Lecture 3

AST5770

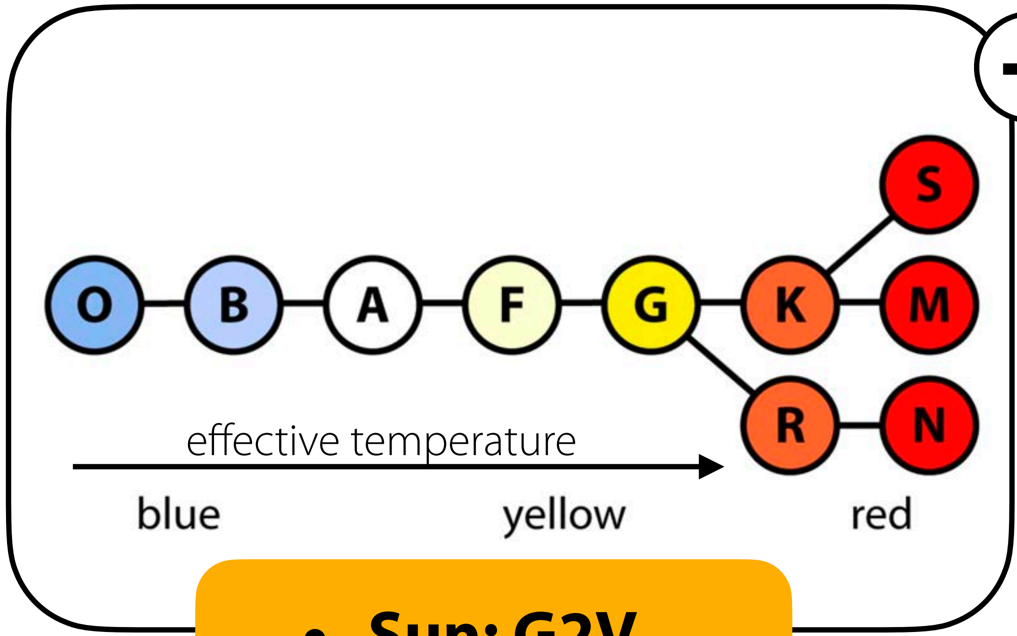
Solar and stellar physics

Sven Wedemeyer, University of Oslo, 2023

Recap

Spectral classification and stellar populations

- Harvard spectral classification /Morgan–Keenan system



+

effective temperature →

0 1 2 3 4 5 6 7 8 9

One decimal possible!

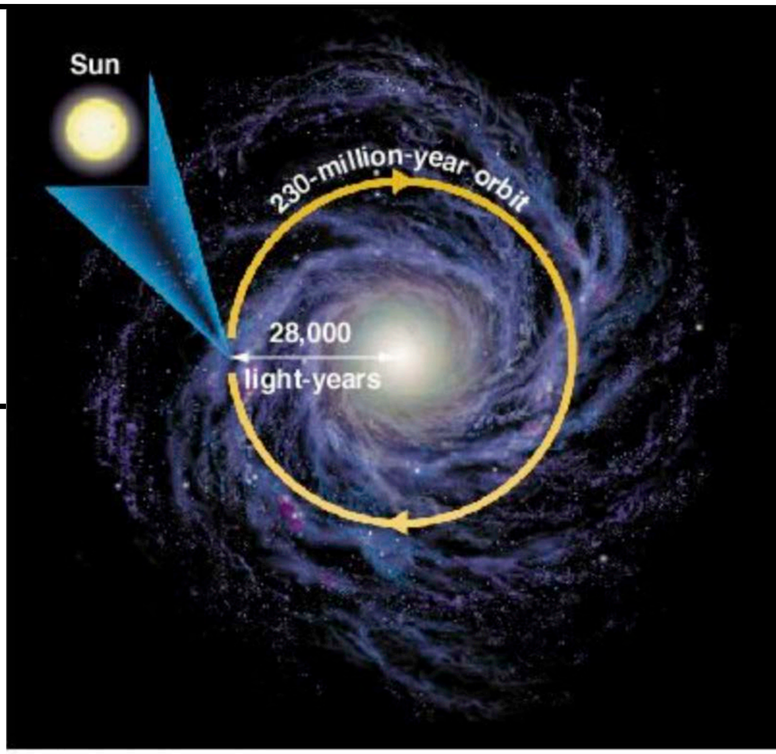
+

Ia - 0	hypergiants
I	supergiants
II	bright giants
III	giants
IV	subgiants
V	main sequence (dwarfs)
VI	subdwarfs

- **Sun: G2V**

- **Population I:** “recent” stars, high metallicity
- **Population II:** old stars, low metallicity
- **Population III:** first stars in the universe (very low metal content)

- Total number of stars in the Milky Way only known roughly: **100 - 400 billion stars**

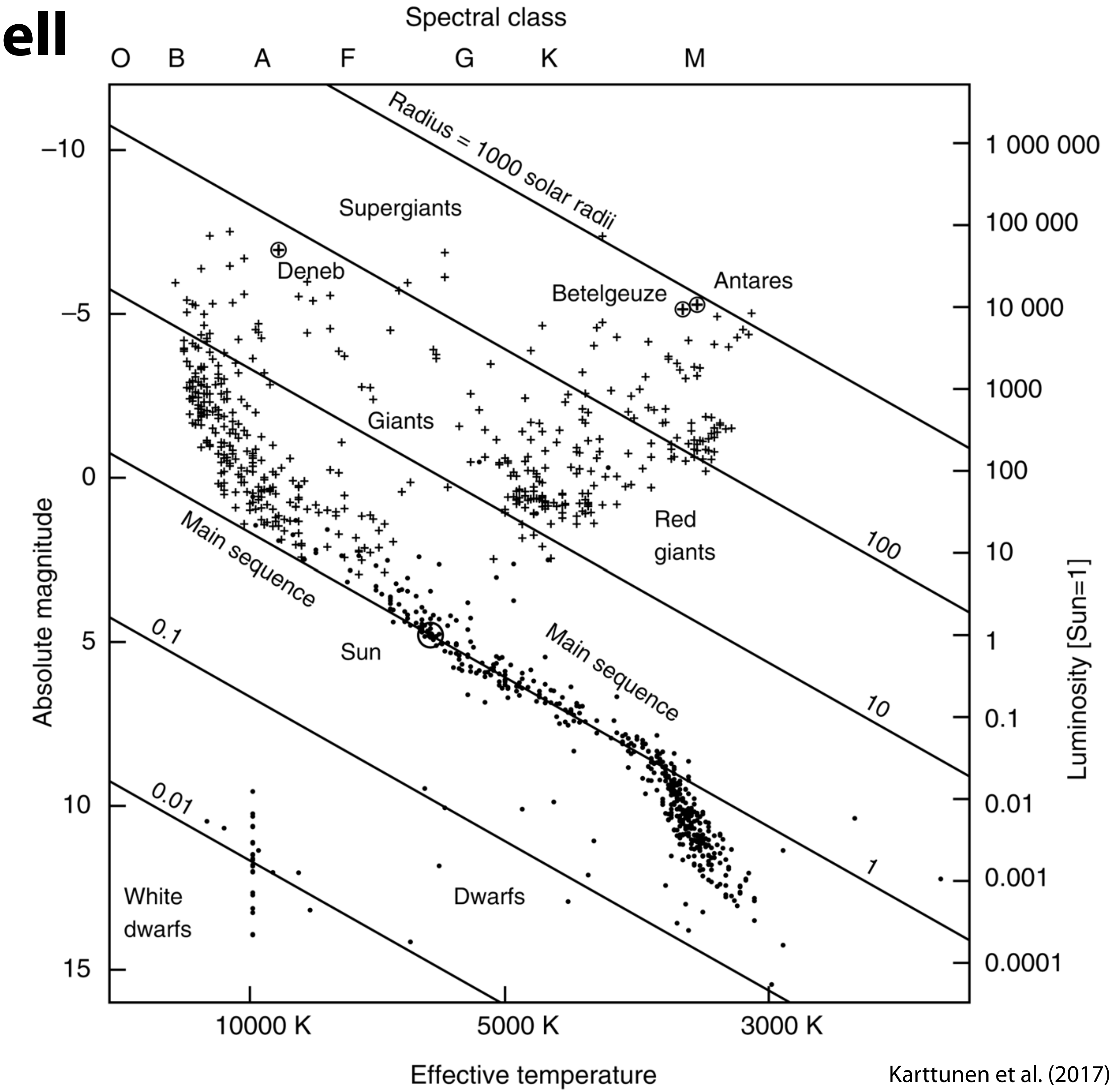


Recap

Hertzsprung-Russell diagram

- Stars clearly not distributed randomly
- Connections between
  - T_{eff}/Spectral class
  - Luminosity/absolute brightness
  - Radius

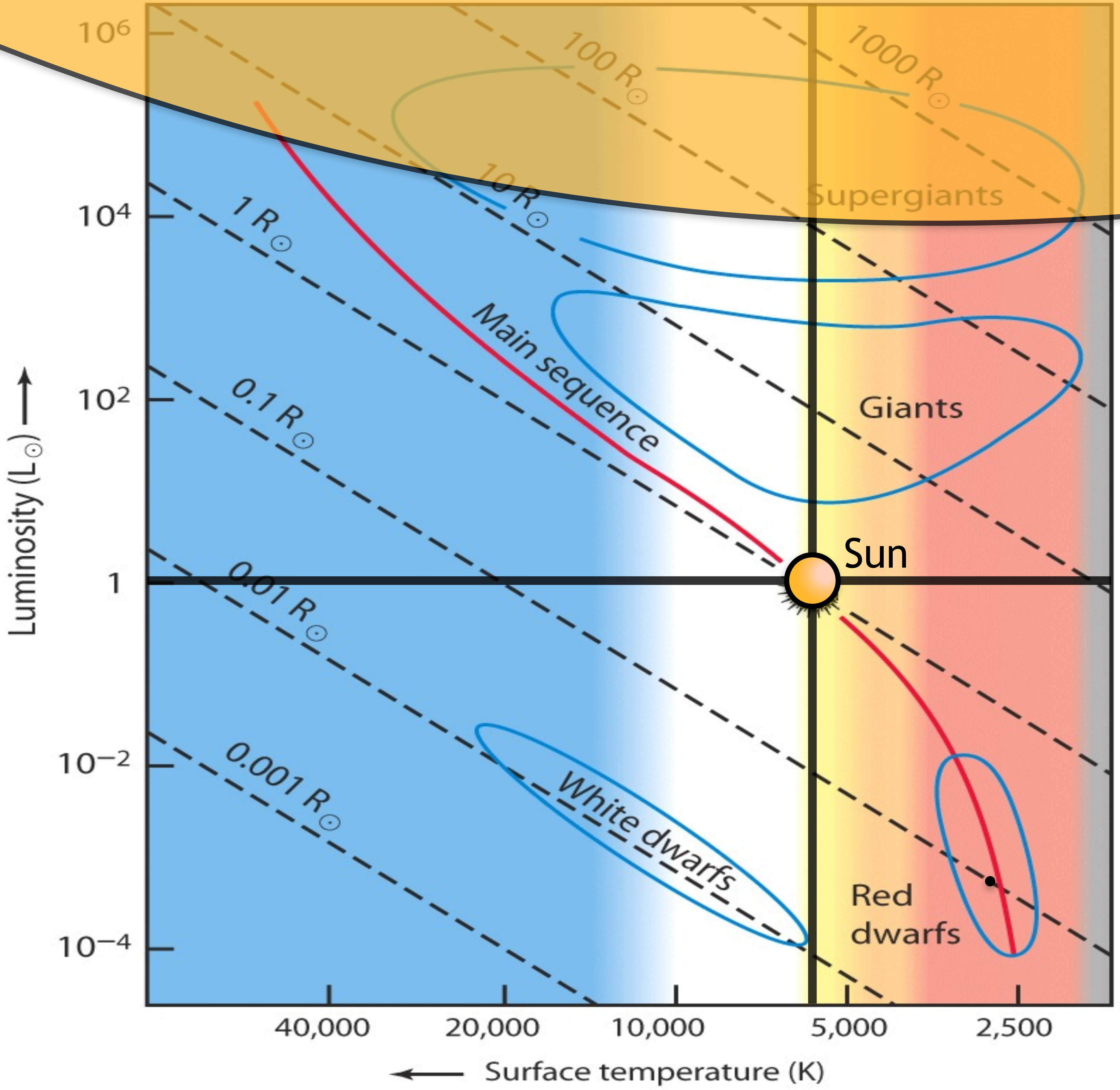
➡ Constraints for models of structure and evolution of stars



Karttunen et al. (2017).

Physical stellar parameters

$L \propto R^2\ T_{\mathrm{eff}}^{\ 4}$



Super giant

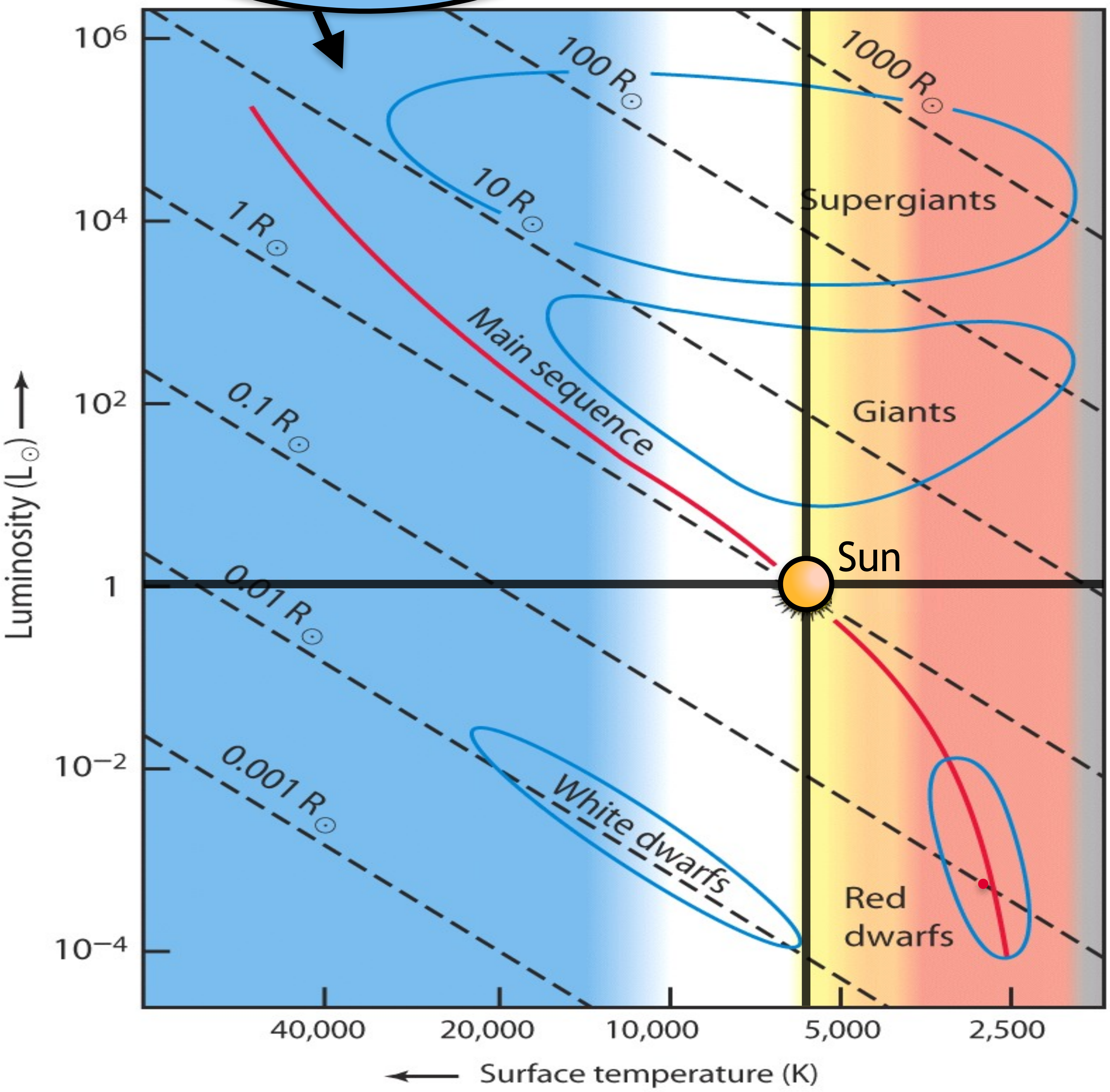
$R = 100\ R_\odot$

$T_{\mathrm{eff}} = T_{\mathrm{eff},\odot} = 5770\ \mathrm{K}$

➡ $L = 100^2\ L_\odot = 10\,000\ L_\odot$

Ph llar parameters

$L \propto R^2\ T_{eff}^{\ 4}$



Spectral type O6

$R = 18\ R_\odot$

$T_{eff} = 38\,000$ K

$= 6.3\ T_{eff,\odot}$

➡ $L = 520\,000\ L_\odot$

Red dwarf star

$R = 0.1\ R_\odot$

$T_{eff} = 0.5\ T_{eff,\odot} = 2885$ K

➡ $L = 0.1^2\ 0.5^4\ L_\odot$

$= 0.0006\ L_\odot$

$= 0.06\ \%\ L_\odot$

Physical stellar parameters

Effective temperature

- Notes:
  - Spectrum of a star can – in first approximation – be described with a black body spectrum for an effective temperature T_{eff}
  - real stellar spectra deviate from blackbody curve
  - effective temperature of a star is defined over the integral
  - Sun: $T_{\mathrm{eff},\odot} \approx 5770$ K.
  - Typical values for other **main sequence stars** :
    - from ~ 2 200 K for the coolest red dwarf stars
    - to up to ~ 45 000 K for the hottest O-type stars.
  - **White dwarfs** can exhibit much higher temperatures of up to ~ 2×10^5 K (basically "exposed" stellar core remnants)

Physical stellar parameters

Gravity acceleration

$$g = \frac{GM}{R^2}$$

mass M,
radius R ,
gravity constant G

- Commonly specified in logarithmic form as **log g** in cgs units.

