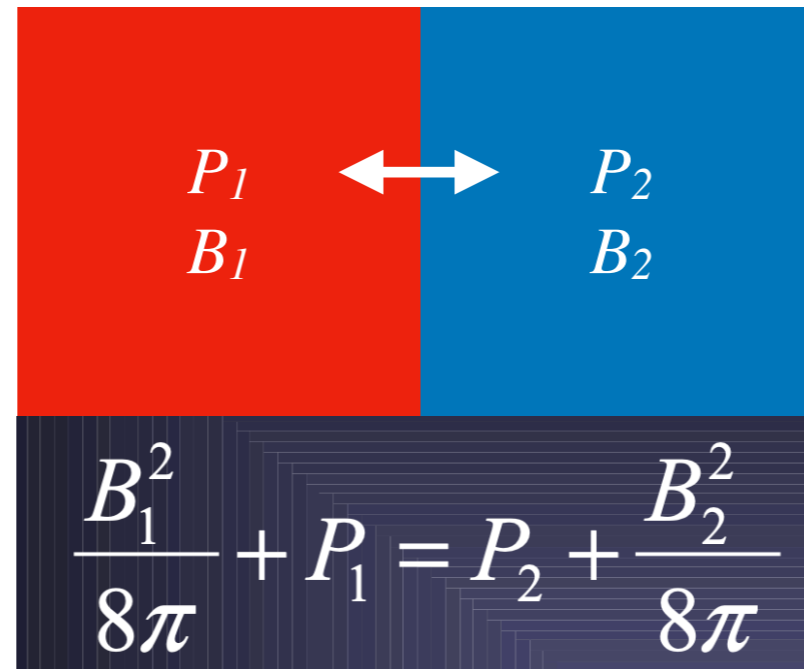
Magnetic pressure



- **Pressure balance** between two different plasma domains (1 and 2):
- If $B_2 = 0$:
 - Extreme case: Component 1 is evacuated: $P_1 = 0$
 - Sets a maximum field strength B_{eq}
 - If $B > B_{eq}$: Not in pressure balance, overpressure in component 1, tends to expand

Magnetism

Magnetic pressure

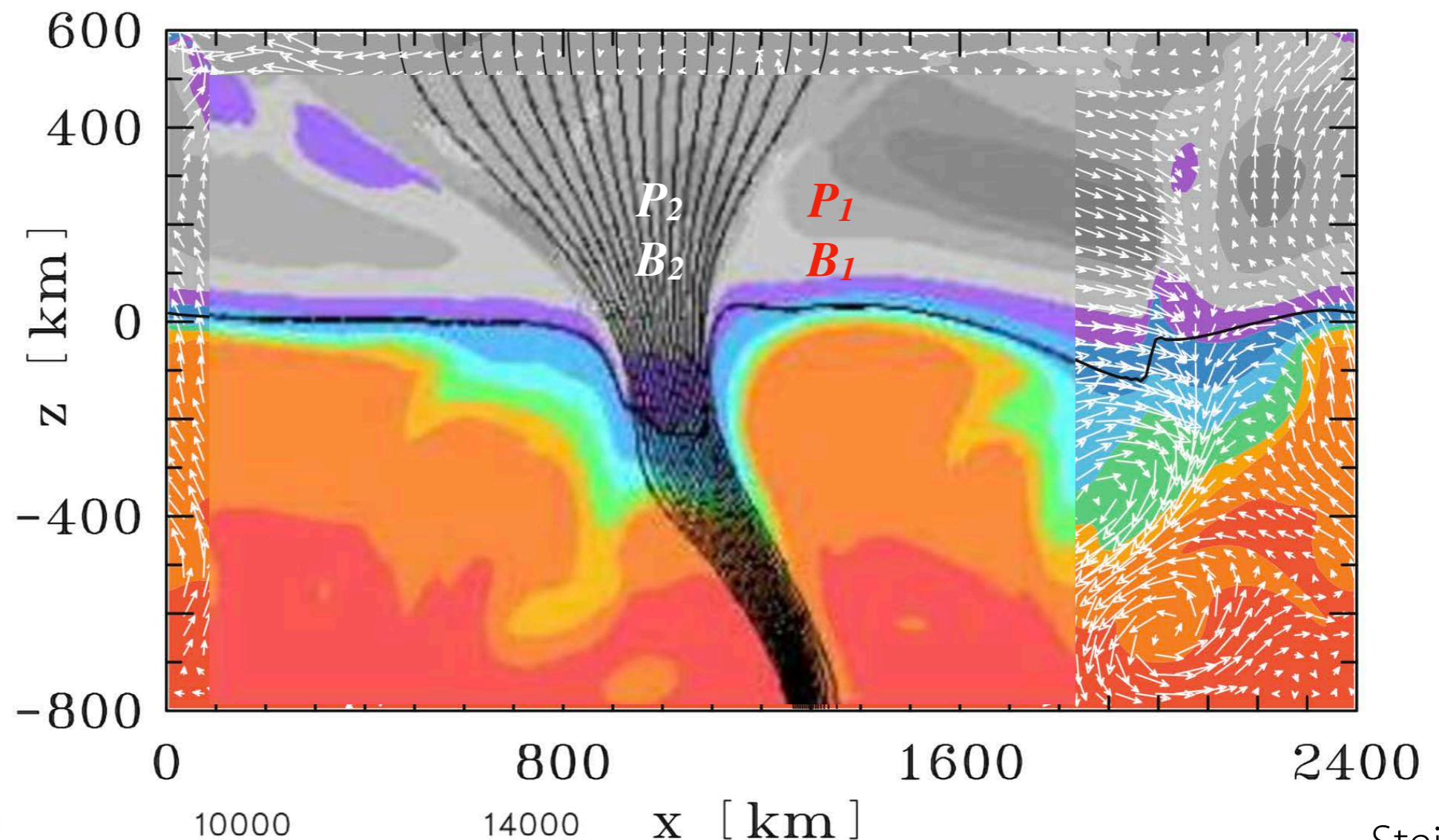


- **Pressure balance** between two different plasma domains (1 and 2)
- If $B_2 = 0$ and $T_1 = T_2$, then also $\rho_1 < \rho_2$ (Equation of state!)
 - ➔ Magnetic features are buoyant compared to the surrounding gas.
- In the convection zone:
 - Lower density inside magnetic flux bundles compared to surrounding plasma
 - Magnetic flux bundle becomes buoyant and rises towards the surface (down the gradient) unless stopped by other forces

Magnetism

Magnetic pressure

- Additional contributions from magnetic pressure inside magnetic flux concentration
- Pressure balance \implies lower gas pressure inside the flux concentration than outside
- Gas pressure of surrounding drops with height \implies Magnetic structure funnels out (wine-glass shape)



