# AST5770

## Solar and stellar physics

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# Practical information

## Data / material for assignments

- Data: `/mn/stornext/d9/svenwe/lecture/AST5770/data`
- Templates: `/mn/stornext/d9/svenwe/lecture/AST5770/assignment`

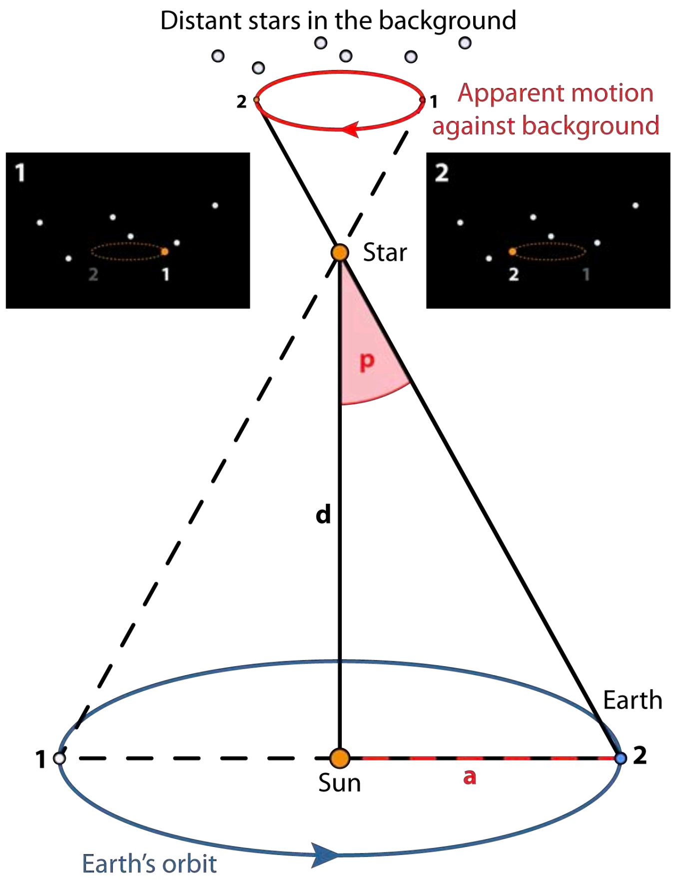
- **Everybody please check: Can you access these directories?**

- On the course webpage
  - Templates
  - Submission dates and requirements
  - A document with information about data & assignments
  - Evaluation matrix

# Recap

- **What is a star: gas + self-gravity + luminous + nuclear fusion**

- **Distances to the stars**
    - Sun at 8 light-min = 1 AU ≈ $150 \times 10^6$ km
    - Nearest star (after the Sun) at 4.3 ly



- **Measuring distances via parallactic angles (parallax)**
    - Apparent motion of a star against (far away) background as seen over a year (Earth's orbit!)
    - Distance d [pc] = 1/p ["] (p: parallax)
    - Definition of parsec: 1 pc = 206265 AU = 3.26 ly
    - Gaia mission mapped billions of stars in the Milky Way

- **Apparent size in the sky**
    - Sun ~ 1/2 degree
    - Interferometric imaging begins to slightly resolved huge nearby stars
    - Most source remain point sources for now

**Betelgeuse ~ 50 milliarcsec**

ALMA (ESO/NAOJ/NRAO)/ E. O'Gorman/P. Kervella

# Recap

- **Radiative flux F** (energy radiated per time unit through an area)
- **Radiative flux <u>density</u> $F_\lambda$ , $F_\nu$** : F per <u>wavelength or frequency unit</u>

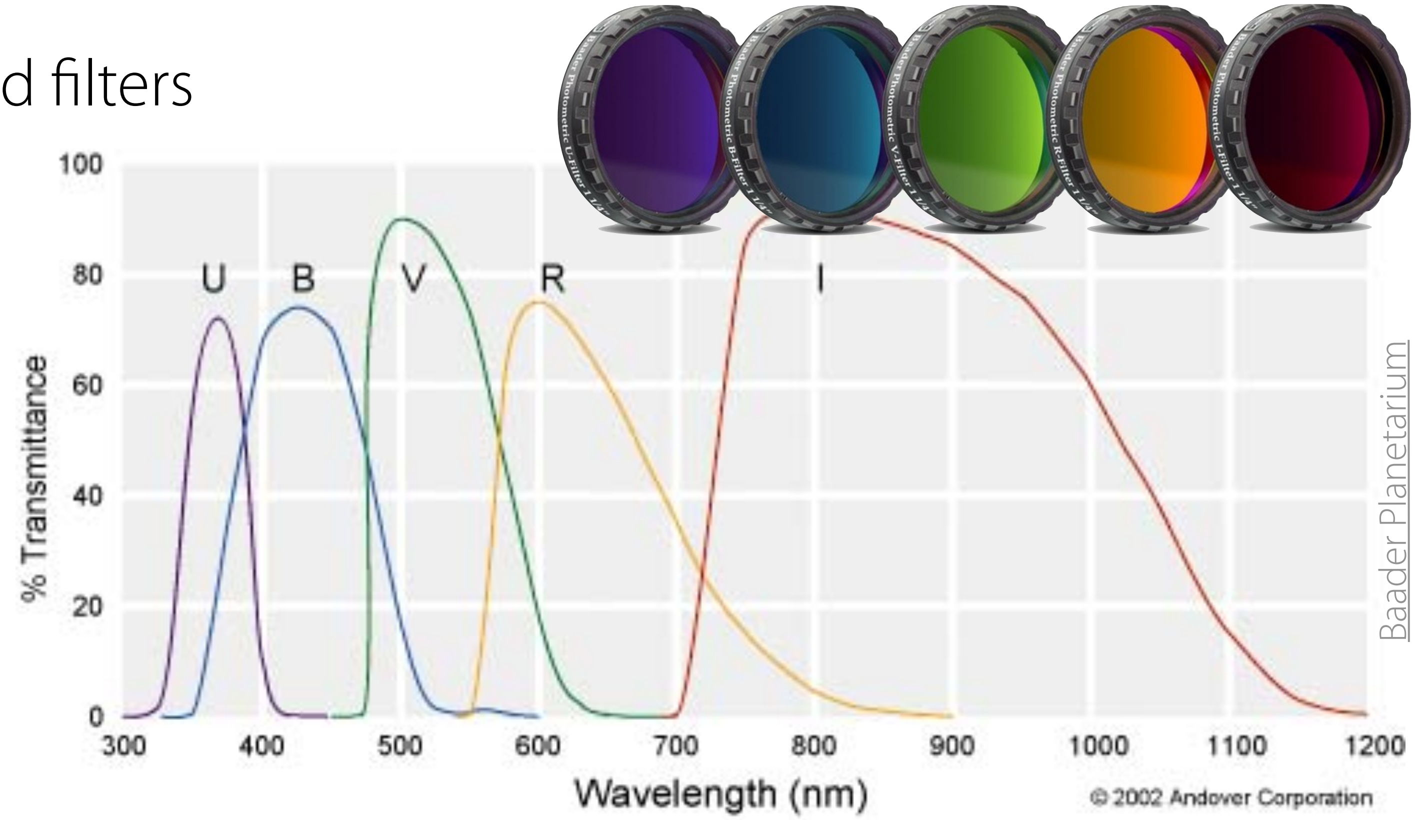
- **Bolometric = over all wavelengths**

- **Brightness** (related to the flux)
    - **Apparent brightness m** - as in the sky
    - **Absolute brightness M** - as if at a distance of 10 pc
    - Both measured on a logarithmic scale in units of magnitudes

- **Colour:** Brightness measured with standard filters
    - Brightness measured in a selected filter marked with corresponding index or as capital letter, e.g. $m_V = V$

- **Colour index:** difference of brightness measured in different bands, e.g. $m_B - m_V = B - V$

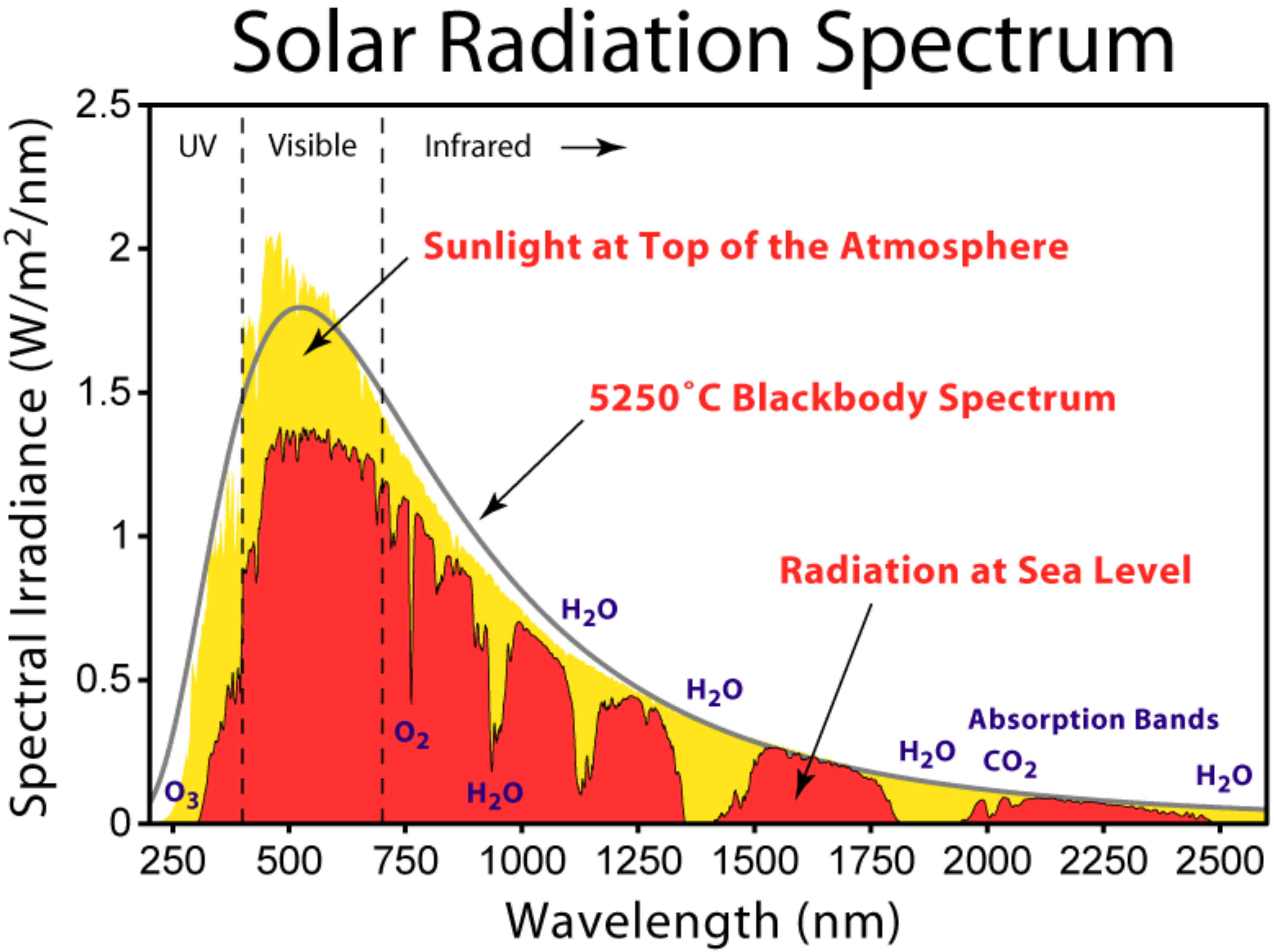
Johnson/Bessell UBVRI filters



Baader Planetarium

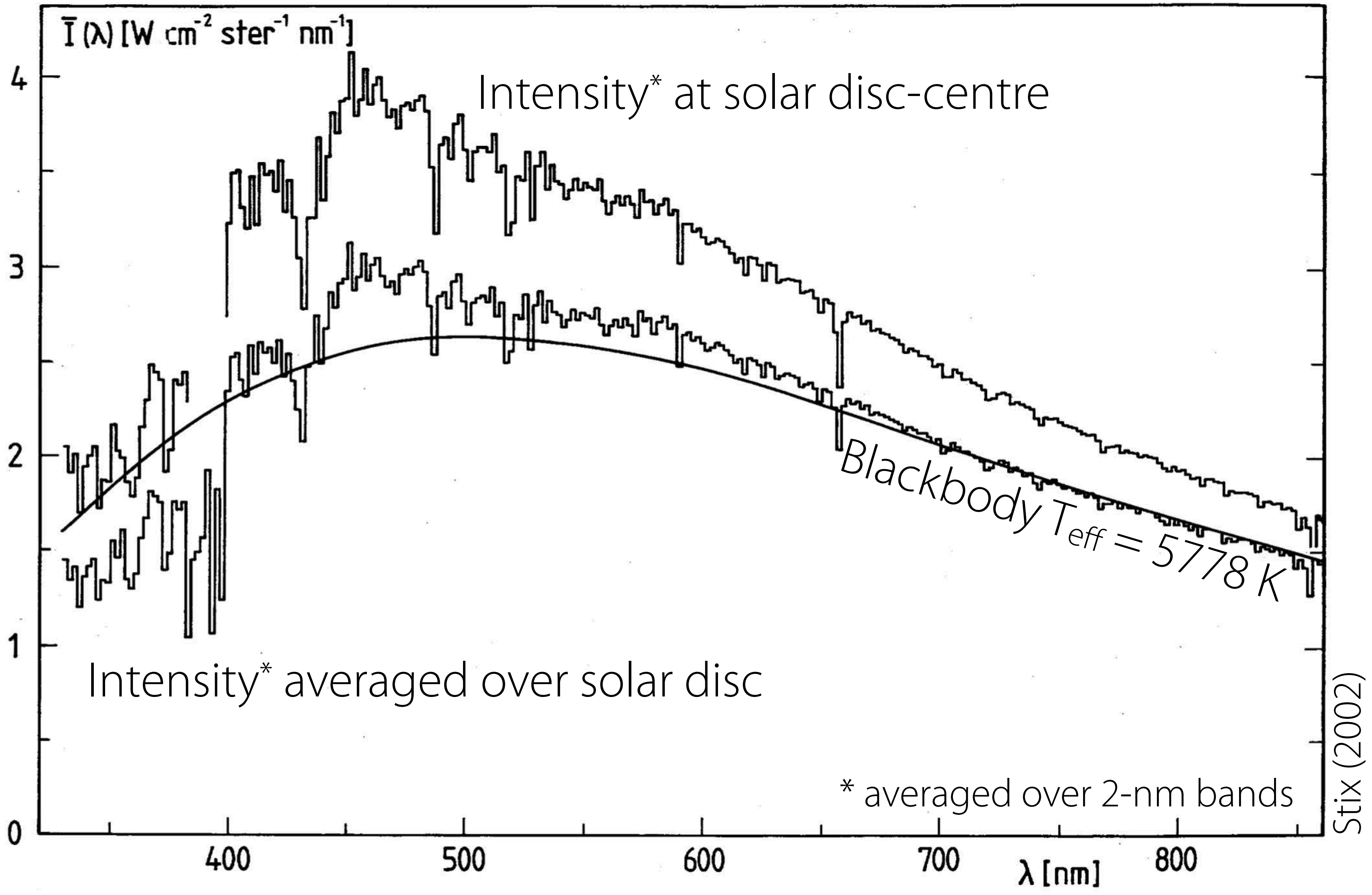
# Observational stellar parameters

## Stellar spectrum



# Observational stellar parameters

## Stellar spectrum



Stix (2002)

- Measured Sun’s intensity as function of wavelength
- The black body spectrum for an effective temperature of Teff = 5778 K
- Data from Neckel and Labs (1984) - Also provided for your project assignment.