- Sun: $g_\odot = 274\ \mathrm{m s^{-2}} = 2.74 \times 10^4\ \mathrm{cm\ s^{-2}}$ — $(\log g)_\odot = 4.44$

- Large range of stellar radii + dependence $g \propto R^{-2}$,

➡ g varies over 8 orders of magnitude from white dwarfs to supergiants

Giants and supergiants:	$g \leq 1 \times 10^{-2}\,\mathrm{m\,s^{-2}}$	$\log g \sim 0$
Main sequence:	$g \sim 2 \times 10^{2}\,\mathrm{m\,s^{-2}}$	$\log g \sim 4.3 - 4.5$
White dwarfs:	$g \sim 1 \times 10^{6}\,\mathrm{m\,s^{-2}}$	$\log g \sim 8$

Physical stellar parameters

Chemical composition

- Most stars (in particular main sequence stars) consist primarily of **hydrogen and helium**.
- The elements heavier than helium are commonly called **metals** in astrophysics, sometimes abbreviated as M.
- **Astronomical abundance scale**: logarithmic scale log ε relative to the Sun
  - Hydrogen as origin of scale with log ε (H) = 12 and all other element relative to hydrogen:

$$A(\mathrm{El}) = \log \varepsilon = log(n_{\mathrm{El}}/n_{\mathrm{H}}) + 12$$

n: number density of element

- The **metallicity** is the relative content of the metals, **M/H**, with respect to the Sun:

$$\varepsilon = \frac{(\mathrm{M/H})_{\mathrm{star}}}{(\mathrm{M/H})_{\odot}} \qquad \text{or} \qquad \log \varepsilon = [\mathrm{M/H}]$$

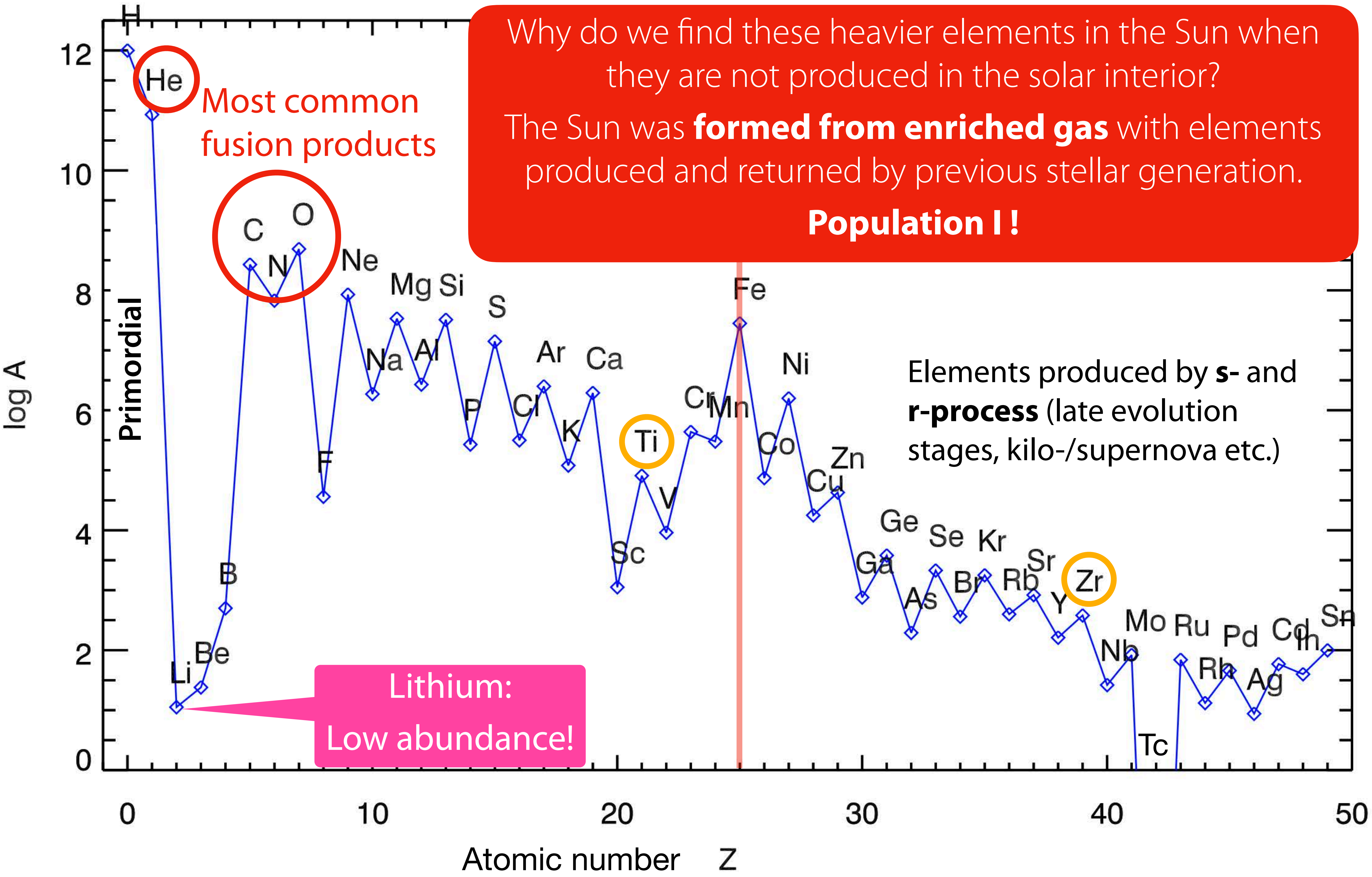
- Iron abundance relatively easy to measure due to large number of spectral lines (and as an important nucleosynthesis product) — often used as a representative metal

$$[\mathrm{Fe/H}] = \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{star}} - \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{sun}}$$

- Also: Cosmo-chemical scale with silicon (Si) as reference but less common for stars

Physical stellar parameters

Chemical composition — Solar abundances



Most common fusion products

Why do we find these heavier elements in the Sun when they are not produced in the solar interior?

The Sun was **formed from enriched gas** with elements produced and returned by previous stellar generation.

Population I !

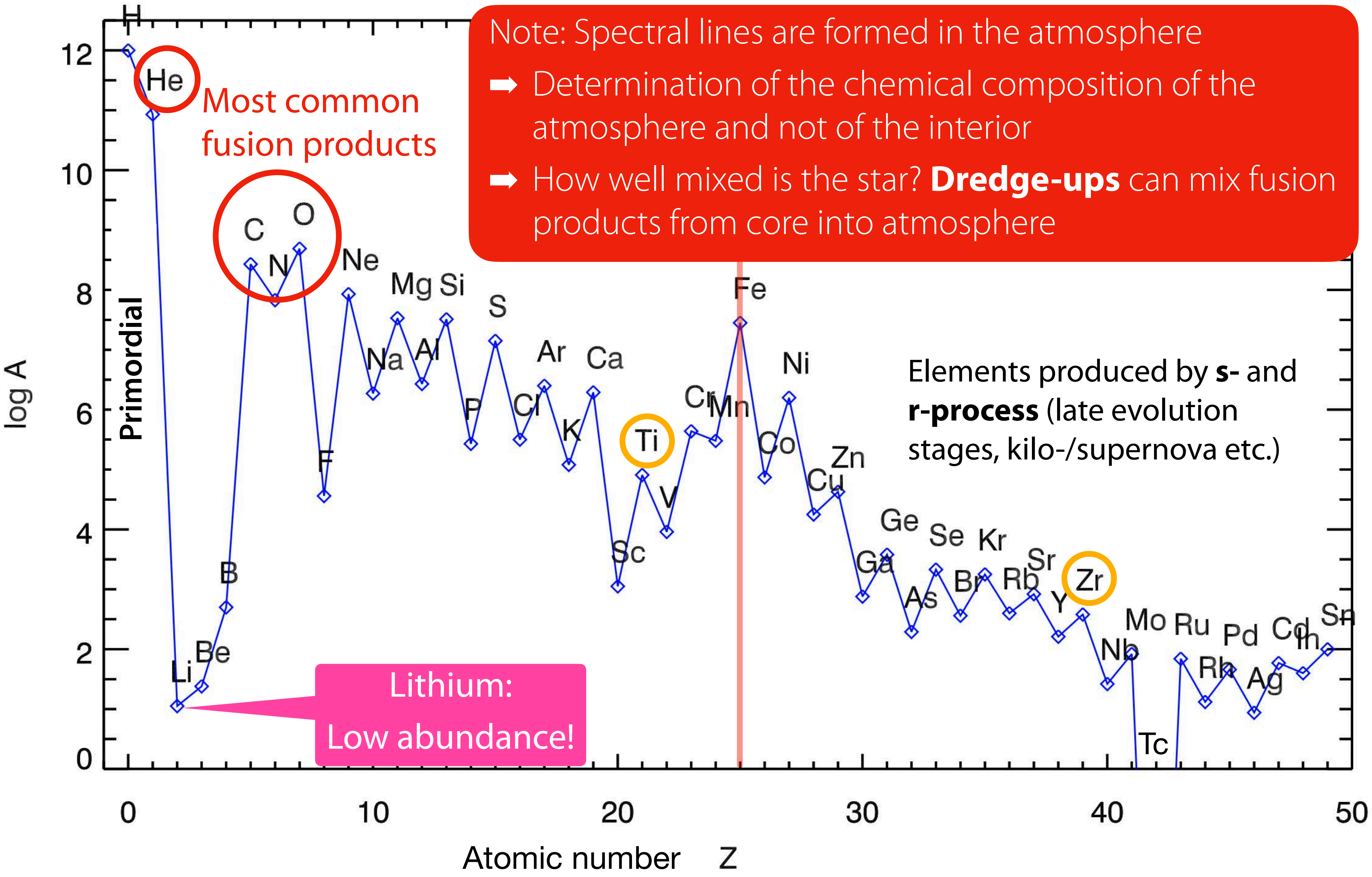
Elements produced by **s-** and **r-process** (late evolution stages, kilo-/supernova etc.)

Lithium:

Low abundance!

Physical stellar parameters

Chemical composition — Solar abundances



Most common fusion products

Note: Spectral lines are formed in the atmosphere

➡ Determination of the chemical composition of the atmosphere and not of the interior

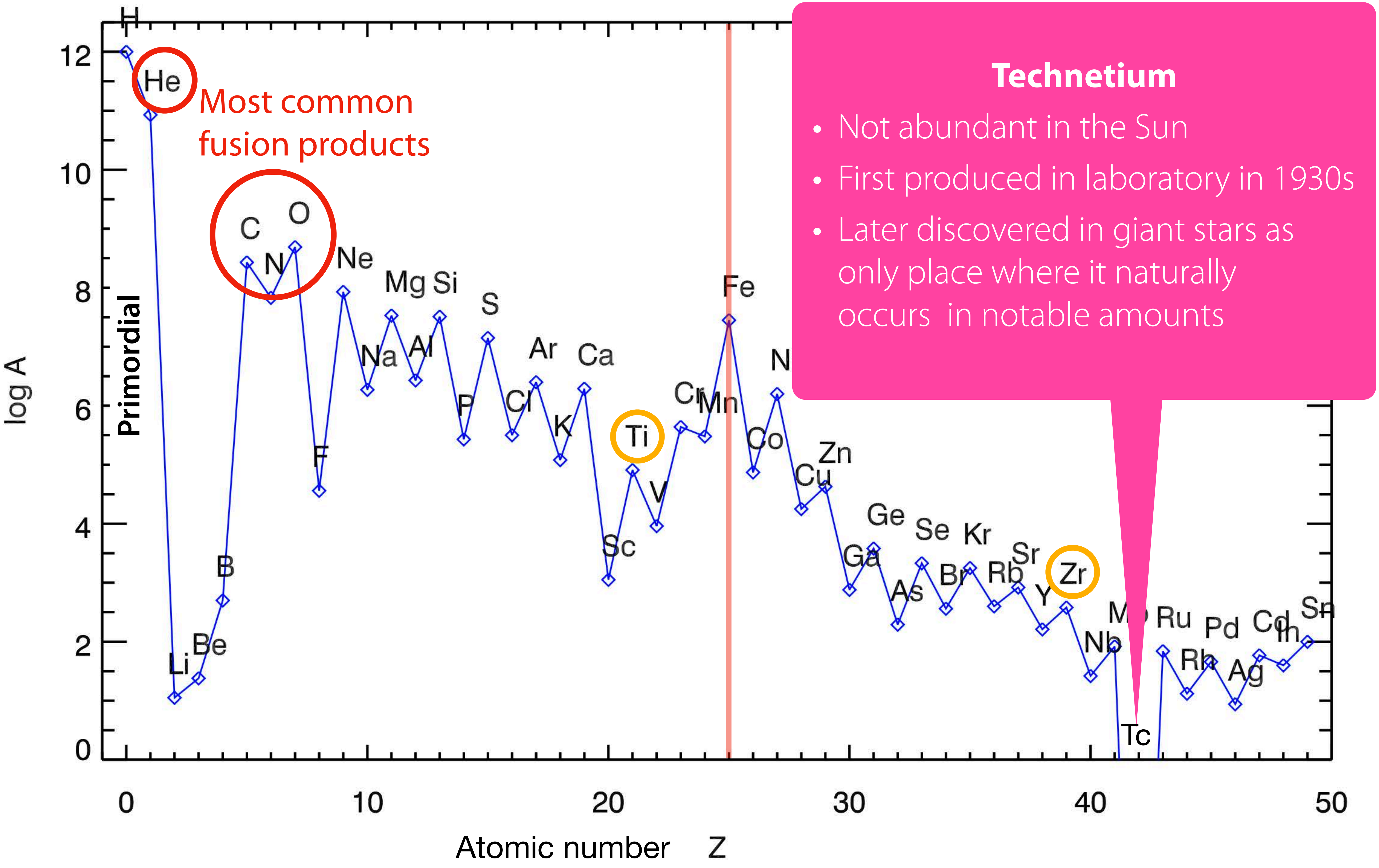
➡ How well mixed is the star? **Dredge-ups** can mix fusion products from core into atmosphere

Elements produced by **s-** and **r-process** (late evolution stages, kilo-/supernova etc.)