- $B_2 > B_1$

- ➔ $P_2 < P_1$

- ➔ $\rho_2 < \rho_1$

Magnetism

Plasma-Beta

- Plasma- β describes the ratio of thermal to magnetic pressure

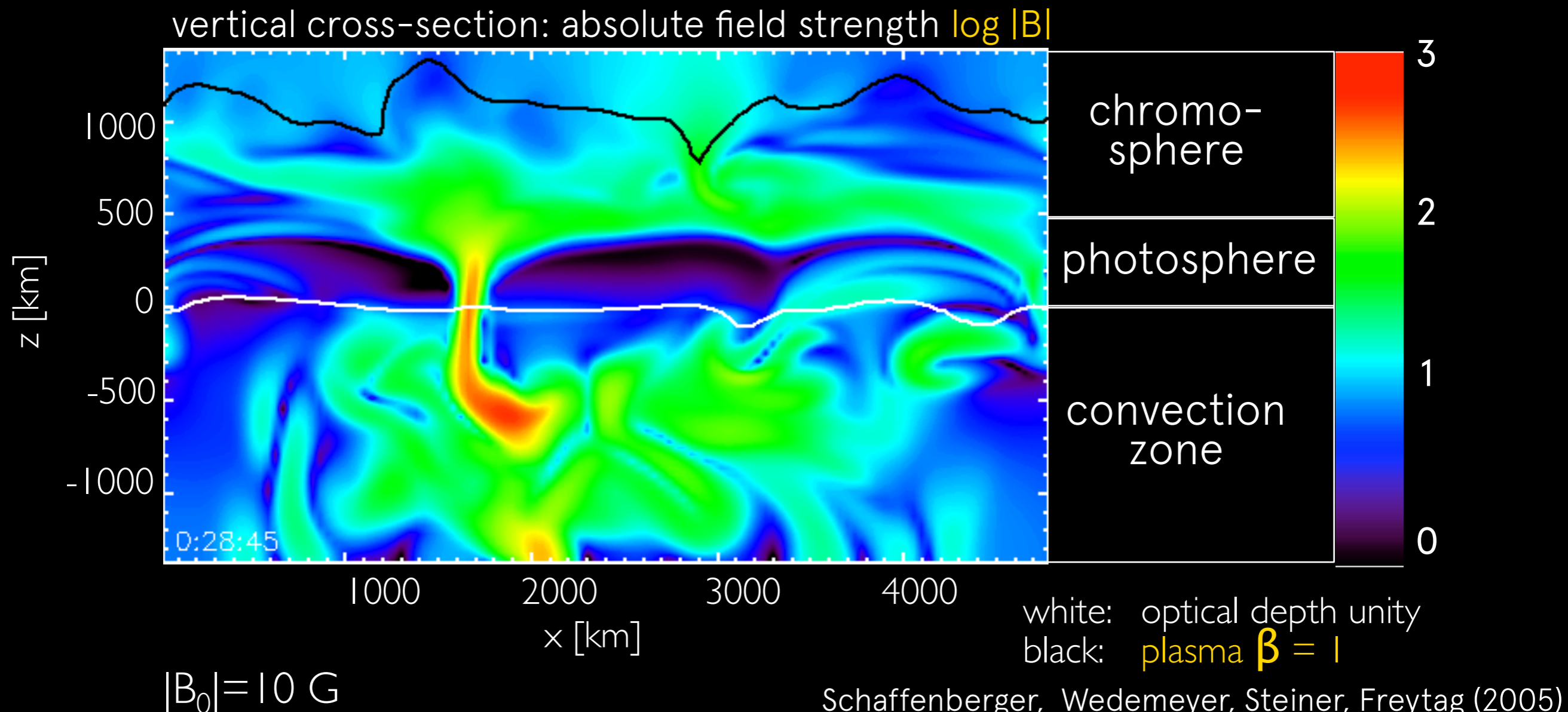
$$\beta = \frac{P_g}{P_m} = \frac{8\pi P_g}{B^2}$$

- **$\beta < 1$: Magnetic field dominates** and dictates the dynamics of the gas
- **$\beta > 1$: Thermal gas dynamics dominate** and forces the field to follow
 - The magnetic field is **frozen-in**.
- β is a local quantity but the typical range of values changes with radius:
 - Convection zone: $\beta > 1$
 - Lower atmosphere (outside strong magnetic field concentrations): $\beta > 1$
 - Chromosphere: transition to $\beta < 1$
 - Corona: $\beta \ll 1$

Magnetism

Plasma-Beta

- Magnetic field in chromosphere is highly dynamic
 - Propagating shock waves compress magnetic field
 - Fast moving filaments of enhanced field



Magnetism

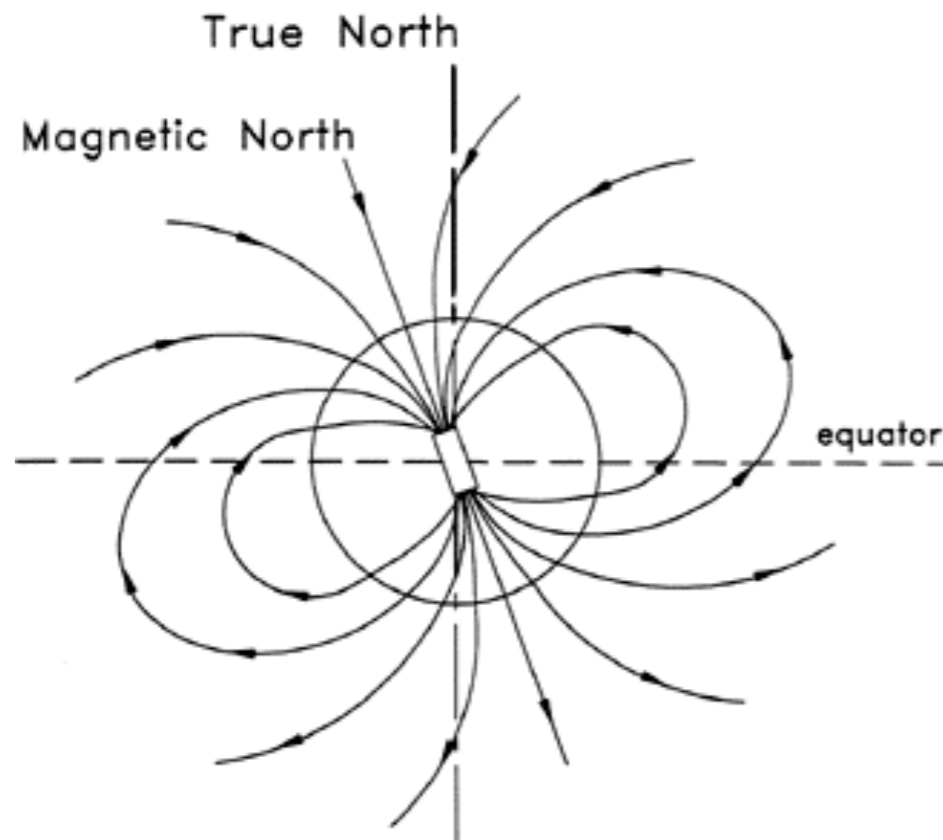
CO⁵BOLD
(close-up)

- Complicated field structure with rotating and/or swaying subgroups
- Continuous reorganisation of structure
- More complicated than individual “flux tubes”