# Observational stellar parameters

## Blackbody spectrum

- Radiation flux density (spectrum) resembles a **blackbody** spectrum, which is given by the Planck function **$B_\nu$ (T)**.
  - $B_\nu$ (T): spectral density of electromagnetic radiation emitted by a blackbody in thermal equilibrium at a given temperature T .

➡ The flux density $F_\nu$ of a blackbody:

$$F_\nu = \pi B_\nu(T)$$

➡ Bolometric flux by integration over all frequencies:

$$F = \int_0^\infty F_\lambda d\lambda = \int_0^\infty F_\nu d\nu = \int_0^\infty \pi B_\nu(T) d\nu = \sigma T^4 \Rightarrow F = \sigma T_{\rm eff}^4$$

- Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k_{\rm B}^4}{15 h^3 c^2} = 5.67 \times 10^{-8} \mathrm{W\,m^{-2}\,K^{-4}} = 5.67 \times 10^{-5} \mathrm{erg\,s^{-1}\,cm^{-2}\,K^{-4}}$$

# Observational stellar parameters

## Stefan-Boltzmann law

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# Observational stellar parameters

## Stefan-Boltzmann law

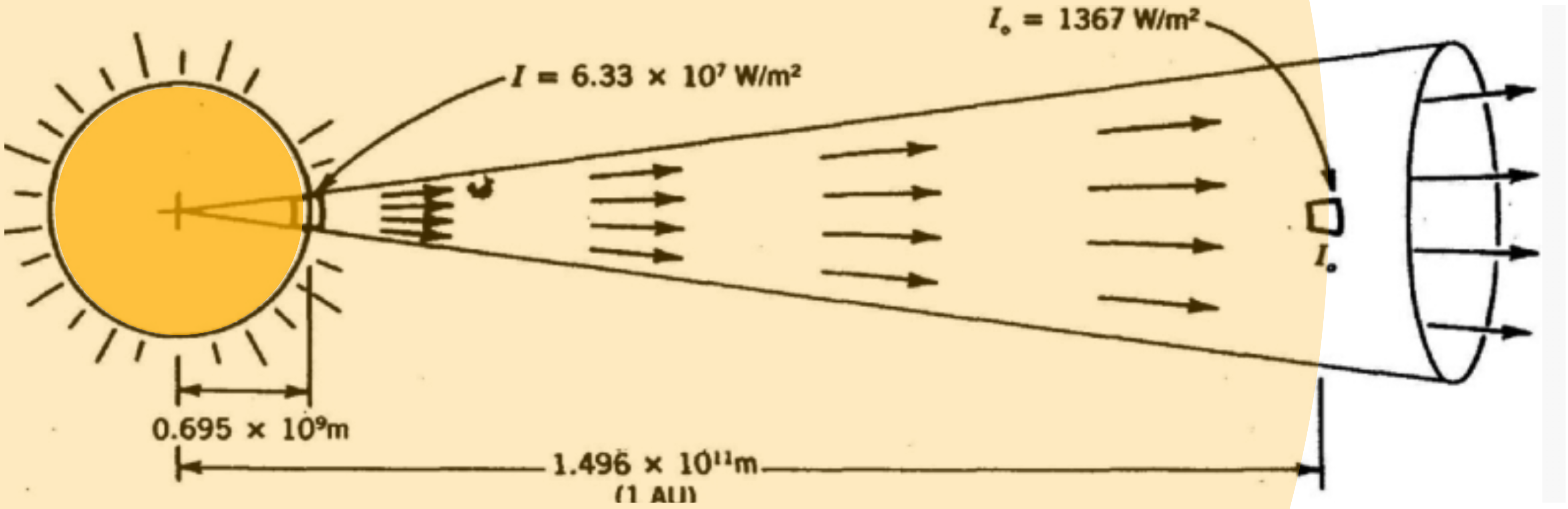
- **Example: Sun**
- Bolometric flux density of the Sun measured just outside Earth's atmosphere by a satellite:

➡ **solar constant** = 1.36 kW $m^{-2}$

  - Radiation emitted from the Sun's "surface" at radius 1 $R_\odot$
  - Diluted over a sphere with radius d = 1 AU with surface area $A = 4\pi\, d^2$
- Correction for "dilution" effect with the factor $d^2/R_\odot^2$ gives flux density at Sun's surface (1$R_\odot$):

$\mathbf{F_\odot = 6.3 \cdot 10^7\ Wm^{-2}}$

- Stefan-Boltzmann law: $F_\odot = \sigma\, T_{\mathrm{eff},\odot}^4$ —> effective temperature of the Sun: $\mathbf{T_{eff,\odot} \approx 5770\ K}$



# Stellar parameters

## Luminosity

- **Total radiative energy output of a star**
  given by flux that emerges across the total surface of a star.
- Assumption: star is spherical with radius R

➡ Surface area $A = 4\pi R^2$

➡ Luminosity of a star $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$

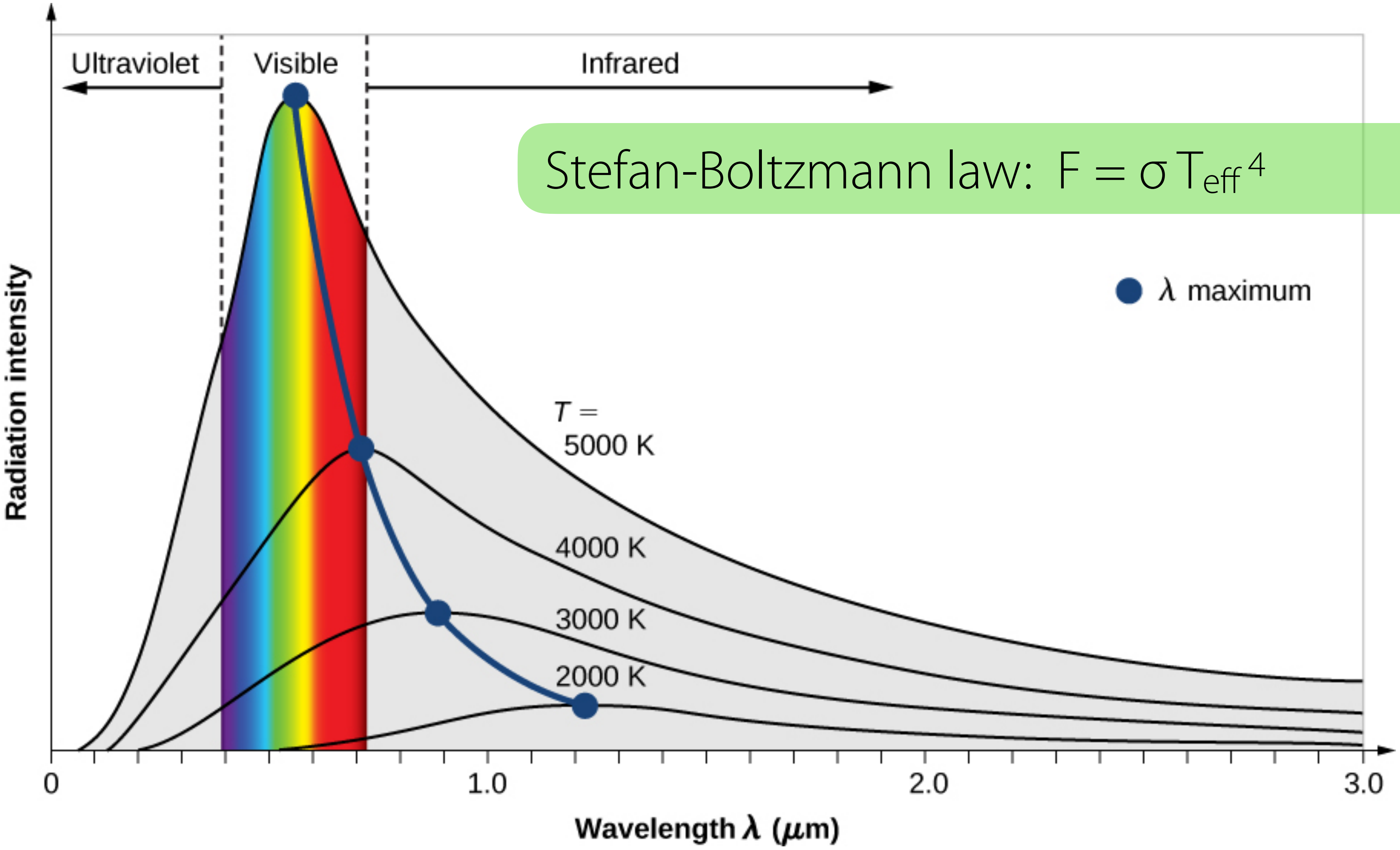
Stefan-Boltzmann law:
$F = \sigma T_{\mathrm{eff}}^4$

- Units: W (SI) or erg s–1 (cgs)
- Convenient to use the bolometric luminosity of the Sun as unit

$$L_\odot = 3.84 \times 10^{26}\,\mathrm{W}$$

# Stellar spectra

- **Blackbody spectra** for different temperatures



Stefan-Boltzmann law: $F = \sigma T_{\mathrm{eff}}^4$

- Increasing $T_{\mathrm{eff}}$

➡ Bolometric flux F (integral under the curve) increases with $T_{\mathrm{eff}}^4$

➡ Wavelength of peak becomes shorter: Wien's displacement law

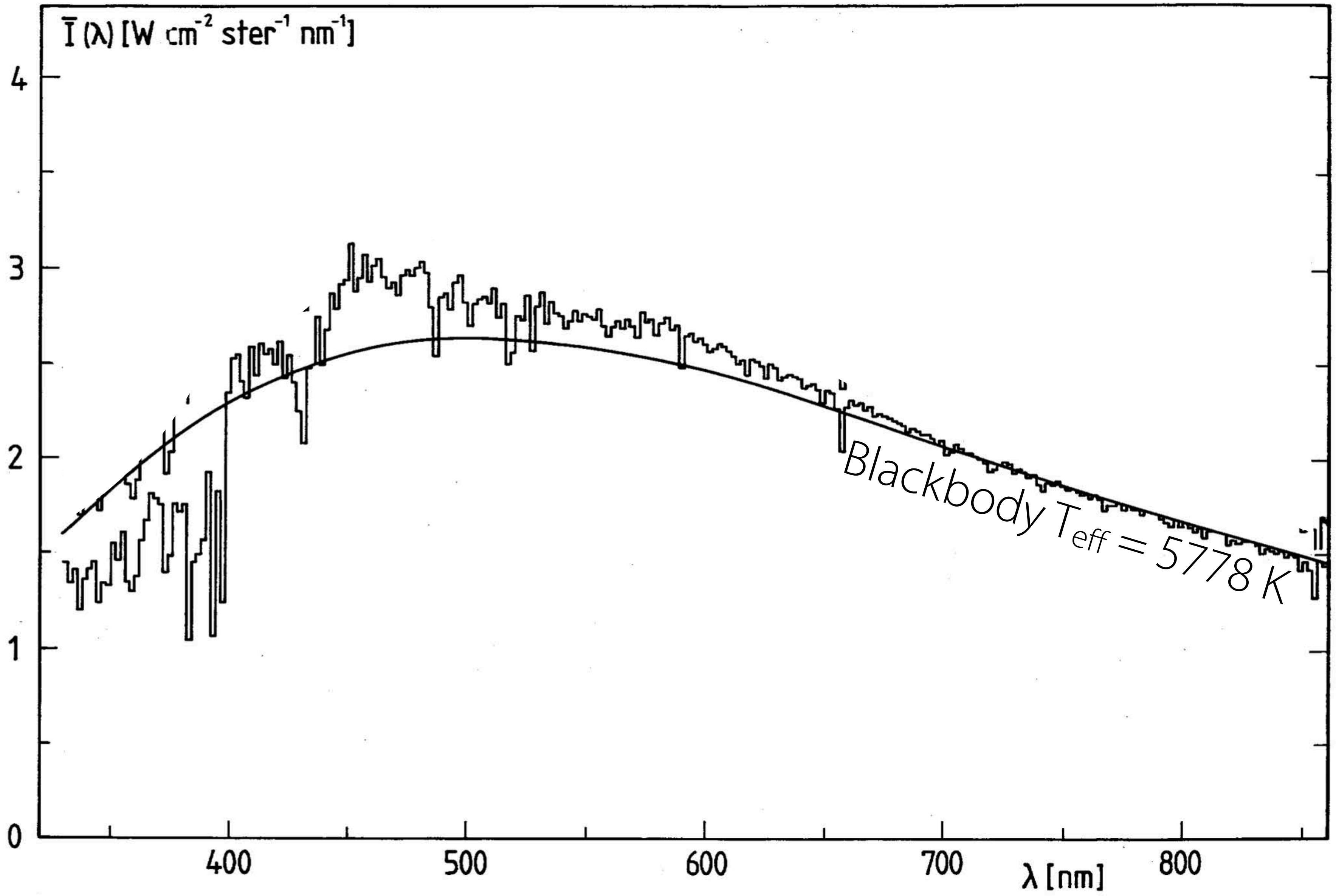
$$\lambda_{\max} = \frac{b}{T}$$

Wien's displacement constant

$$(b = 2.89777 \times 10^{-3}\,\mathrm{m} \cdot \mathrm{K} \approx 2900\,\mu\mathrm{m} \cdot \mathrm{K})$$

