AST5770

Solar and stellar physics

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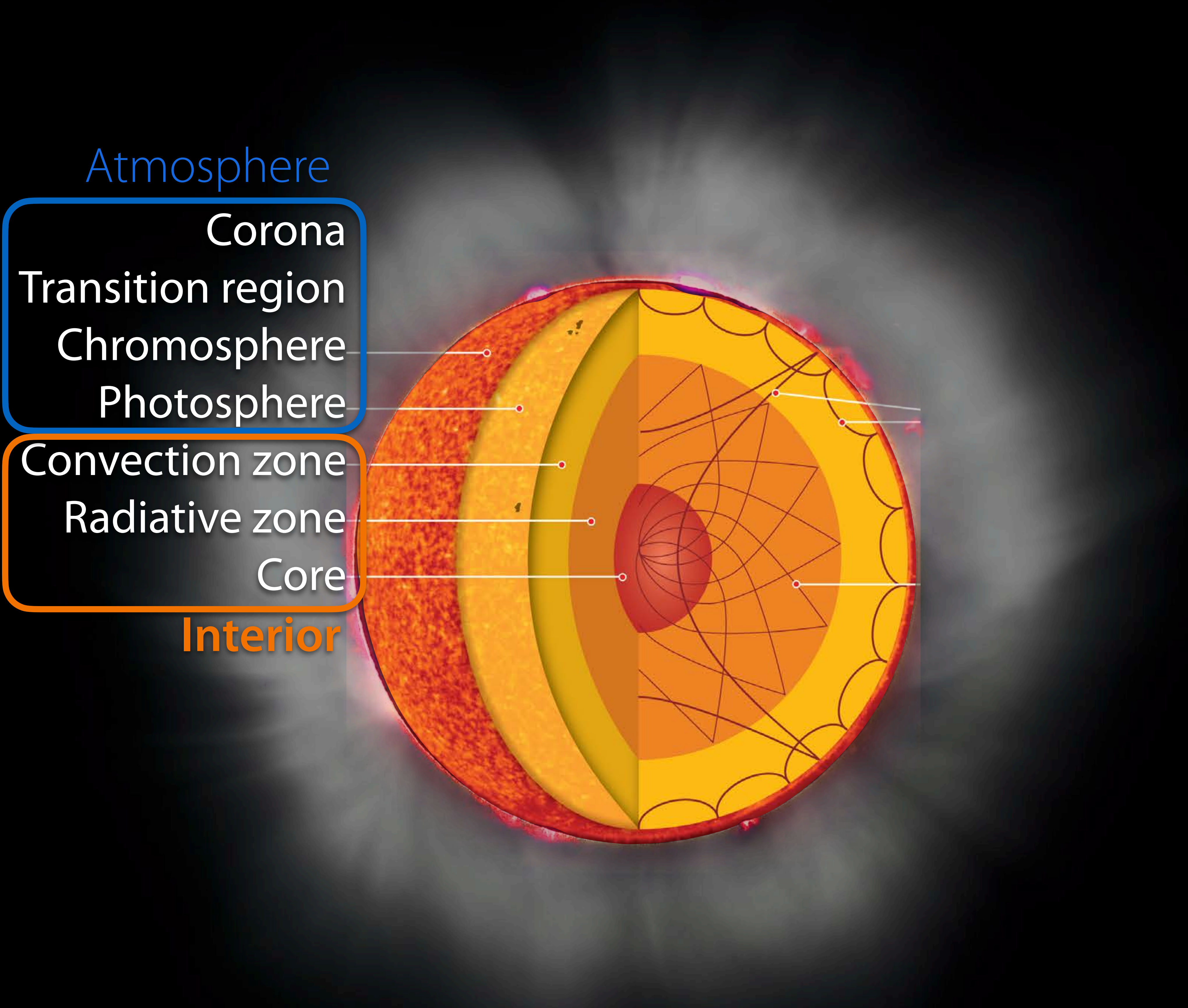
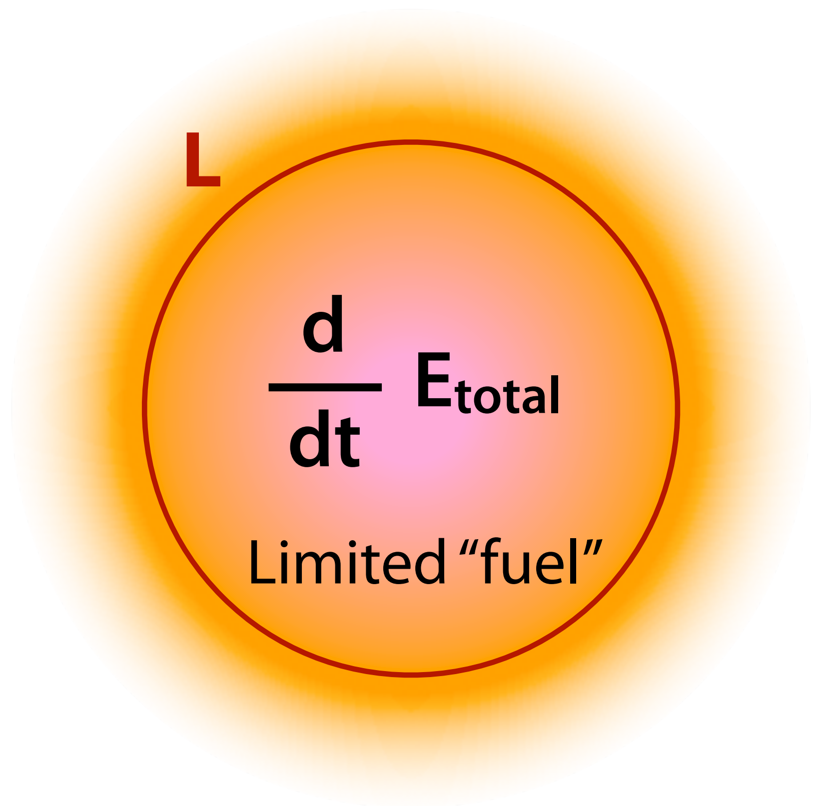
Stellar structure — The Sun



Stellar interior

Overview



- Stars radiate (luminosity)
  - **Conservation of energy** (whole system)

  ➡ Total energy of a star decreases with time

  ➡ How is the required energy set free?

  ➡ How is the energy transported to the surface?

- The physical properties of interior structure determined by
  - Temperature T
  - Pressure P
  - Chemical composition μ

  Equation of state

  Opacity

- Gradients d/dr decisive for energy transport and in return the stratification

- Stellar structure due to **balance** and will change as function of time

Energy reservoirs

Overview

- There are different **energy reservoirs** available:
  - Potential energy E_p (all mass elements of a star).
  - Thermal energy E_t (kinetic energy of all particles)
  - Nuclear energy E_n (set free by nuclear reactions, e.g., fusion)
  - Chemical energy (set free during chemical reactions, atoms combing, typically insignificant under stellar conditions)

Remember

- Dynamical time scale t_d
- Thermal time scale t_t
- Nuclear time scale t_n

- **Total energy** $E_{total} = E_n + E_t + E_p$

➡ Decreases as a star emits energy as radiation at the surface.

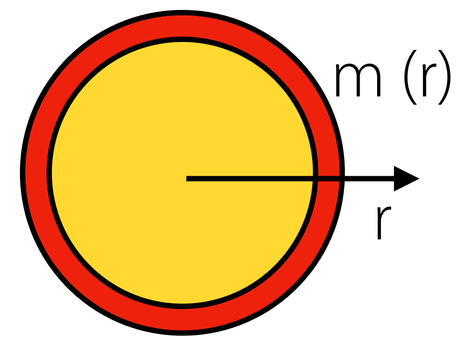
➡ Resulting luminosity of a star = temporal change of the star's energy content:

$$L = - \frac{d}{dt} E_{total}$$

Energy reservoirs

Gravitational potential

- **Gravitational potential energy** for a system of two particles: $$U = -\frac{G m_1 m_2}{r}$$
- **Contraction**: masses closer together
  ➡ potential energy becomes more negative.



  - Instead now integration over thin shells of thickness dr

$$U = -G \int_0^R \frac{m(r) 4\pi r^2 \rho}{r} dr \quad = -\frac{3GM^2}{5R}$$

  - Apply **virial theorem:**
    total energy =1/2 (time-averaged) potential energy →

$$E_{\text{total}} = \frac{3GM^2}{10R} \quad \frac{3}{5}\frac{GM}{R}$$

- **Virial theorem:** relates the average total kinetic and average potential energy of a gravitationally bound system and thus to internal thermodynamical quantities (e.g, temperature, pressure, density)

$$\langle T \rangle = -\frac{1}{2}\langle U \rangle \quad \longrightarrow \quad \frac{1}{2} v^2 = \frac{3}{2}\frac{k_B T}{m_p} = \frac{3}{5}\frac{GM}{R}$$

T: total kinetic energy of the system U: potential energy of the system.

M: stellar mass R: stellar radius
v: particle velocity T: temperature
G: Newton's constant
k_B: Boltzmann constant
m_p: proton mass

Energy reservoirs

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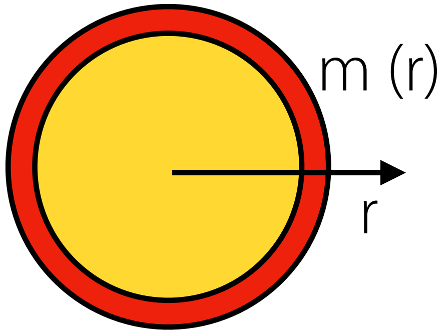


- Instead now integration over thin shells of thickness dr

$$U = -G \int_0^R \frac{m(r) 4\pi r^2 \rho}{r} dr \quad = -\frac{3GM^2}{5R}$$

- Apply **virial theorem:**
  total energy =1/2 (time-averaged) potential energy → $E_{total} = \frac{3GM^2}{10R}$

- **Example: Contraction of the Sun at current luminosity**

$$t \approx \frac{E_{total}}{L_\odot} \approx \frac{1.1\ 10^{41}\ \text{J}}{3.84\ 10^{26}\ \text{W}} \approx 2.9\ 10^{14}\ \text{s} \approx 9\ 10^6\ \text{yr}$$

(Very simplified estimate!)

- Proposed as energy source for the Sun by Kelvin and Helmholtz in late 19th century (before fusion was known)
- **Dismissed** due to clear geological and biological indications of much higher age

Energy reservoirs

Chemical energy

- Chemical reactions: based on interactions of orbital electrons in atoms.
  - Typical energy differences between atomic orbitals ΔE ~10 eV.

- **Example:** Assume the Sun is pure hydrogen

➡ Total number of H atoms in the Sun: $n = \frac{M_{Sun}}{m_H} = \frac{1.99 \times 10^{30}}{1.67 \times 10^{-27}} = 1.19 \times 10^{57}$

  - Assume <u>all</u> atoms release ΔE ~ 10 eV each due to chemical reactions

$$L = -\frac{d}{dt} E_{total}$$

➡ Total released chemical energy: $\approx 10^{58}\,eV = 10^{39}\,J$

➡ **How long could the Sun radiate at current luminosity?**

$$t \approx \frac{E_{chem}}{L_\odot} \approx \frac{10^{39}\ \mathrm{J}}{3.84\ 10^{26}\ \mathrm{W}} \approx 10^5\ \mathrm{yr}$$

➡ Purely based on chemical energy: only 100,000 years
(~100 times less than for the gravitational potential energy)

➡ Obviously not a viable energy source.