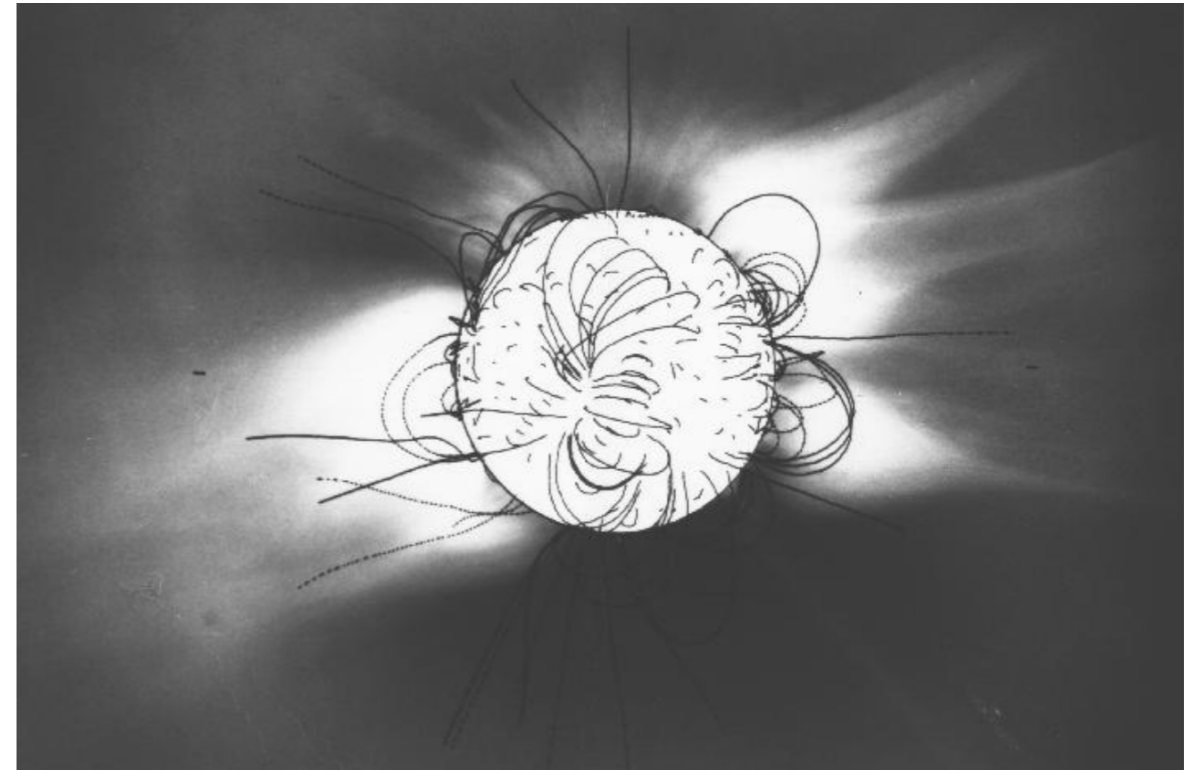
Magnetism

Global magnetic field configurations

Earth — dipole field



Sun — more complicated

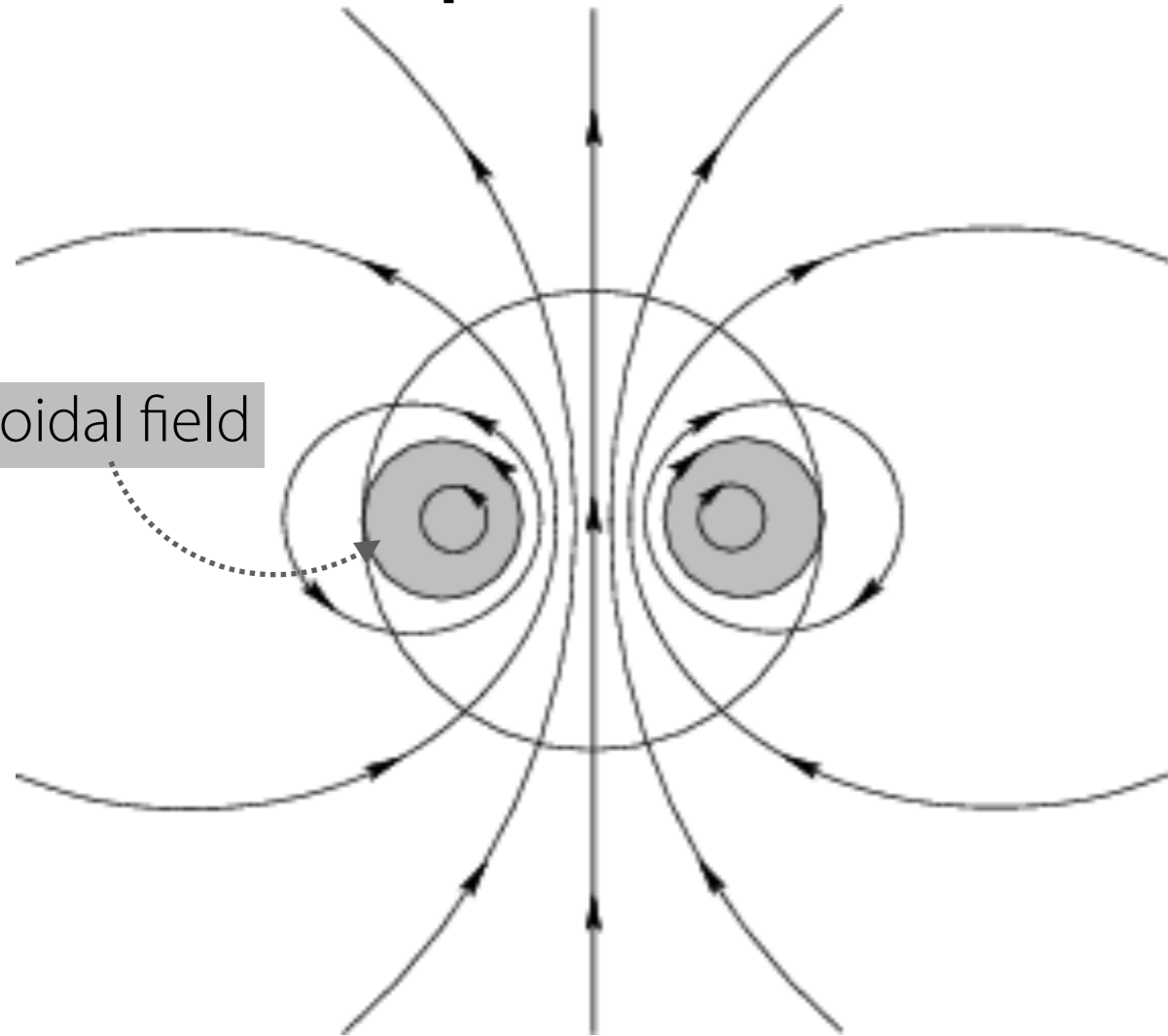


- Like Earth, also the Sun is permeated by a dipole field but with much more complicated additional field geometry that changes over time (solar cycle)

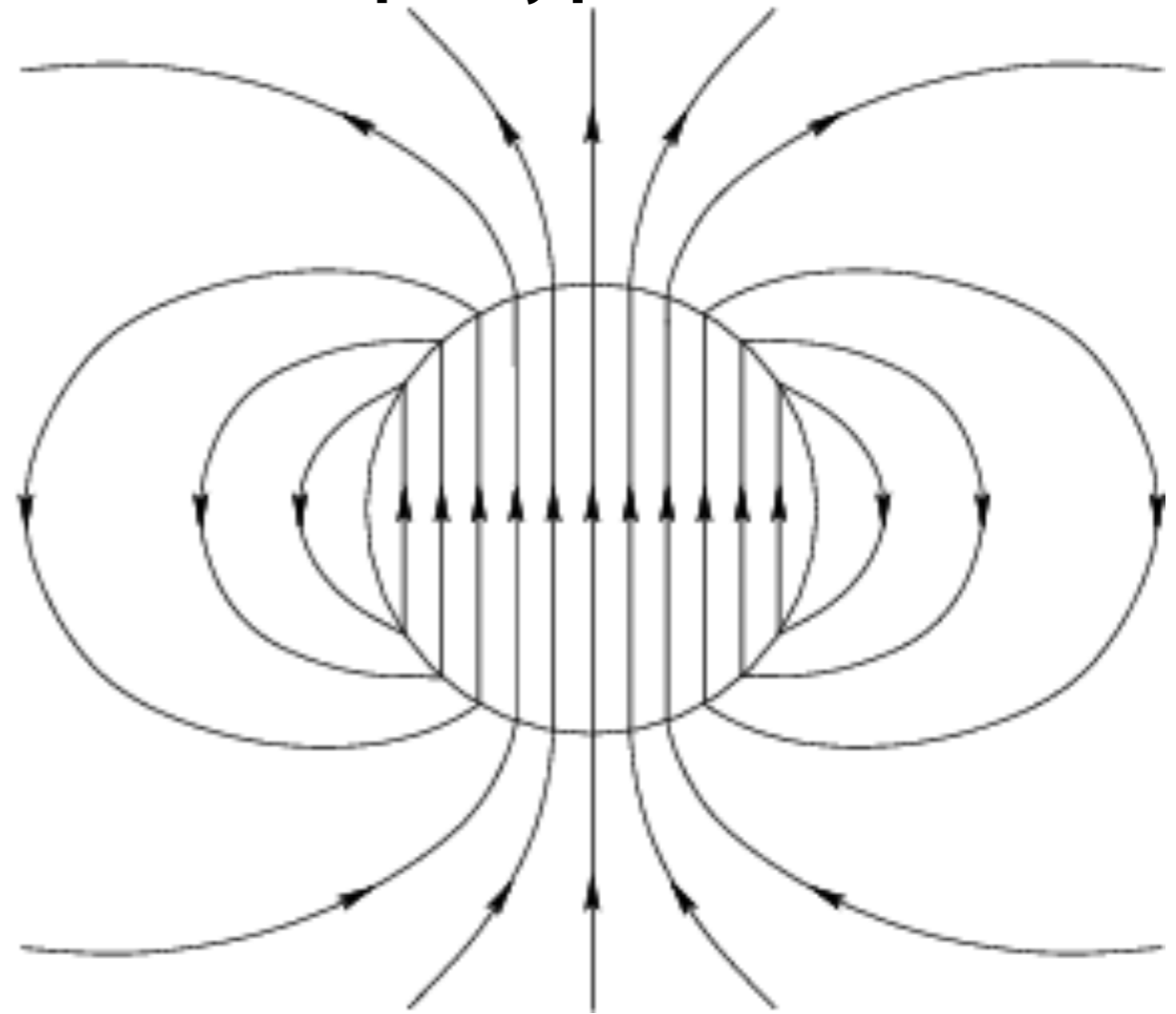
Magnetism

Global magnetic field configurations

mixed poloidal-toroidal field



purely poloidal field



Braithwaite & Spruit 2004

- Note — in the hypothetical case of a non-rotating star: the left config. is stable, the right one not