Interactive app:
https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html

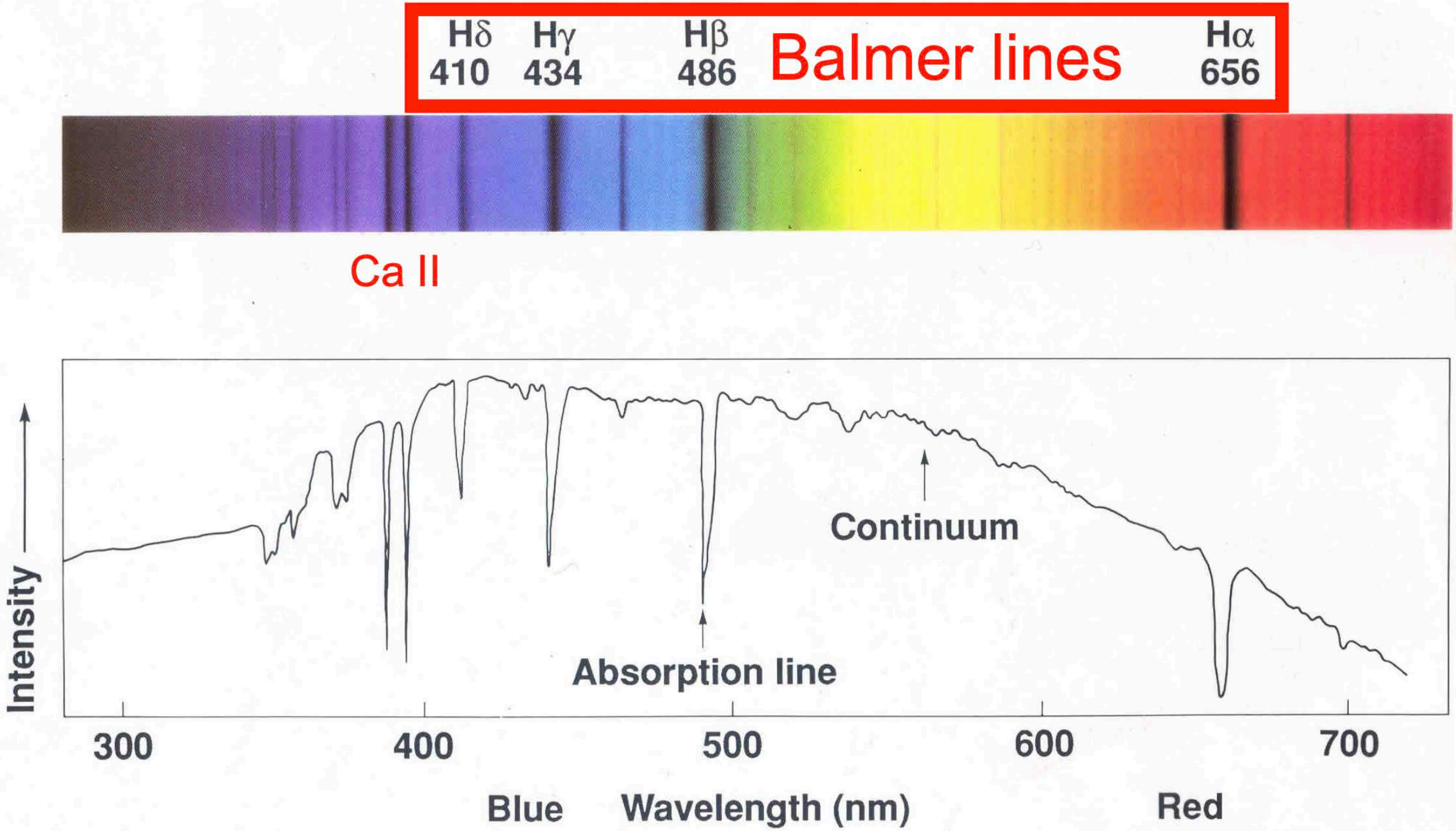
# Stellar spectra



Stix (2002)

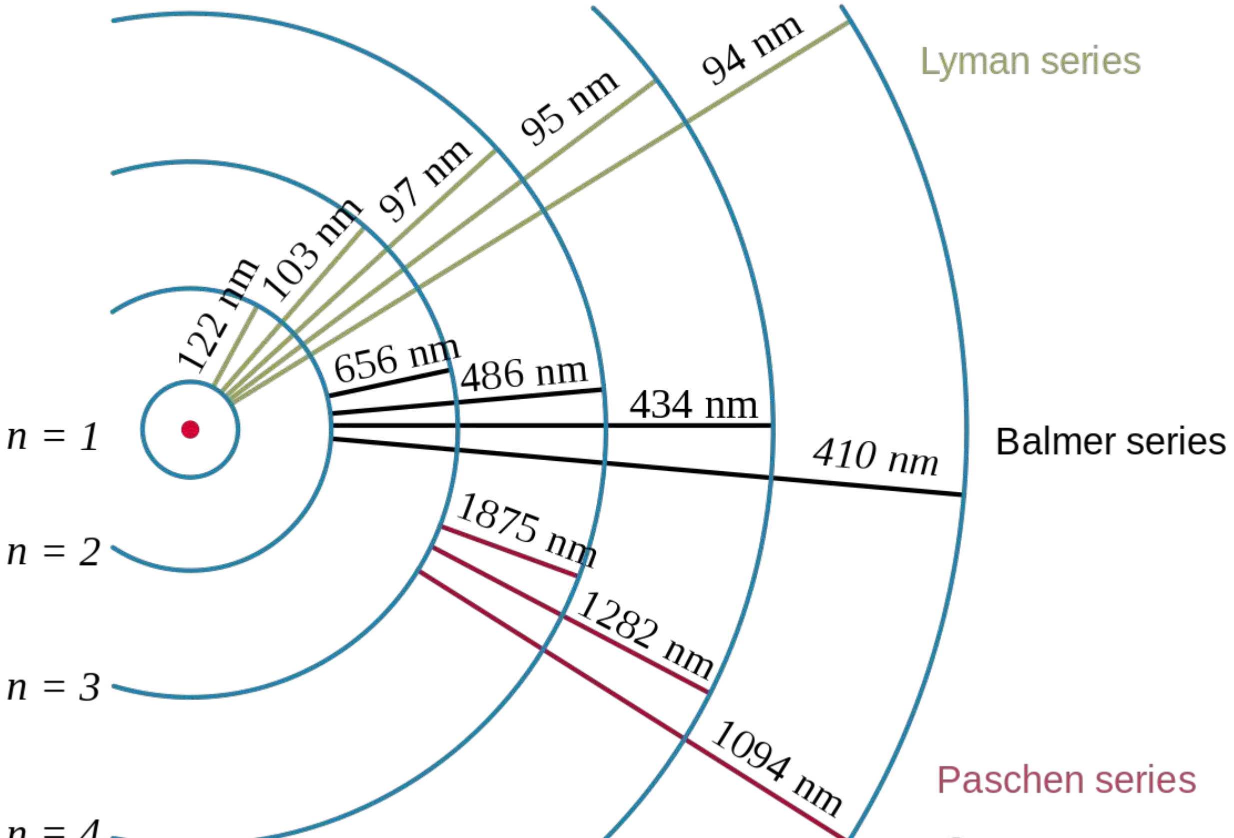
- A real stellar spectrum …
  - … is reasonably well described at longer wavelengths
  - … **deviates** in particular at shorter wavelengths and/or in wavelength regions with many (absorption) lines!

# Observational stellar parameters

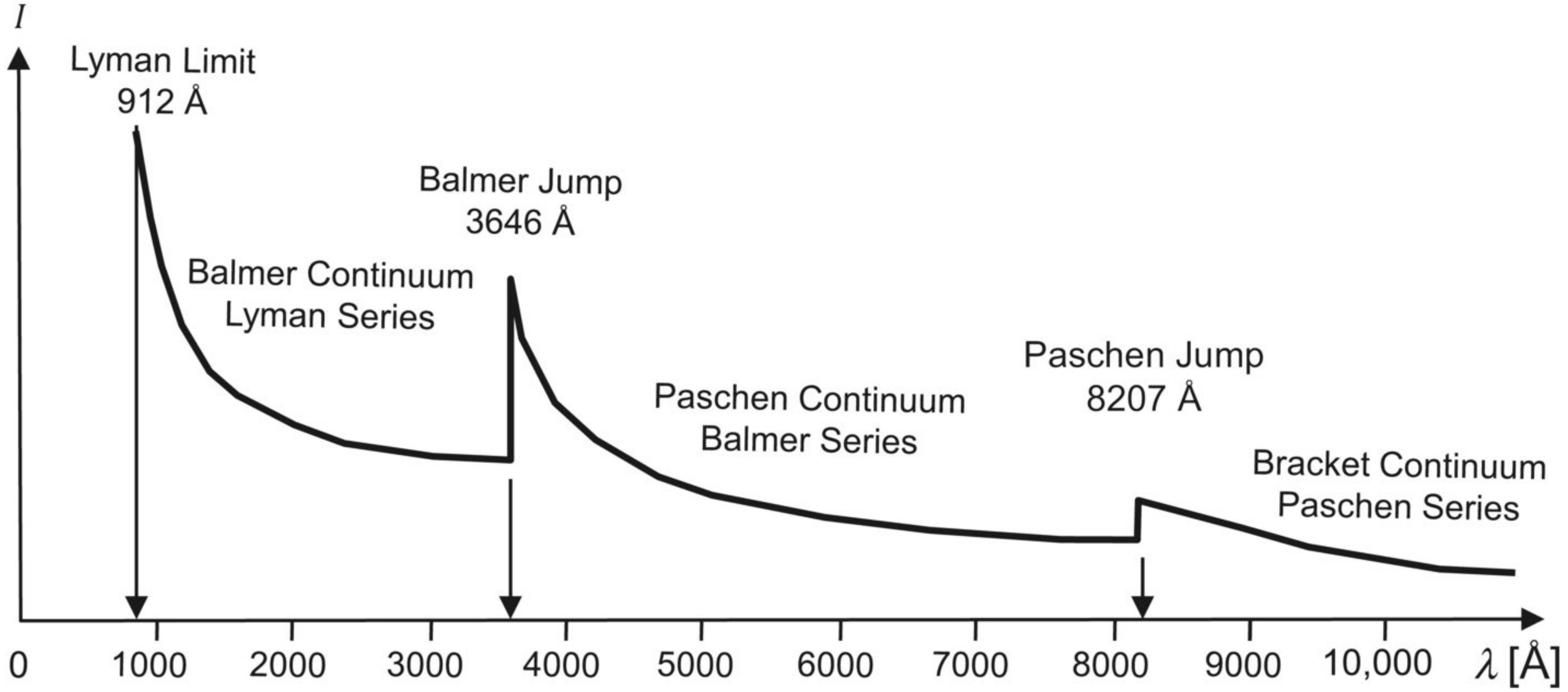


# Stellar spectra

## Hydrogen lines and continua



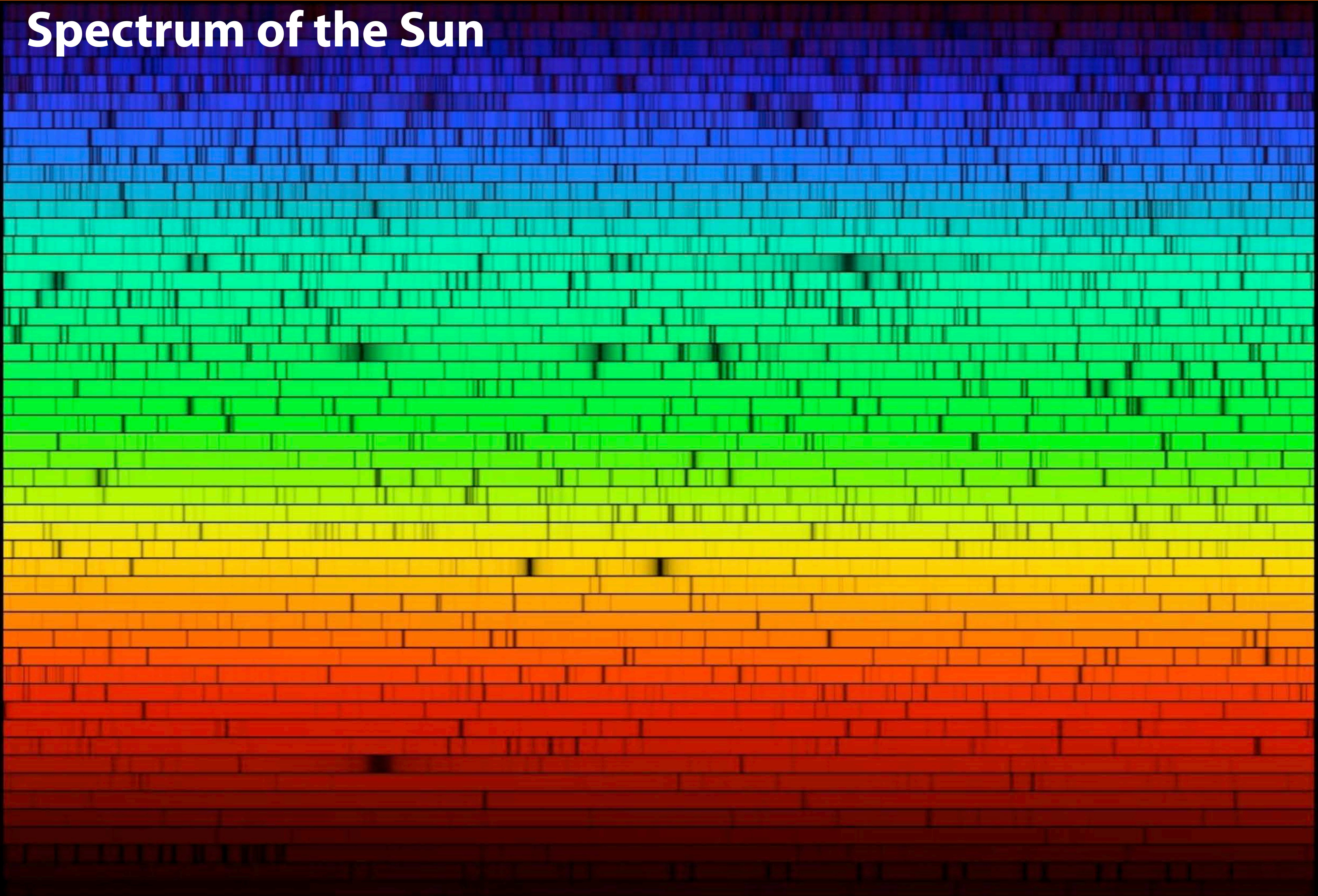
- Energy of photon $E = h\nu$
- Photon absorbed at matching energy differences in the atom
- Hydrogen energy levels: $E_n \propto n^{-2}$
- Rydberg formula $\lambda^{-1} \propto (n_1^{-2} - n_2^{-2})$



➡ Notable deviations from blackbody!