Lithium:
Low abundance!

Physical stellar parameters

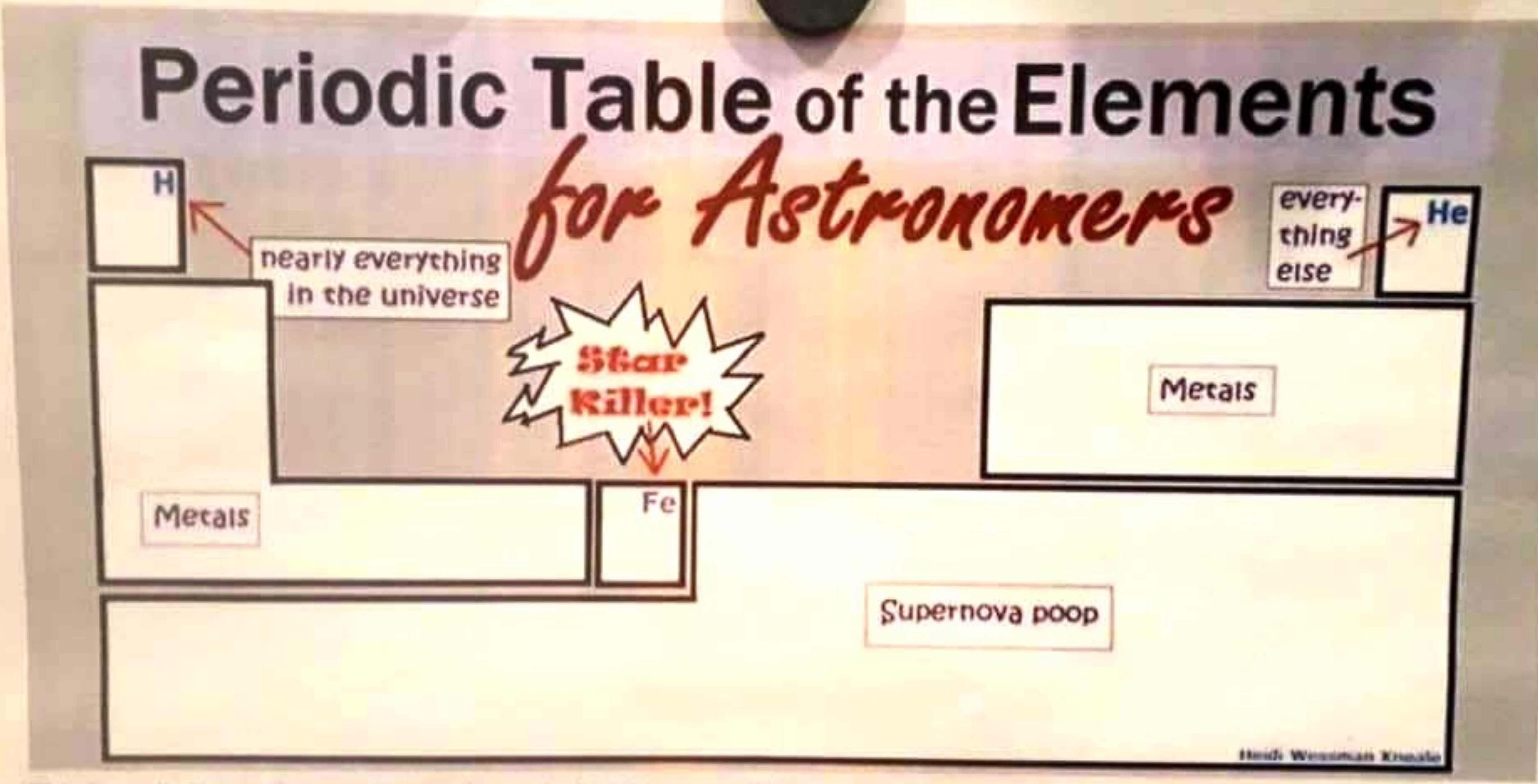
Chemical composition — Solar abundances



Most common fusion products

Technetium

- Not abundant in the Sun
- First produced in laboratory in 1930s
- Later discovered in giant stars as only place where it naturally occurs in notable amounts



This is really how astronomers see the periodic table according to astronomer and author **Heidi Weissman Kneale** (Supplied: Heidi Weissman Kneale (@heidikneale))

Physical stellar parameters

Addition: Broadband photometry with GAIA

Instrument

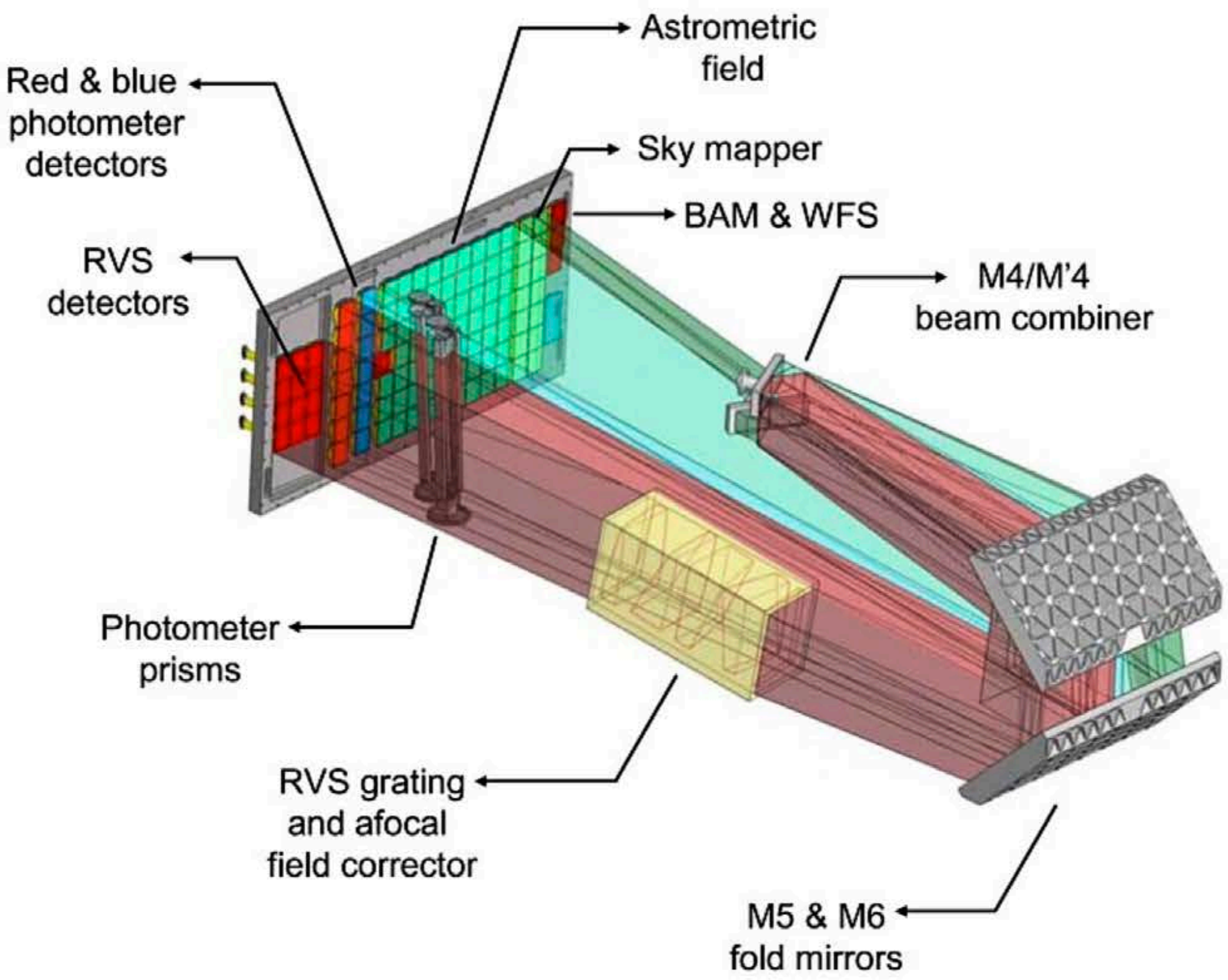


Fig. 2. On their way to the BP/RP and RVS sections of the focal plane, light from the two *Gaia* telescopes is dispersed in wavelength. Picture courtesy of EADS-Astrium.

BP - **RP** = **Color index**

blue **Red**

Passbands

G passband

Transmitted wavelength ranges

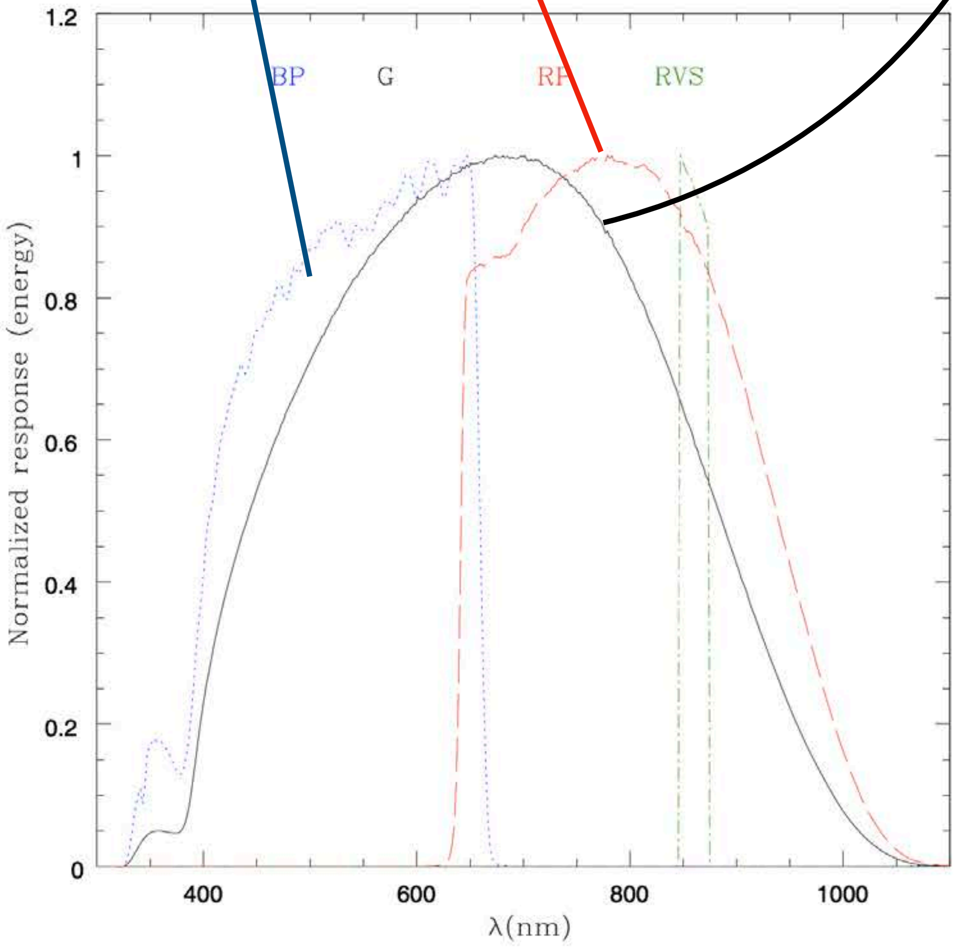


Fig. 3. *Gaia G* (solid line), G_{BP} (dotted line), G_{RP} (dashed line) and G_{RVS} (dot-dashed line) normalised passbands.

Jordi et al. (2010)

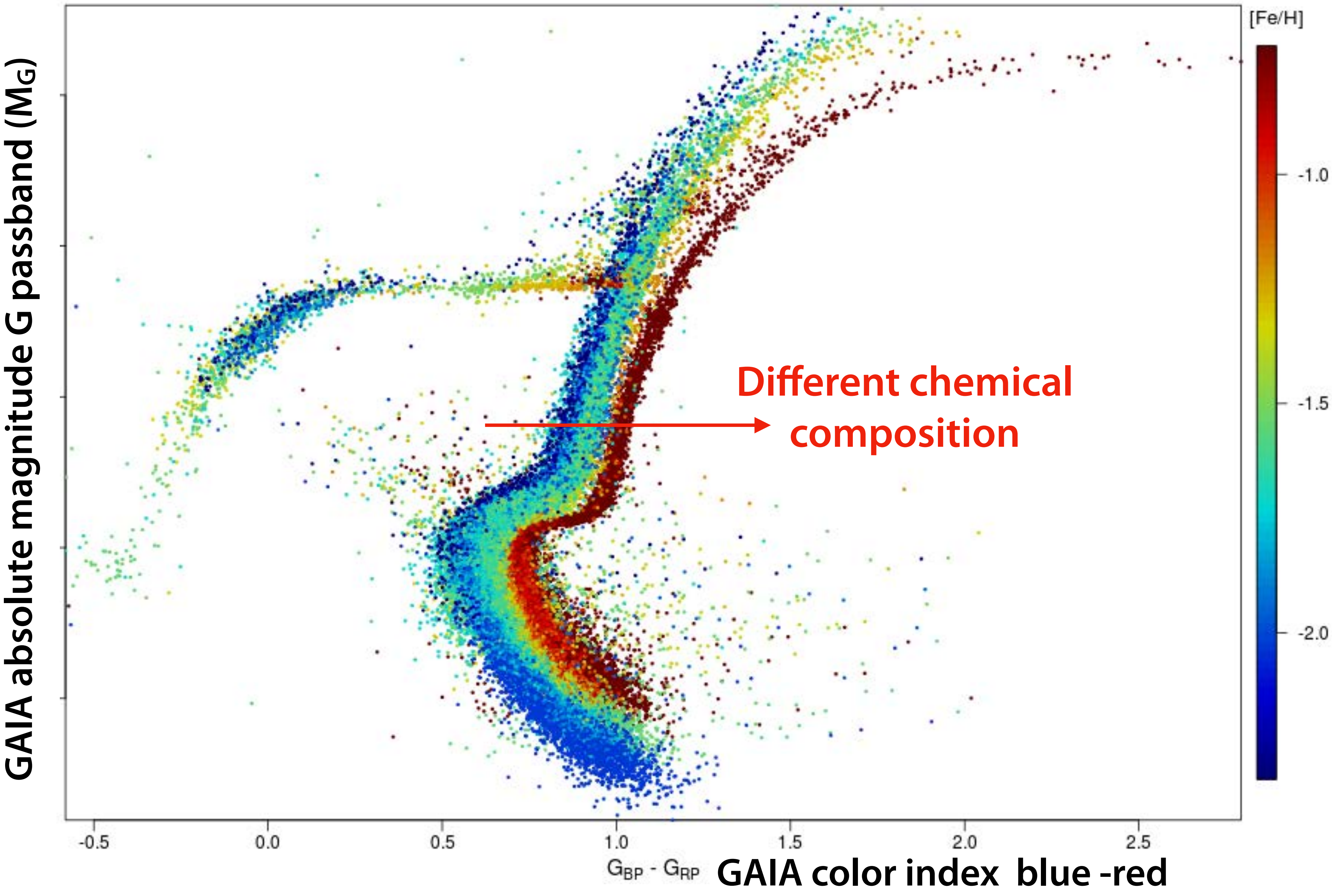
Physical stellar parameters

Chemical composition

- HRD for 14 globular clusters: Groups of stars with different metallicity

➡ Distinct subsets in the HRD

Metallicity
(log!)



