AST5770

Solar and stellar physics

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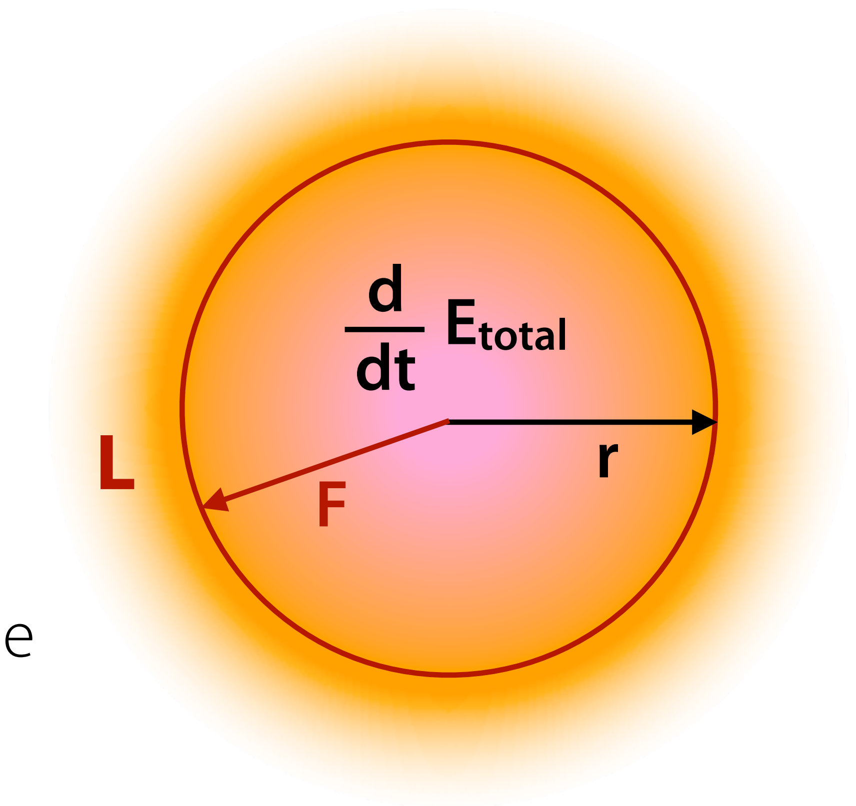
Recap

Energy transport

- **Energy flux** (outward) $F = F_{\mathrm{R}} + F_{\mathrm{C}} = L/4\pi r^2$
  - Conduction is negligible in the Sun
  - The contributions of convection and radiation change as function of radius
  - **Luminosity**: energy flux arriving and being emitted from the surface at radius r
  - **Flux driven by a temperature gradient**



- **Energy transport mechanisms**
  - **Diffusion** — microscopic
    - Radiative energy transport (photons carry energy)
    - Conduction (particle collisions, not important in the solar interior)
  - **Convection** — macroscopic bulk motion of gas

➡ (net) energy flux:

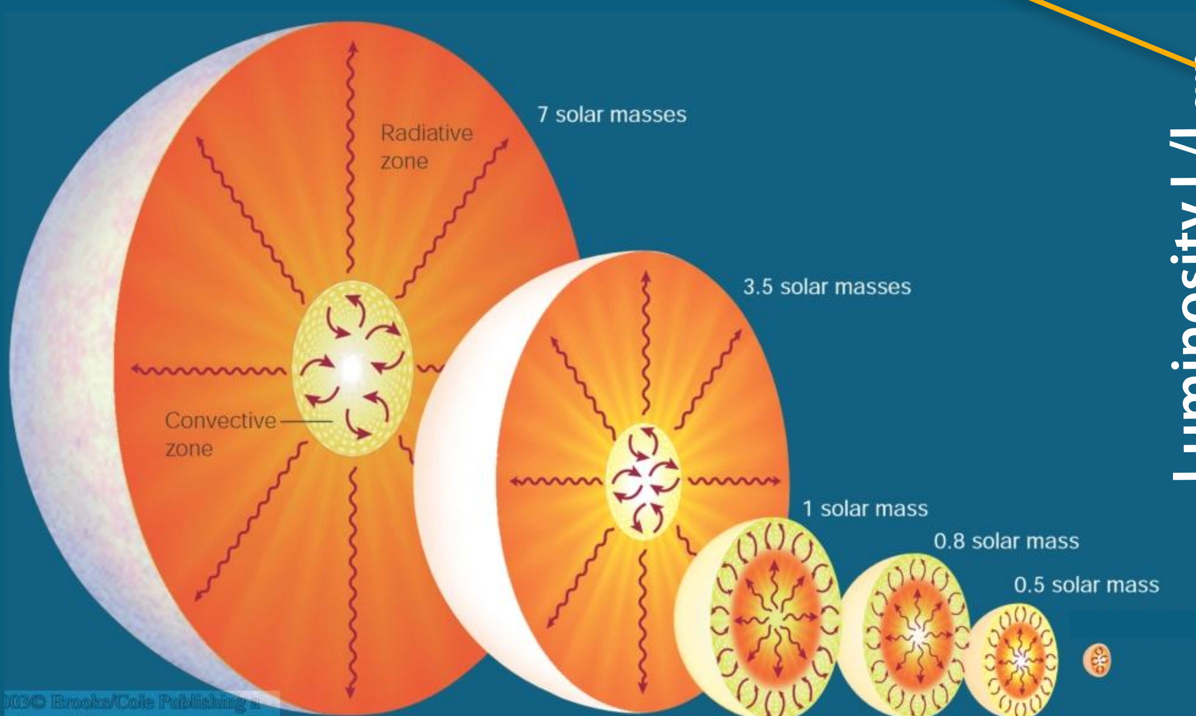
$$\boldsymbol{F} = -D\,\nabla U,$$

U: Energy density
D: Diffusion coefficient

Recap

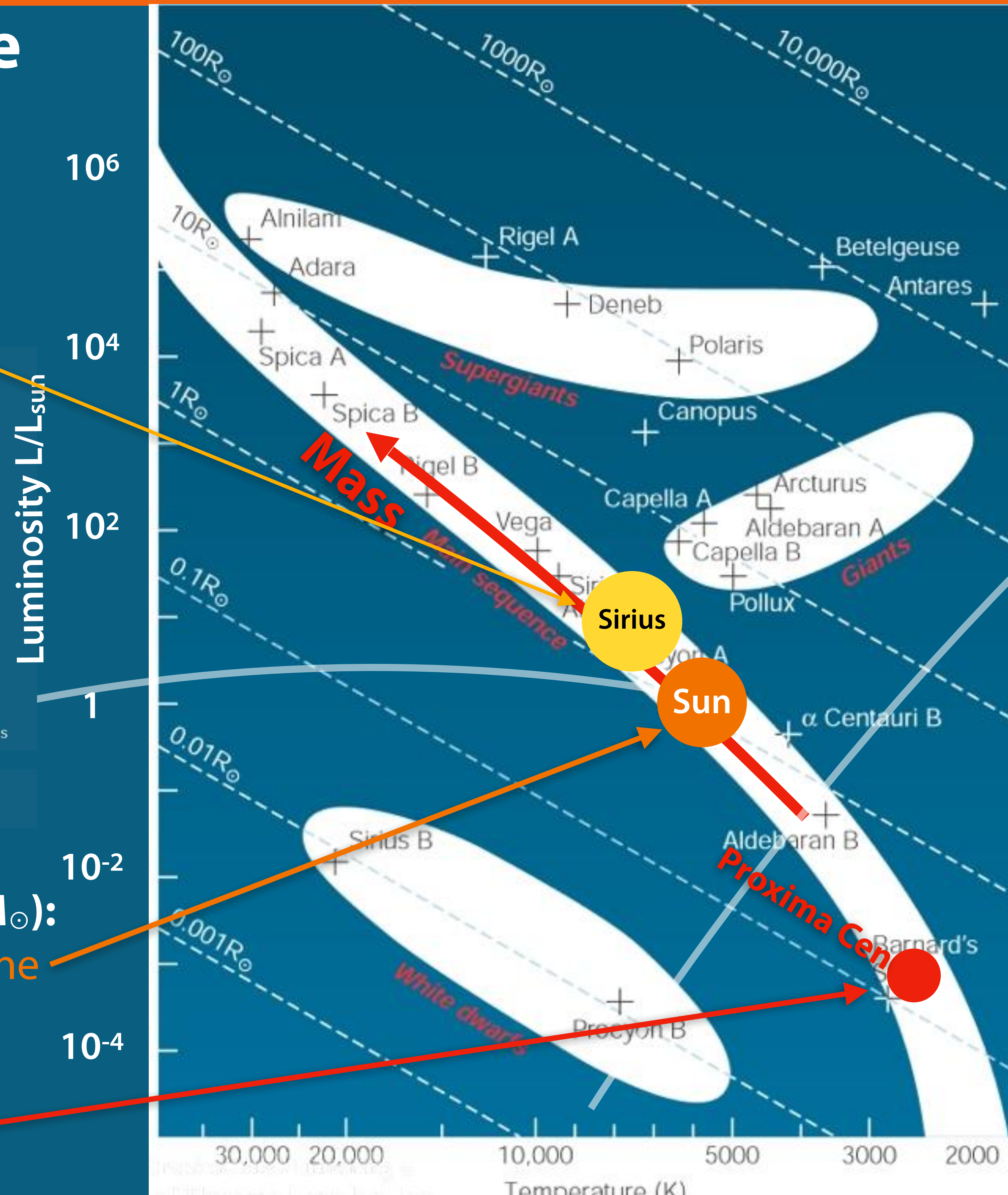
Along the main sequence

Early-type main sequence (> 1M$_\odot$)
Inner convective zone,
outer radiative zone



Late-type main sequence(~1M$_\odot$ — 0.4 M$_\odot$):
Inner radiative zone, outer convective zone

(Very) low-mass stars (M<0.3-0.4 M$_\odot$:
Fully convective interior



Recap

Radiative energy transport

- **mean free path** of a photon = distance over which the intensity decreases by a factor of e

$$\ell_{\rm ph} = \frac{1}{\kappa\rho}$$

κ: opacity, ρ: mass density

➡ See radiative transfer equation

- If mean free path of photons short, radiative energy transport as diffusion process

➡ Radiative transfer handled with **diffusion approximation**

➡ Radiative energy flux

$$\boldsymbol{F}_{\rm rad} = -K_{\rm rad}\,\nabla T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\nabla T.$$

- With $F_{rad} = l / 4\pi r^2$ in spherical geometry (radius r):

$$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi a c T^3}\frac{l}{r^2}$$

- With the equation for mass conservation $\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \implies$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$$

➡ **Temperature gradient required to carry the entire luminosity *l* by radiation.**

- A region with this gradient = in radiative equilibrium ($\implies$ radiative zone).

Energy transport

Radiative energy transport

- **Temperature gradient for radiative energy transport** $$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$$

- **Note**:
  - Valid only if conditions for **LTE** are fulfilled (requires short mean free paths, much shorter than the radius, $l_{ph} \ll R$)
  - Not valid if l_{ph} becomes much longer (at the surface, near photosphere where photons escape into space)

    ➡ Diffusion approximation is no longer valid

    ➡ Solution of full equations of radiative transfer necessary!

- **In practice (in simulations):**
  - Stellar interiors can be handled with the diffusion approximation up to some depth below the surface.

    ➡ Computationally much cheaper!
  - Surface-near layers + atmosphere to be treated with full radiative transfer equations!

    ➡ Computationally much more demanding!