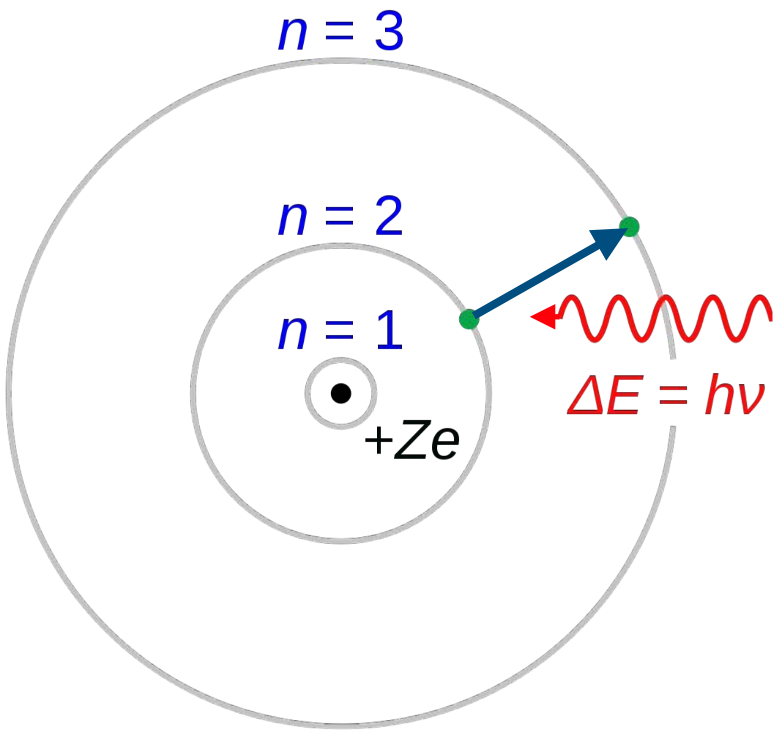
# Stellar spectra

## Spectrum of the Sun



# Spectral classification

## Line strength

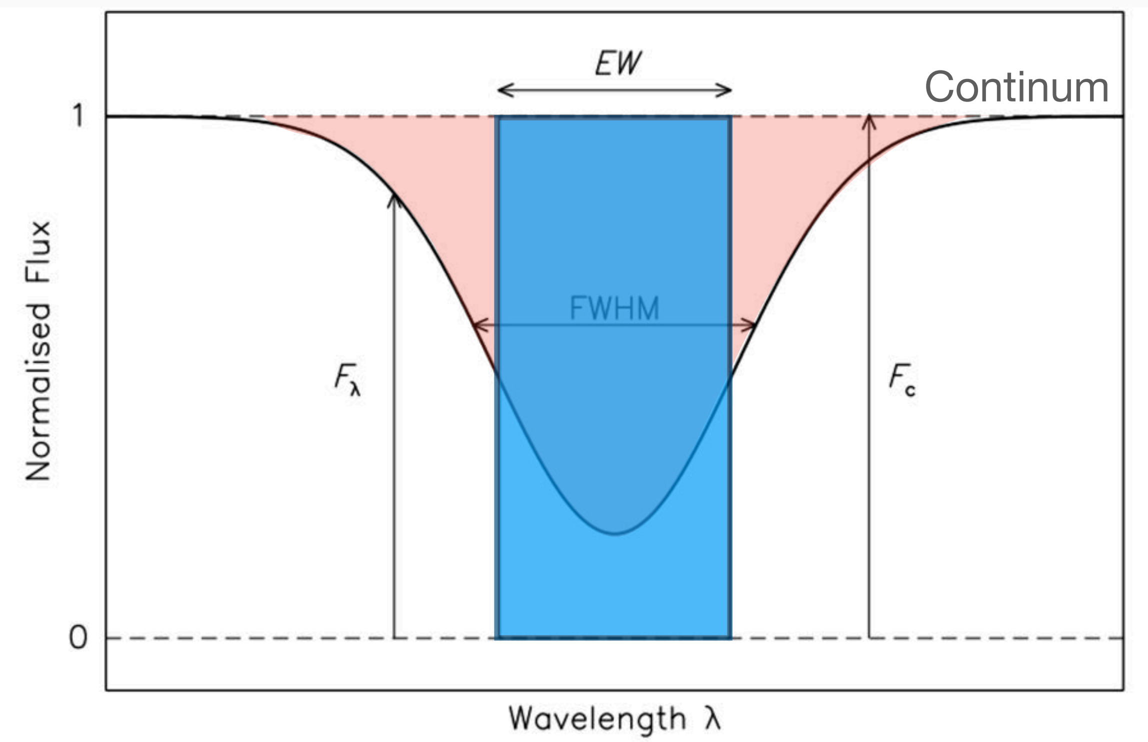


- **Strength of a spectral line** depends on the number of atoms/molecules with electrons in the starting orbit for the spectral line under consideration
- **Example: H-α** line at 656.3nm in absorption, n=2 → n=3

  ➡ H atoms needed that have electrons in level n=2,

  ➡ Must already have absorbed a photon to raise n=1 → n=2

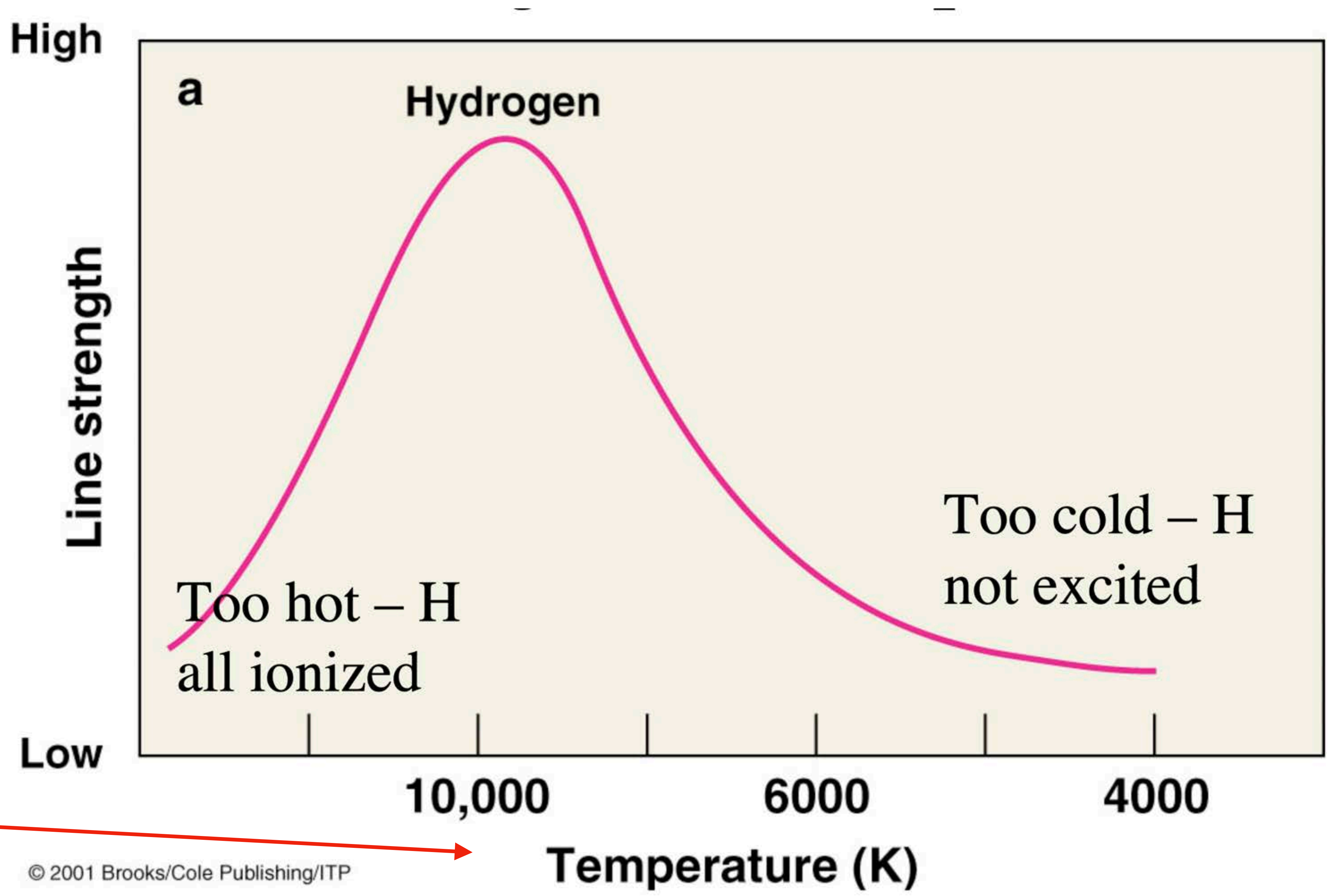
  ➡ Requires high enough temperature
- Level populations and thus strength of a spectral line depends thermodynamic properties of the gas (and chemical abundance)
- Line strength measured as **equivalent width**
  - Area between spectral line and continuum and reshape into rectangle that extends from 1 to 0
  - Width of covered area



# Spectral classification

## Line strength

- Strength of a spectral line depends on chemical abundance of the element and thermodynamic properties such as the gas **temperature**



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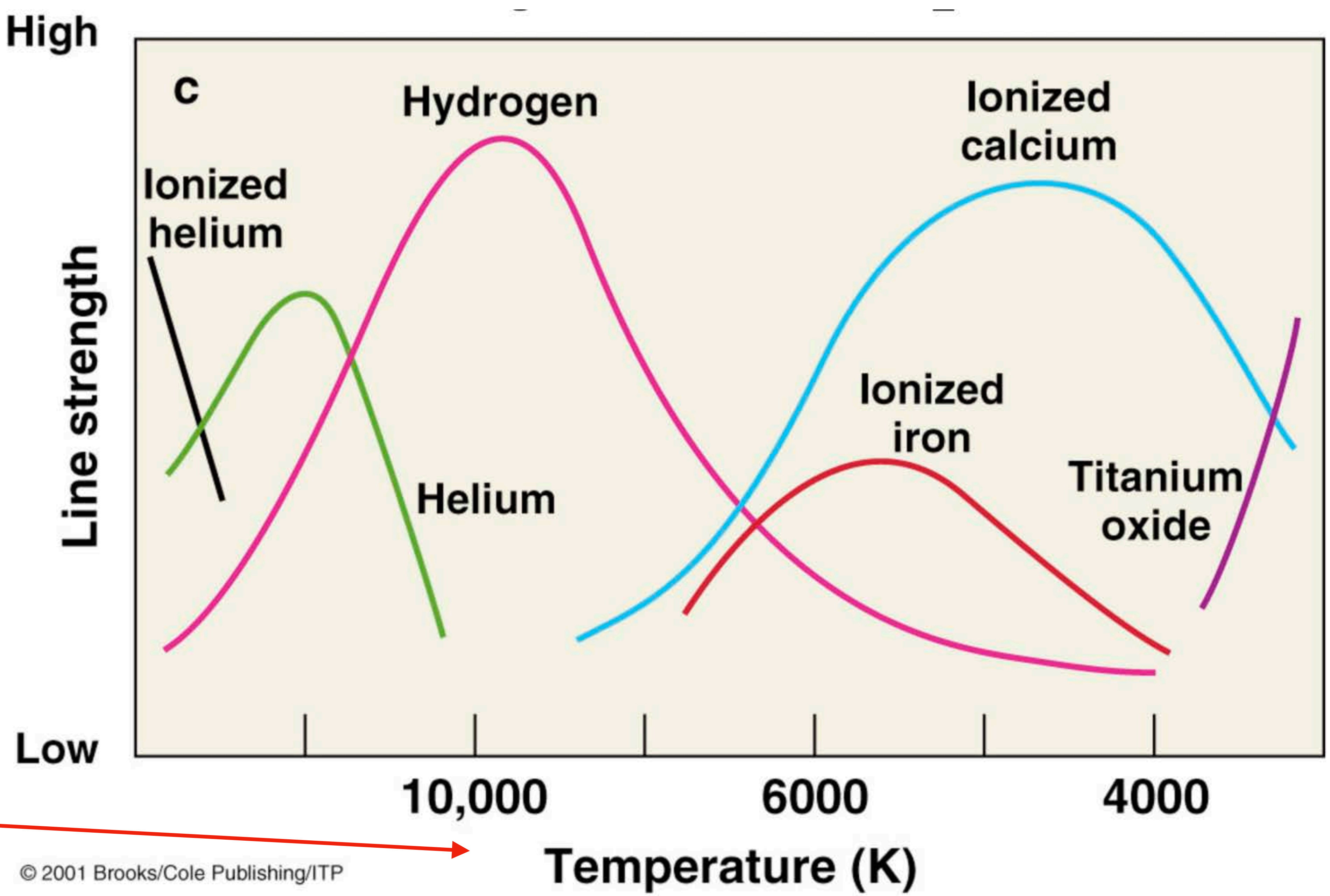
Reversed axis!

# Spectral classification

## Line strength

- Analysis of many spectral lines of different elements (and ionisation stages) allows to determine the gas temperature



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Reversed axis!