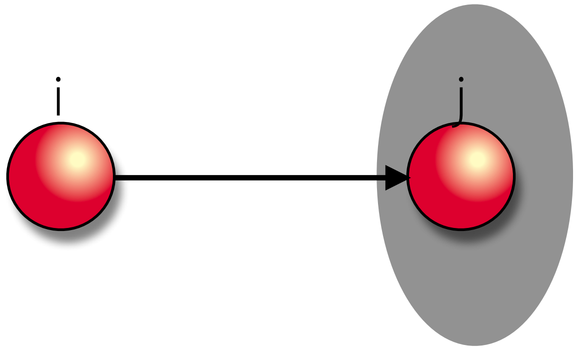
Energy reservoirs

Nuclear energy

- **Wanted: Energy production rate as function of temperature, density, chemical composition**
- Needed:
  1. What is the probability of a certain type of reaction to occur?

     ➡ Cross-section and number densities of reactants!

     ➡ Reaction rates
  2. How much energy is released in each reaction?



- Energy production rate must be related to the rate $\mathbf{r_{i,j}}$ of collisions between two particles (i,j)
  - Amount of energy released per unit mass if each reaction releases an energy $\mathbf{\Lambda}$:

$$\varepsilon_{ij} = \left(\frac{\Lambda}{\rho}\right) r_{ij}$$

ρ: mass density

Energy reservoirs

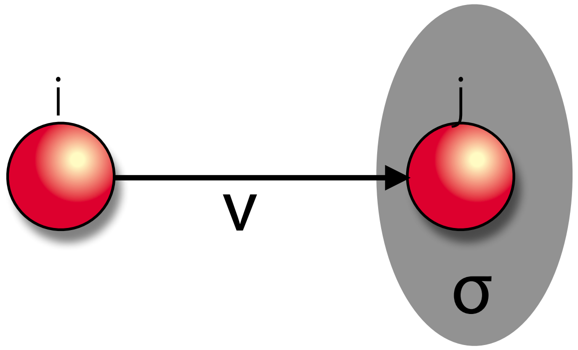
Nuclear energy — Reaction rates

- Rate proportional to number of i-j pairs in the volume.

➡ Reaction rate per second and cm³ :

$$r_{ij} = \sigma\, v\, n_i\, n_j$$

v: relative velocity between particles
σ: cross-section



- Interaction between particles of same species: divide rate by 2:

$$r_{ij} = \frac{\sigma\, v\, n_i\, n_j}{1+\delta_{ij}}$$

Energy reservoirs

Nuclear energy — Reaction rates

- Rate proportional to number of i-j pairs in the volume.

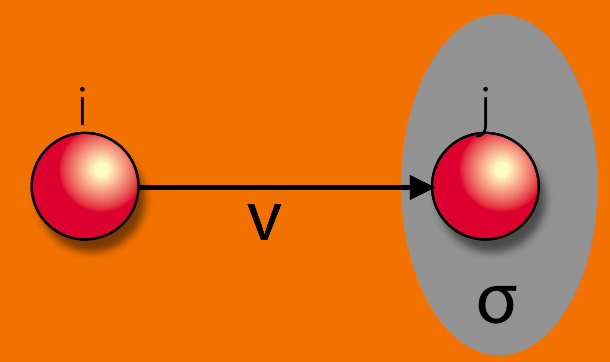
➡ Reaction rate per second and cm^3 :

$$r_{ij} = \sigma\, v\, n_i\, n_j$$

v: relative velocity between particles
σ: **cross-section**

- Interaction between particles of same species: divide rate by 2:

$$r_{ij} = \frac{1}{1+\delta_{ij}}\, n_i\, n_j\, \sigma\, v = \frac{1}{1+\delta_{ij}}\, \frac{X_i\, \rho}{A_i\, m_u}\, \frac{X_j\, \rho}{A_j\, m_u}\, \sigma\, v$$



In this context wanted:
$\langle \sigma\, v \rangle$

- **Number density n_i:** number of particles per cm^3

$$n_i = \frac{N_i}{V} = \frac{X_i\, \rho}{m_i}$$

$$\mathbf{m_i} \approx A_i \cdot m_u$$

- Abundance of species i: $$Y_i = \frac{X_i}{A_i}$$

Particles=nuclei of species i
N_i: Absolute number of particles of species I
V: Volume
ρ : mass density (g/cm3)
m_i: mass of nucleus of species i
X_i : fraction of total mass of gas consisting of particle type i
A_i : atomic weight
m_u: atomic mass unit

Use cgs units…

Energy reservoirs

Nuclear energy — recap: fusion

- **Coulomb barrier** due to electric charges of particles

$$U_C = \frac{1}{4\pi\varepsilon_0}\frac{Z_i Z_j e^2}{d}$$

(Coulomb potential energy)

- Particle energy needed for reaction to become possible:

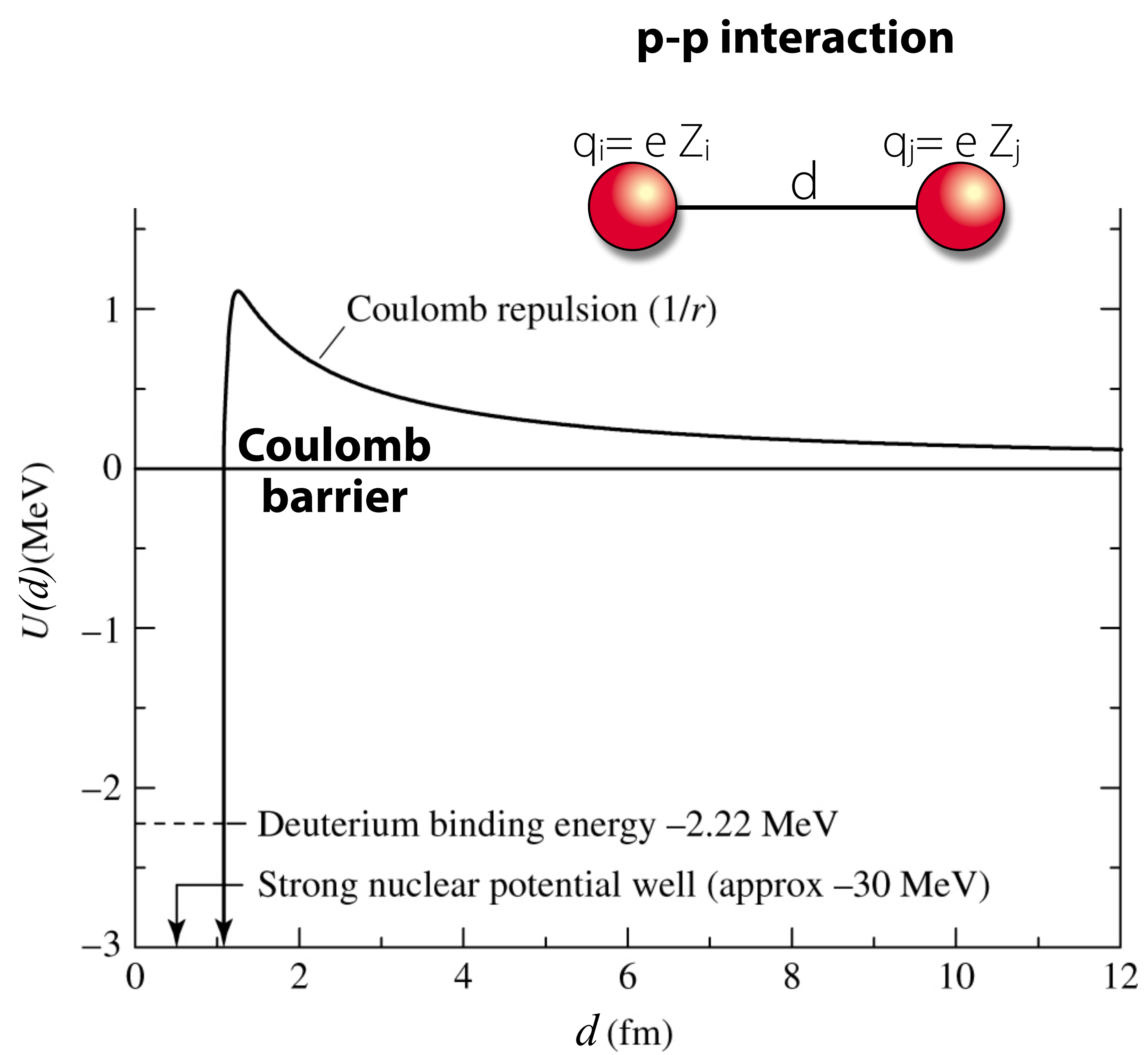
$$E = 3/2\,kT \geq U_c$$

- Equivalently: Required particle velocity :

$$v \;>>\; \sqrt{3kT/(2m)}$$

Mean thermal velocity

- Quantummechanical **tunneling** "through" Coulomb barrier!



Energy reservoirs

Nuclear energy — recap: fusion

- **Coulomb barrier** due to electric charges

$$U_C = \frac{1}{4\pi\varepsilon_0}\frac{Z_i Z_j e^2}{d}$$

(Coulomb potential energy)