Energy transport

Radiative energy transport

- In hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}m} = -\frac{Gm}{4\pi r^4}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}$$

$$\frac{\mathrm{d}T}{\mathrm{d}m} = \frac{\mathrm{d}P}{\mathrm{d}m}\cdot\frac{\mathrm{d}T}{\mathrm{d}P} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\cdot\frac{\mathrm{d}\log T}{\mathrm{d}\log P}$$

- **Radiative temperature gradient**

$$\nabla_{\mathrm{rad}} = \left(\frac{\mathrm{d}\log T}{\mathrm{d}\log P}\right)_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa l P}{m T^4}$$

  - Describes logarithmic variation of T with depth for a star in hydrostatic equilibrium and pure radiative energy transport (with pressure as depth coordinate)

Energy transport

Radiative energy transport

- **Frequency-dependence:** Radiative flux in frequency interval *[ν, ν + dν]*: $F_\nu d\nu$

➡ $\boldsymbol{F}_\nu = -D_\nu \nabla U_\nu = -D_\nu \frac{\partial U_\nu}{\partial T} \nabla T$ with $D_\nu = \frac{1}{3} c \ell_\nu = \frac{c}{3\kappa_\nu \rho}$

➡ Integral over all frequencies: $\boldsymbol{F} = -\left[\frac{c}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T}\,\mathrm{d}\nu\right]\nabla T.$

➡ Can be written (as before) as $\boldsymbol{F} = -K_{rad}\, \nabla T$

but radiative conductivity needs to look like this now: $K_{\mathrm{rad}} = \frac{c}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T}\,\mathrm{d}\nu.$

➡ Proper average of opacity κ_ν needed! $\frac{1}{\kappa} = \frac{1}{4aT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T}\,\mathrm{d}\nu.$

$4aT^3 = \int_0^\infty (\partial U_\nu/\partial T)\,\mathrm{d}\nu$

➡ Energy density U_ν in same frequency interval proportional to **Planck function**! $U_\nu \propto B_\nu$

Energy transport

Radiative energy transport

- **Rosseland mean absorption coefficient**

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT}\, d\nu}{\int_0^\infty \frac{dB_\nu}{dT}\, d\nu}$$

Used for the radiative temperature gradient

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac} \frac{\kappa l}{r^4 T^3}$$

Energy transport

Radiative energy transport

- **<u>Rosseland</u> mean absorption coefficient**

$$\frac{1}{\kappa_R} = \frac{\int\limits_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int\limits_0^\infty \frac{dB_\nu}{dT} d\nu}$$

- Weighted with $1/\kappa_\nu$ $\Longrightarrow$ More energy transported at frequencies where the matter is more transparent.
- Weighted with dB_ν/dT $\Longrightarrow$ More energy transported at frequencies where the radiation field is more temperature-dependent (stronger gradients).
- κ essentially as an inverse conduction coefficient!

Energy transport

Conductive energy transport

- Collisions between the gas particles (ions and electrons) can also transport heat.
- Energy flux due to heat conduction (equivalently to radiative energy flux)