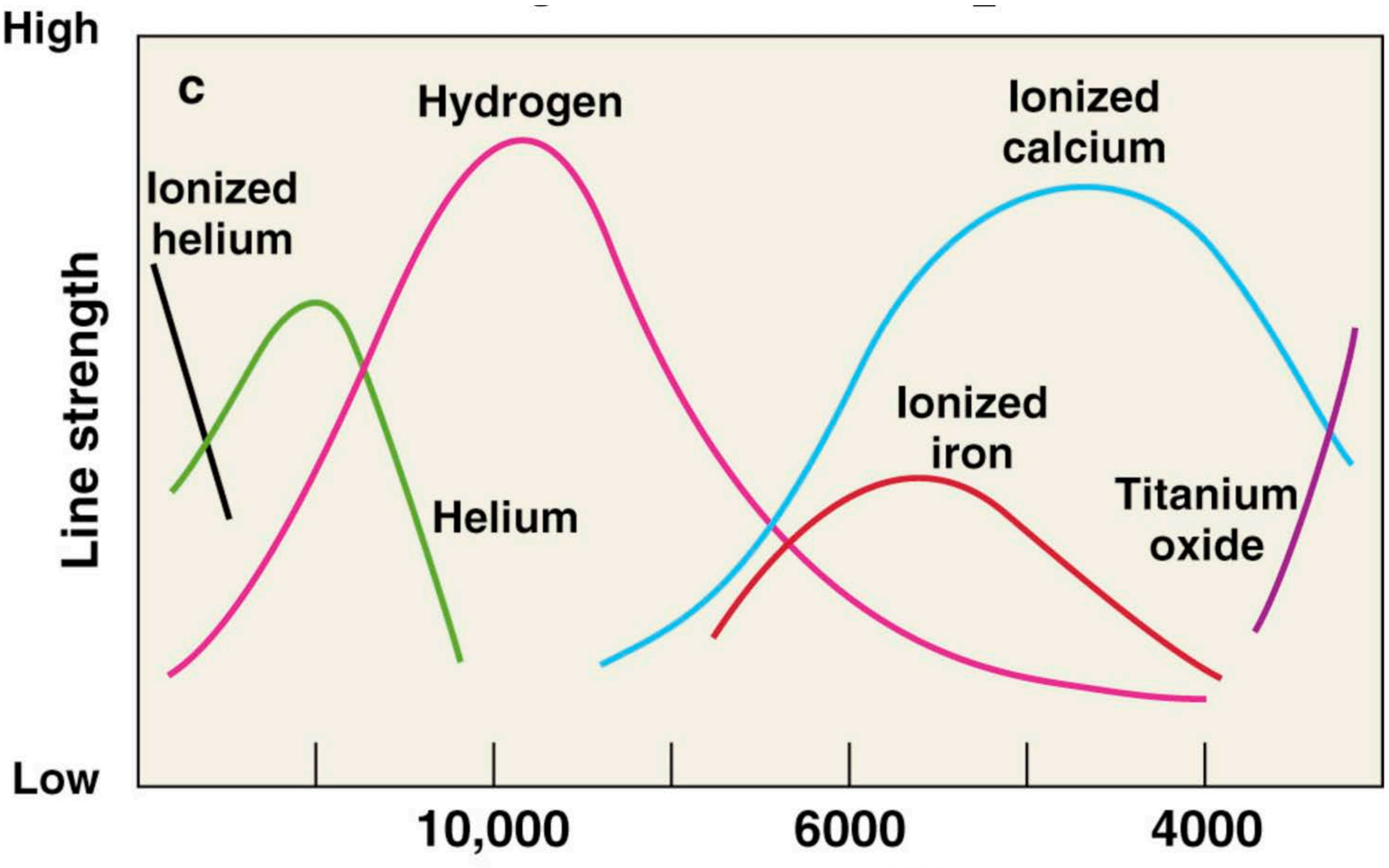
# Spectral classification

## Line strength

- Saha equation: Ionization degree for any gas in thermal equilibrium

$$\frac{n_{i+1} n_e}{n_i} \propto \exp\left[-\frac{(\epsilon_{i+1} - \epsilon_i)}{k_B T}\right]$$

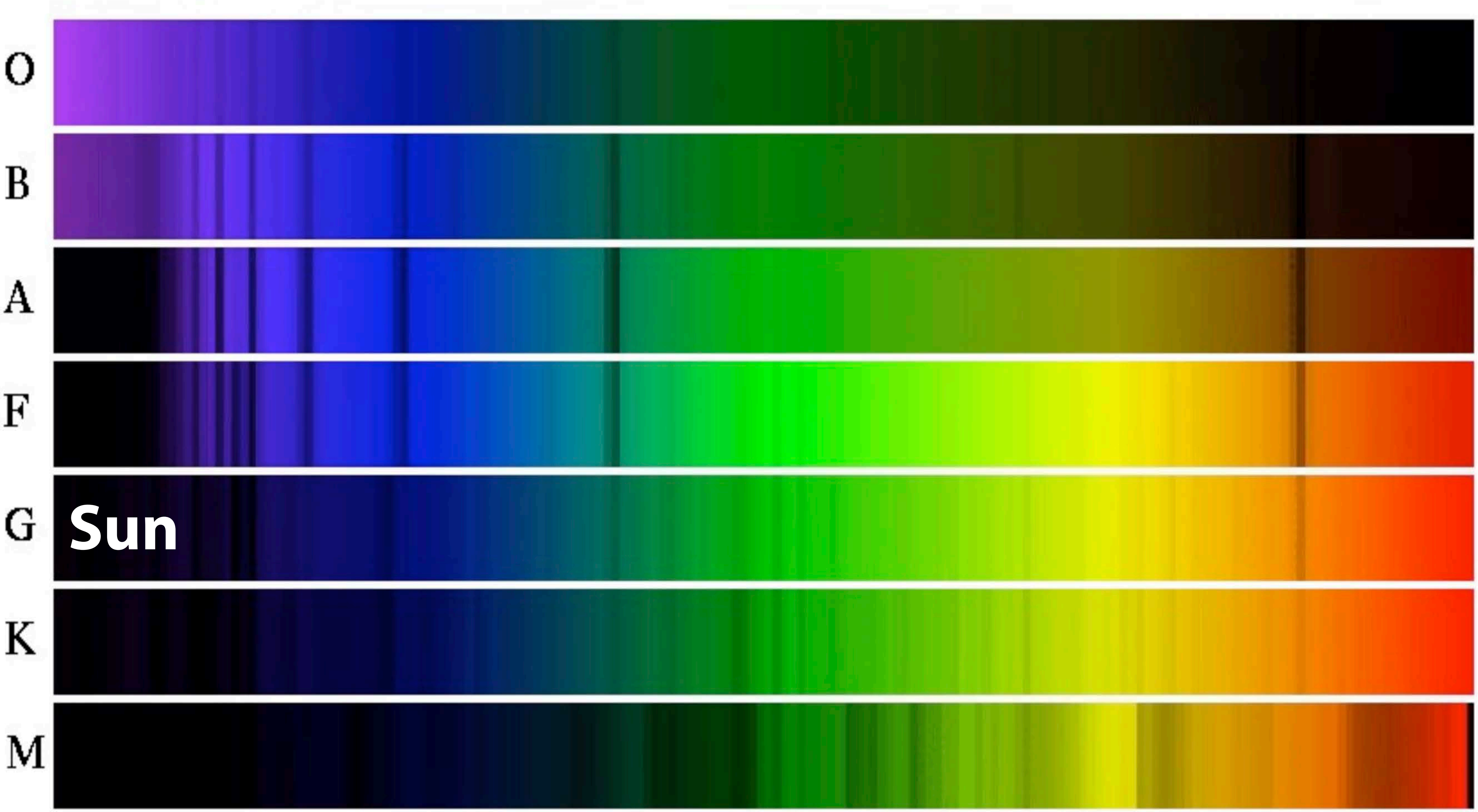
n: Number density of atoms in the i-th ion state (i electrons removed)
ε: ionisation energy



# Spectral classification

- Spectral types defined according to occurrence and strength of spectral lines

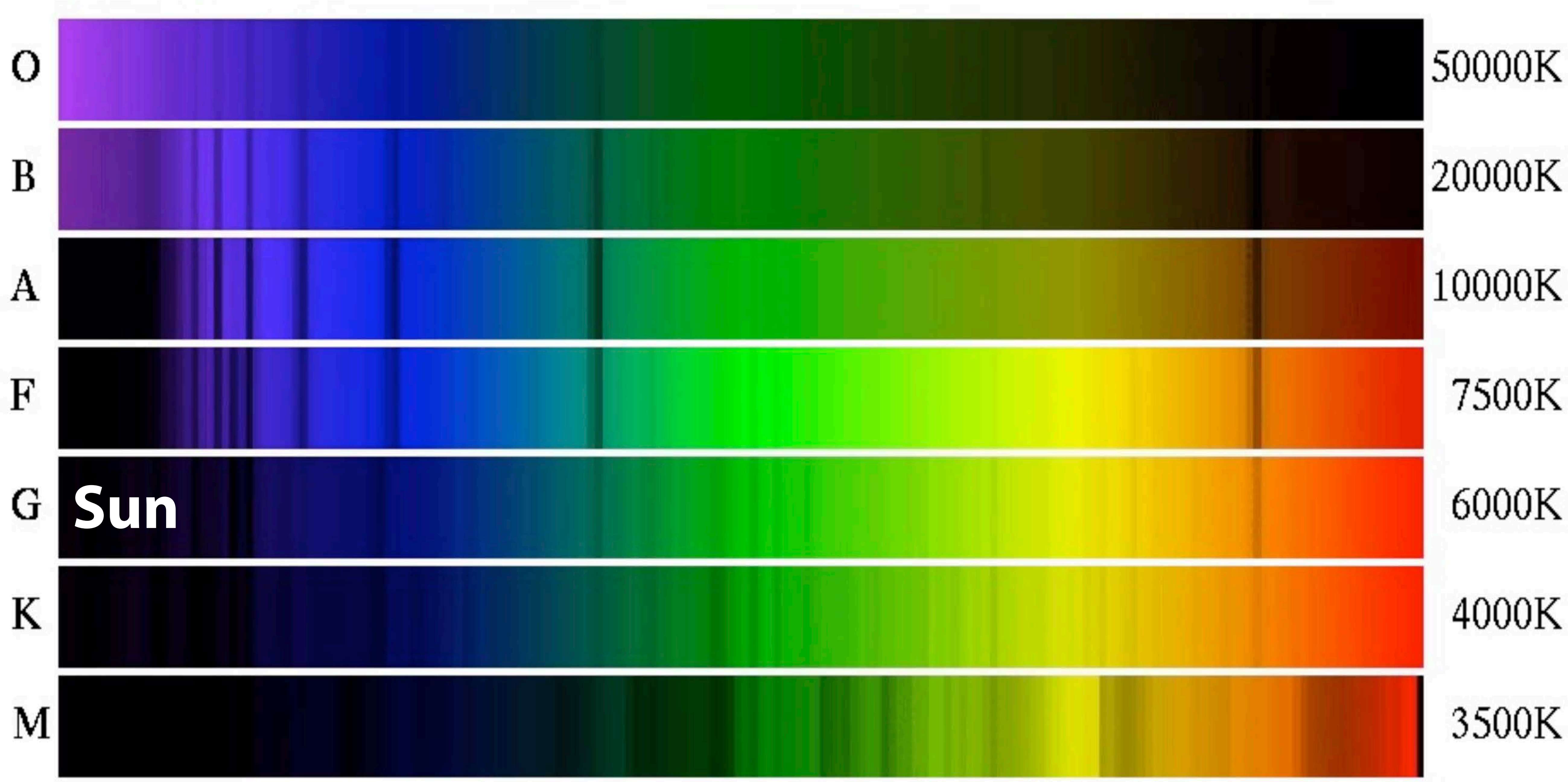
➡ Sorting the different types into a sequence



# Spectral classification

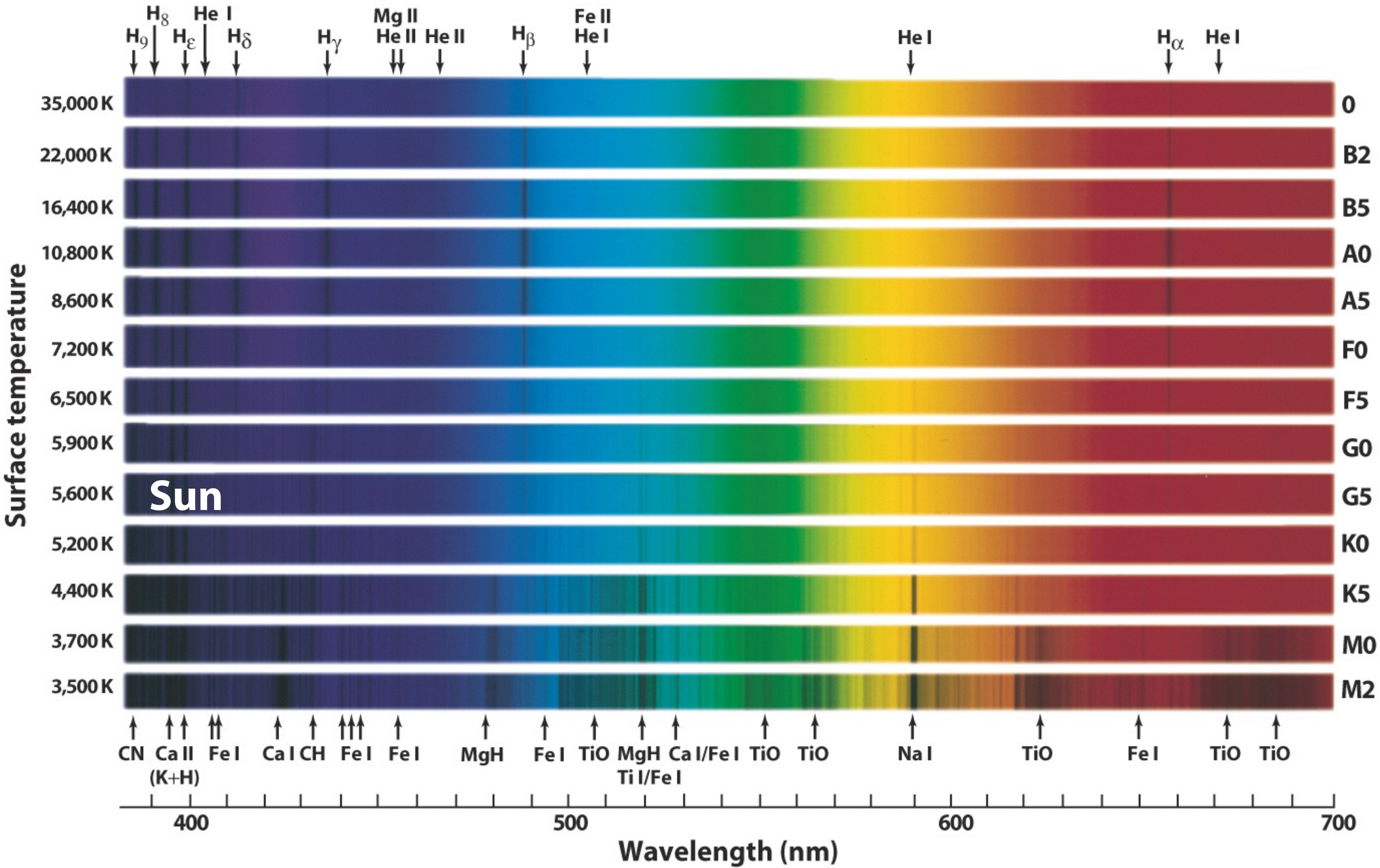
- Measuring brightnesses, colour indices, spectral lines (+Stefan-Boltzmann law)