Magnetism

Take aways

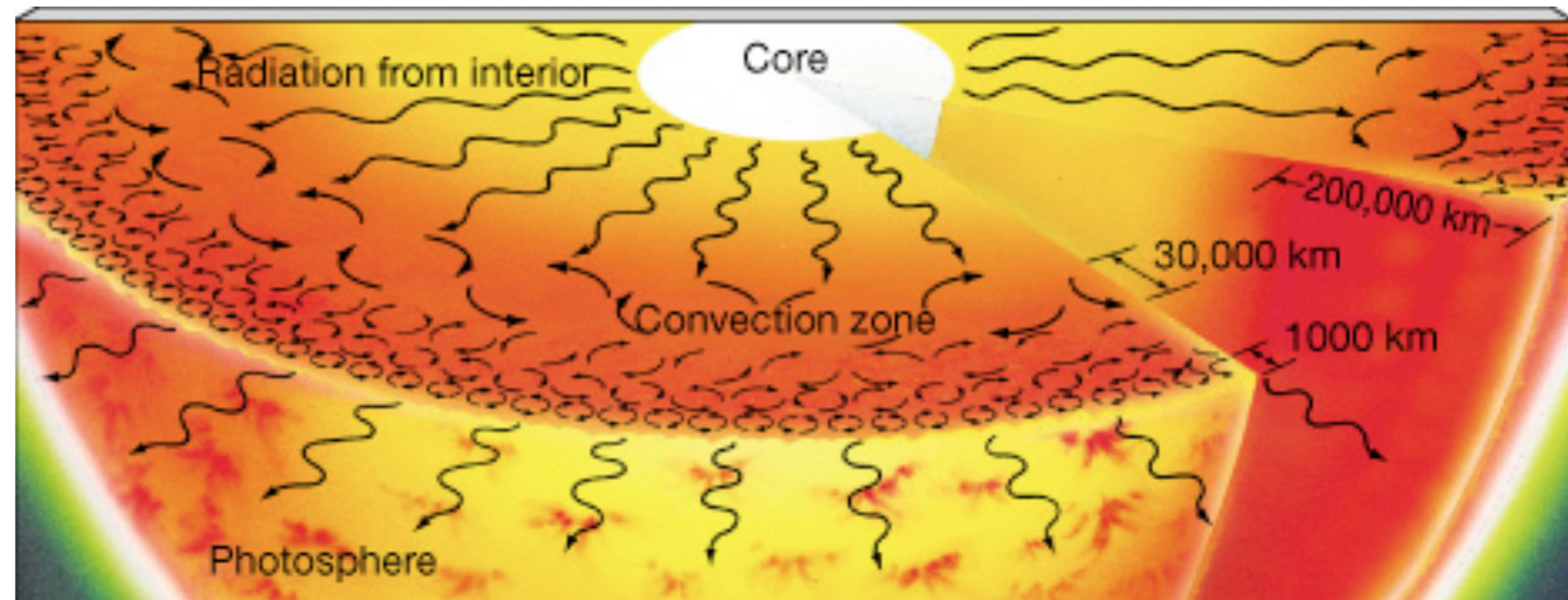
- Ionised gas (plasma) in motion — electric and magnetic fields need to be considered
- Magnetic pressure arises — impacts structure and dynamics of the plasma
 - Higher magnetic field means lower thermal pressure and lower density with respect to the surrounding
- Plasma- β parameter = ratio of thermal to magnetic pressure
 - **$\beta < 1$: Magnetic field dominates** and dictates the dynamics of the gas
 - **$\beta > 1$: Thermal gas dynamics dominate** and forces the field to follow
 - The magnetic field is **frozen-in**.

The solar dynamo

Dynamo

Overview

- Interior of the Sun: plasma (ionized gas) — charged particles
- Convection moves around the plasma (turbulence)
 - ➔ Moving charged particles generate electric currents
 - ➔ Electric currents generate magnetic fields (via Ampere's law).
 - ➔ Changing magnetic fields change, induce electric currents (Faraday's law).
 - ➔ **Self-reinforcing dynamo process**
- Continuous generation of magnetic dipole fields
- Convection currents **stretch and twist** the magnetic field lines, increases magnetic tension (*analogy for magnetic field lines: rubber bands*)
- Magnetic field gets stronger in some locations and/or orientation of field varied



Dynamo

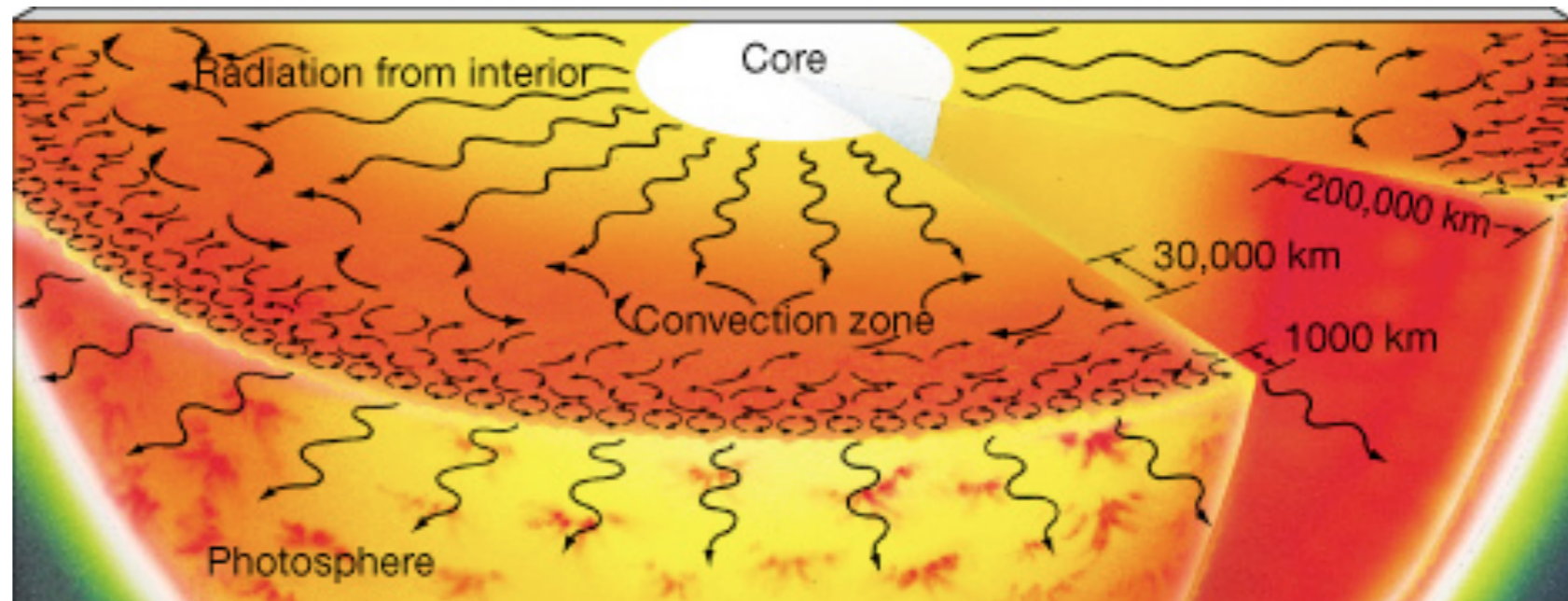
Overview

- Requirements for an efficient dynamo
 - **Properties of the flows** in the solar interior
 - convection
 - differential rotation
 - meridional flow
 - ★ **Tachocline:** Strong radial change in rotation speed, exhibits a strong radial shear
- Plasma motions must convert meridional (poloidal) magnetic field into an azimuthal (toroidal) magnetic field, and vice versa.
- **Induction** has to overcome magnetic diffusion (large magnetic Reynolds number)
- Magnetic field decays on time scales much shorter than the Sun's life time
- ➔ **New field is generated** continuously

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times (\vec{u} \times \vec{B})$$