$$\boldsymbol{F}_{cd} = -K_{cd}\,\nabla T$$

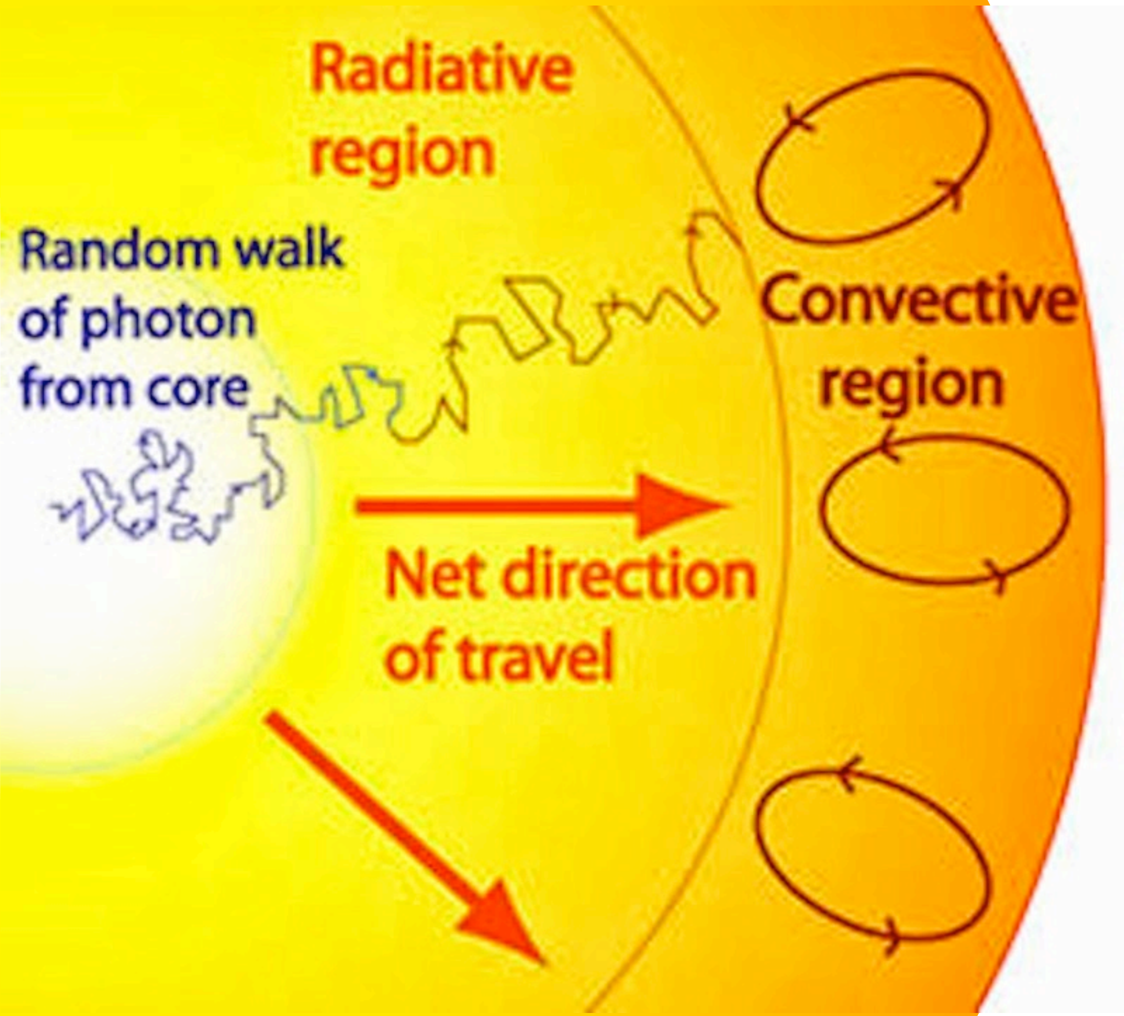
- Conductive and radiative energy flux can be combined:

$$\boldsymbol{F} = \boldsymbol{F}_{\rm rad} + \boldsymbol{F}_{\rm cd} = -(K_{\rm rad} + K_{\rm cd})\,\nabla T$$

- Define an equivalent conductive opacity $K_{\rm cd} = \dfrac{4acT^3}{3\kappa_{\rm cd}\,\rho}$
- Combined energy flux $\boldsymbol{F} = -\dfrac{4acT^3}{3\kappa\rho}\boldsymbol{\nabla}T$ with $\dfrac{1}{\kappa} = \dfrac{1}{\kappa_{\rm rad}} + \dfrac{1}{\kappa_{\rm cd}}$
- Transport mechanism with largest flux dominates
(= mechanism for which the plasma is more transparent)

Energy transport

Radiative energy transport



- Time for a photon to travel from centre to surface without interaction: ~2s
- **But: mean free path of photons very small!**
- In the dense solar interior: Mean free path of a photon only 10^{-2} m
  - Random walk, photon absorbed and re-emitted ~10^{22} times before reaching surface

  ➡ Time ~ thermal timescale of the Sun **~ 2 10^{7} yr**

  ➡ Observed radiation due to fusion reactions (on average) tens of millions of years ago.