➡ Spectral types can sorted into a sequence as function of temperature



# Stellar spectra

- Brightnesses, colour indices + **occurrence and strength of spectral lines for different ionisation stages**
  ➡ Information about temperature and chemical composition
  ➡ Sorting the different types into a sequence
  ➡ Spectral types can sorted into a sequence as function of temperature

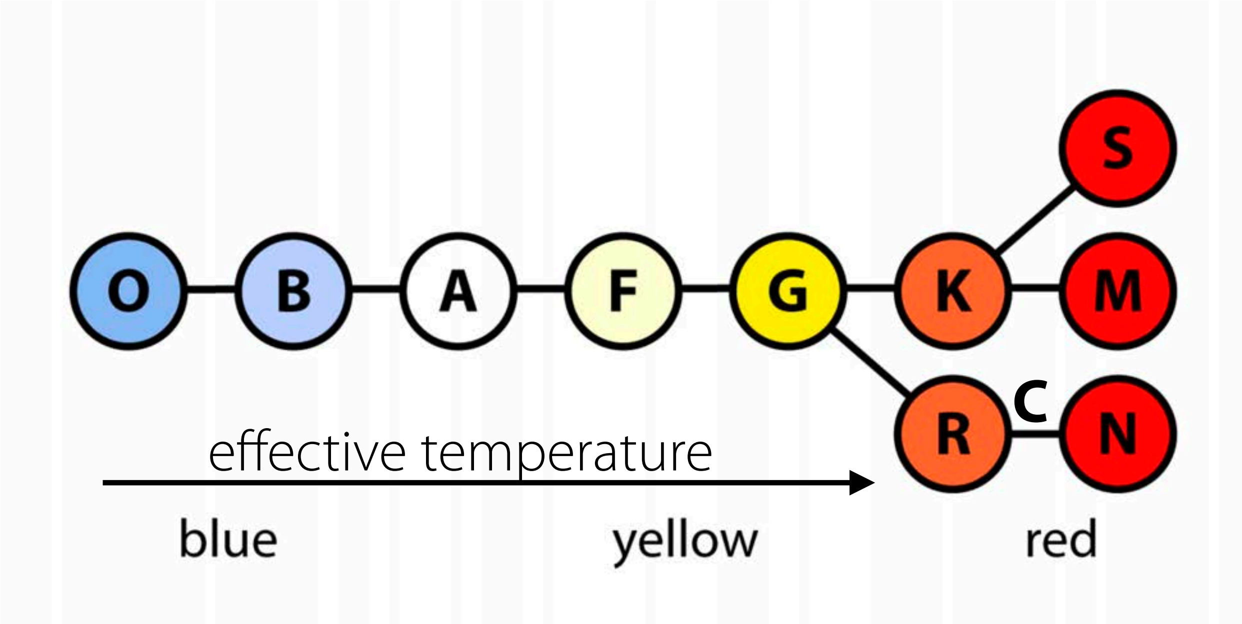


# Spectral classification

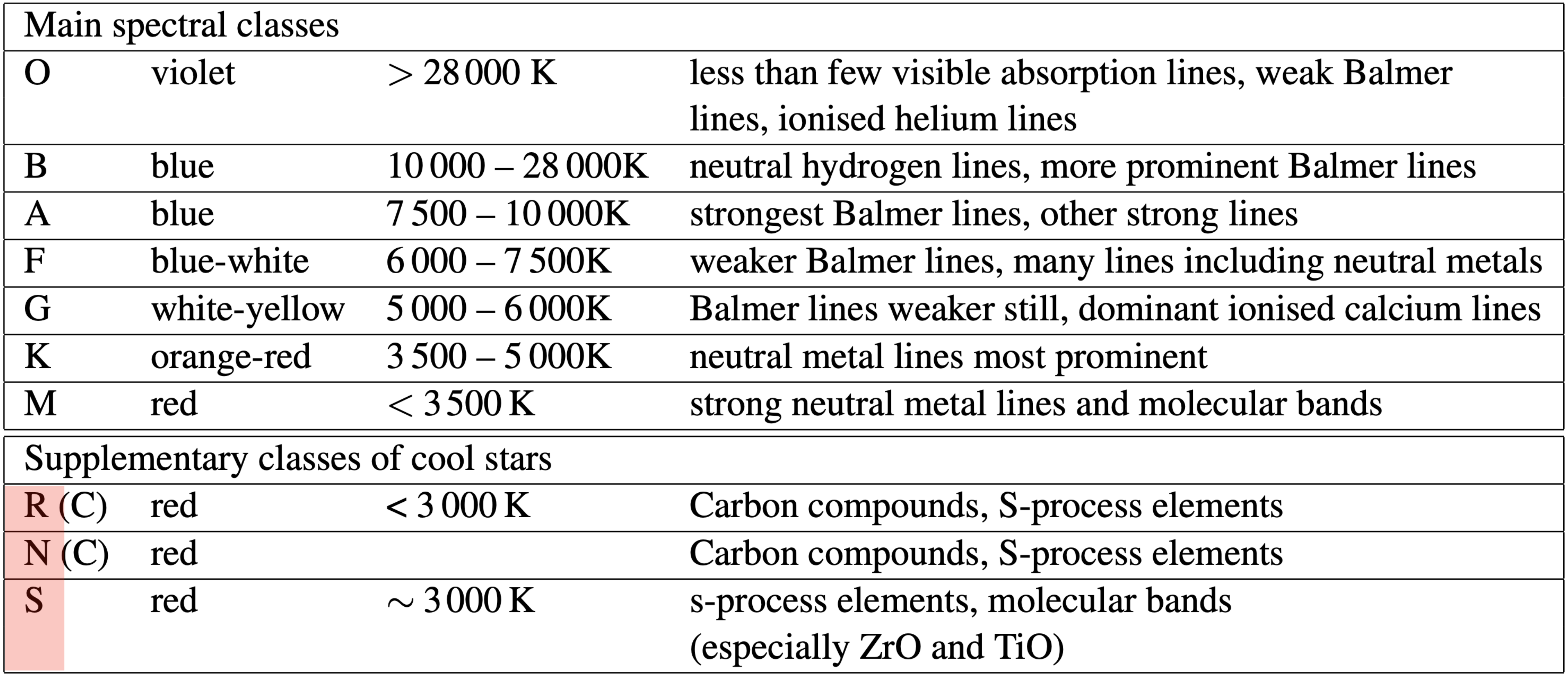
- Different classification schemes
- **Harvard spectral classification**
- Further developed and extended

➡ **Morgan–Keenan system**

- A star is classified by a
  - Spectral class
  - Decimal sub-division (0-9) with effective temperature decreasing with in increasing digit
  - Luminosity class



# Spectral classification



| | | | |
|---|---|---|---|
| Main spectral classes | | | |
| O | violet | > 28 000 K | less than few visible absorption lines, weak Balmer lines, ionised helium lines |
| B | blue | 10 000 – 28 000K | neutral hydrogen lines, more prominent Balmer lines |
| A | blue | 7 500 – 10 000K | strongest Balmer lines, other strong lines |
| F | blue-white | 6 000 – 7 500K | weaker Balmer lines, many lines including neutral metals |
| G | white-yellow | 5 000 – 6 000K | Balmer lines weaker still, dominant ionised calcium lines |
| K | orange-red | 3 500 – 5 000K | neutral metal lines most prominent |
| M | red | < 3 500 K | strong neutral metal lines and molecular bands |
| Supplementary classes of cool stars | | | |
| R (C) | red | < 3 000 K | Carbon compounds, S-process elements |
| N (C) | red | | Carbon compounds, S-process elements |
| S | red | ∼ 3 000 K | s-process elements, molecular bands (especially ZrO and TiO) |

- **R,N or C-type: "carbon stars"** — red giant stars and Asymptotic Giant Branch (AGB) stars.
  - regular M-type giant stars have more oxygen and carbon -> referred to as "oxygen-rich" stars.
- **S-type stars:** carbon and oxygen are approximately equally abundant
  - prominently spectral features due to the s-process elements (e.g. zirconium monoxide (ZrO).

# Spectral classification

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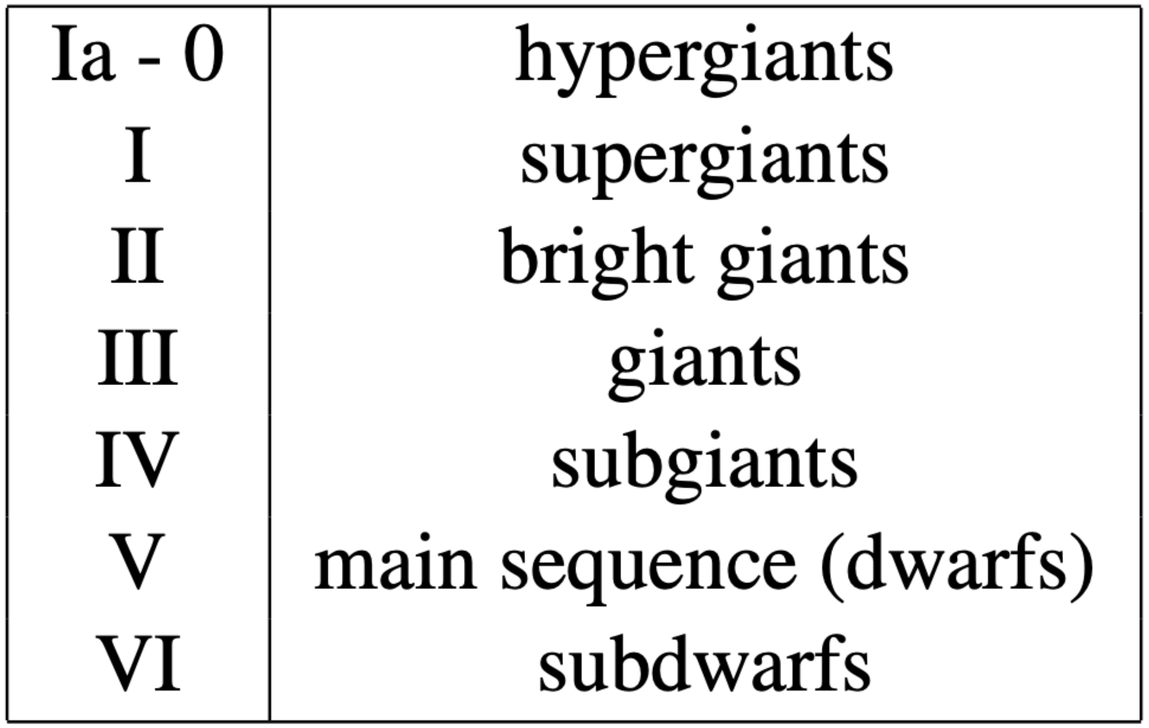
| Very low mass /sub-stellar spectral classes (mostly brown dwarfs) | | | |
|---|---|---|---|
| L | IR | 1 500 – 2 500 K | lines of alkali metals (e.g. N) and metallic compounds (e.g. FeH) |
| Y | IR | 800 – 1 500 K | methane absorption lines |
| T | IR | < 800 K | water and ammonia lines |

# Spectral classification

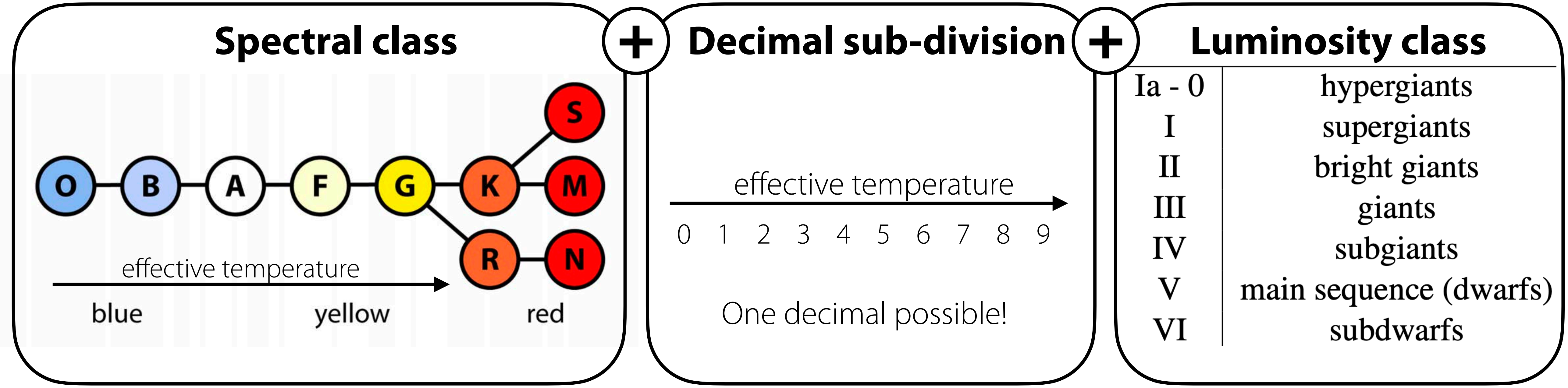
## Luminosity class



| | |
|---|---|
| Ia - 0 | hypergiants |
| I | supergiants |
| II | bright giants |
| III | giants |
| IV | subgiants |
| V | main sequence (dwarfs) |
| VI | subdwarfs |

# Spectral classification

- A star is classified by a



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- **Harvard spectral classification**
- Further developed and extended

➡ **Morgan–Keenan system**

- **Examples**

| | |
|---|---|
| Sun | G2V |
| Sirius A | A0V |
| Proxima Cen | M5.5V |
| Betelgeuse | M1I |
| Aldebaran | K5III |