Magnetic
diffusivity

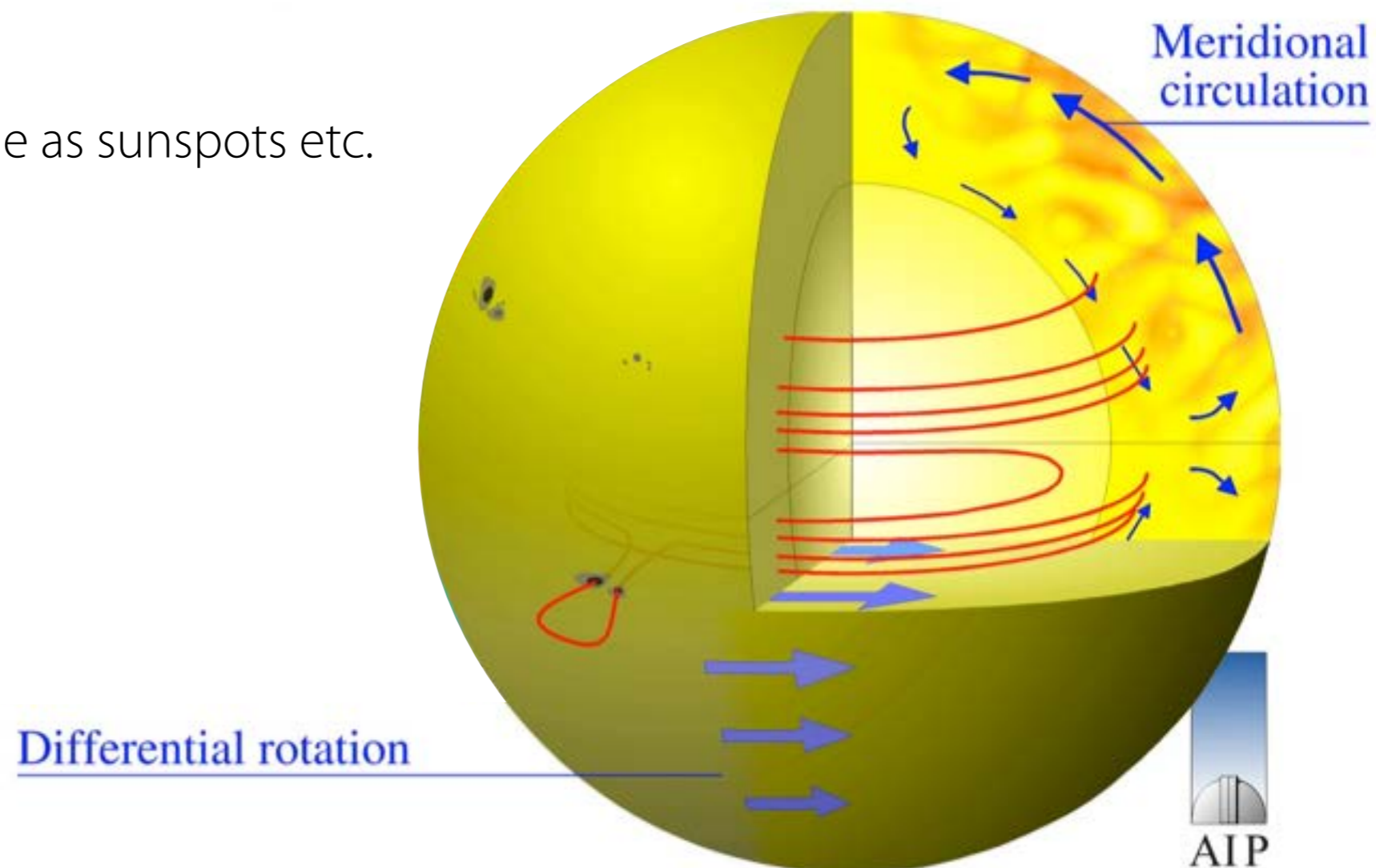
Velocity



Dynamo

Overview

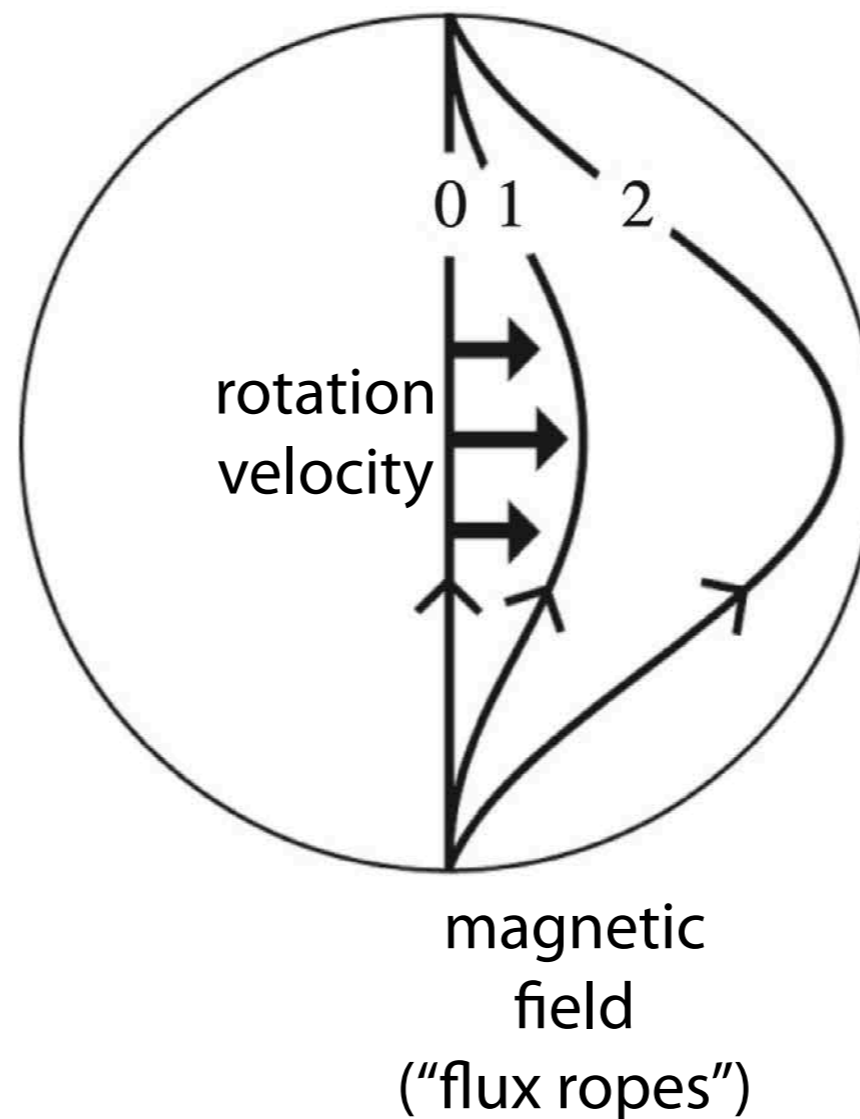
- Current understanding: Magnetic field generated by a dynamo located near the bottom of the convection zone (overshoot layer, tachocline)
- Produces toroidal flux bundles
- Once magnetic field sufficiently strong, flux bundles become buoyant (Parker instability)
 - ➡ Rise towards surface
 - ➡ Break through surface, visible as sunspots etc.



Dynamo

Ω -effect (Omega effect)

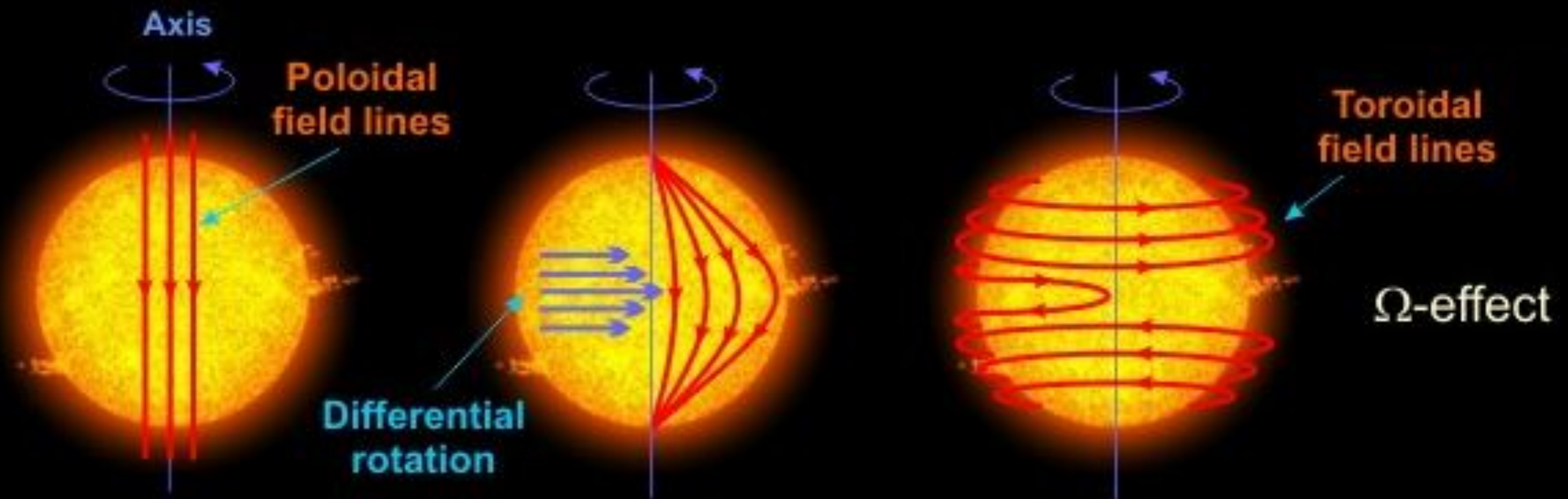
- Omega effect converts initially meridional (poloidal) magnetic field into azimuthal (toroidal) magnetic field due to **differential rotation**



Dynamo

Ω -effect (Omega effect)

- Omega effect converts initially meridional (poloidal) magnetic field into azimuthal (toroidal) magnetic field
- ➔ Initial meridional magnetic field is twisted and coiled around the Sun due to **differential rotation**
- ➔ Creates magnetic flux strands in the azimuthal (toroidal) direction in shallow depths and low latitudes