  - Net direction towards surface due to gradients (opacity)

Energy transport

Radiative energy transport

- **Radiation pressure** $P_{\rm rad} = {}^1/_3 \, a \, T^4$
- Outward force that must be smaller than gravitational force in order to maintain hydrostatic equilibrium (HE)!

$$\left|\frac{dP_{\rm rad}}{dr}\right| < \left|\left(\frac{dP}{dr}\right)_{\rm HE}\right| \quad \Rightarrow \quad \frac{\kappa\rho}{4\pi c}\frac{l}{r^2} < \frac{Gm\rho}{r^2}$$

➡ Upper limit to the local luminosity: (local) **Eddington luminosity:** Maximum energy flux that can be carried by radiation

$$l < \frac{4\pi c G m}{\kappa} = l_{\rm Edd}$$

➡ Can get violated by intense nuclear burning

➡ In these situations, radiative energy transport insufficient for maintaining hydrostatic equilibrium

- At the surface (m=M): $L < L_{\rm Edd} = \frac{4\pi c G M}{\kappa}$

  (κ ↑ In photosphere)

Condition violated?

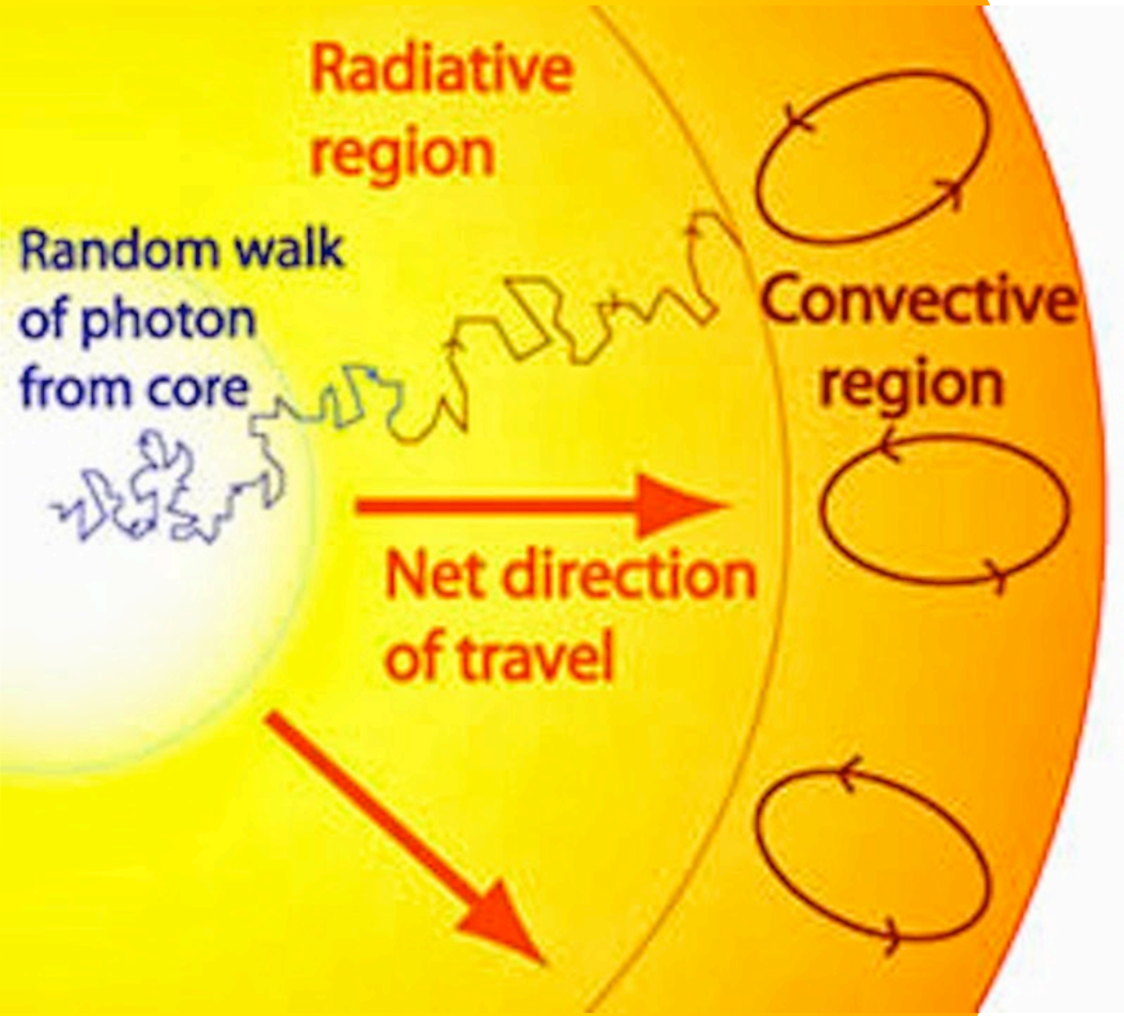
➡ No hydrostatic equilibrium!