Different chemical composition

Credits: ESA/Gaia/DPAC, Carine Babusiaux et al. A&A 616, A10 (2018) "Gaia Data Release 2: Observational Hertzsprung-Russell diagrams"

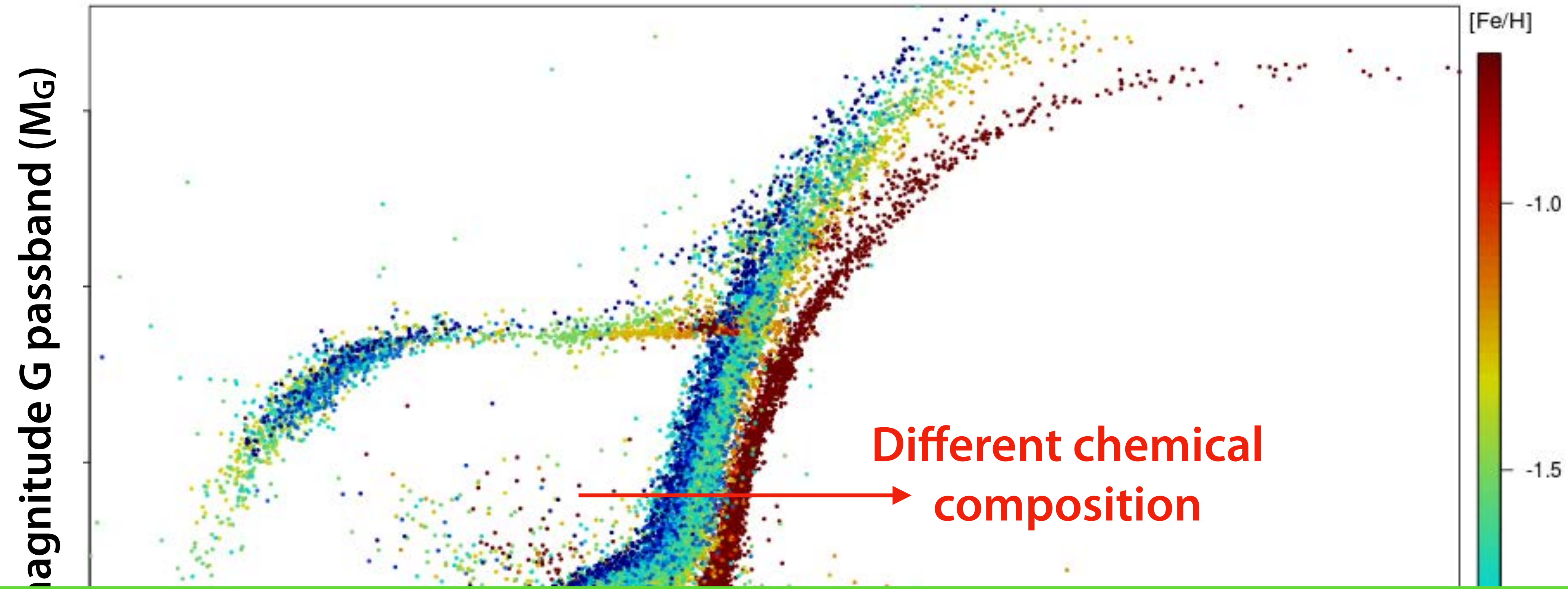
Physical stellar parameters

Chemical composition

- HRD for 14 globular clusters: Groups of stars with different metallicity

➡ Distinct subsets in the HRD

Metallicity
(log!)



Different chemical composition

- In general (all stars) range in metallicity: **-4.5 < [Fe/H] < +1.0**
- Note the logarithm
  - **[Fe/H] = 0.0: Solar metallicity**, relative abundance of metals (Fe) like in the Sun
  - [Fe/H] = +1.0: metals (Fe) 10 times more abundant in this star than in the Sun
  - [Fe/H] = -2.0: metals (Fe) 100 times less abundant in this star than in the Sun

Physical stellar parameters

Chemical composition

- Chemical composition of **stellar interior not directly observable**
- Specified only as mass fractions of hydrogen (X), helium (Y), and all heavier elements (Z):

$$X+Y+Z=1$$

- Mass fractions vary with locations, in particular in radial direction from the core to the atmosphere (element segregation, fusion, etc.)
- Sun (based on models): $X = 0.73$, $Y = 0.25$, $Z = 0.02$.

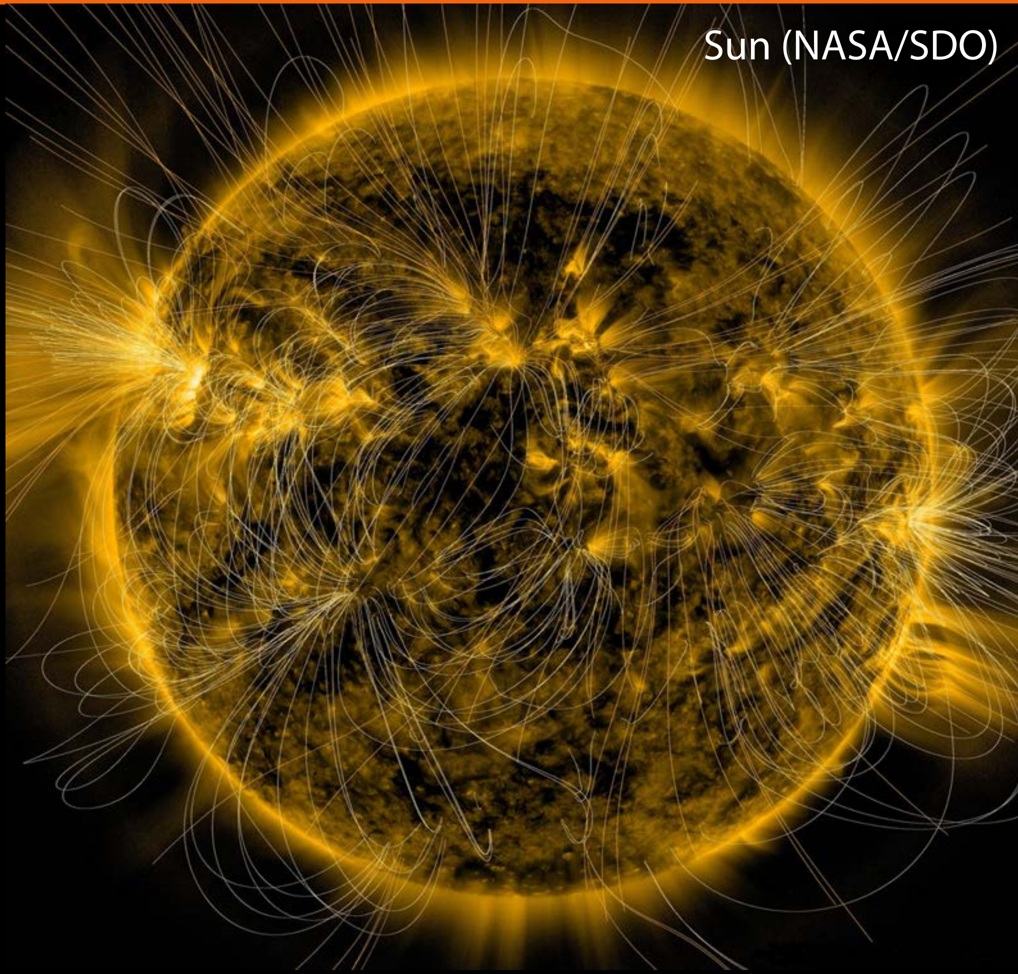
- Numbers above refer to **number densities**, i.e. the number of particles per volume.
- Different atomic weight of elements/nuclei result in different values for the mass fraction!
- In terms of mass, the Sun consists of ~ 90 % H, ~ 10 % He, and only 0.1 % for the remaining elements.
- Relative content (i.e. the **abundance**) of heavier elements in the Sun is roughly the same as for Earth (formed from the same material in the protostellar cloud)

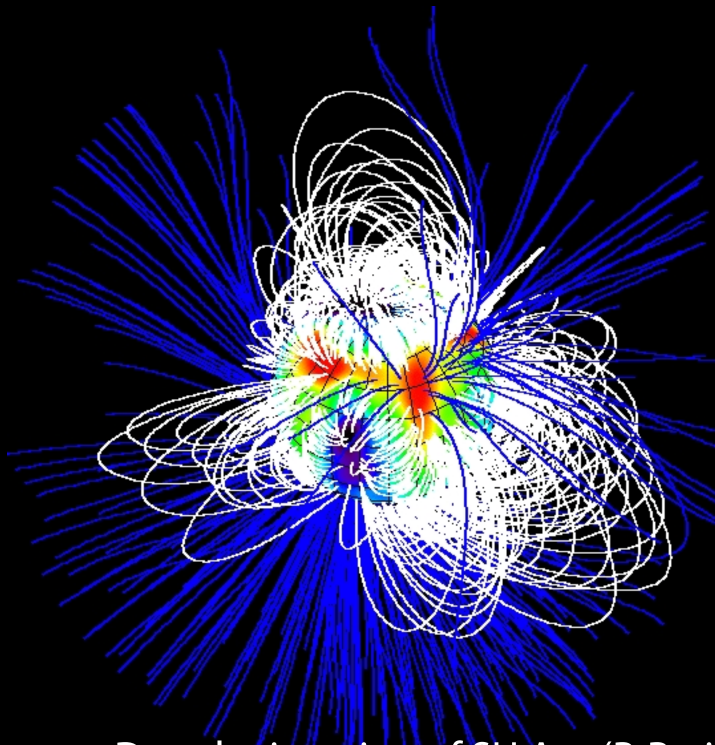
Physical stellar parameters

Magnetic field (B)

- Measurement via Zeeman effect in spectral lines
- **Sun**
  - Magnetic field very **inhomogeneous**/structured
  - In atmosphere, strongest: **Sunspots** B =2000–3000 G
  - Photosphere on average B ≈ 100–300 G
  - Min. in Quiet Sun and coronal holes: B < 1 G
  - Magn. field strength lower in upper atmosphere
- **Stars**
  - Only observed as point source but time series allow reconstruct of magnetic field as the star rotates

  ➡ Presence of starspots can be inferred
  - Average field strengths of several 1000 G are detected (up to 6000 G and higher in M-type dwarf stars, compare to White Dwarfs: 10^4 — 10^9 G)
- **Solar/stellar cycles:** Magnetic field changes


Sun (NASA/SDO)


Zeeman Doppler imaging of SU Aur (P. Petit)

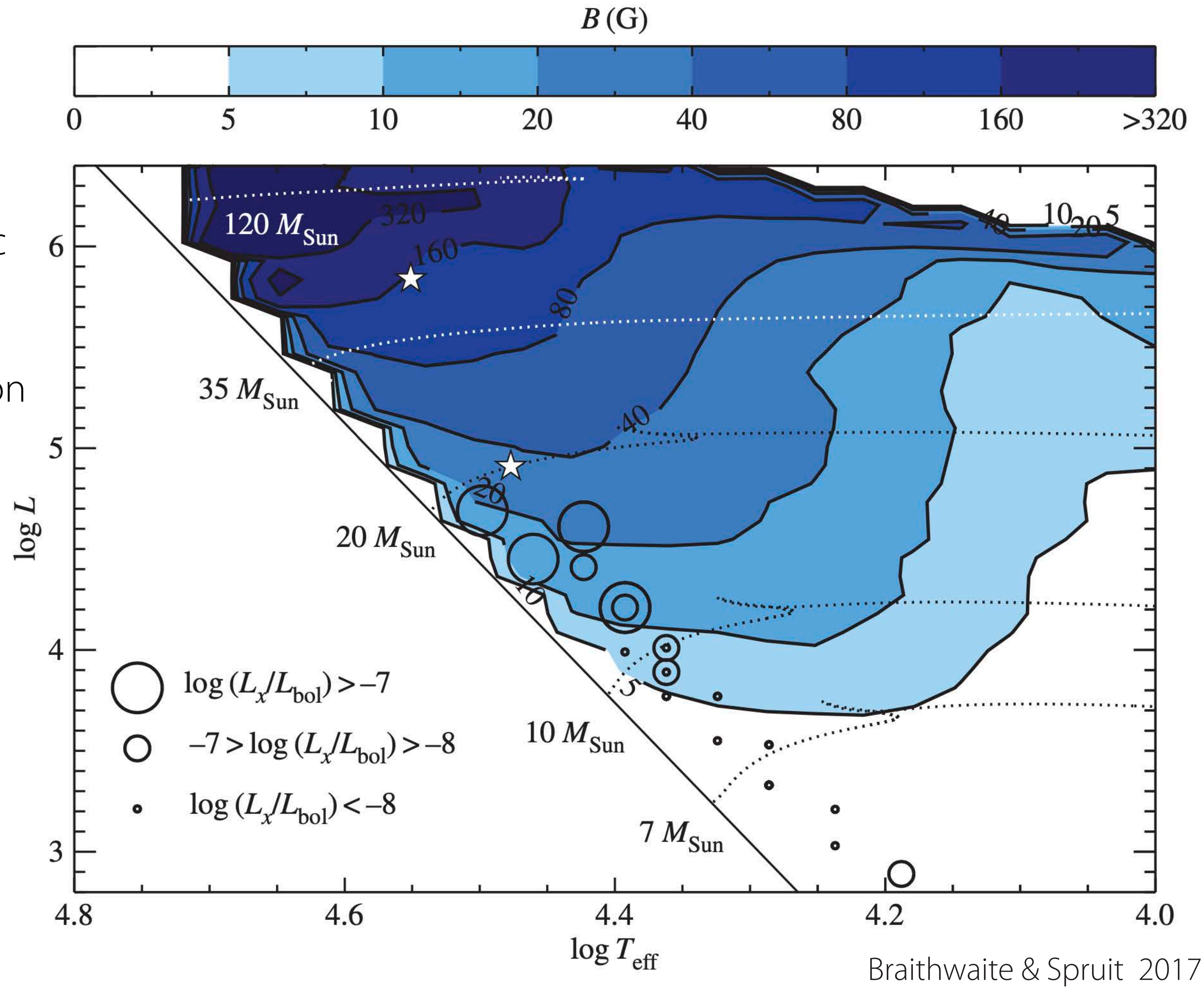
Physical stellar parameters

Magnetic field (B)