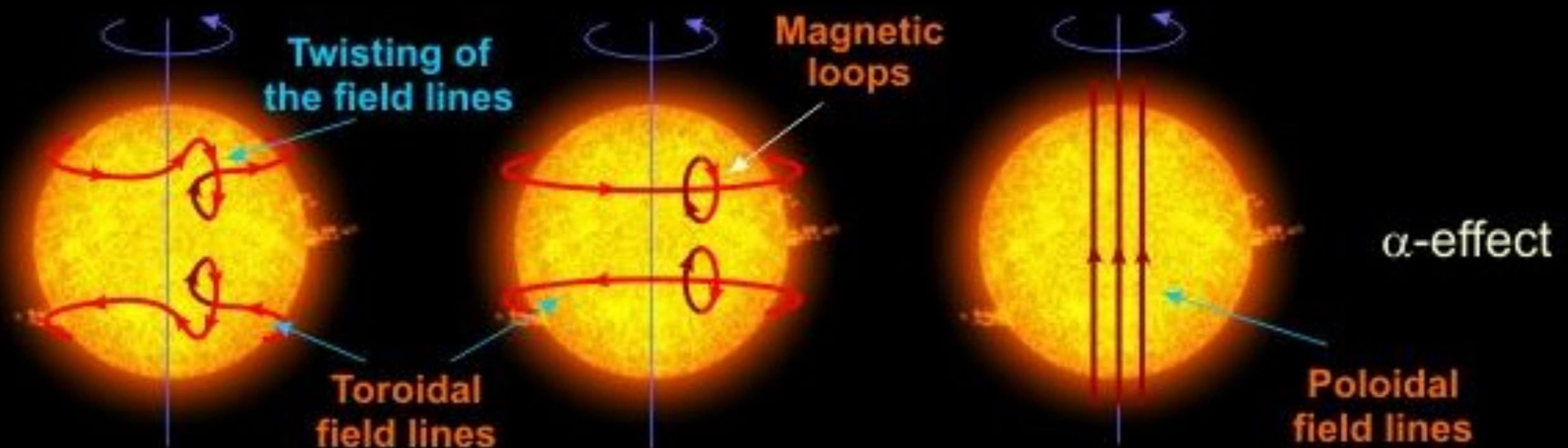


Dynamo

α -effect (Alpha effect)

(Parker 1955, Babcock 1961)

- Alpha effect converts an azimuthal (toroidal) magnetic field into a meridional (poloidal) magnetic field.
- Precise mechanism still not fully understood
- Most likely due to interaction between
 - velocity field of the plasma
 - rotation of the sun
 - toroidal magnetic field
 - Coriolis force acting on rising flux tubes



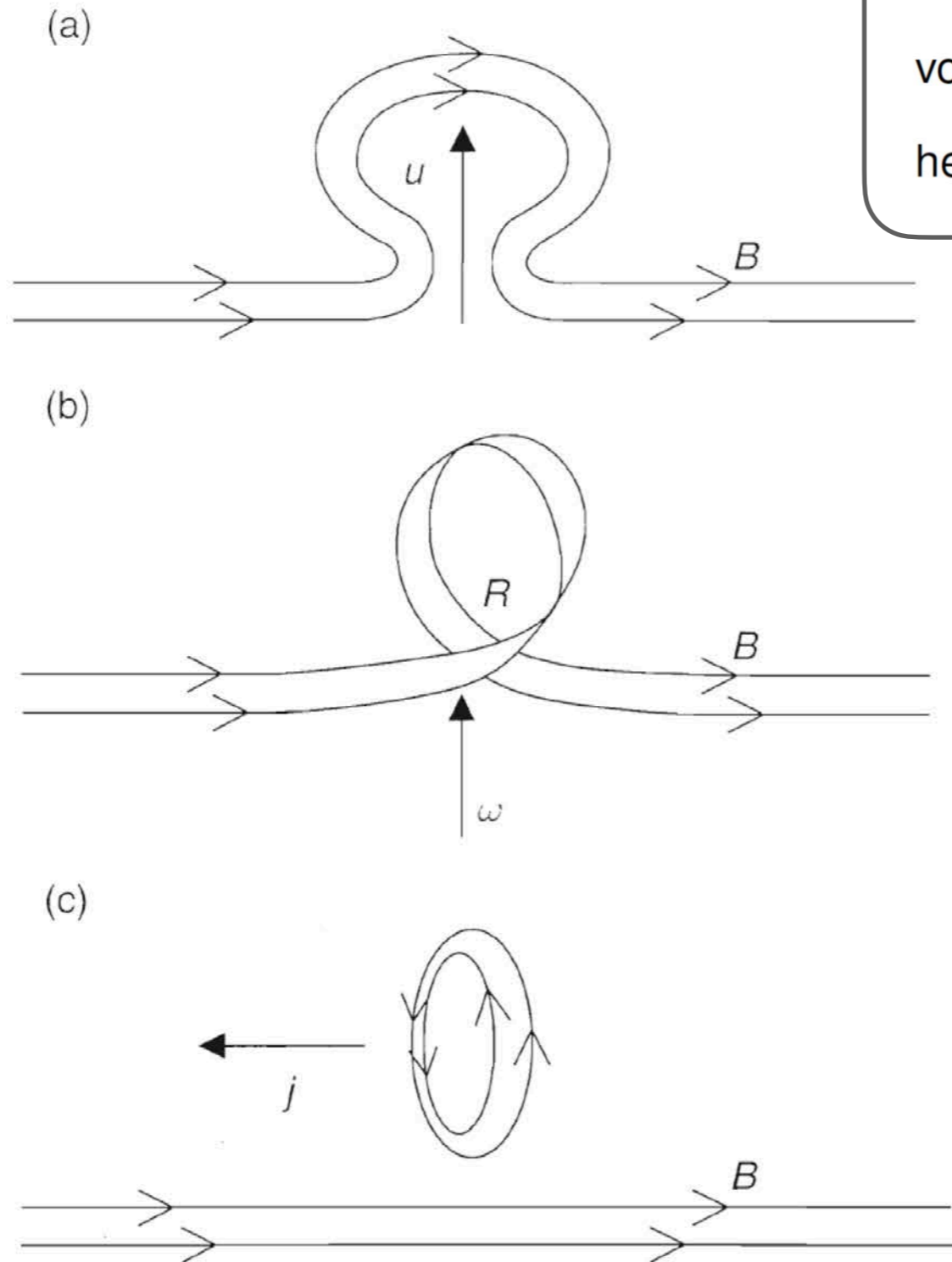
Dynamo

α -effect (Alpha effect)

- Sphere of hot plasma rotating at an angular velocity $\boldsymbol{\omega}$.
- Convection: Plasma “bubbles” rise upwards with velocity \boldsymbol{v} .
- High gas pressure (deep) in the convection zone: $P_g \gg P_m \Rightarrow \beta \gg 1$
 - ➔ **High plasma-beta** conditions, magnetic field frozen in
 - ➔ Toroidal magnetic field gets partially dragged along by the moving plasma bubbles
- Solar rotation induces **Coriolis force** ($\boldsymbol{\omega} \times \boldsymbol{v}$) on plasma bubbles
- Bubbles and with it the frozen-in magnetic field twists as it moves upwards and expands.
- Note: Signs of both the Coriolis force and toroidal magnetic field are reversed in the northern versus the southern hemisphere!
 - ➔ Small-scale magnetic field loops of the same polarity in both hemispheres
- Small-scale loops gradually merge due to magnetic diffusivity
 - ➔ Generates a large scale poloidal magnetic field (Parker 1955).

Dynamo

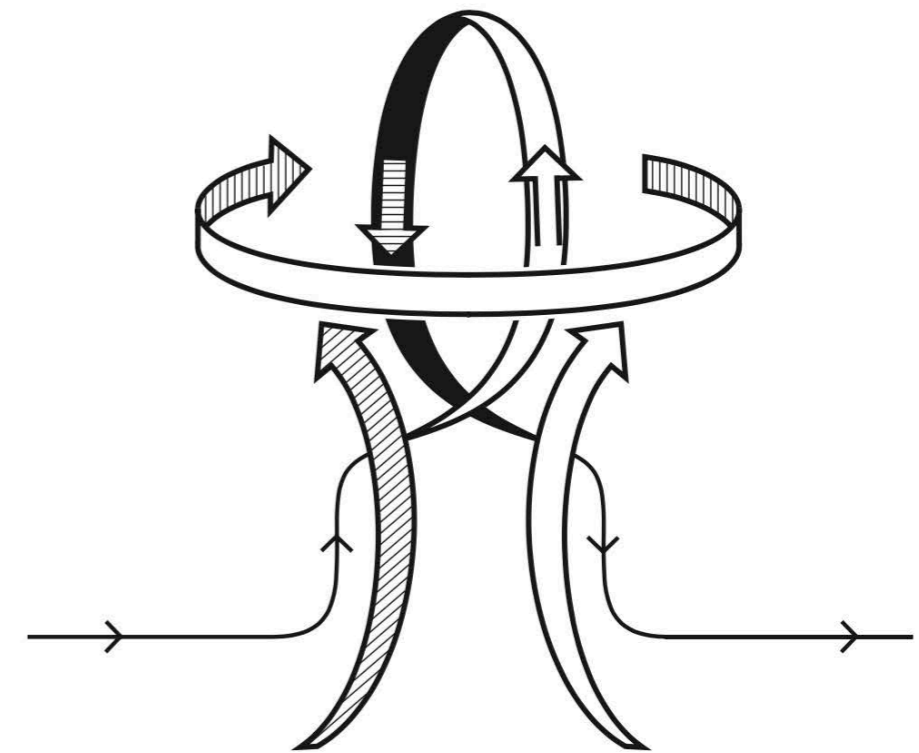
α -effect (Alpha effect)