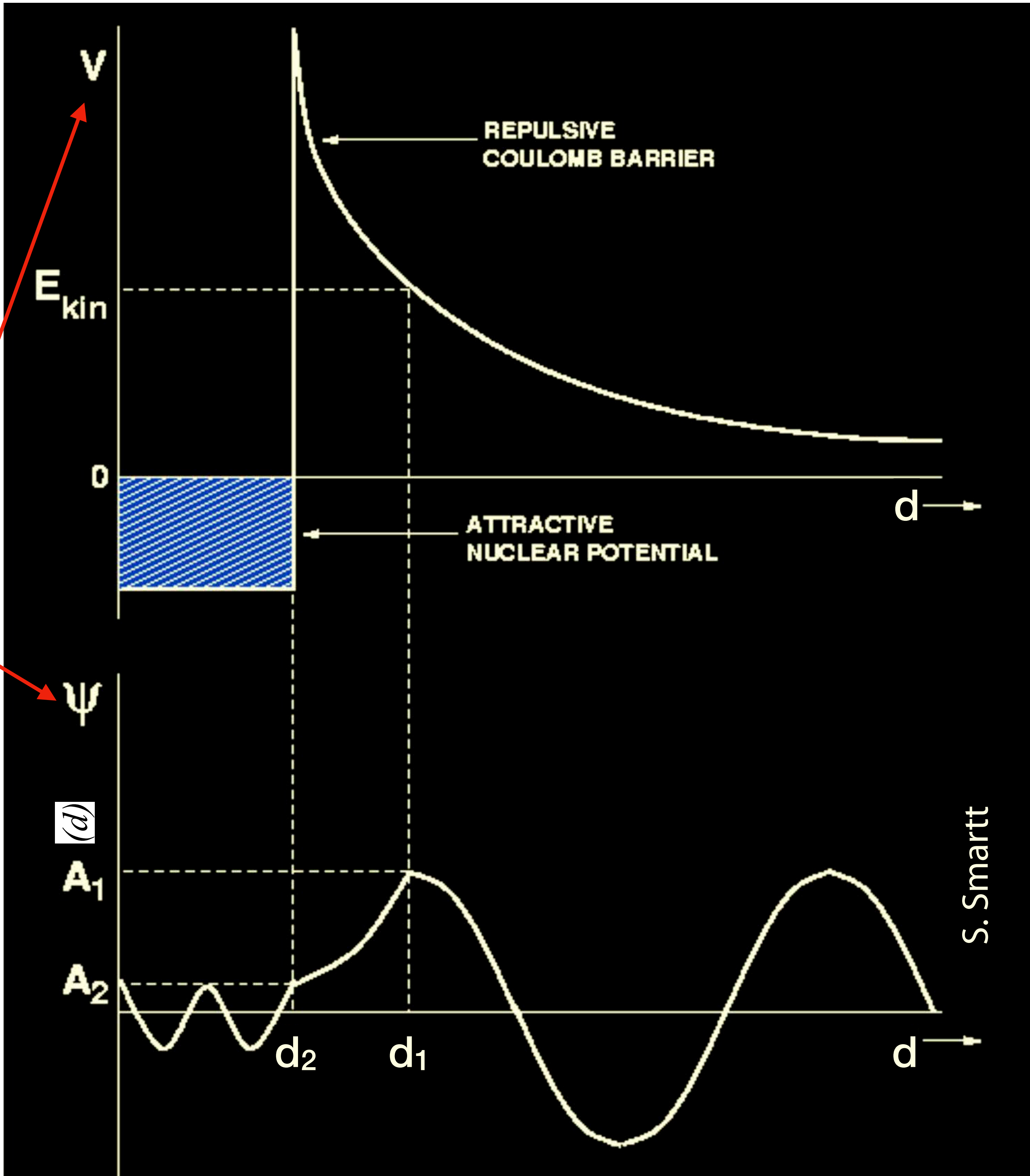
Penetration probability $\propto e^{\frac{-\pi Z_i Z_j e^2}{\varepsilon_0 h v}}$

(Gamow)

- Quantummechanical **tunneling** "through" Coulomb barrier!

V **(potential energy)**

Wave function



Energy reservoirs

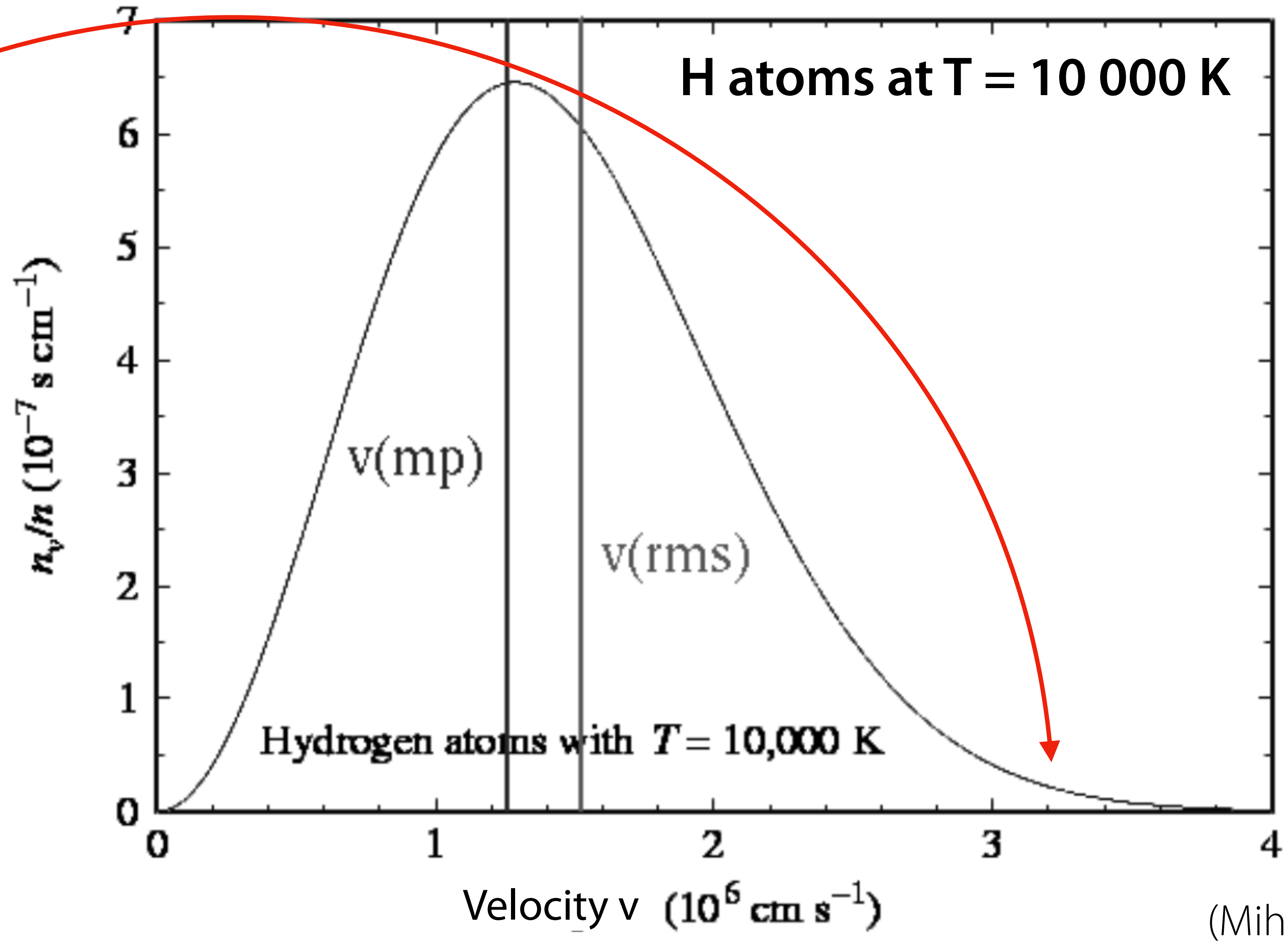
Nuclear energy — recap: fusion

- Distribution of particle velocities: **Maxwellian** velocity distribution

$$P(v)\,dv = 4\pi\left[\frac{\mu}{2\pi kT}\right]^{3/2} e^{-m\,v^2/(2kT)}\,v^2\,dv$$

reduced mass $\mu = m_1 m_2/(m_1 + m_2)$

- **Note the high-velocity tail!**



(Mihos)

Energy reservoirs

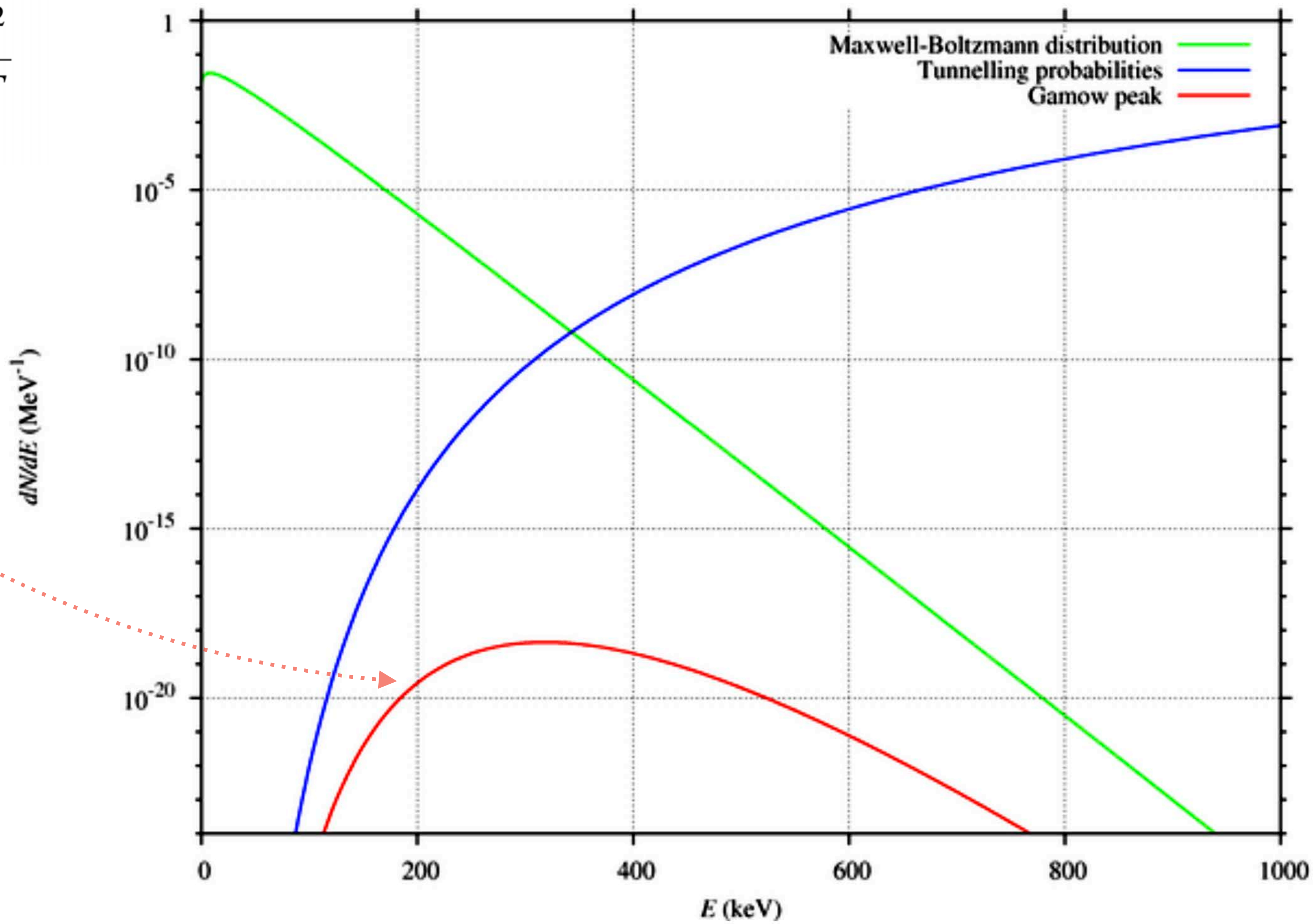
Nuclear energy — recap: fusion

- Combining tail of velocity distribution and tunnelling probability

➡ Probability that this reaction occurs

$$\propto e^{\frac{-\pi Z_i Z_j e^2}{\varepsilon_0 h v}} e^{-\frac{mv^2}{2kT}}$$

➡ Gamow peak



Energy reservoirs

Nuclear energy — Reaction rates

- **Conclusion**: Reaction rates depend on
  - Number of reactants (expressed as **chemical composition and mass density**)
  - **Temperature** (via cross-section, reaction probability)