# Spectral classification

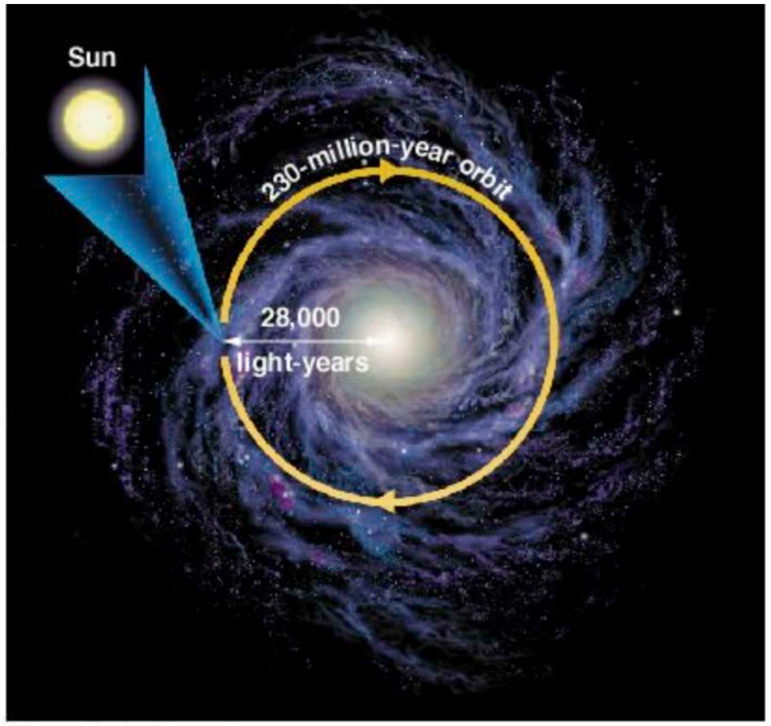
## Additional classification

- Special spectral types
  - **W** for Wolf-Rayet stars with no hydrogen lines in their spectra
  - **D** for white dwarfs
  - …
- **Extra information** can be added to the spectral type of a star if it differs from the other regular types / show **peculiarities**
  - e: presence of pronounced emission lines
  - v: variable spectral features.
  - …
  - Example: M5.5Ve
- Please note that
  - the accuracy of a spectral classification depends on the quality of the available data
  - a spectral classification can change as a star changes

# Spectral classification

## Stellar populations

- Observations show that decreasing **metal content** correlated with increasing age of stars.
- Stars (in our galaxy) can be further divided into populations according to their chemical composition or metallicity
  - **Population I:** "recent" stars, high metallicity
  - **Population II:** old stars, low metallicity
  - **Population III:** first stars in the universe (very low metal content)
- Originally, pop I+II, pop III added in 1978

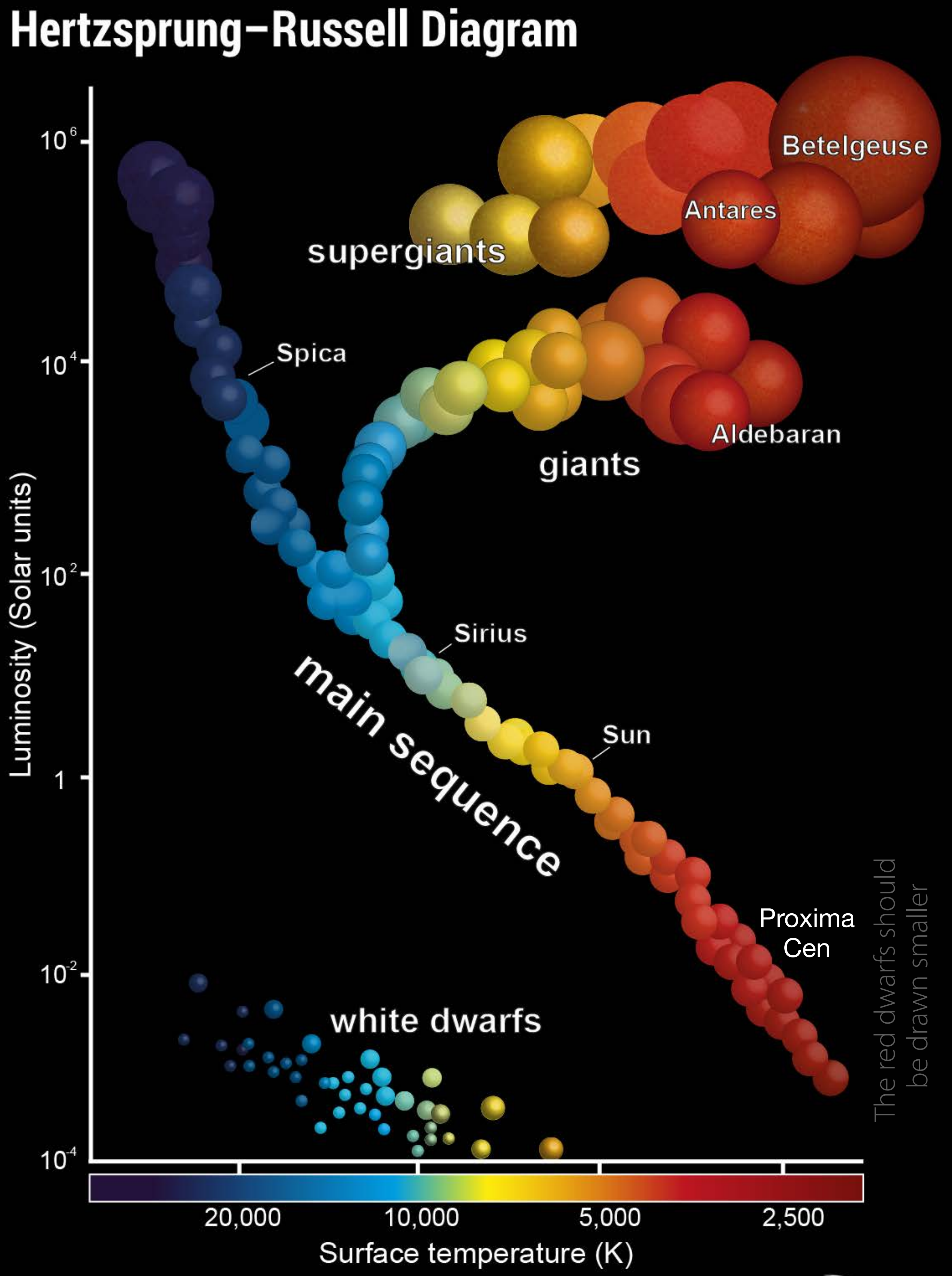


- Total number of stars in the Milky Way only known roughly: **100 - 400 billion stars**

# Stellar classification

## Hertzsprung-Russell diagram

- Stars not randomly distributed
- Distribution yields important clues for stellar structure and evolution
- Different stellar types and evolution stages:
  - Main sequence
  - Giants and supergiants
  - Dwarf stars
  - …



The red dwarfs should be drawn smaller

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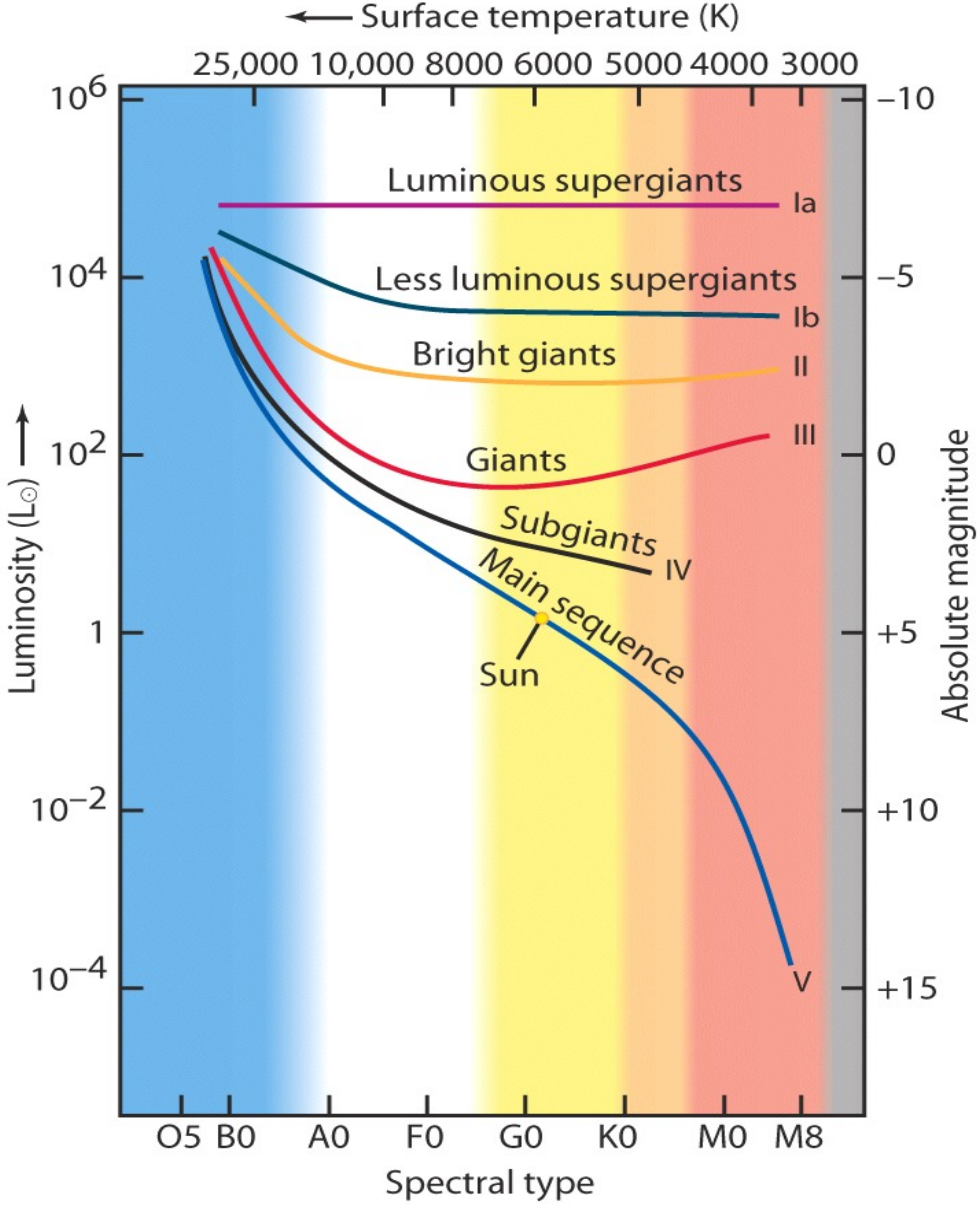
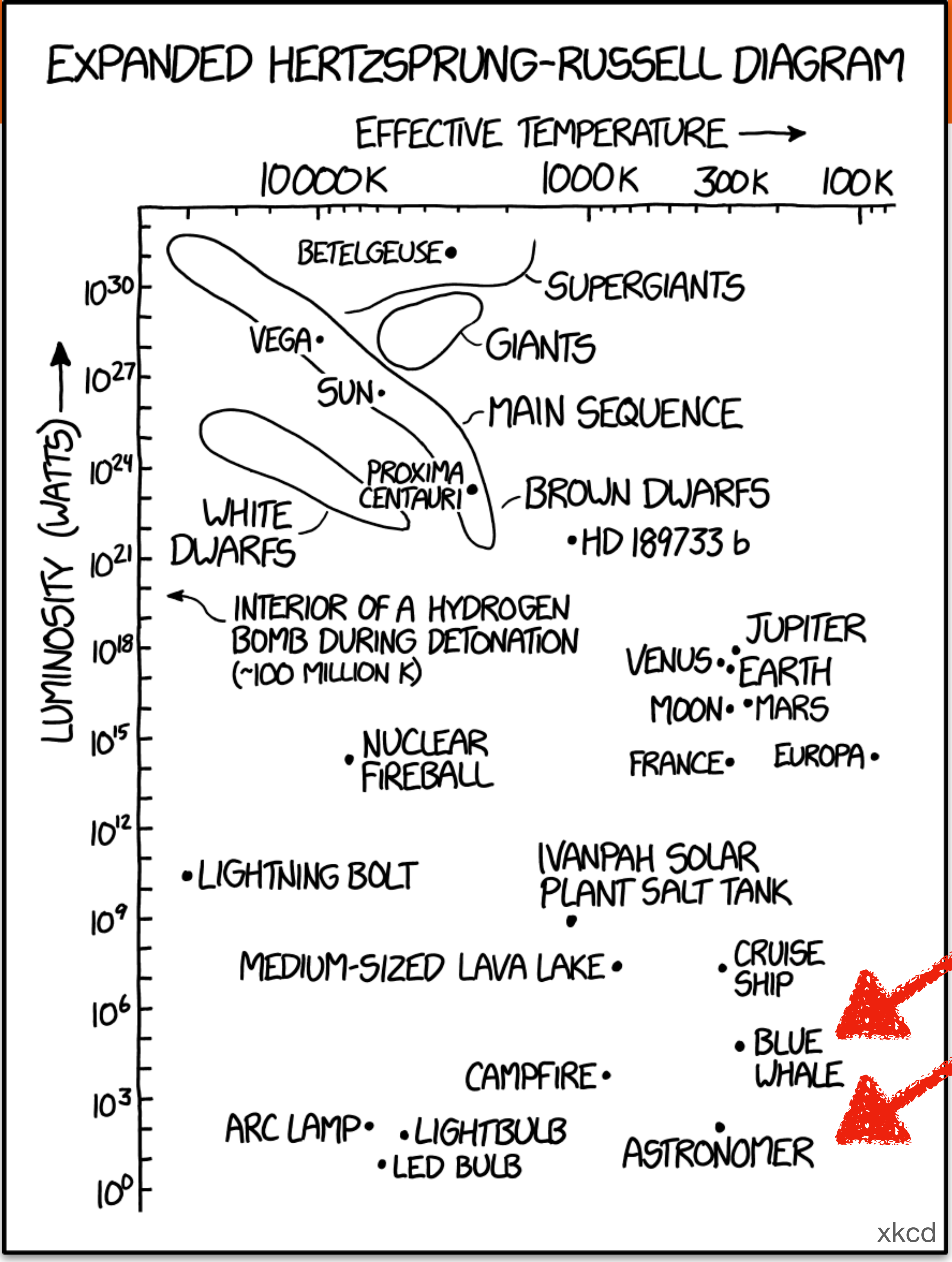
# Stellar classification

## Hertzsprung-Russell diagram



# Stellar classification

## Hertzsprung-Russell diagram



# Physical stellar parameters

- The fundamental (global) parameters that describe a star are
  - **mass M,**
  - **radius R,**
  - **luminosity L.**
- They are commonly expressed in units of the solar values $M_\odot$, $R_\odot$, and $L_\odot$
- The parameters will change with time. **Age** of a star also an important parameter.