velocity \mathbf{u}

vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$

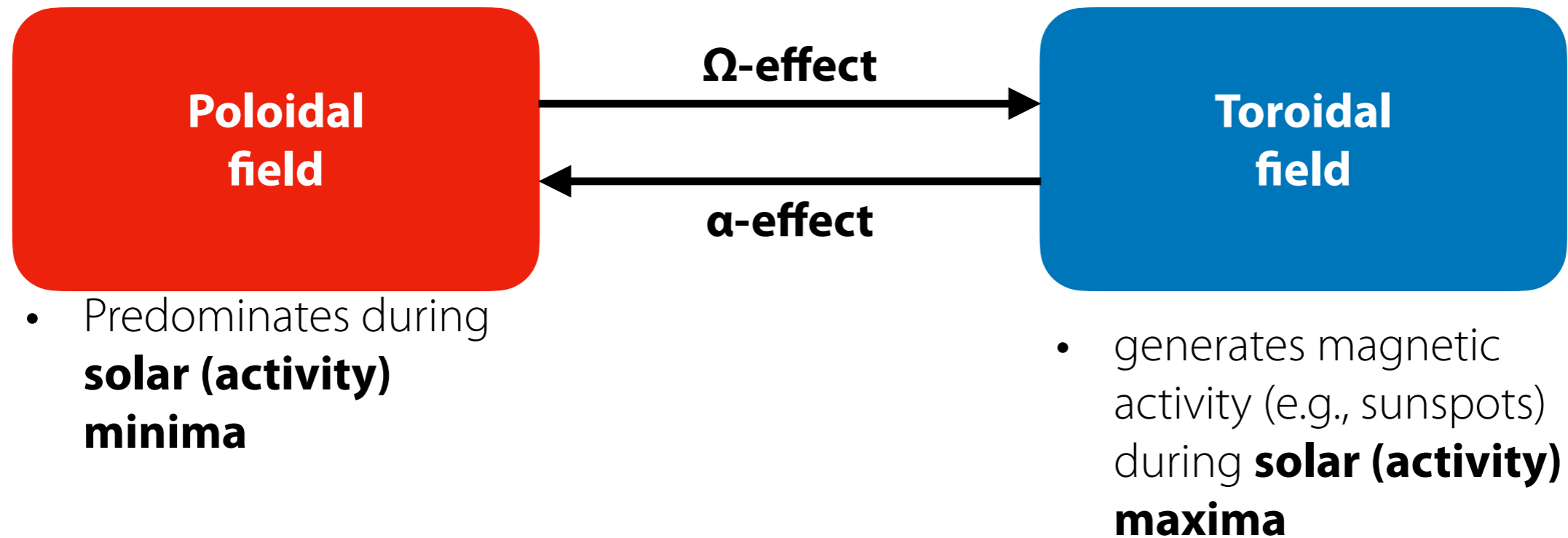
helicity $H = \mathbf{u} \cdot \boldsymbol{\omega}$



Rising and twisting
converts toroidal into
poloidal flux

Dynamo

α and Ω

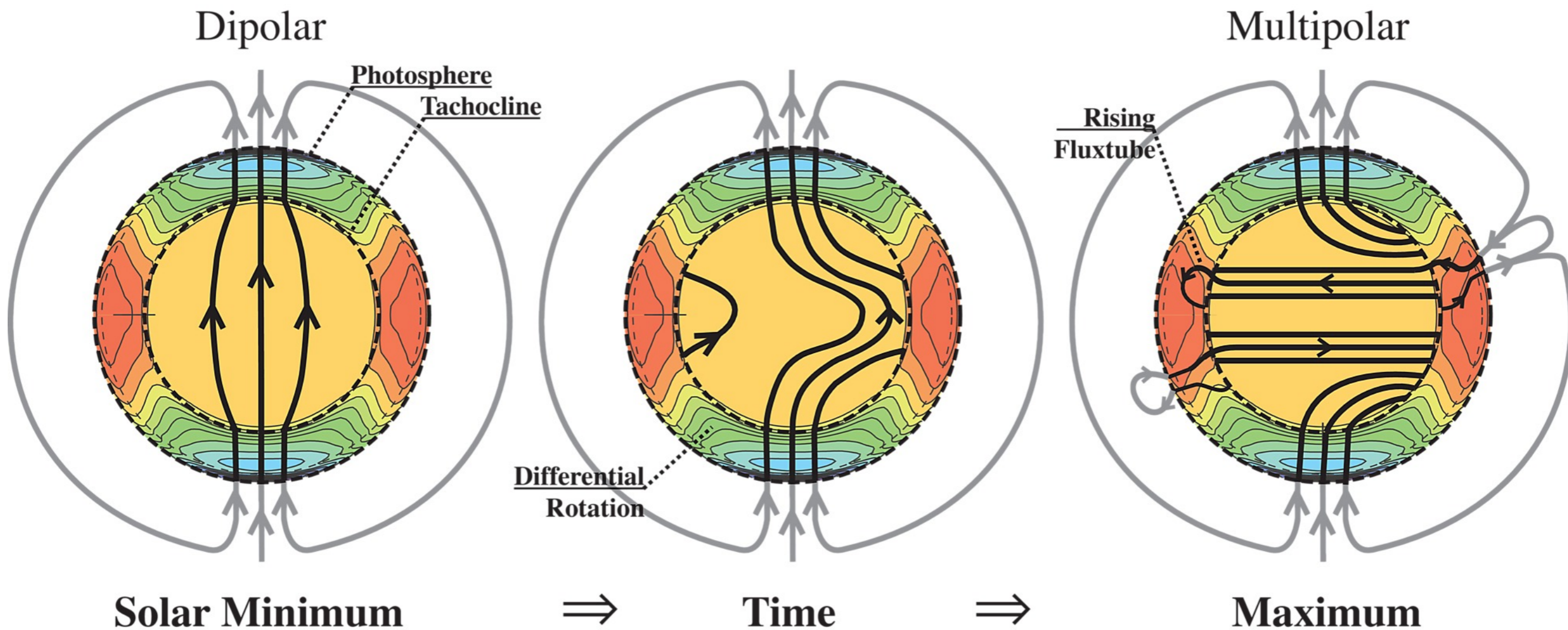


- **Solar cycle:** Change between these extreme configurations, forming a solar activity minimum
 - One cycle period ~ 11 year
 - Global polarity of the Sun's magnetic field (N-S) swaps during that period
 - Complete cycle back to the same polarity = $2 \times 11 \text{ yr} = 22 \text{ yr}$