- Magnetic field generation via dynamo

➡ We expect magnetic field strengths to depend on stellar structure and rotation

Massive stars



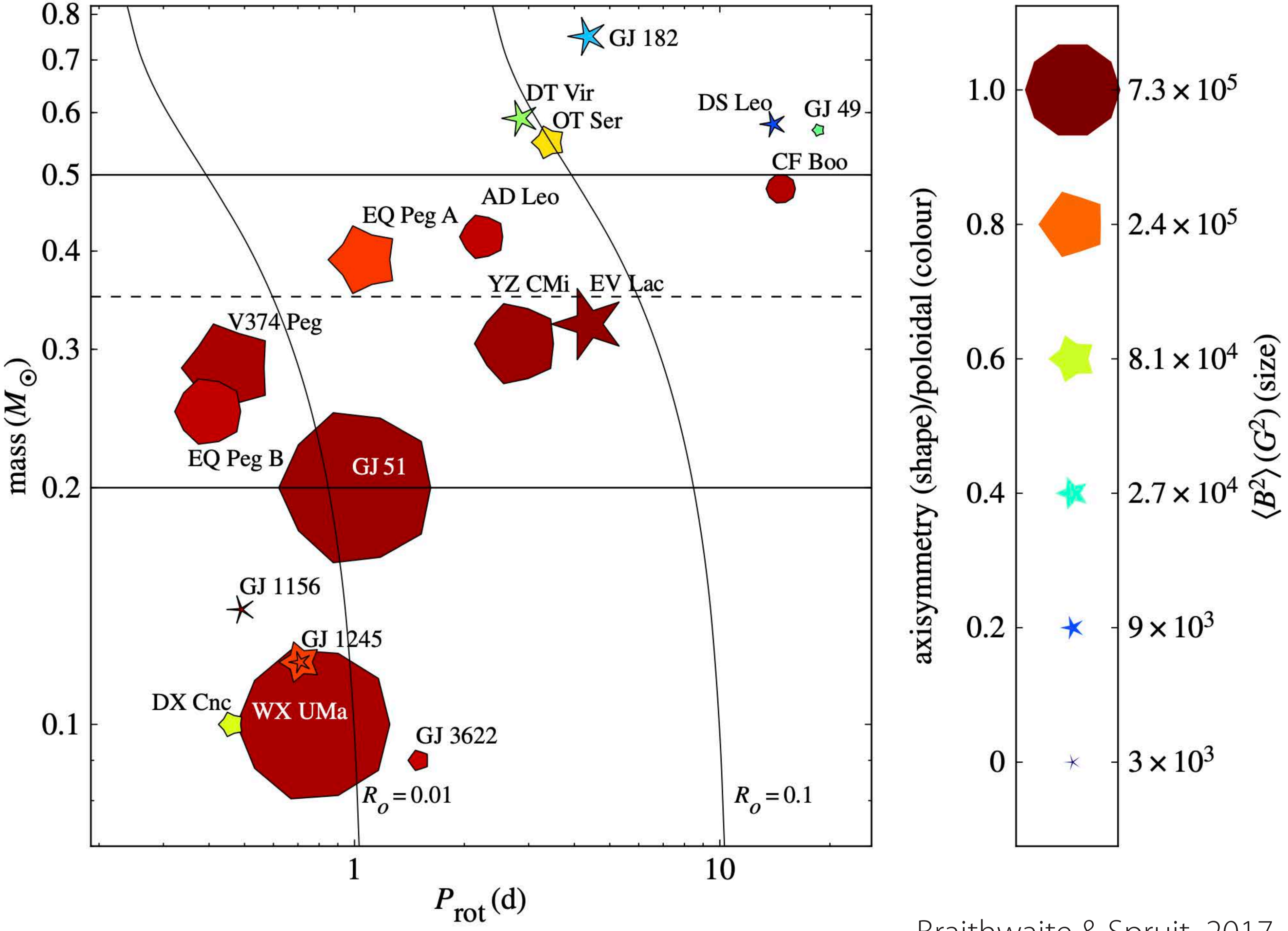
Braithwaite & Spruit 2017

Physical stellar parameters

Magnetic field (B)

- Rotation rate of a star important for magnetic field generation via dynamo

Red M-type dwarf stars



Braithwaite & Spruit 2017

Relations between fundamental parameters

Mass-luminosity relation

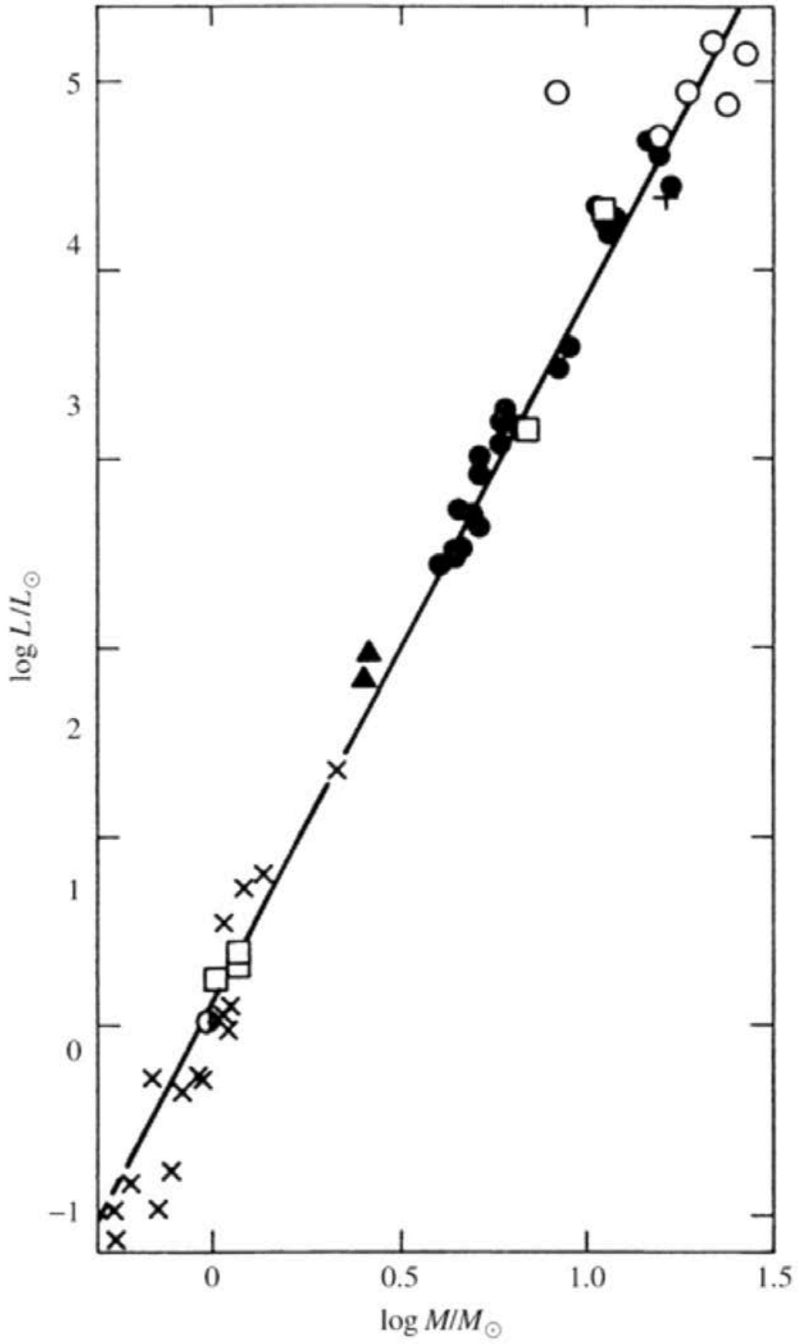
- Observations of main-sequence stars imply a relation of L and mass M of a star is related as n

$$L / L_\odot = a \, (M/M_\odot)^{n}$$

- Exponent n (and factor a)
  - same for subsets of stars with similar internal structure / nuclear power source; varies for different stellar types accordingly.

$M/M_\odot$	< 0.43	0.43 - 2	2 — 55	> 55
a	0,23	1	1,4	$3.2\ 10^4$
n	2,3	4	3.5	1

- The mass-luminosity relation can be derived by using aforementioned equations for g and L:

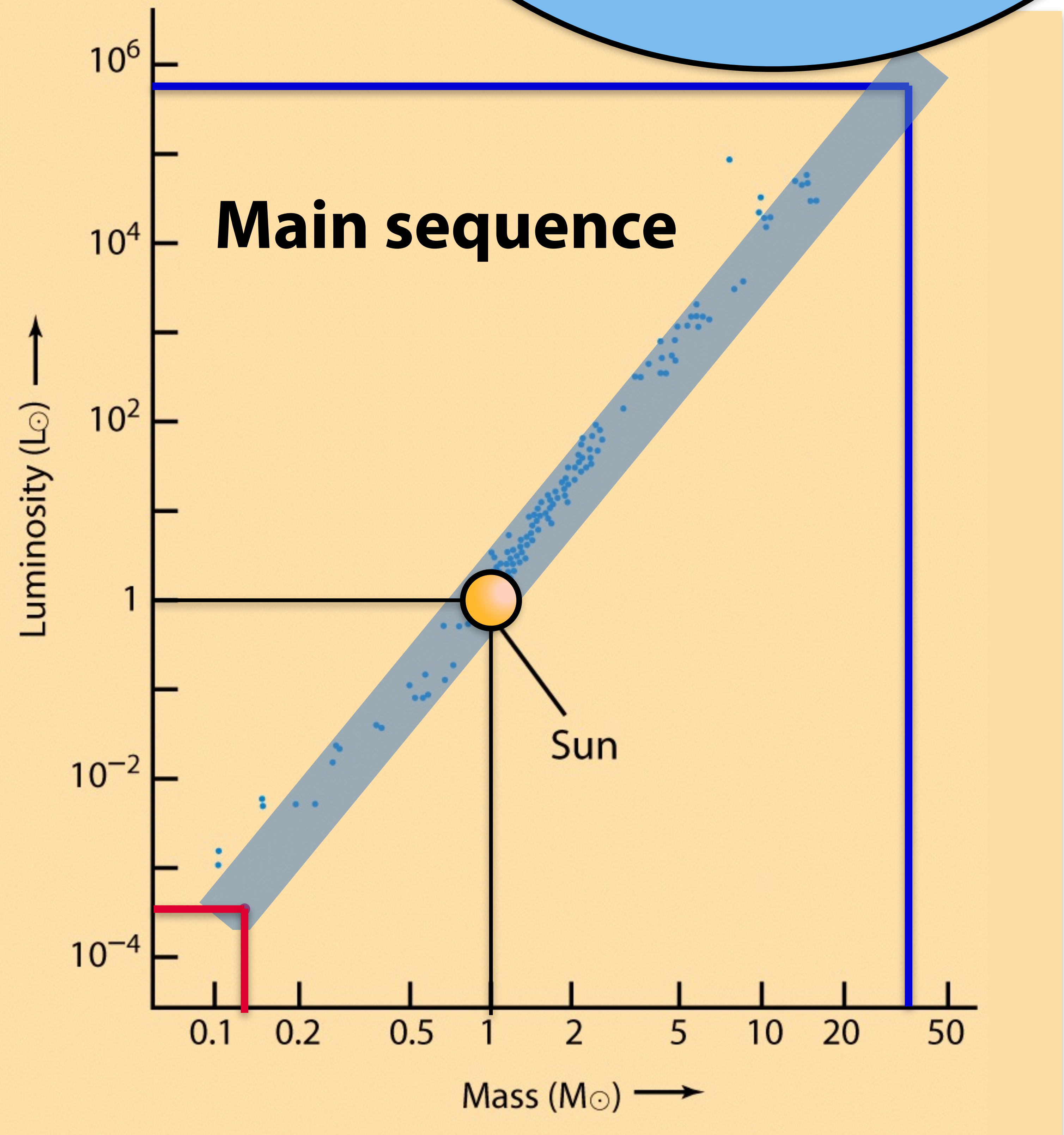


$$g = \frac{GM}{R^2} \quad \Rightarrow \quad M = \frac{gR^2}{G} \qquad \text{and} \qquad L = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad \Rightarrow \quad \frac{M}{L} = \frac{1}{4\pi G\sigma}\,\frac{g}{T_{\text{eff}}^4}$$

➡ The ratio M/L can thus be derived from the parameters Teff and g, which can be determined from stellar spectra.

Relati ntal parameters

Mass-lumi



$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^n$$

Spectral class O6

$R = 18\ R_\odot$

$T_{\mathrm{eff}} = 38\,000\ \mathrm{K}$

$= 6.3\ T_{\mathrm{eff},\odot}$

➡ $L = 520\,000\ L_\odot$

➡ $M = 40\ M_\odot$

Red dwarf star

$R = 0.1\ R_\odot$

$T_{\mathrm{eff}} = 0.5\ T_{\mathrm{eff},\odot} = 2885\ \mathrm{K}$

➡ $L = 6\ 10^{-4}\ L_\odot$

➡ $L = 0.06\ \%\ L_\odot$

➡ $M = 0.12\ M_\odot$

Relations between fundamental parameters

Mass-radius relation

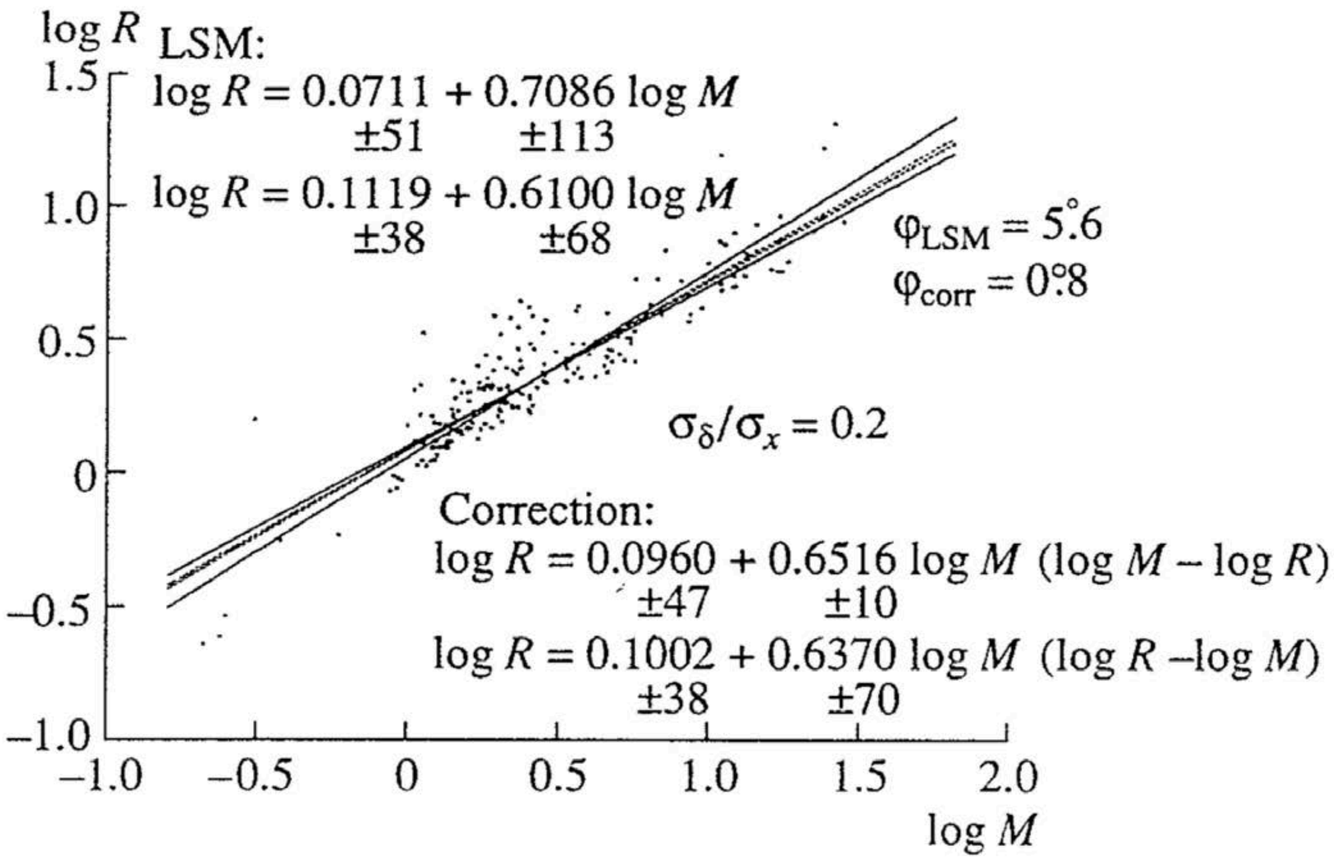
- Mass and radius related via the gravitational acceleration at the surface (which can be estimated from spectral lines)

$$g = \frac{GM}{R^2}$$

- Empirically for main sequence stars with $M \geq M_\odot$

$$R/R_\odot = (M/M_\odot)^{0.6}$$

- More precise relations can be found when considering internal structure of stars in detail



Gorda & Svechnikov (1998)