- For simplicity: Approximate reaction rates as **power laws**

$$r_{ij} \approx r_0 X_i X_j \rho^{1+\alpha} T^{\beta}$$

$$\Rightarrow \varepsilon_{ij} = \left(\frac{\Lambda}{\rho}\right) r_{ij} \approx \varepsilon_0 X_i X_j \rho^{\alpha} T^{\beta}$$

- Parameter **α**: Depends on details of reaction (how many reactants are involved)
  - For two-body interactions (i.e. p+p collisions), $\alpha \sim 1$
- Parameter **β**: Temperature dependence

➡ Fusion reactions known to very sensitively dependent on temperature

➡ β can have a wide range of values

Energy reservoirs

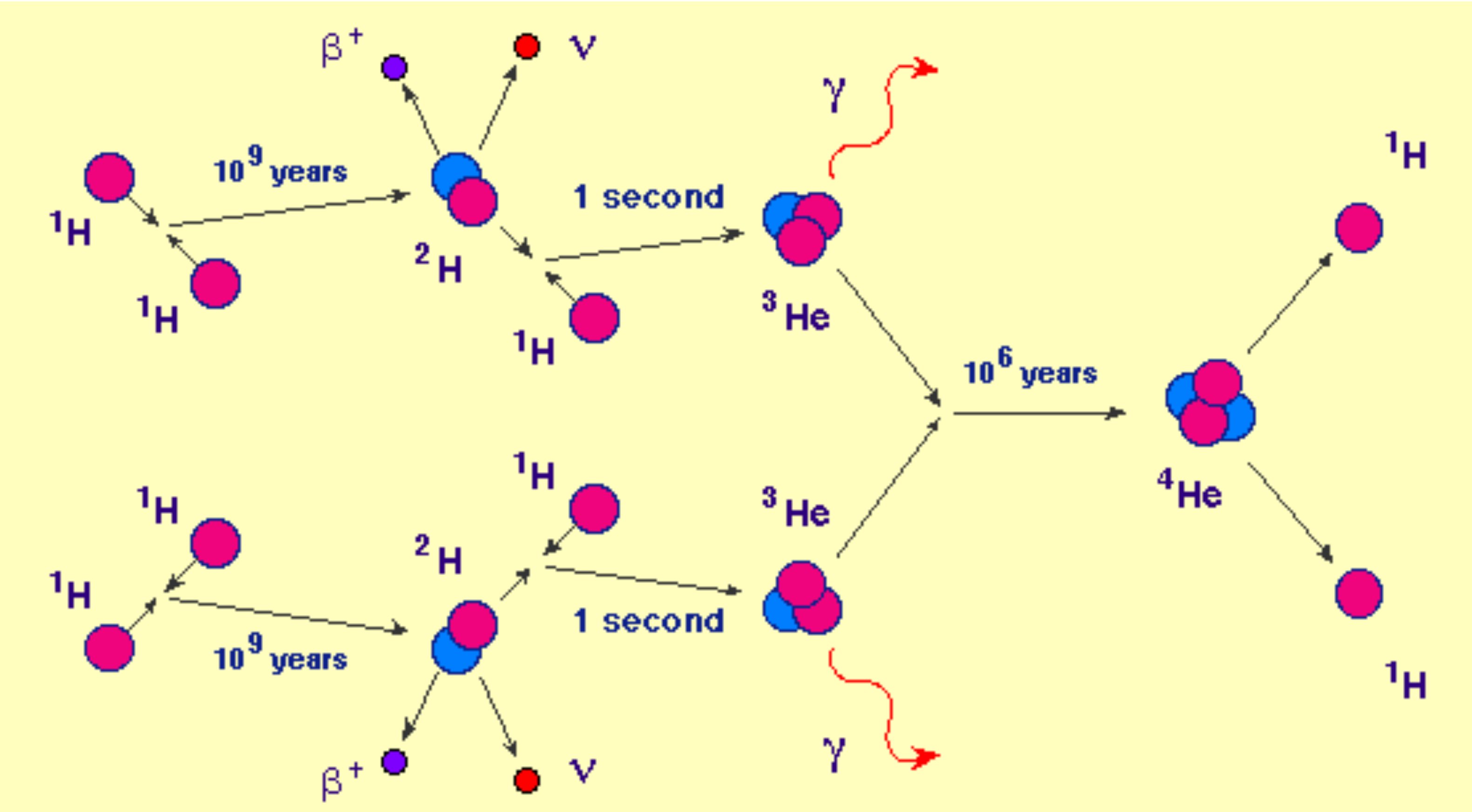
Nuclear energy — Hydrogen burning

- Different fusion reactions with different chains, branches, steps
- Individual reactions reaction rates (and a reaction being relevant) depends on the local plasma properties such as temperature, density, and abundance of the required reactants
  - Will change as function of radius from centre to surface

- **Example:** Hydrogen burning in the Sun (and solar-like stars):
  - Mostly pp chain with three different branches with different relative occurrence

- **PP1 in the Sun:**

Step 1	${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \upsilon_e$	10^9 yr
Step 2	${}^2_1H + {}^1_1H \rightarrow {}^3_2He + \gamma$	~1s
Step 3	${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1H$	10^6 yr



Energy reservoirs

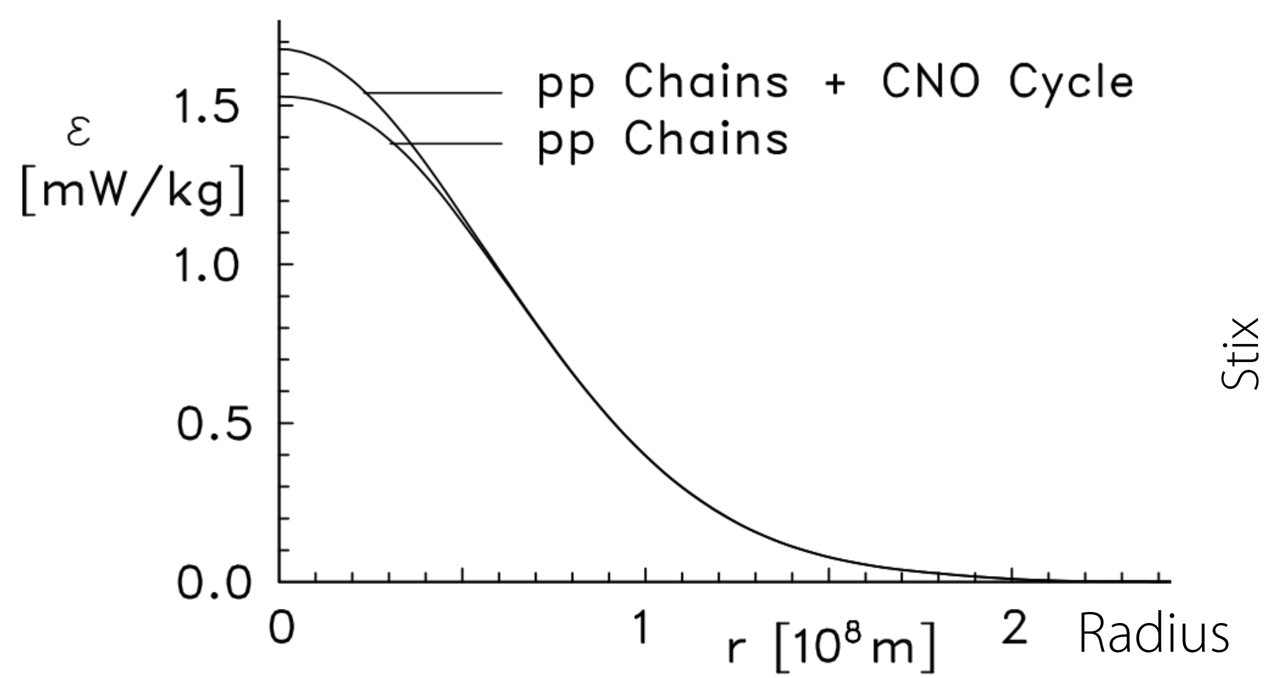
Nuclear energy — Hydrogen burning

PP1	PP2	PP3	CNO
${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu_e$ ${}^2_1H + {}^1_1H \rightarrow {}^3_2He + \gamma$ ${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1H$	${}^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$ ${}^7_4Be + e^- \rightarrow {}^7_3Li + \nu_e$ ${}^7_3Li + {}^1_1H \rightarrow 2{}^4_2He$	${}^7_4Be + {}^1_1H \rightarrow {}^8_5B + \gamma$ ${}^8_5B \rightarrow {}^8_4Be + e^+ + \nu_e$ ${}^8_4Be \rightarrow 2{}^4_2He$	${}^{12}_6C + {}^1_1H \rightarrow {}^{13}_7N + \gamma$ ${}^{13}_7N \rightarrow {}^{13}_6C + e^+ + \nu_e$ ${}^{13}_6C + {}^1_1H \rightarrow {}^{14}_7N + \gamma$ ${}^{14}_7N + {}^1_1H \rightarrow {}^{15}_8O + \gamma$ ${}^{15}_8O \rightarrow {}^{15}_7N + e^+ + \nu_e$ ${}^{15}_7N + {}^1_1H \rightarrow {}^{12}_6C + {}^4_2He$
69 %	31 %	<0.3%	
$T < 1.4\ 10^7$ K	$T > 1.4\ 10^7$ K	$T > 3\ 10^7$ K	

Relative occurrence in the Sun. **PP1 dominates in the current Sun.**

Temperature at which this branch dominates

- Overall nuclear energy production rate ε, either in total or by fusion chain derived as sum over all involved reactions