Dynamo

Solar cycle — change of magnetic field configuration

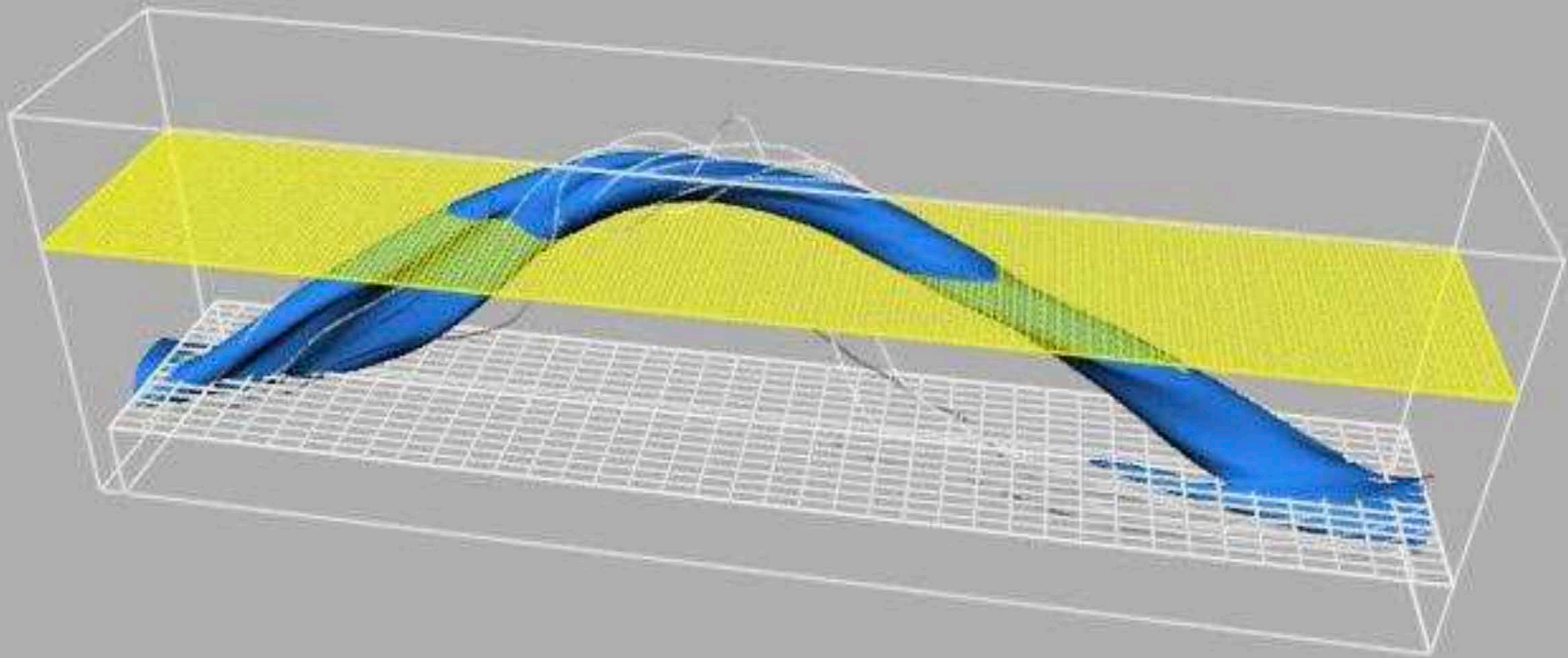
- Below tachocline: Rotation as solid body
- Above tachocline: differential rotation — faster rotation near equator, slower at poles
- Magnetic dipole field (poloidal) at solar minimum
- Over time: differential rotation shears magnetic field at the tachocline, drags it along the equator, converts into toriodal configuration.



Dynamo

Emergence of a magnetic flux tube

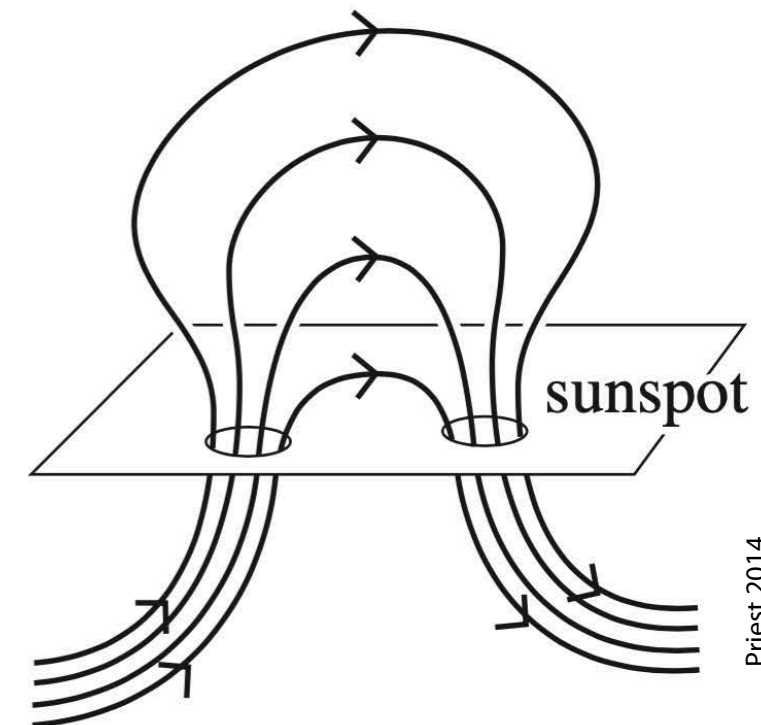
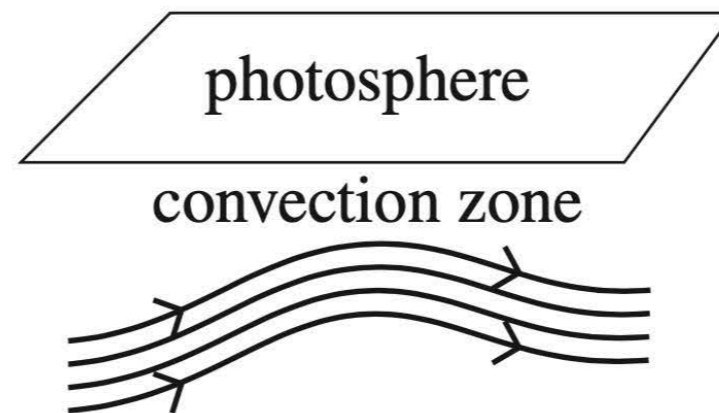
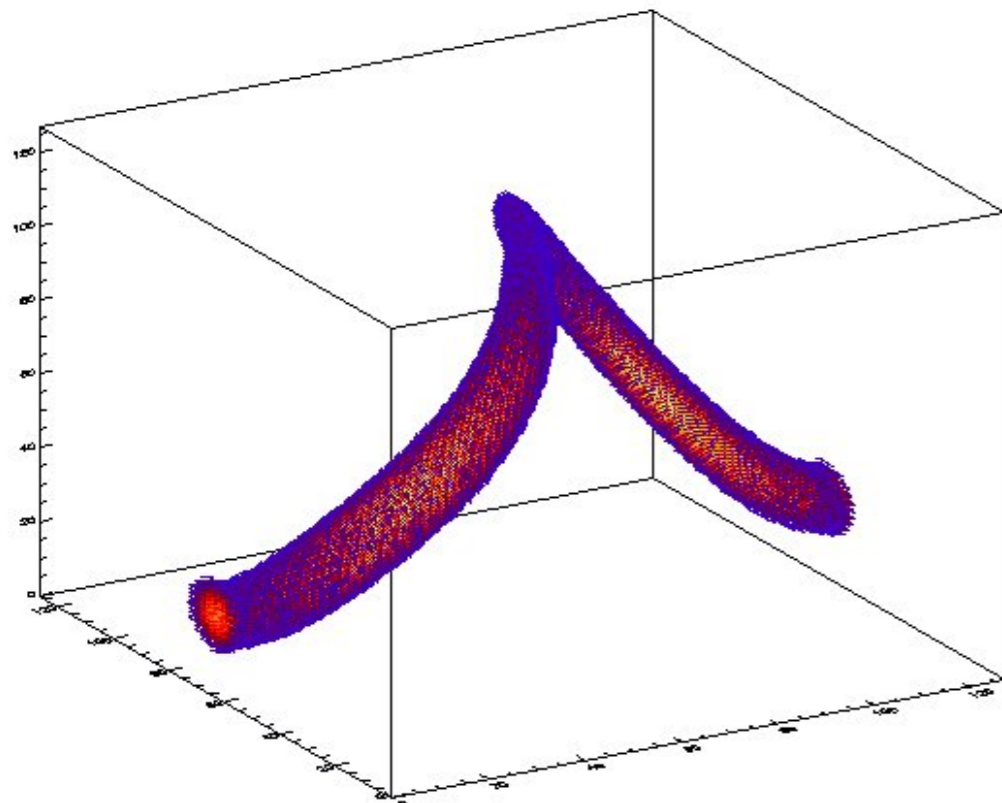
- Magnetic field generated mainly in the tachocline near bottom of convection zone
- **Magnetic pressure** inside flux rope — lower density inside than in the surrounding plasma
➔ Magnetic flux rope becomes **buoyant, rises upwards** (Parker instability)



Dynamo

Emergence of a magnetic flux tube

- Magnetic flux rope rises to surface due to its buoyancy (Parker instability)
- Flux rope reaches surface eventually
- The two points where the loop breaks through the surface are sunspots of opposite polarity
- Flux rope produces a bipolar active region at the surface
 - In reality often more complicated topology (sunspot groups)
- While rising, the magnetic flux structure can become twisted



Dynamo

Magnetic fields at the surface — Active Regions

2012 March - Sunspot evolution

HMI CONTINUUM NOAA 1429



Dynamo

Magnetic fields at the surface — Active Regions

Backyard Video Astronomy by Paolo Porcellana

Earth

NOAA 1785 Sunspot Evolution



Dynamo

Magnetic fields at the surface — Active Regions

- Magnetogram (HMI)