➡ Gas accelerated outwards due to radiative pressure.

➡ Can potentially lead to mass loss!

Energy transport

Radiative energy transport



- Note: At high temperature all atom are completely ionised
  ➡ Photons move through a plasma consisting of free electrons and atomic nuclei (incl. protons)
- Towards surface (in solar-like stars): Not all atoms are completely ionised anymore
  ➡ Changes in mean molecular weight and gradients
  ➡ Convection becomes the dominant mode of energy transport subject to stability criterion
- Note: Radiative energy transport the "default" in convectively stable regions

Energy transport

Convection

$$F = F_{\mathrm{rad}} + F_{\mathrm{c}} + F_{\mathrm{cd}} = L / 4\pi r^2$$

$$F_{\mathrm{rad}} \propto \nabla T$$

- Transport for larger energy flux (luminosity) requires larger temperature gradient required.
- But: Upper limit to ∇T in stellar interior
  - ➡ Stratification becomes unstable if limit exceeded
  - ➡ (Additional) dominating energy transport via convection

Energy transport

Surface convection — granulation

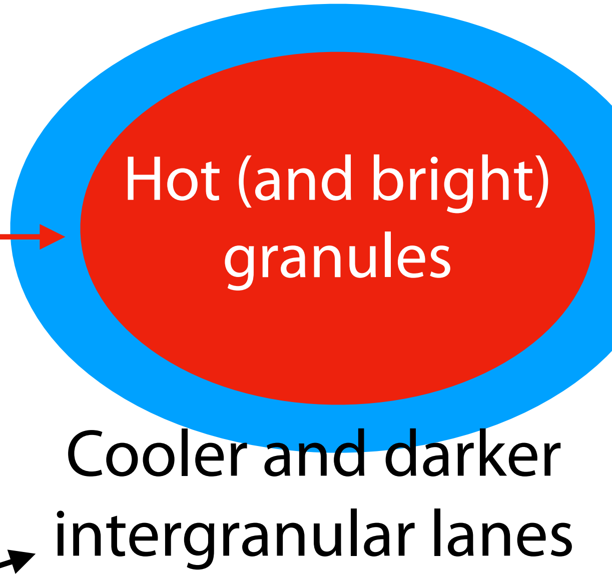
- Density, temperature decreases with radius
- Eventually plasma transparent enough (longer mean free path)