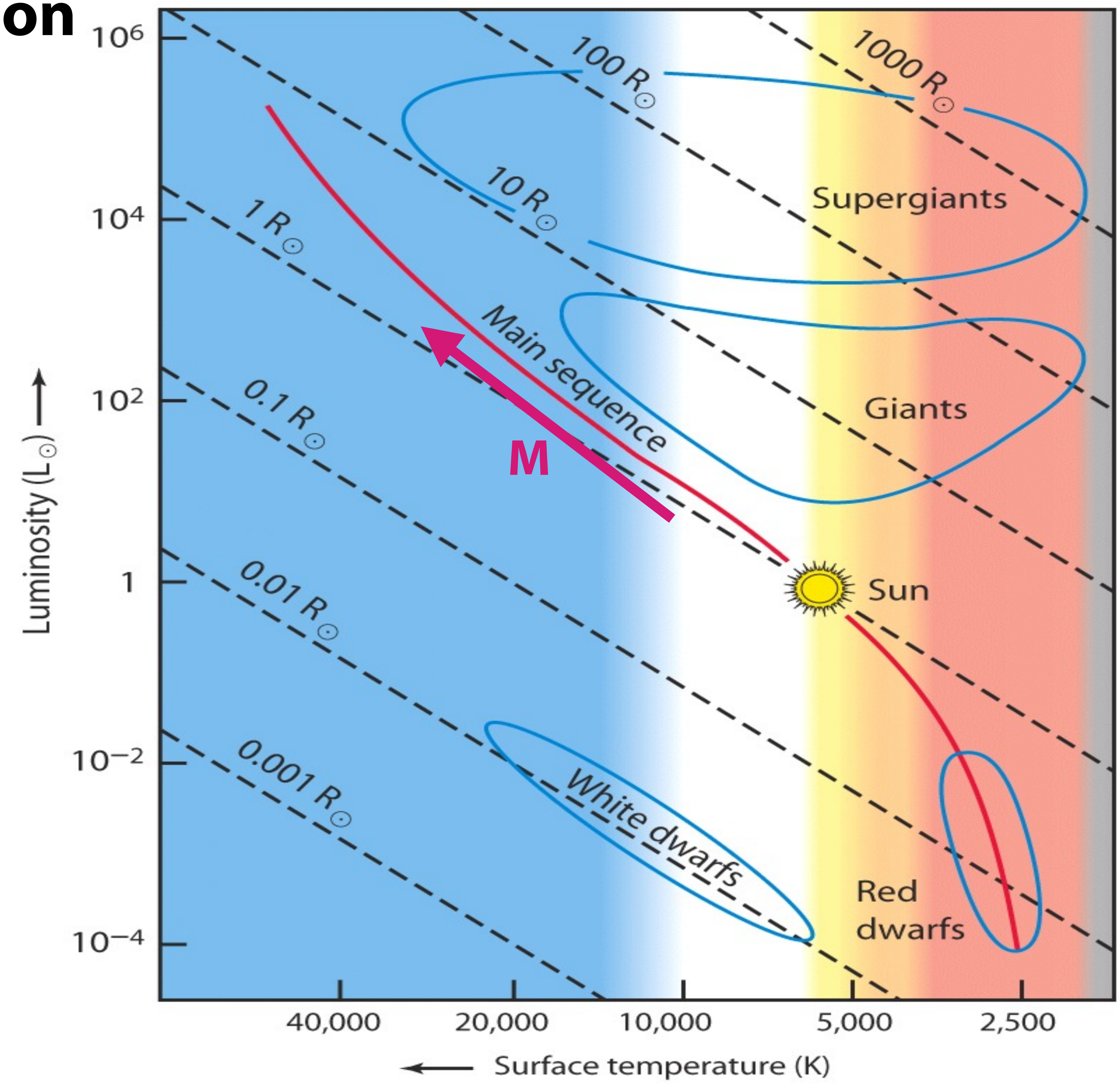
Relations between fundamental parameters

Mass-radius relation

$$R/R_\odot = (M/M_\odot)^{0.6}$$

$$M \geq M_\odot$$

- Careful: Masses are not typically given in HRD as relations dependent on stellar structure in more complicated way than for main sequences



Relations between fundamental parameters

Mass-temperature relation

$$\boxed{R/R_\odot = (M/M_\odot)^{0.6}} \text{ and } \boxed{L/L_\odot = (M/M_\odot)^{3.5}} \text{ and } \boxed{L = 4\pi R^2 \sigma T_\text{eff}^4}$$

$$\boxed{T_\text{eff}^4 \propto M^{2.3}}$$

➡ Along main sequence: **Effective temperature(roughly) proportional to the stellar mass**

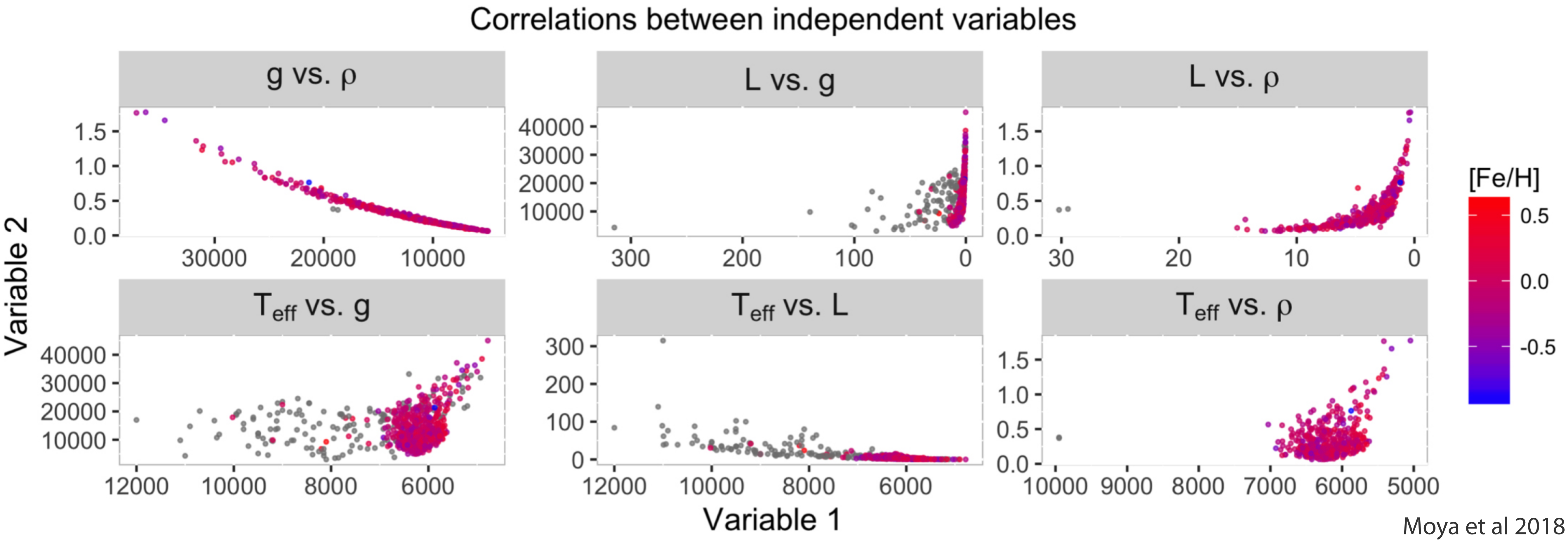
- Please note: Empirical relations based on subgroups of stars

  ➡ To be applied and interpreted within limits

  ➡ Detailed physical consideration of stellar structure superior.

Relations between fundamental parameters

Empirical relations



Moya et al 2018

- Please note: Empirical relations based on subgroups of stars

➡ To be applied and interpreted within limits

➡ Detailed physical consideration of stellar structure superior.

➡ Often it is more than just two parameters that are related!

Physical stellar parameters

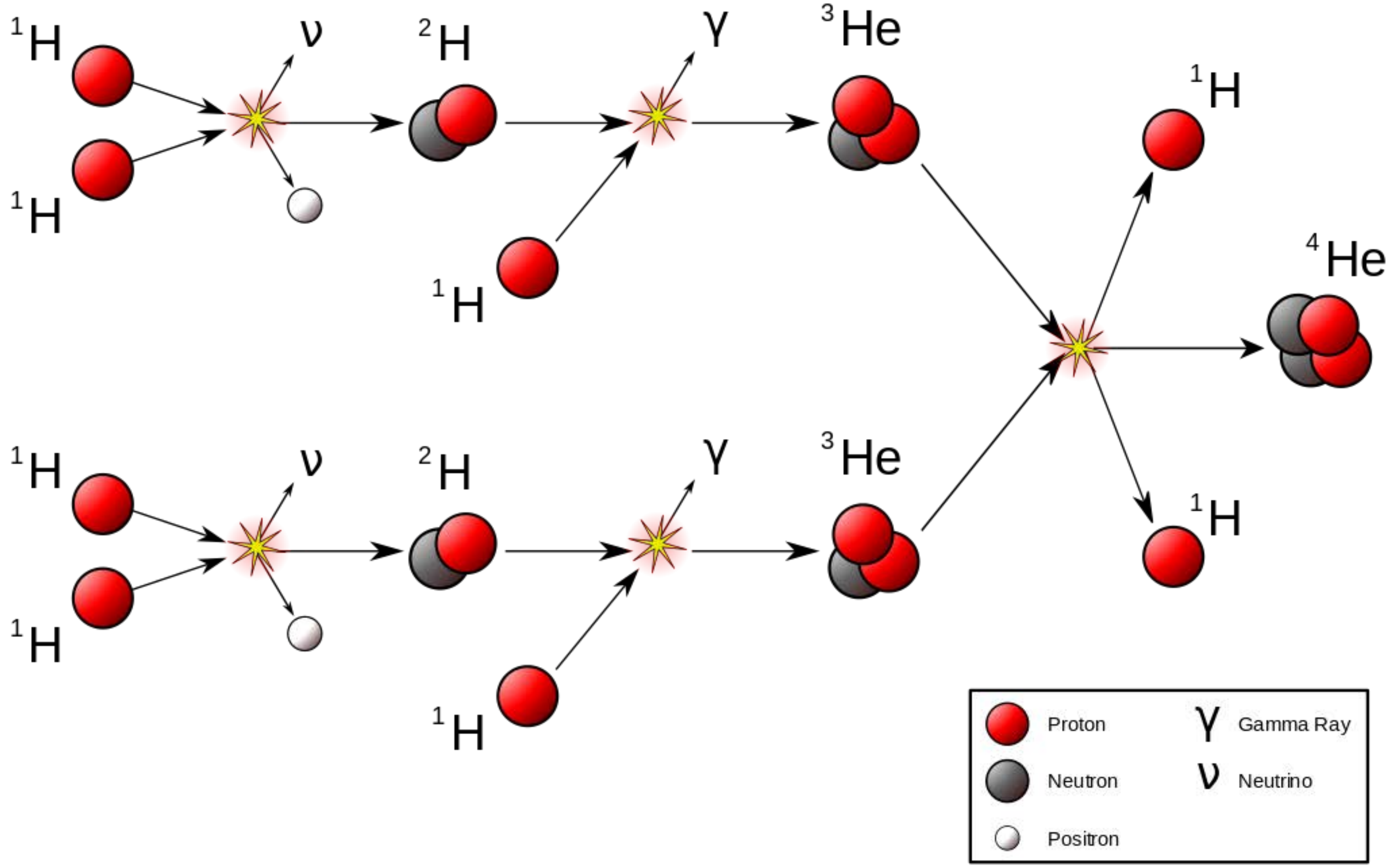
Lifetime of the Sun on the main sequence

- In solar interior: Thermonuclear fusion (here pp chain)
- 4 hydrogen nuclei (protons) → 1 helium nucleus (alpha particle)
- Mass difference $\Delta m = 4\, m_p - m_\alpha > 0$ corresponds then to released energy

$$\Delta E = \Delta m\, c^2$$

- **Energy released per fusion reaction:**

$$E_{FUS} \sim 4.3 \times 10^{-12}\ \mathrm{Ws}$$



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- **Energy released per fusion reaction:**

  $E_{FUS} \sim 4.3 \times 10^{-12}$ **Ws**

Solar luminosity

$L_\odot \sim 3.9 \times 10^{26}$ **W**

- Fusion reactions (pp) in the Sun per second: $N_{FUS} \sim L_\odot / E_{FUS} \sim \underline{10^{38}\ s^{-1}}$

Physical stellar parameters

Lifetime of the Sun on the main sequence

- Mass "lost" via fusion per second

  $\Delta M = N_{FUS} \times 4\, m_p \sim 6.7 \times 10^{11}$ kg/s (m$_p$: proton mass)

- Solar mass $M_\odot = 2 \times 10^{30}$ kg
- Solar lifetime if **all mass** is "used" at constant rate via H→He fusion (pp chain)
- $\tau = M_\odot / \Delta M = 2 \times 10^{30}$ kg / 6.7×10^{11} kg/s $= 3 \times 10^{18}$s $= 9 \times 10^{10}$ år
  90 billion year

But we know: **Sun's lifetime ~ 10^{10} yr!**

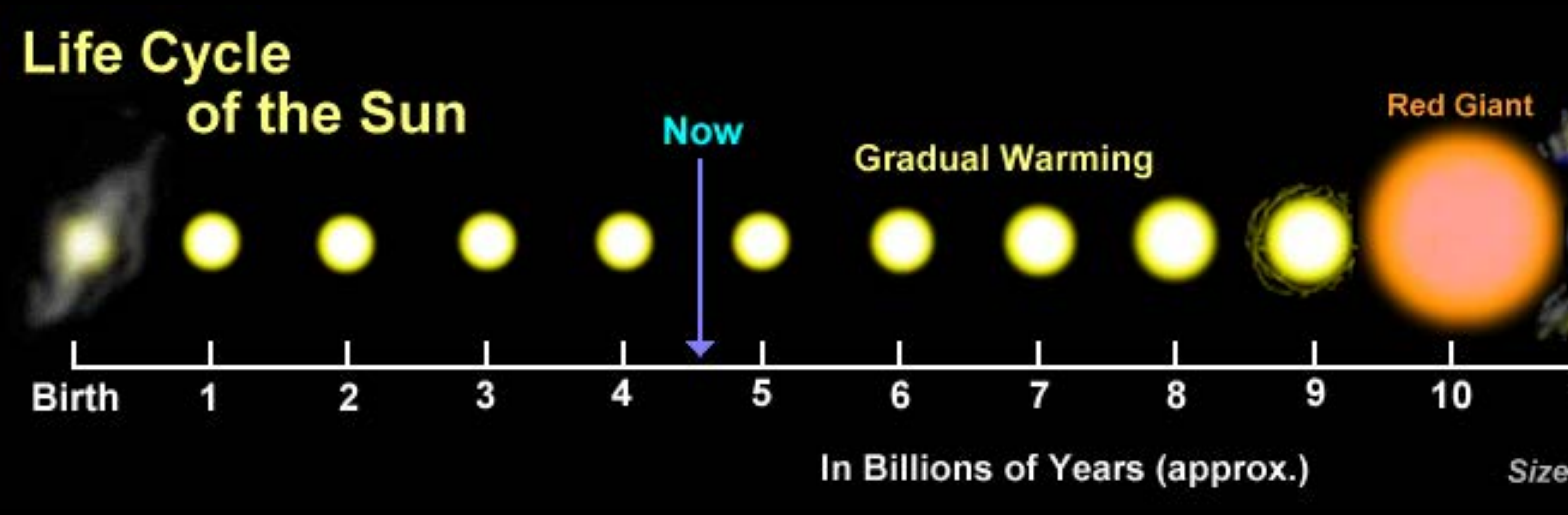
➡ Corresponds to 10% of solar mass.

➡ Explanation: H-He fusion only in solar core (and also not constant rate)

➡ More detailed evolution models needed.