Energy reservoirs

Nuclear energy

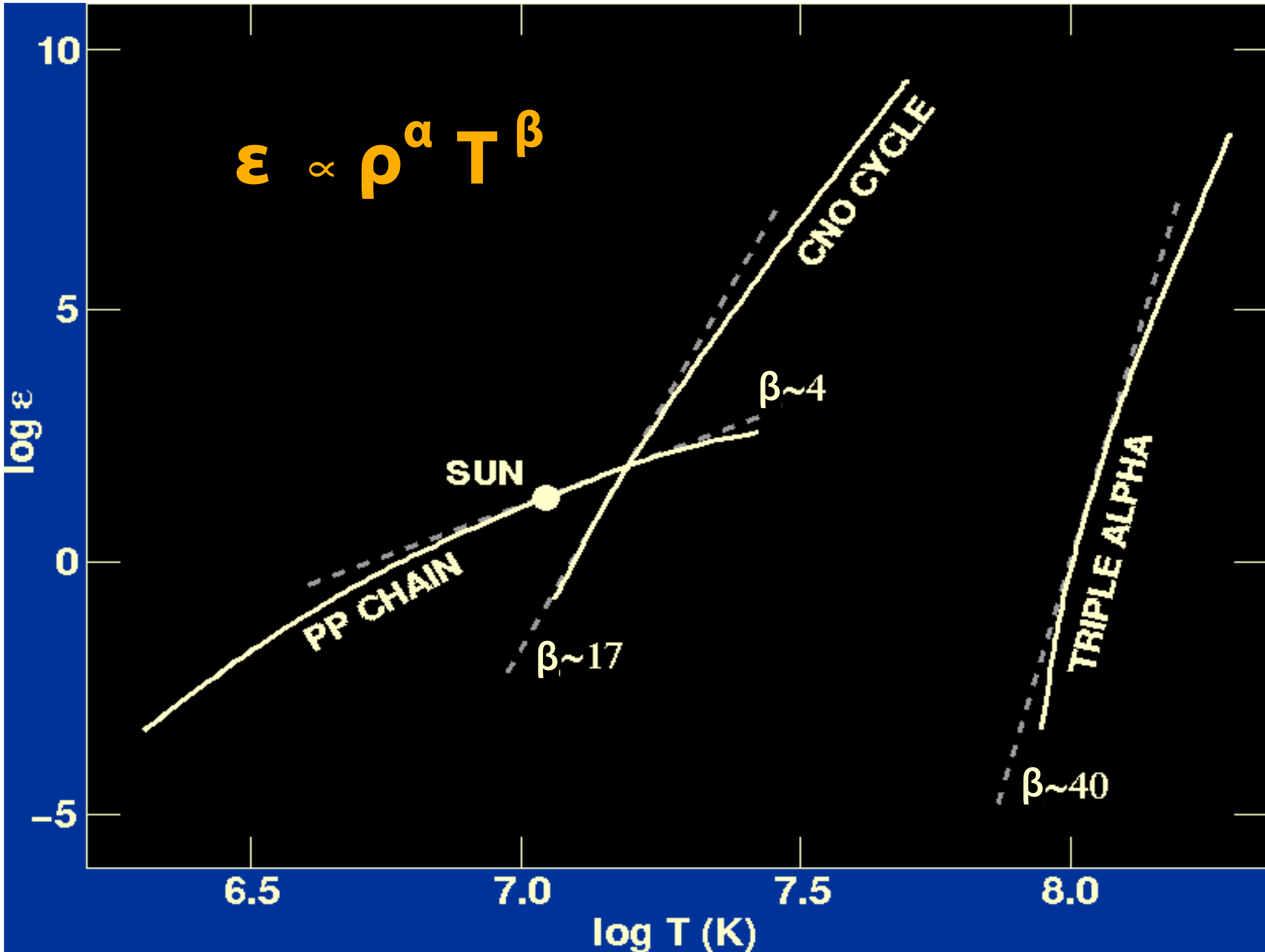
	H-burning		**He-burning**
	pp chain	**CNO cycle**	**Triple-α**
Energy released per complete reaction chain	26.0 MeV plus ~0.73 MeV as neutrinos	25 MeV	7.3 MeV
Overall contribution in the current Sun	99 %	1 %	~0
Energy production rate	$\varepsilon \propto \rho T^{4}$	$\varepsilon \propto \rho T^{17}$	$\varepsilon \propto \rho T^{40}$

$$^{4}He + ^{4}He \rightarrow ^{8}Be$$
$$^{8}Be + ^{4}He \rightarrow ^{12}C + \gamma$$

Energy reservoirs

Nuclear energy production rates

- **Nuclear energy production rate depends sensitively on temperature (increasing β)**
- Fusion reactions involve successively heavier elements
  - in ascending order: the PP chain, the CNO cycle and the triple-alpha reaction)
- Higher fusion reactions become more temperature dependent and require **higher temperatures** to operate (larger Coulomb barrier to overcome for heavier, more positively charged nuclei)
- Dependence on density
  - linear ($\alpha=1$) for two-particle reactions (pp chain, CNO cycle …)
  - quadratic ($\alpha=2$) for three-particle reactions (e.g., triple-alpha process)



Energy reservoirs

Higher burning stages

- Further burning stages require higher central temperature
- Achieved at progressively larger stellar masses (e.g. Carbon burning needs M > 4MS)
- Examples of higher burning stages:

At T ~ 6 10^8 K	$O_8^{16} + He_2^4 \rightarrow Ne_{10}^{20}$ $Ne_{10}^{20} + He_2^4 \rightarrow$ Mg24 $Mg^{24} + He_2^4 \rightarrow Si_{14}^{28}$	4.7 MeV 9.3 MeV 10.0 MeV
At T ~10^9 K	$C_{12} + C_{12} \rightarrow Mg^{24}$ $O_{16} + O_{16} \rightarrow S_{32}$ $Mg^{24} + S_{32} \rightarrow Fe_{56}$	14 MeV 16 MeV END OF FUSION

Energy reservoirs

Higher burning stages

- Further burning stages require higher central temperature
- Achieved at progressively larger stellar masses (e.g. Carbon burning needs M > 4MS)
- Examples of higher burning stages:

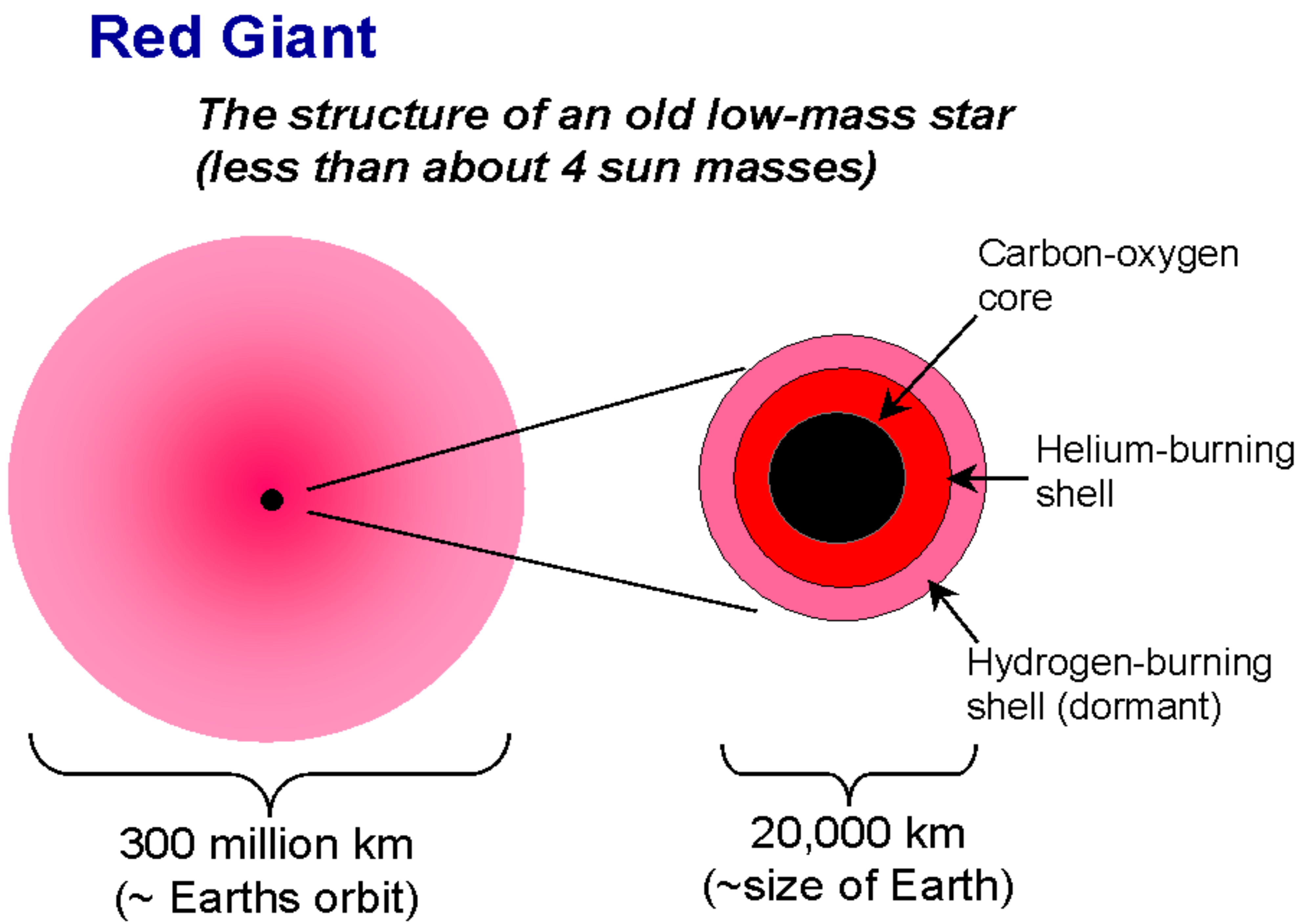
Nuclear Fuel	Process	$T_{threshold}$ 10^6K	Products	Energy per nucleon (Mev)
H	PP	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C,O	0.61
C	C+C	600	O,Ne,Ma,Mg	0.54
O	O+O	1000	Mg,S,P,Si	~0.3
Si	Nuc eq.	3000	Co,Fe,Ni	<0.18

S. Smartt

Energy reservoirs

Higher burning stages

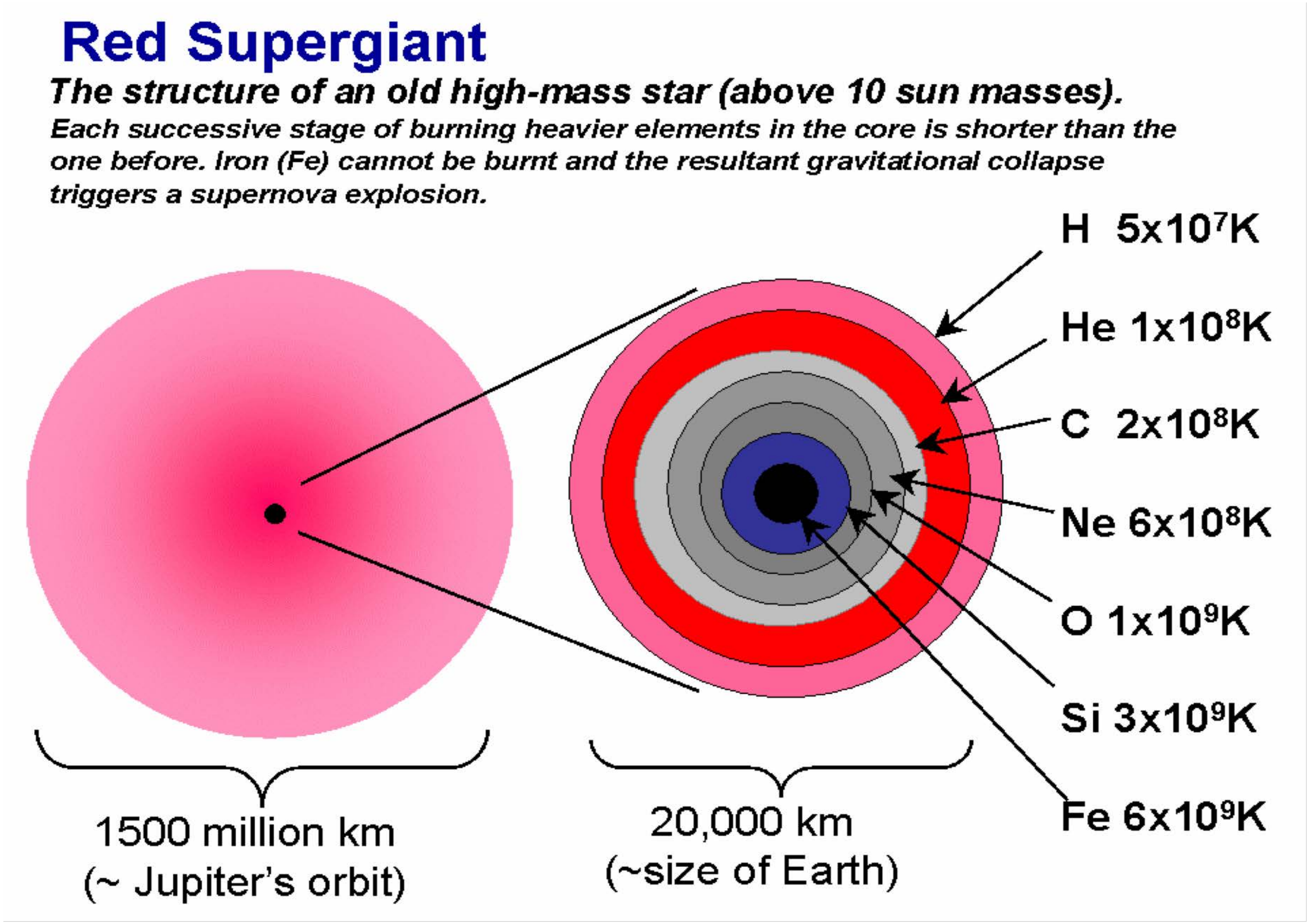
- Shell burning



Energy reservoirs

Higher burning stages

- Shell burning



Energy transport

"Follow the energy"