- **Stellar atmosphere** (layer from where we receive most of the observable information) is characterised by the following parameters:
  - **effective temperature $T_{eff}$**
  - **gravity acceleration g**
  - **chemical composition** (expressed as metallicity)
  - **magnetic field strength**
    (although the magnetic field is typically difficult to be expressed by just one parameter)

- Often stellar properties can only be derived with **significant uncertainties**,

# Physical stellar parameters

## Mass

- According to our definition of a star, nuclear fusion in its interior is required.

  ➡ Minimum mass of a star $M_{min} \approx 0.08 M_\odot$.

  ➡ Objects with $M_{min} < 0.08 M_\odot$ (but more mass than planets): brown dwarfs ($M_{bd} < 0.08 M_\odot$).

- Highest masses $M > 100 M_\odot$
  - Known examples with up to ~ $250 M_\odot$

- Number of stars with a certain mass decreases strongly with mass!

  ➡ only few very massive stars but very many low-mass stars.

  ➡ very massive stars are therefore typically far away

- Strong stellar winds and outflowing gas result in clouds surrounding these stars can make the determination of the stellar mass less reliable.

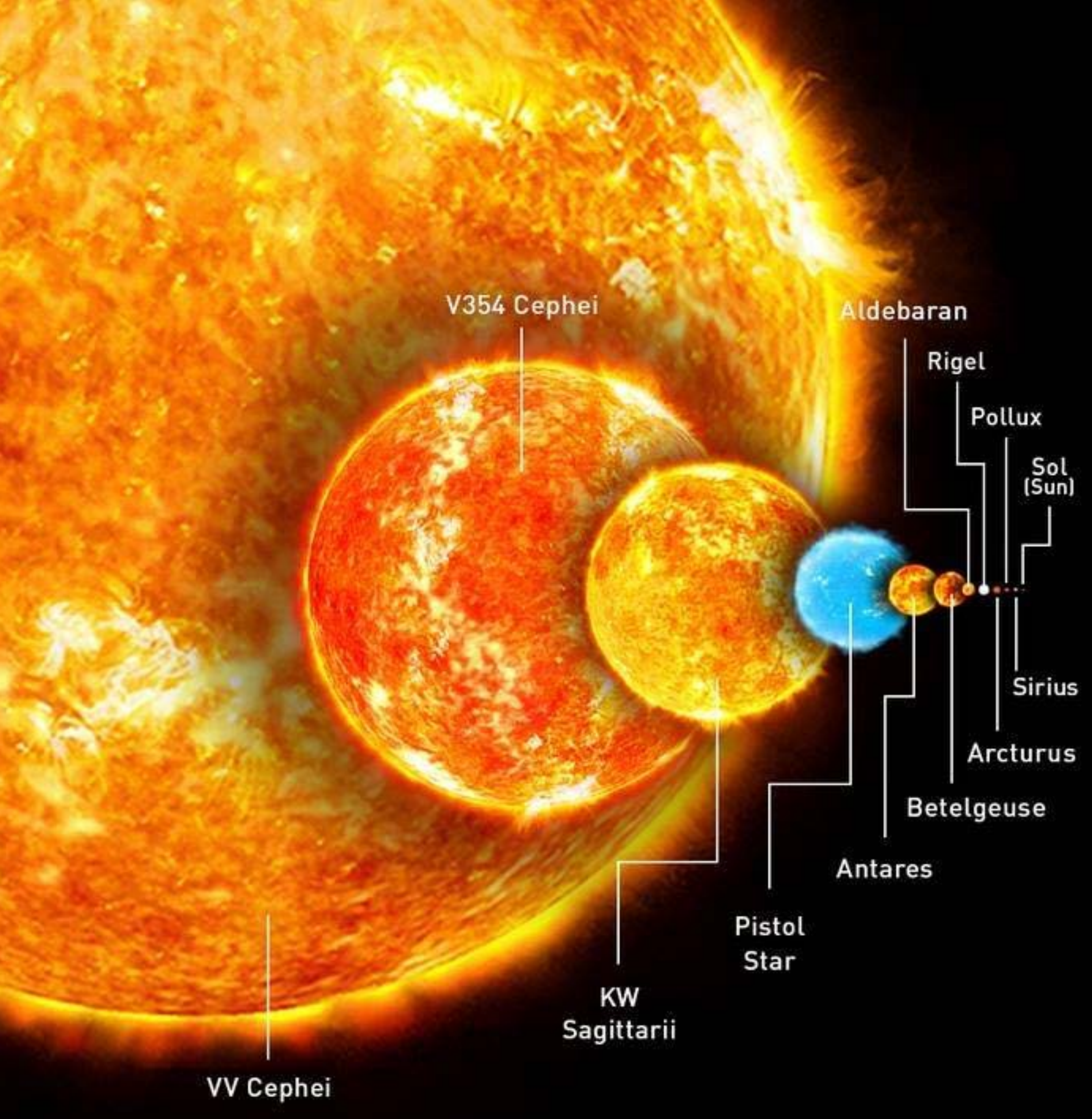
# Physical stellar parameters

## Radius

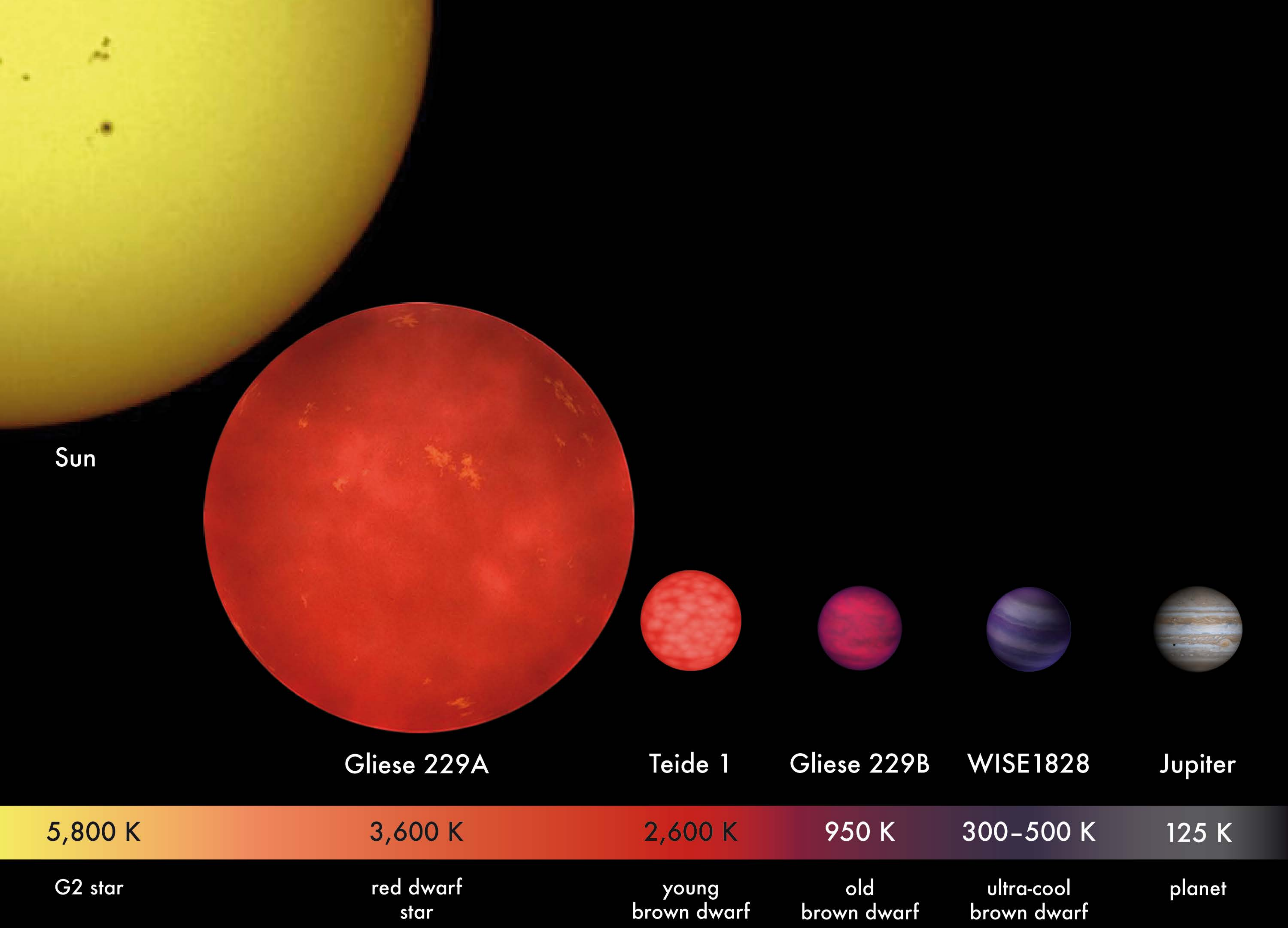
- **Main sequence stars**
  - Typical values: 0.1 $R_\odot$ to ~ 25 $R_\odot$.
  - Radii increase as function of effective temperature along the main sequence
  - Red dwarfs at the cool end being much smaller than the Sun
  - Hot main sequence stars being much larger than the Sun.

- **Red giants, supergiants, …**
  - Diameters larger than the orbit of Mars.
  - Examples: Antares (680 – 800 $R_\odot$), Betelgeuse (900 $R_\odot$), and Mu Cephei (972 – 1,260 $R_\odot$).
  - Largest stars: radii currently estimated to up ~ 2000 $R_\odot$ .

- **White dwarfs**
  - $R < 0.02\ R_\odot$



Careful: Scale might not be accurate (anymore)



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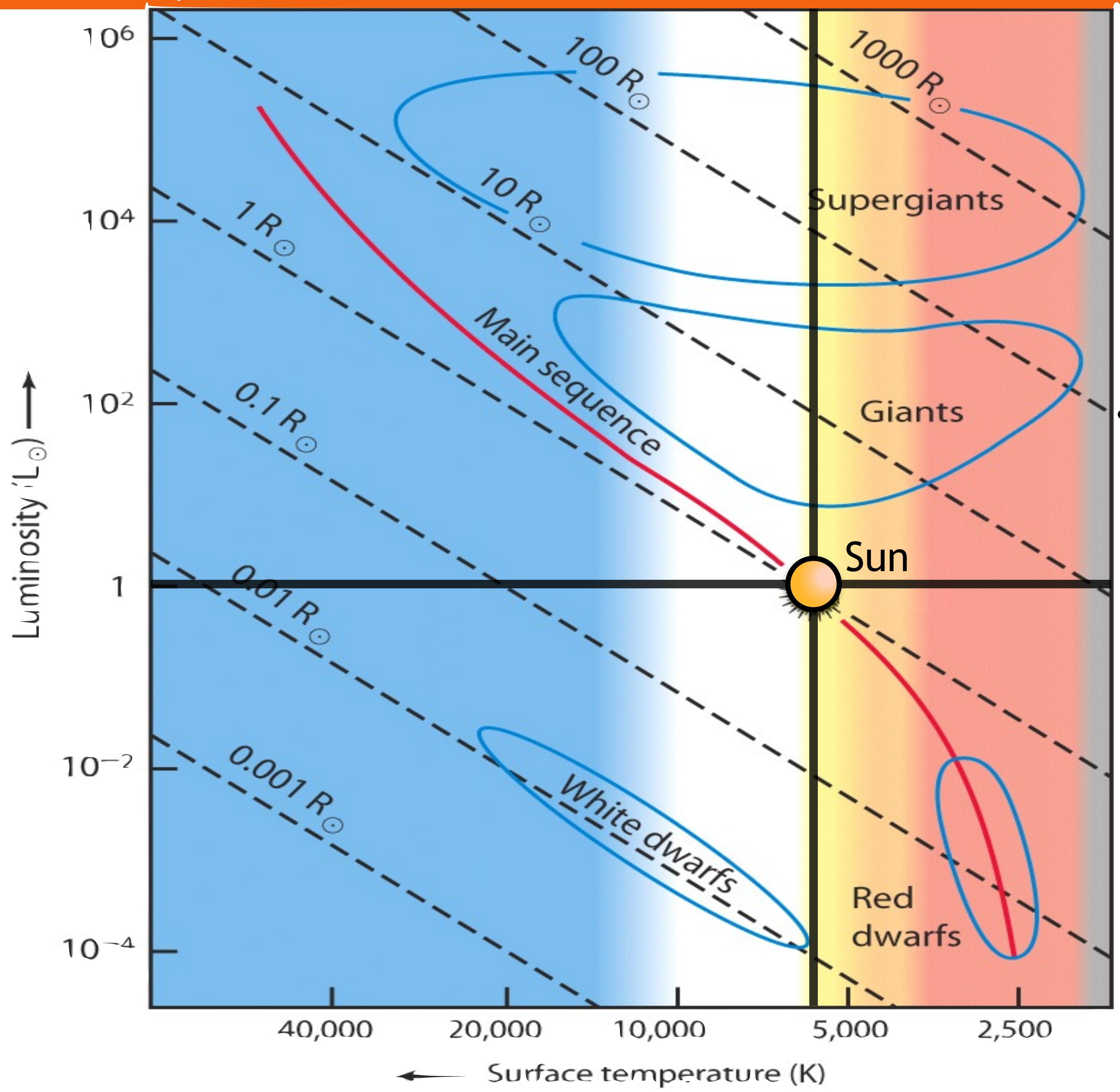
# Physical stellar parameters



- Lines of constant radius

# Physical stellar parameters

## **Luminosity** ➡ $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$

- The bolometric luminosity of stars spans r many orders of magnitude: $10^{-4}\,L_\odot$ — $10^6\,L_\odot$

- Depends to **$4^{th}$ power** on $T_{\mathrm{eff}}$

➡ Small difference in $T_{\mathrm{eff}}$ results in a large change in L!
(Same true for uncertainties)

- **Example 1**: blue-white supergiant Deneb (α Cyg) — one of the brightest stars in the sky:
$L \sim 60\,000$ — $200\,000 L_\odot$.
  ➡ Large uncertainty is due to the poorly known distance!

- **Example 2**: Red supergiant Betelgeuse $L \approx 100\,000 L_\odot$