Physical stellar parameters

Lifetime of the Sun on the main sequence



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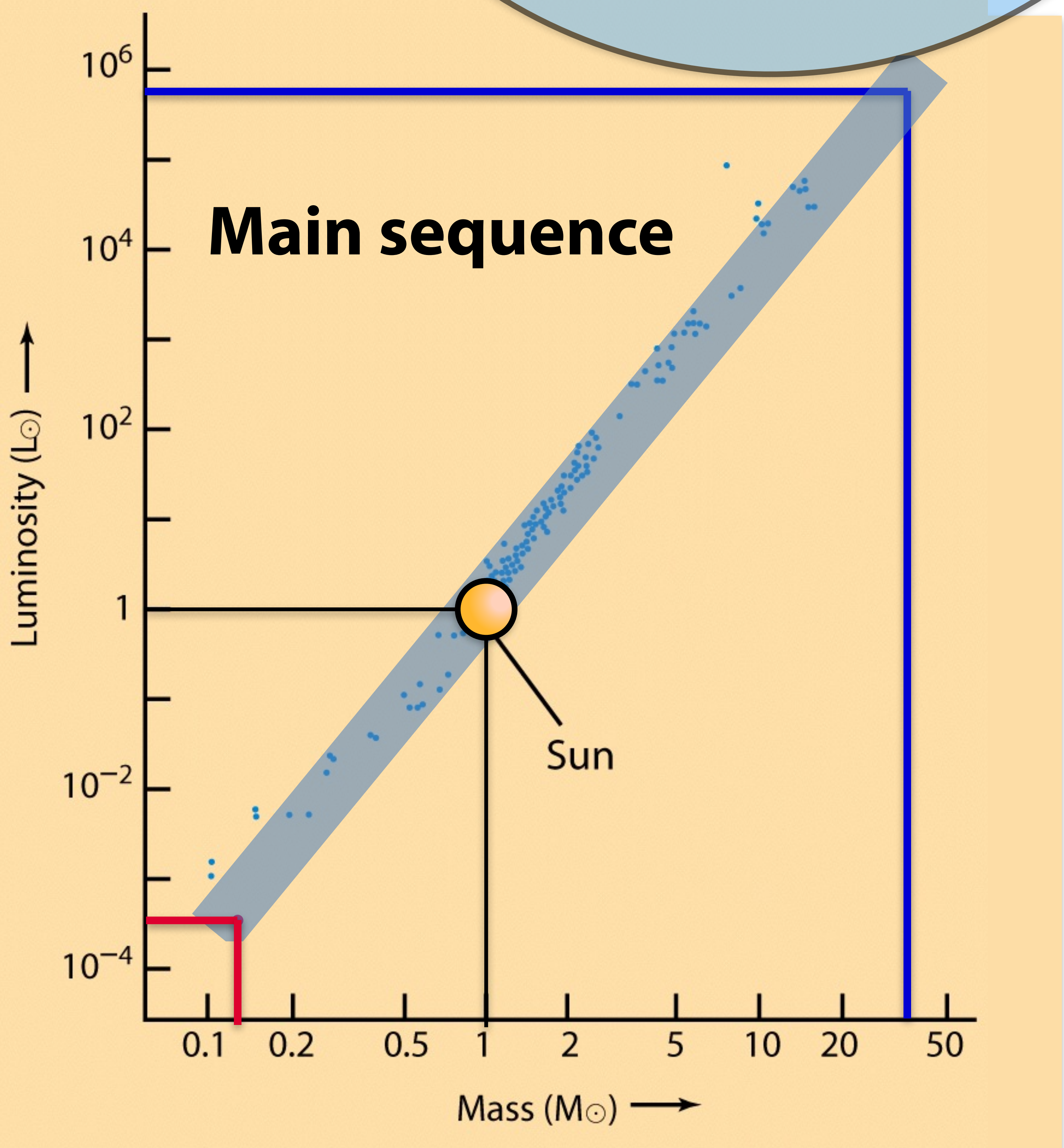
Fundamental parameters

Lifetime on main sequence



$$\boxed{\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}}$$

- Energy set free by fusion
- Stellar **mass** sets absolute limit
- **Luminosity** gives energy loss rate

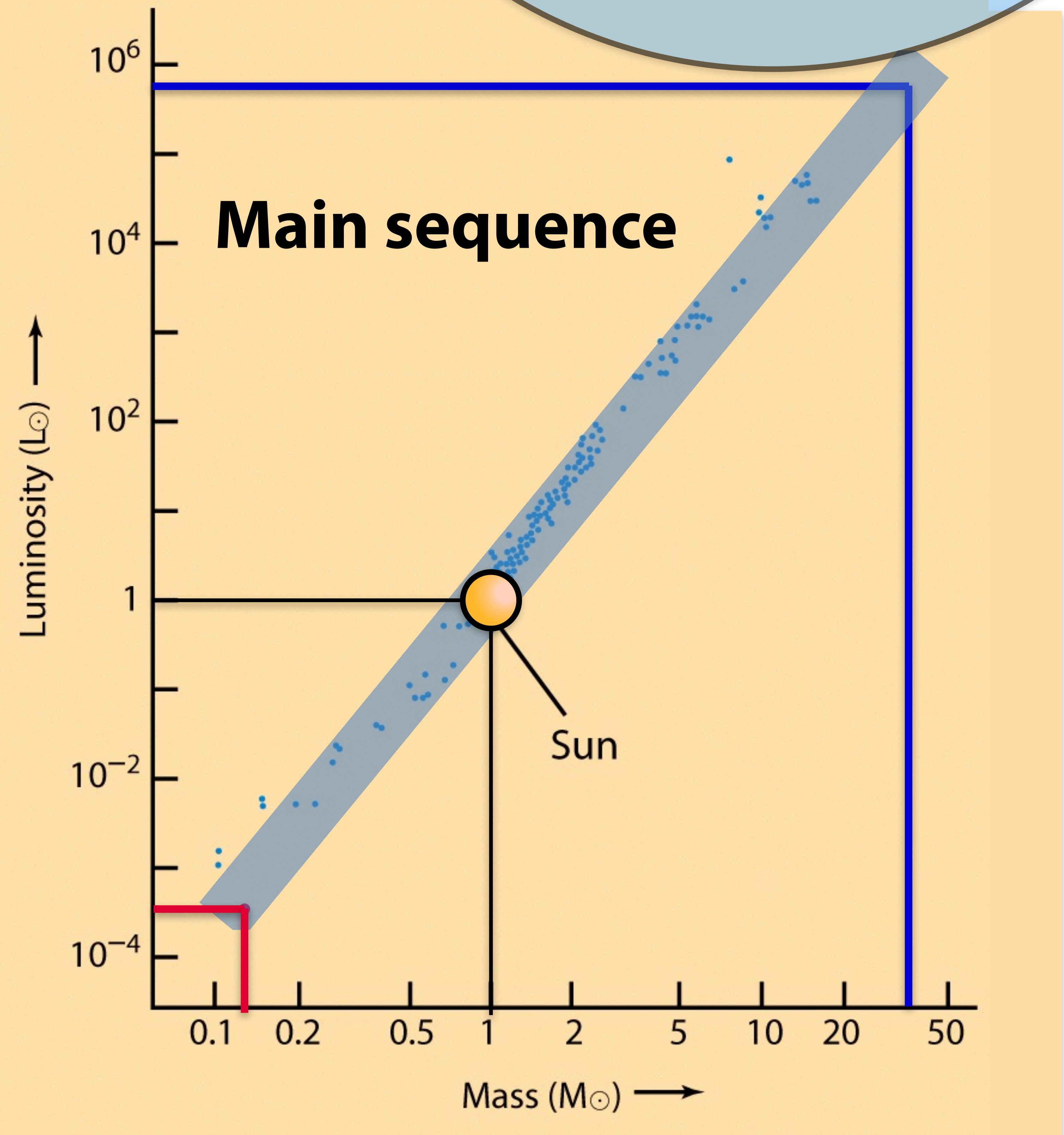
- Simplifying assumption for an order of magnitude estimate: Energy conversion via fusion with constant rate set at constant luminosity

➡Max. time at “constant burn”

$$\tau \propto \mathbf{M / L = \ \ M^{n-1}}$$

Fundamental parameters

Lifetime on main sequence



$$\boxed{\tau \propto \mathbf{M / L}}$$

- Energy set free by fusion
- Stellar **mass** sets absolute limit
- **Luminosity** gives energy loss rate

➡Max. time at "constant burn" M / L

Spectral class O6

$R = 18\ R_{\odot}$, $T_{\mathrm{eff}} = 38\,000$ K , $M = 40\ M_{\odot}$

➡ $L = 520\,000\ L_{\odot}$

➡ Compared to Sun

$M[M_{\odot}]/L[L_{\odot}] = 40 / 520\,000 = 8\ 10^{-5}$

➡ 13000 shorter time

Red dwarf star

$R = 0.1\ R_{\odot}$, $T_{\mathrm{eff}} = 2885$ K, $M = 0.12\ M_{\odot}$

➡$L = 6\ 10^{-4}\ L_{\odot}$

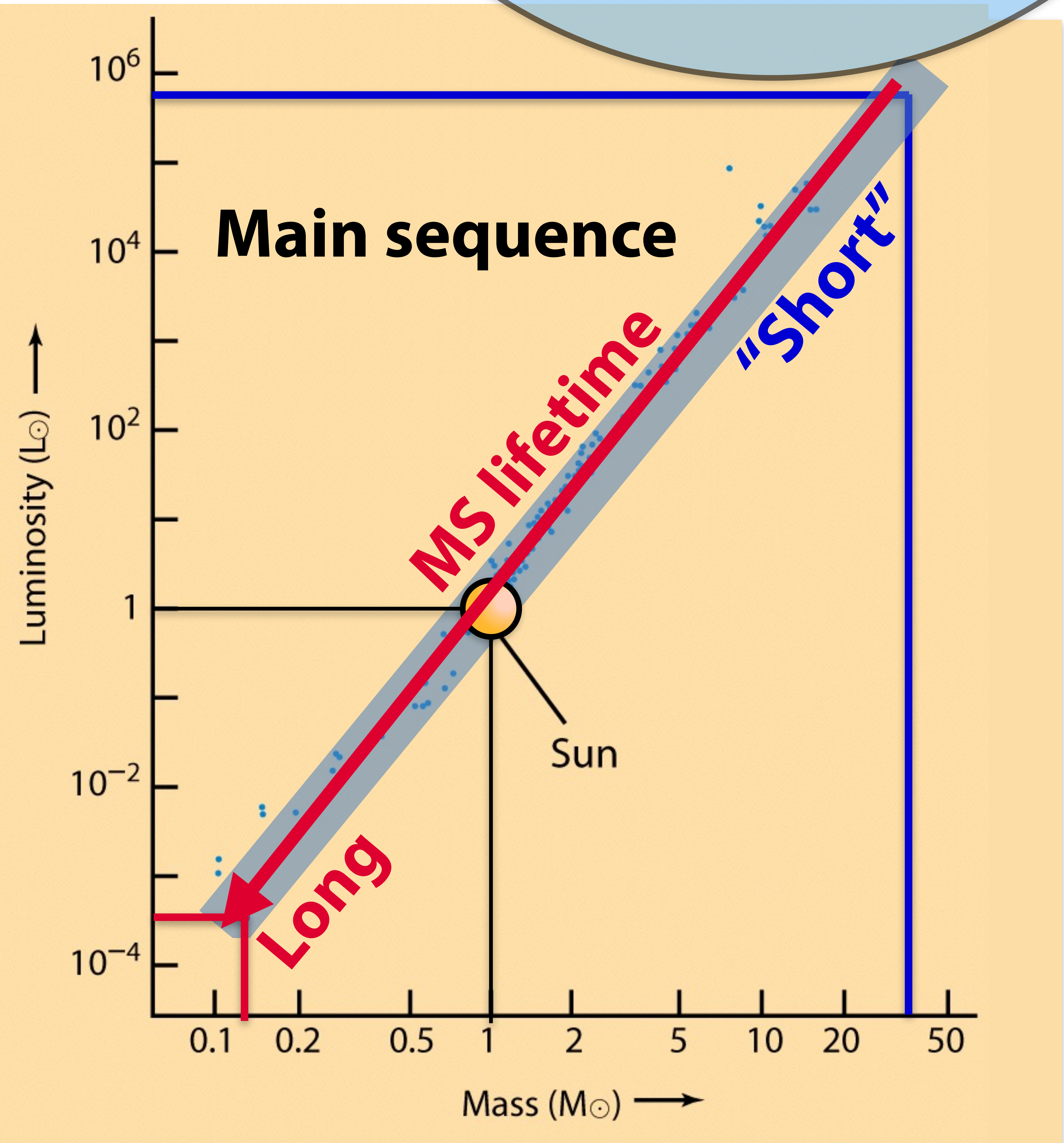
➡ Compared to Sun

$M[M_{\odot}]/L[L_{\odot}] = 0.12 / 6\ 10^{-4} = 200$

➡ 200 times longer

Fundamental parameters

Lifetime on main sequence



$$\boxed{\tau \propto \mathbf{M / L}}$$

- Lifetime of Sun on main sequence on the order of 10^{10} yr

Spectral class O6

➡ $\tau \sim 8\ 10^{-5}\ \ 10^{10}$ yr ~ 800 000 yr

➡ "Only" less than 1 million years.

Red dwarf star

➡ $\tau \sim 200\ \ 10^{10}$ yr ~ $2\ 10^{12}$ yr yr

➡ 2000 billion years

➡ Age of universe "only" ~14 billion years!

Evolutionary Time Scales

Overview

- **Time scales are a fundamentally important in physics as they highlight which physical process is important in which context.**

  ➡ Allows to focus on what is important and derive "lean" and yet accurate models.

- **Changes in a star** occur on different time scales at different evolutionary phases.
  - Most important:
    - Nuclear time scale t_n
    - Thermal time scale t_t
    - Dynamical or freefall time scale t_d.
  - Usually: $t_d \ll t_t \ll t_n$

Evolutionary Time Scales

Nuclear Time Scale

- **Definition:** time in which a star radiates away all the energy that can be released by nuclear reactions

- **Estimate** (as already done for the Sun) for **hydrogen burning**
  - Time in which all available hydrogen is turned into helium.
  - Theoretical considerations / evolutionary model calculations:

  ➡ ~10 % of total mass of hydrogen in a star can be consumed before other, more rapid evolutionary mechanisms set in.

  ➡ Of these 10% a mass fraction 0.7 % is turned into energy in

$$t_\mathrm{n} \approx \frac{0.007 \times 0.1\, Mc^2}{L}$$

- For the Sun:

$$t_\mathrm{n} \approx \frac{M/M_\odot}{L/L_\odot} \times 10^{10}\ \mathrm{a}$$

- Remember: Mass-luminosity relation with high exponent

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\mathrm{n}$$

➡ $M = 30\ M_\odot$: $t_\mathrm{n} \sim 2$ million yr

Evolutionary Time Scales

Thermal Time Scale

- **Definition:** time in which a star would radiate away all its thermal energy if the nuclear energy production were suddenly turned off

- **Estimate:** time it takes for **radiation from the centre to reach the surface**.
  - Virial theorem: $\langle T \rangle = -\frac{1}{2}\langle U \rangle$

T: total kinetic energy of the system
U: potential energy of the system.