- **Energy is conserved but will be converted between different forms.**
  - **Source:** nuclear fusion as the by far dominant source (for the Sun: in the core)
  - **Transport** outwards
  - **Leaves** the star at the "surface", mostly in form of radiation

- **Energy transport mechanisms:**
  - **Radiation**: Photons carry energy as propagate through the star (emission/absorption).
  - **Convection**: Net rise of buoyant (hot) gas towards surface.
  - **Conduction**: Transfer of kinetic energy between gas particles during collisions

  - The efficiency / contribution of the different mechanisms depends on the local plasma conditions (such as density, opacity)

  - Conduction not important in the solar interior but in the corona!

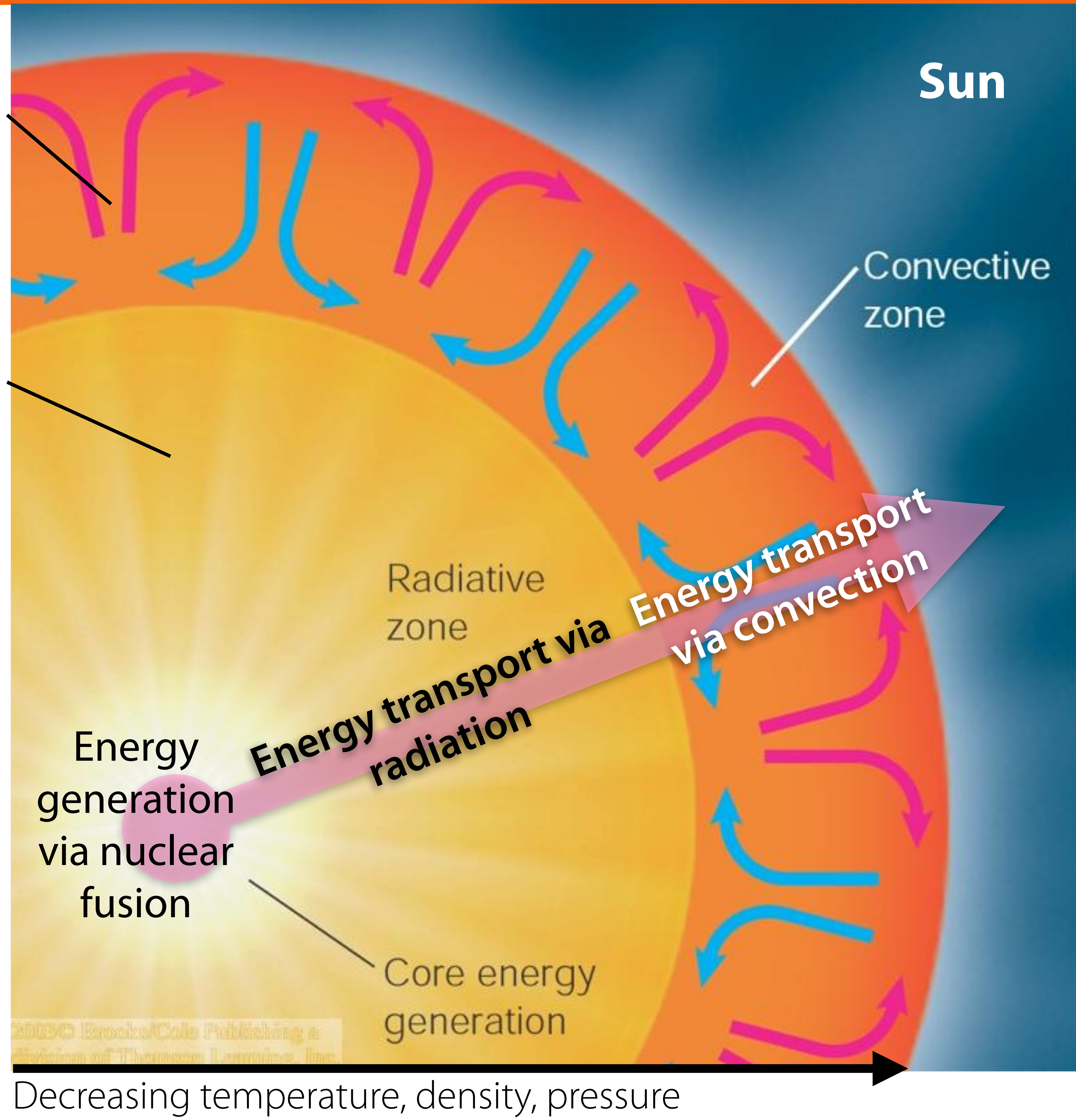
Energy transport

In the Sun and solar-like stars

- At any depth in the Sun:
  - Energy flux **F** defined as the luminosity per unit area.
  - Energy transport by radiation (F_R) and by convection (F_C)
  - Conduction insignificant in solar interior

$$F = F_{\mathrm{R}} + F_{\mathrm{C}} = L/4\pi r^2$$

➡ Basically the same structure for all stars with approx. 1 solar mass or less.



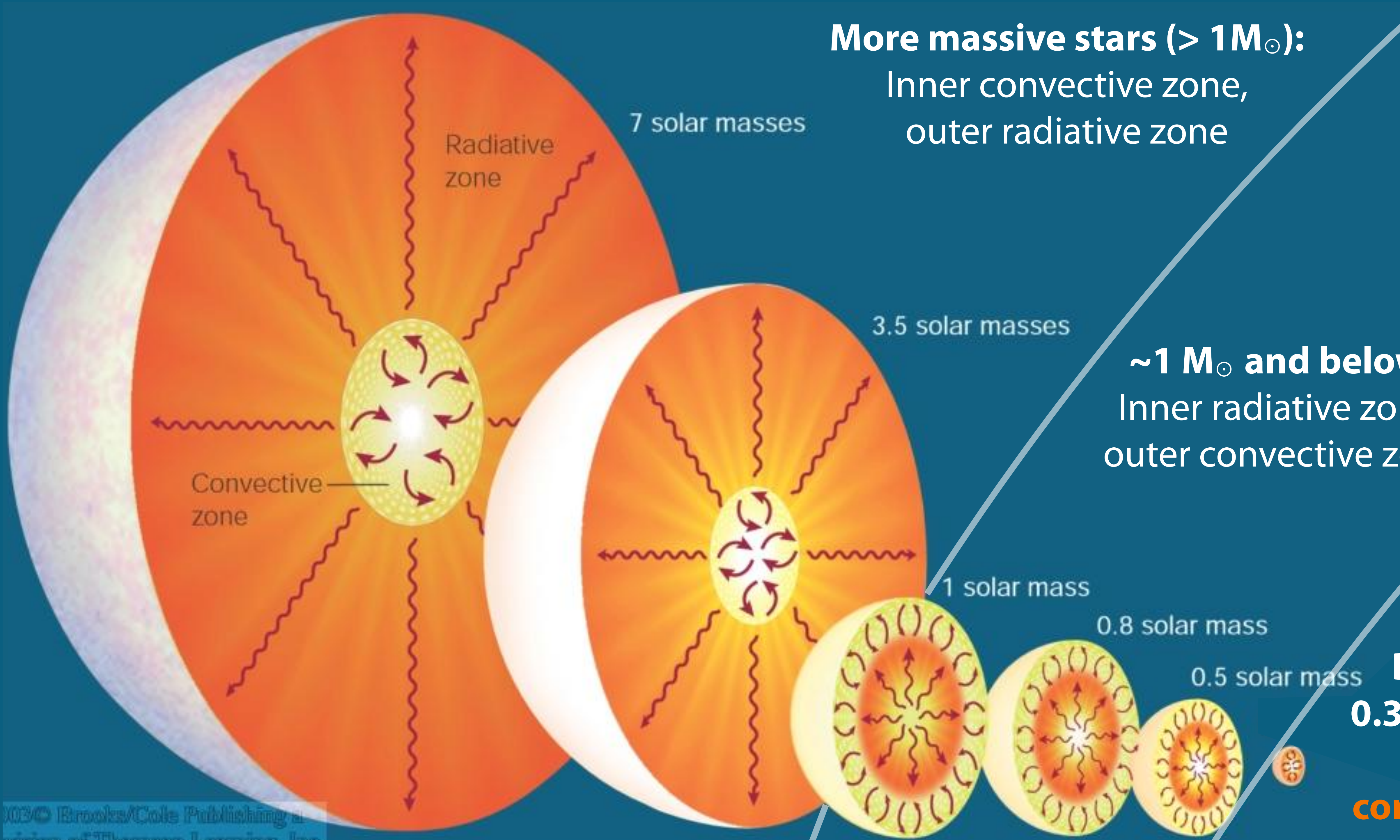
Energy transport

Along the main sequence

More massive stars (> 1M$_\odot$):
Inner convective zone,
outer radiative zone

~1 M$_\odot$ and below:
Inner radiative zone,
outer convective zone

Below 0.3-0.4 M$_\odot$: **Fully convective**

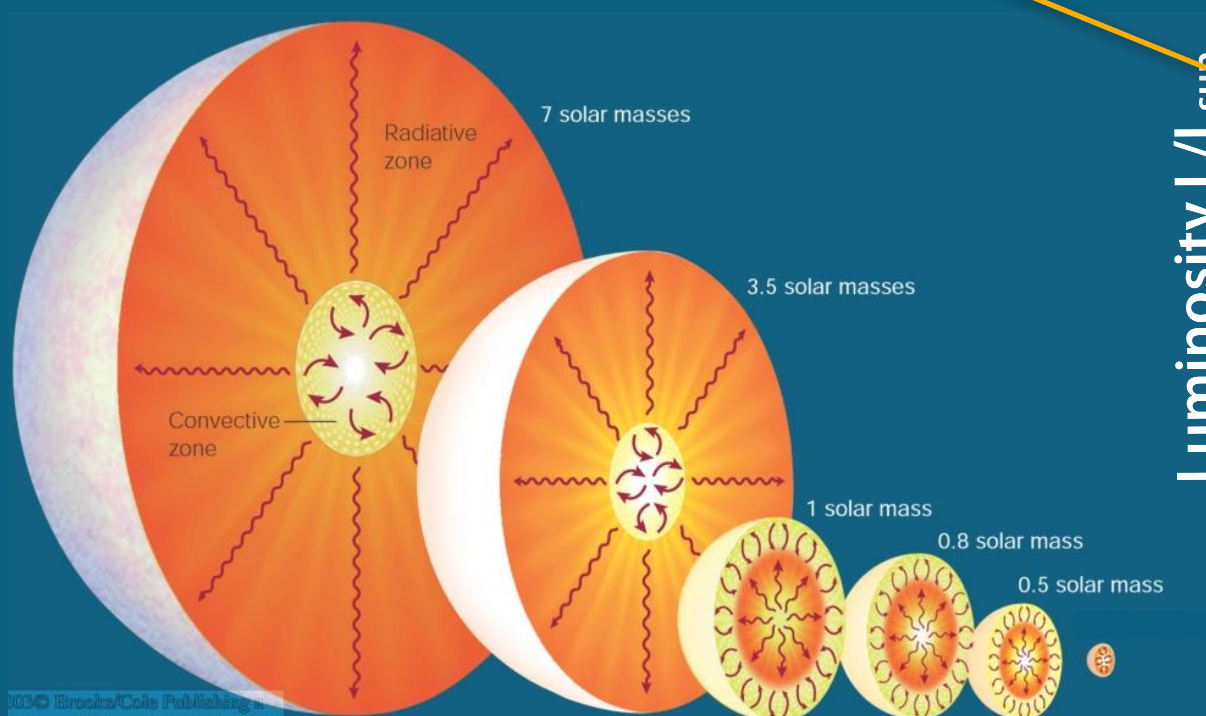


Dominant fusion reaction in stellar core	CNO cycle	pp chain

Energy transport

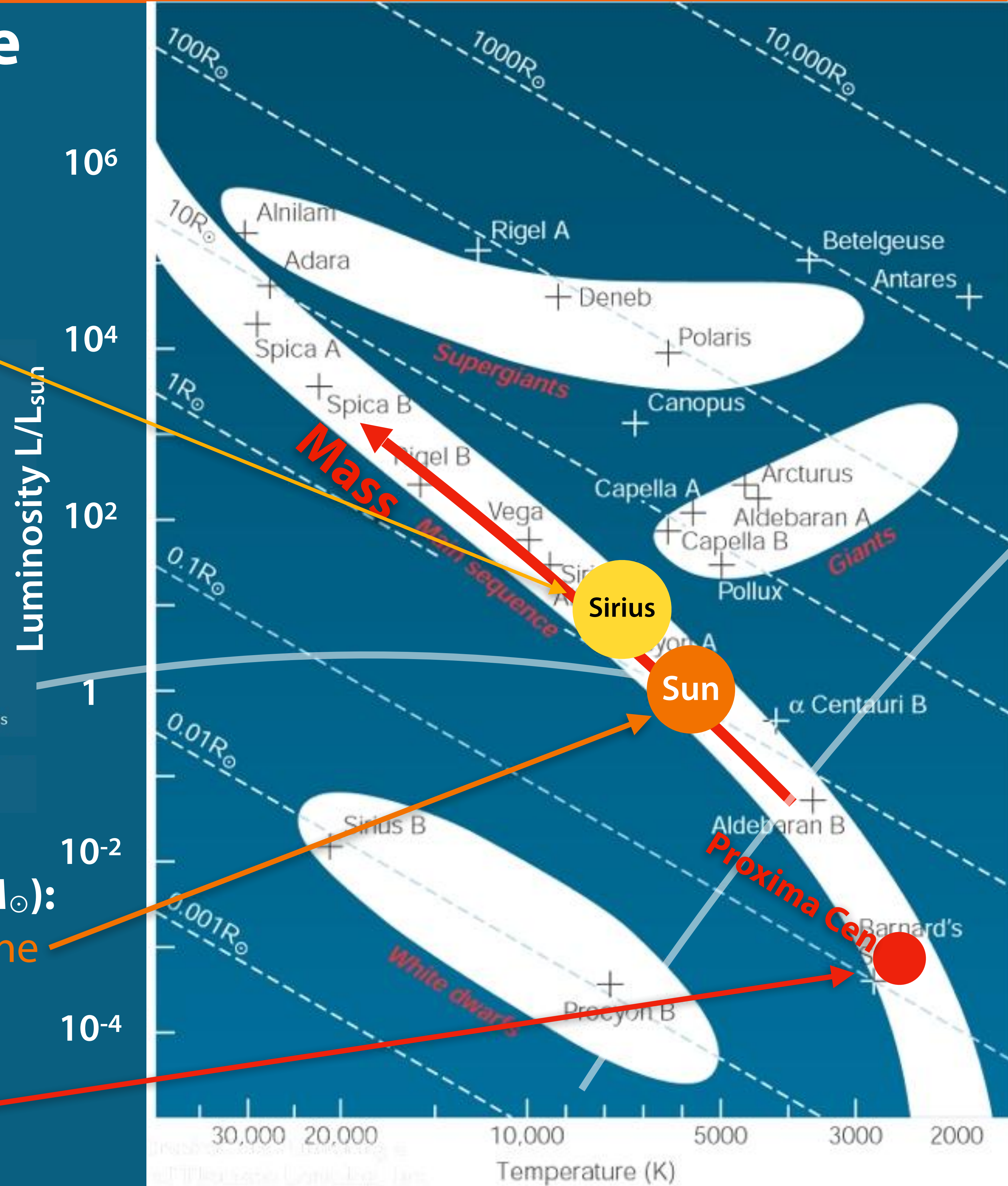
Along the main sequence

Early-type main sequence (> 1M$_\odot$)
Inner convective zone, outer radiative zone



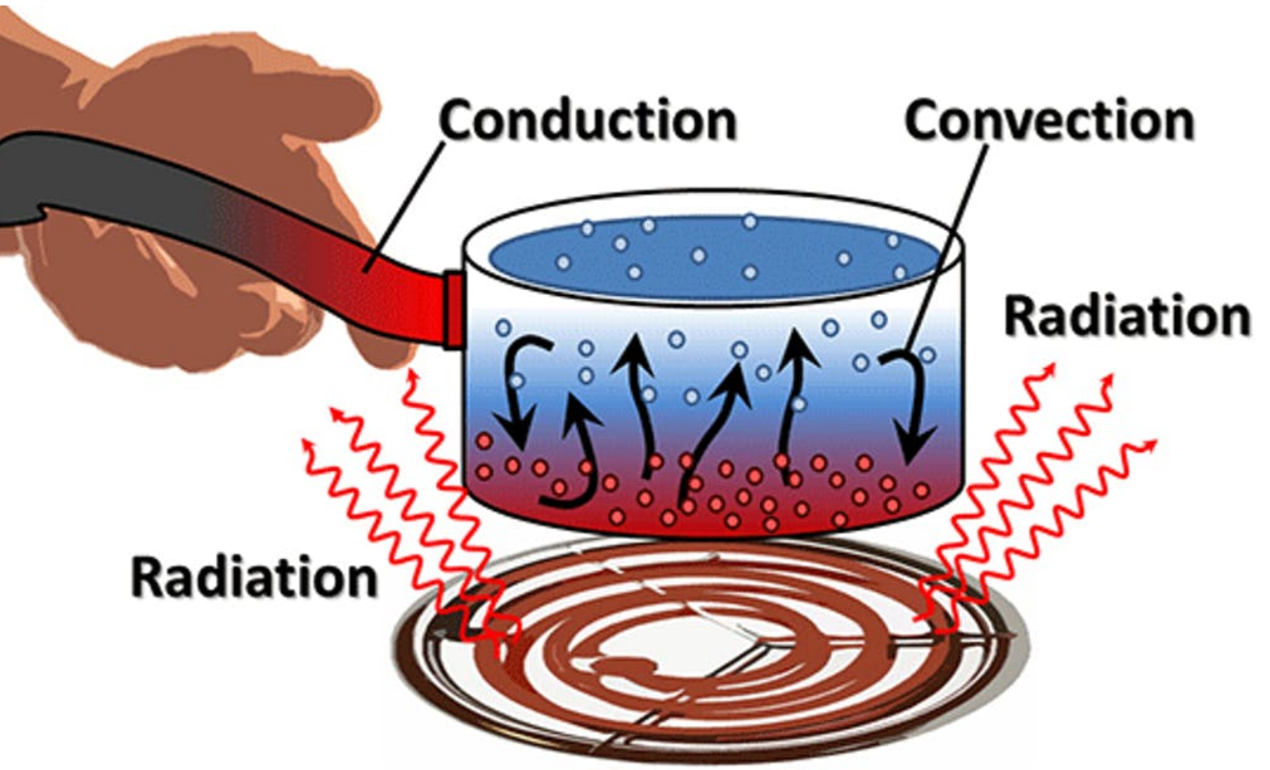
Late-type main sequence(~1M$_\odot$ — 0.4 M$_\odot$):
Inner radiative zone, outer convective zone

(Very) low-mass stars (M<0.3-0.4 M$_\odot$:
Fully convective interior



Energy transport

Overview

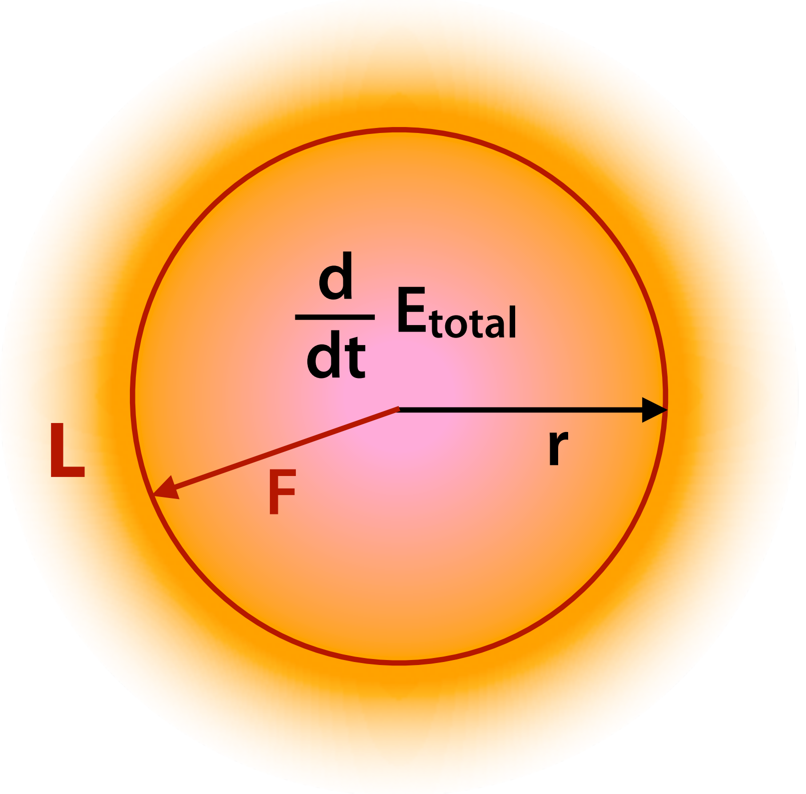


- **Radiation**: Photons carry energy, travels a distance between emission and being absorbed again
- **Conduction**: Particles carry energy, travels a distance between collisions with other particles (during which energy is exchanged)
- **Convection**: mass motion of elements of gas

- **Conditions** for the occurrence of the different modes of energy transport:
  - Conduction and radiation: whenever a temperature gradient is maintained.
  - Convection: only if the temperature gradient exceeds a critical value.