➡ Kinetic energy of the thermal motion of the gas particles equals half of the potential energy.

$$t_{\mathrm{t}} \approx \frac{0.5\,GM^2/R}{L} \approx \frac{(M/M_\odot)^2}{(R/R_\odot)(L/L_\odot)} \times 2 \times 10^7\ \mathrm{a}$$

G: constant of gravity
R: stellar radius

- For the Sun: t_t ~ 20 million yrs ~1/500 t_n (nuclear time scale)

Evolutionary Time Scales

Dynamical Time Scale

- **Definition:** the time it would take a star to collapse if the pressure supporting it against gravity were suddenly removed.
- **Estimate:** time for a particle to **fall freely from the stellar surface to the centre**
  - Equals half of period given by Kepler's third law with stellar radius R corresponding to semimajor axis of the orbit:

$$t_\mathrm{d} = \frac{2\pi}{2}\sqrt{\frac{(R/2)^3}{GM}} \approx \sqrt{\frac{R^3}{GM}}$$

- Shortest time scale of the three!
- For the Sun: $t_\mathrm{d} \sim 30\mathrm{min}$

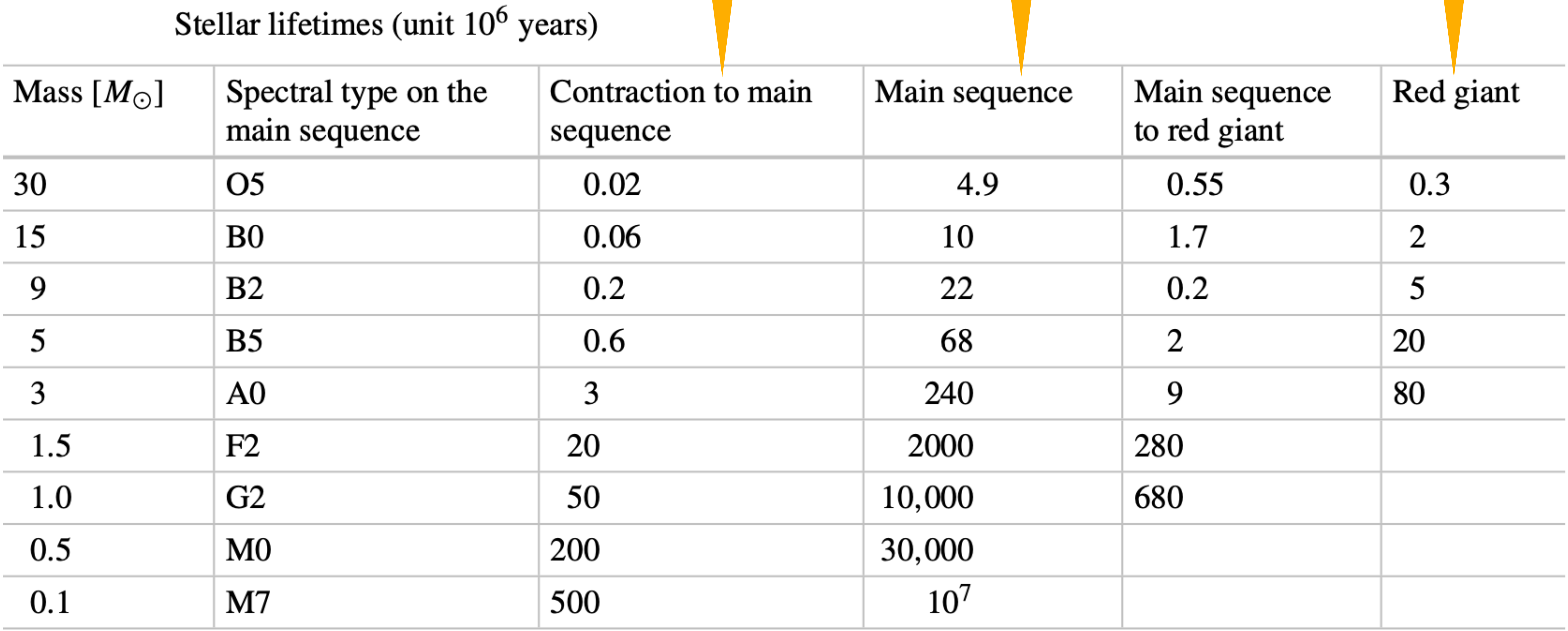
Evolutionary Time Scales

Stellar lifetimes

Dynamical
Time scale
t_d

Nuclear
time scale
Hydrogen
burning
t_n

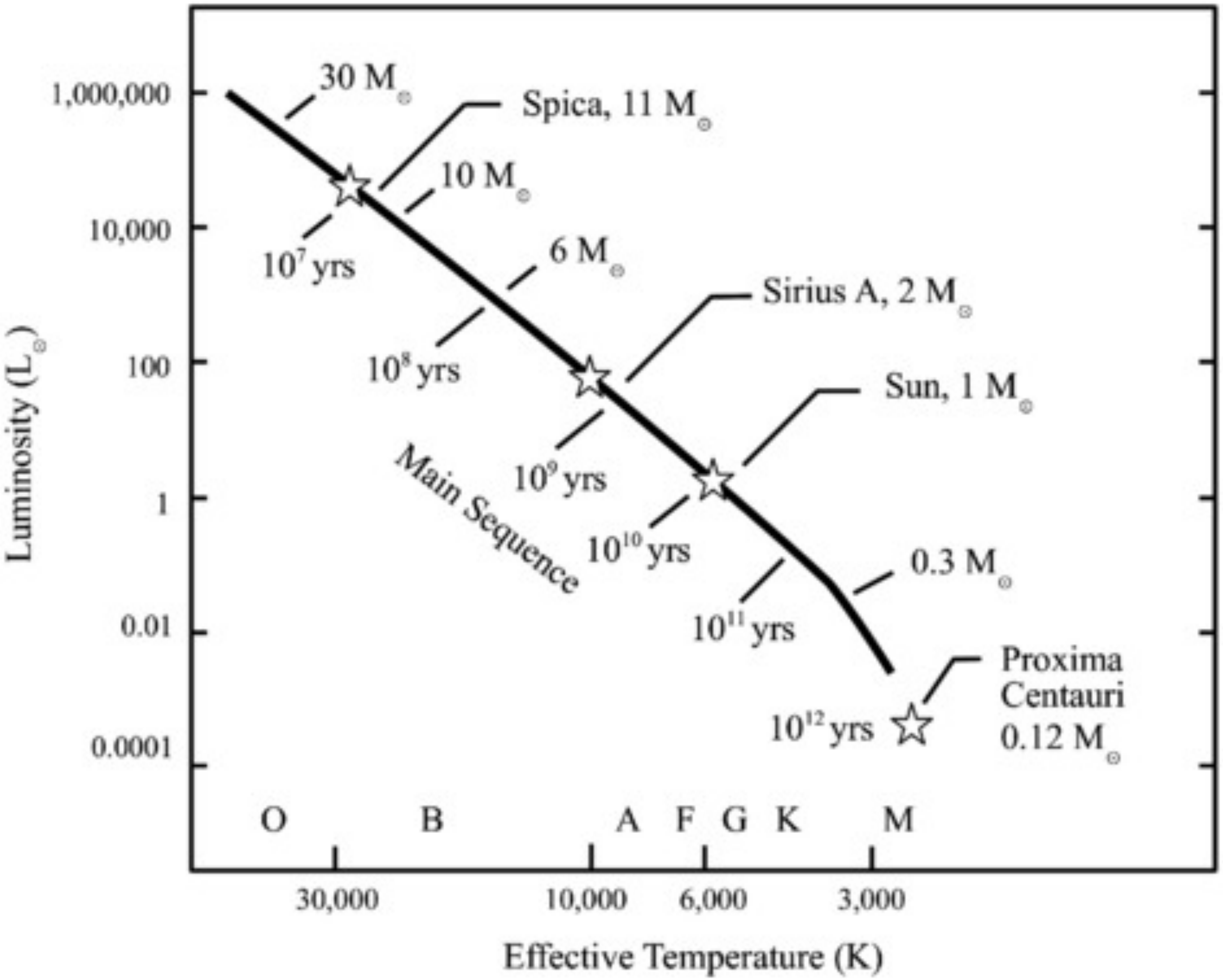
Nuclear
time scale
He burning etc
$t_{n'}$



Stellar lifetimes (unit 10^6 years)

Mass [$M_\odot$]	Spectral type on the main sequence	Contraction to main sequence	Main sequence	Main sequence to red giant	Red giant
30	O5	0.02	4.9	0.55	0.3
15	B0	0.06	10	1.7	2
9	B2	0.2	22	0.2	5
5	B5	0.6	68	2	20
3	A0	3	240	9	80
1.5	F2	20	2000	280	
1.0	G2	50	10,000	680	
0.5	M0	200	30,000		
0.1	M7	500	10^7		

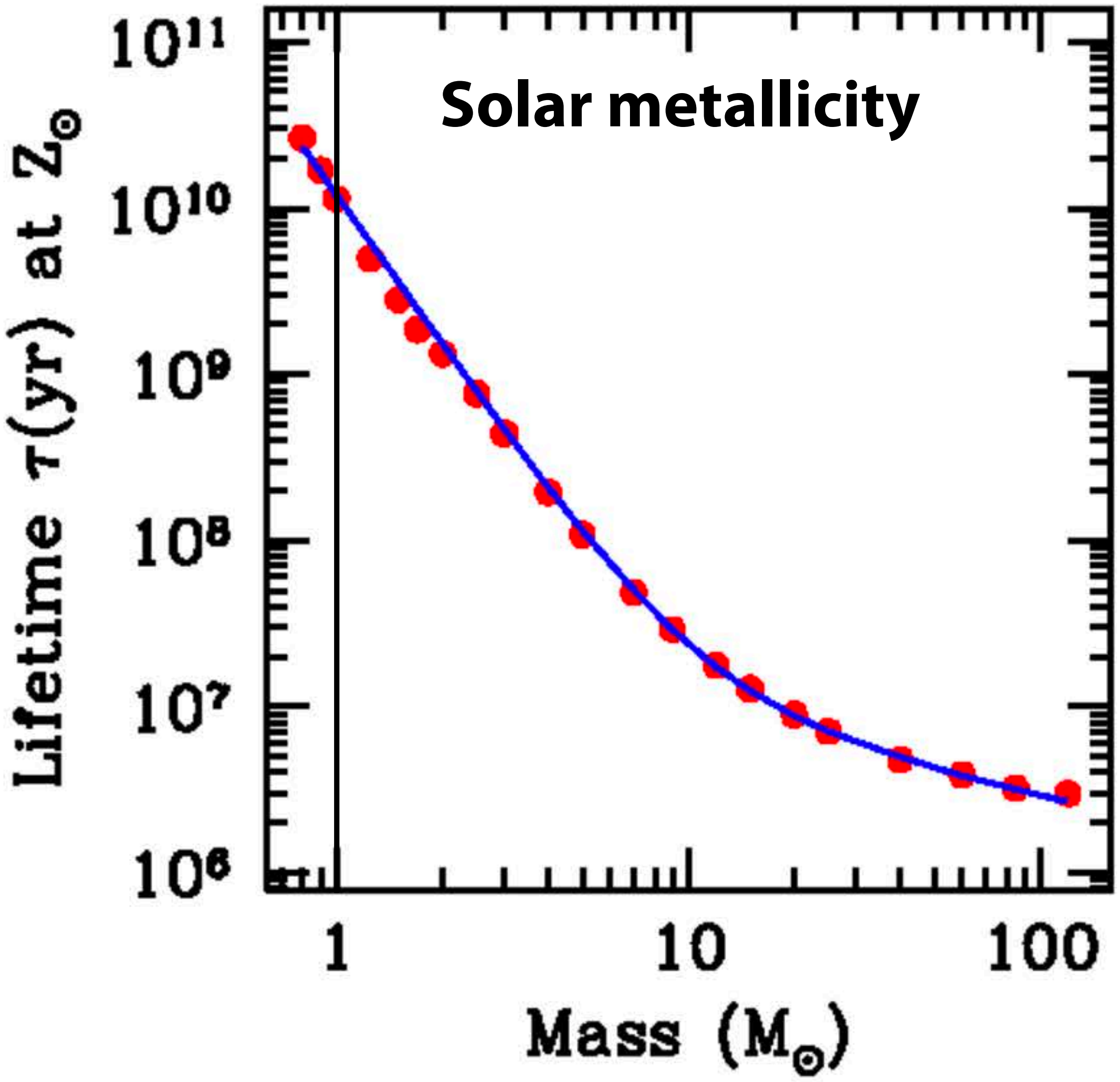
Stellar lifetimes



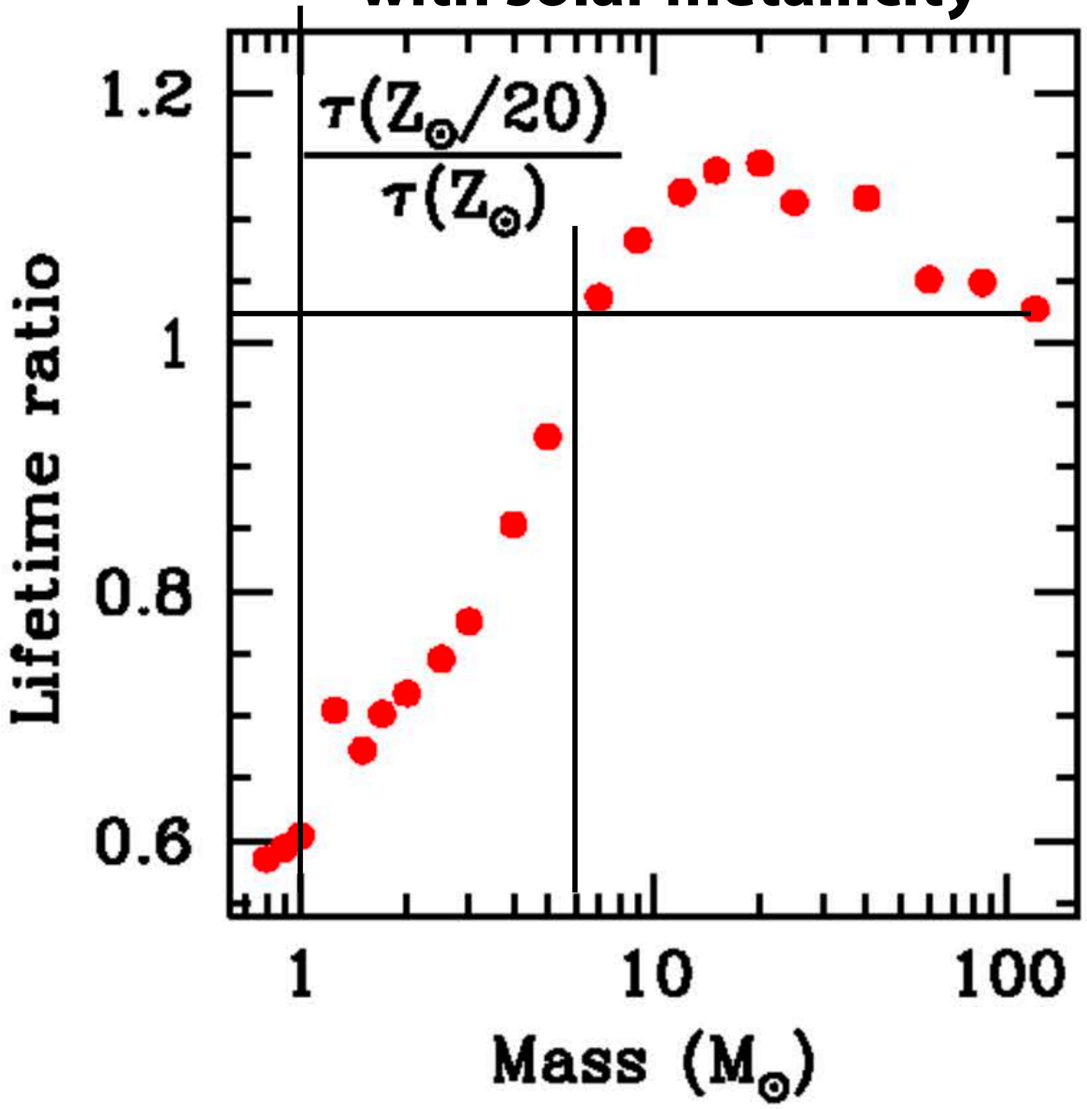
Stellar lifetimes

Impact of metallicity

- Metallicity (chemical composition) of a star influences its evolution (time scales)
- For $Z = 0.05\ Z_\odot$ (lower metallicity):
- $M < 6\ M_\odot$: lifetimes get shorter
- $M > 6\ M_\odot$: lifetimes get longer


Solar metallicity

Lifetimes for stars with low metallicity compared to stars with solar metallicity



Stellar lifetimes

Type	Mass	Temp.	Luminosity	Life time [10^6yr]	Occurrence
O	50	40 000 K	100 000	10	0.00001 %
B	10	20 000 K	1000	100	0.1 %
A	2	8500 K	20	1 000	0.7 %
F	1.5	6500 K	4	3 000	2 %
G	1	5700 K	1	10 000	3.5 %
K	0.8	4500 K	0.2	50 000	8 %
M	0.3	3200 K	0.01	200 000	80 %

- **Main sequence stars** with same mass have typically similar radius, luminosity and temperature (only small slow changes during the time of the main seq. and chemical composition)

- **Low mass stars by far most abundant!**

Stellar lifetimes

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- NOTE: This is the total lifetime of the main series (these stars are not older than the universe)
- No K or M-type dwarf has left the main sequence yet!