Energy transport

Overview



- **Energy flux** (outward) $F = F_{\mathrm{R}} + F_{\mathrm{C}} = L/4\pi r^2$
  - Conduction is negligible in the Sun
  - The contributions of convection and radiation change as function of radius
  - Luminosity: energy flux arriving and being emitted from the surface at radius r
  - Flux driven by a temperature gradient

- Note: **Neutrinos** produced by fusion carry energy
  - Carry comparatively little energy and thus neglected here (strictly speaking, the total energy production due to all sources balances the luminosity and neutrino flux)
  - Neutrinos escape from normal stars essentially without interaction with matter (but that is no longer true for very dense stars/ stellar remnants)
  - Strictly speaking: Energy production rate ε_ν for neutrinos should be taken into account

Energy transport

Reminder

- **Diffusion:** A general concept, time-dependent: **Net transport** of particles or energy
  - Driven by a corresponding gradient — towards equilibrium
  - Random microscopic motion

➡ (net) energy flux: $\boldsymbol{F} = -D\,\boldsymbol{\nabla} U,$

U: Energy density
D: Diffusion coefficient

- Gradient in energy density connected to temperature gradient:

$$\nabla U = (\partial U/\partial T)_V \nabla T = C_V \nabla T$$

➡ Heat conduction: $\boldsymbol{F} = -K\,\nabla T$ with $K = \frac{1}{3}\bar{v}\ell C_V$

$\bar{v}$: avg. velocity
l: mean free path
Cv: specific heat capacity per constant volume
K: conductivity

➡ valid for all particles in LTE, including gas particles but also photons

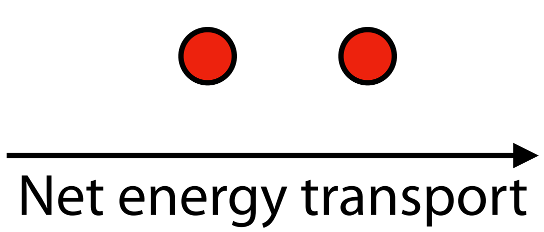
Energy transport

Reminder

Diffusion:

- A general concept, time-dependent: **Net transport** of particles or energy
- Driven by a corresponding gradient — towards equilibrium
- **Random microscopic motion**

- Diffusion of particles: **Conduction** Particles pass on their internal (kinetic/ potential) energy to neighbouring particles without moving over large distances
- **Radiative diffusion** via photons

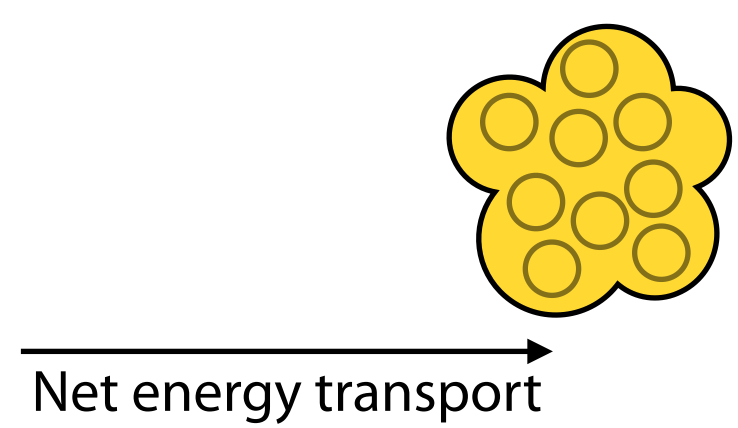


Advection:

- Particles move over longer distances (and transport heat); e.g. as part of a fluid with macroscopic (large-scale) motion

- **Macroscopic (bulk) motion** (particles/mass)

- Convection with macroscopic motion



Energy transport

Diffusive energy transport in stellar interiors

Conduction	vs	Radiation
Gas particles (electrons)		**Photons**
Energy carried by a typical particle: $E = {}^3/_2\, k\, T$	**Comparable**	**Energy** carried by a typical photon: $E = h\, c\, /\, \lambda$
Number density of particles	**>>**	Number density of photons.
Mean free path (between collisions) Typically 10^{-10} m	**<<**	Mean free path before being absorbed or scattered Typically 10^{-2} m

- Smaller number of **photons** is far outweighed by their **much larger mean free path**!

➡ Photons get easier from location with high temperature to one with lower temperature

➡ Larger transport of energy

➡ Radiation is the dominant energy transport mechanism in most stars.

➡ **Conduction negligible in the interiors** of (nearly all) main sequence stars.

⊙ **Conduction relevant in the solar corona**!

Energy transport

Radiative energy transport

- Mean free path of a photon very small in interior of stars
  ➡ Location where photon is emitted and location where photon absorbed have nearly same temperature
  ➡ Conditions of local thermodynamic equilibrium fulfilled
  ➡ Source function = Kirchhoff–Planck function

- **Radiative energy diffusion**

  ➡ $\boldsymbol{F} = -K\,\nabla T$ with $\boldsymbol{K} = \frac{1}{3}\bar{v}\ell\, C_V$

  - Velocity $\bar{\boldsymbol{v}} = c$
  - Energy density $U = aT^4$

  ➡ $C_V = dU/dT = 4\,a\,T^3$ with the radiation constant $a = \frac{8\pi^5 k^4}{15h^3c^3} = 7.56\times 10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}}$.

  ➡ How do we derive the free mean free path?

Energy transport

Radiative energy transport

- Local luminosity $l\ (r)$:
  rate at which energy (as heat) flows outward through a sphere of radius r

- In spherical symmetry: l related to radial energy flux F

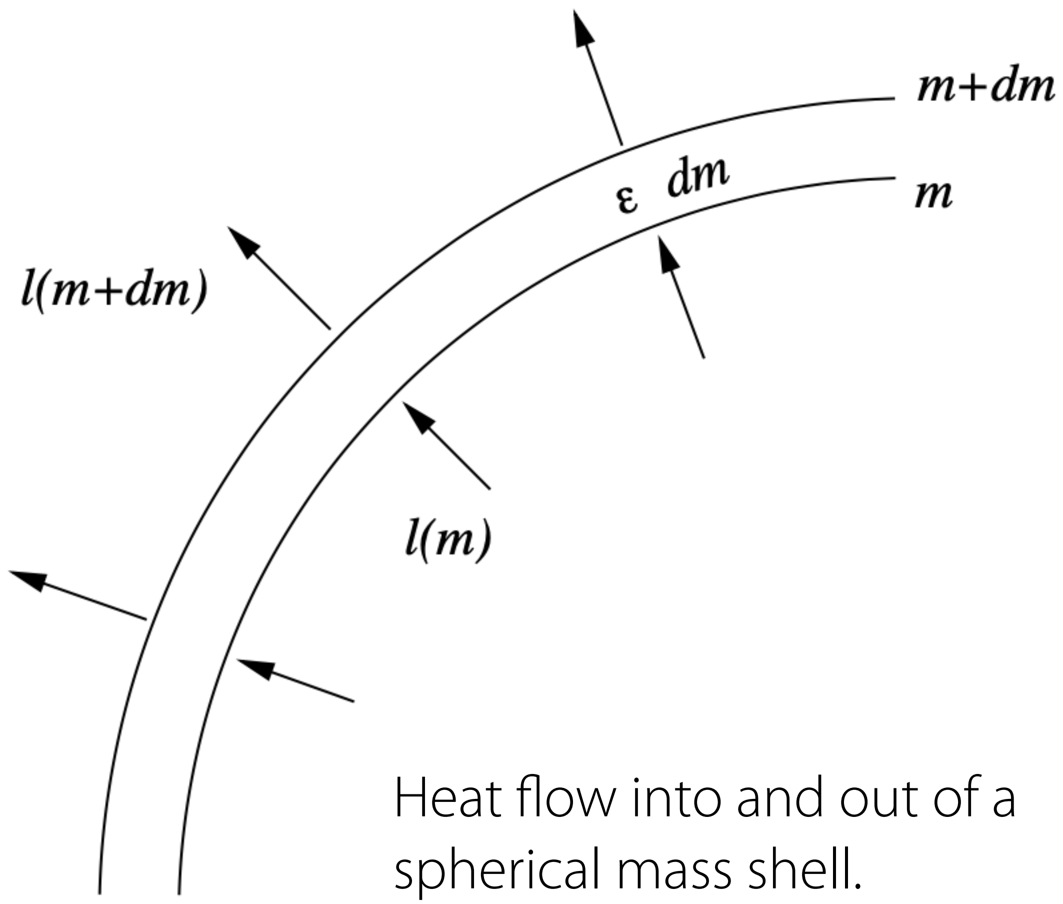
$$l\ (r) = 4\pi r^2\ F$$

  - At the surface: $l = L$
  - At the centre: $l = 0.$

- Normally heat flows outwards, in direction of decreasing temperature (gradient!)

  ➡ l is usually positive

- l negative under special circumstances (e.g. neutrino emission cooling the core)



Heat flow into and out of a spherical mass shell.

Energy transport

Radiative energy transport

- Considered here: Energy transport only by radiation
  - If mean free path of photons short, radiative energy transport as diffusion process
  - ➡ Radiative transfer handled with **diffusion approximation**
- Radiative transfer equation $$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu \rho I_\nu$$

  ➡ Intensity I_ν diminished over distance s (in absence of emission)

  ➡ **mean free path** = distance over which the intensity decreases by a factor of e

$$\ell_{\mathrm{ph}} = \frac{1}{\kappa\rho}$$

κ: opacity

ρ: mass density

➡ Radiative conductivity $$K_{\mathrm{rad}} = \frac{4}{3}\frac{acT^3}{\kappa\rho}$$

➡ Radiative energy flux $$\boldsymbol{F}_{\mathrm{rad}} = -K_{\mathrm{rad}}\,\boldsymbol{\nabla}T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\boldsymbol{\nabla}T.$$

Energy transport

Radiative energy transport

- Radiative energy flux $\boldsymbol{F}_{\rm rad} = -K_{\rm rad}\,\boldsymbol{\nabla} T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\boldsymbol{\nabla} T.$
- With $F_{rad} = l \,/\, 4\pi r^2$ in spherical geometry (radius r): $\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi acT^3}\frac{l}{r^2}$
- With the equation for mass conservation $\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2\rho} \Longrightarrow \frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$

➡ Temperature gradient required to carry the entire luminosity *l* by radiation.

- A region with this gradient = in radiative equilibrium ($\Longrightarrow$ radiative zone).