➡ **Radiation effectively removes heat from rising convective cells** at surface

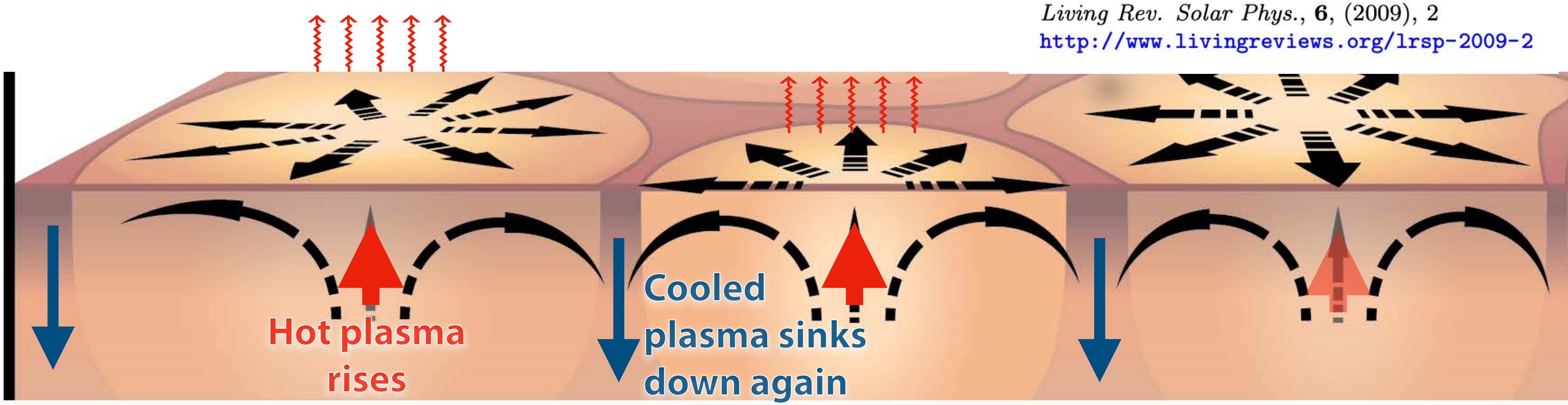
➡ Plasma cools

➡ Advected sideways (pushed away from more upwelling gas below)

➡ Cooled and dense **plasma sinks down again**



Living Rev. Solar Phys., **6**, (2009), 2
http://www.livingreviews.org/lrsp-2009-2



Top of convection zone — energy transport via convection (bulk motion)

Energy transport

Literature: Nordlund, Stein, Asplund
Living Rev. Solar Phys., 6, (2009), 2
http://www.livingreviews.org/lrsp- 2009- 2

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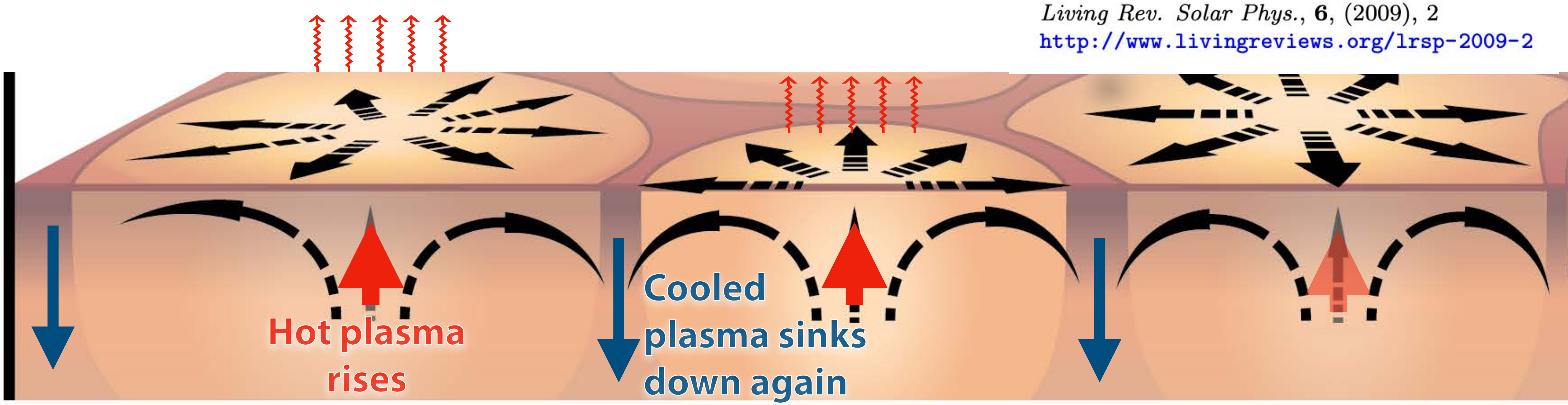
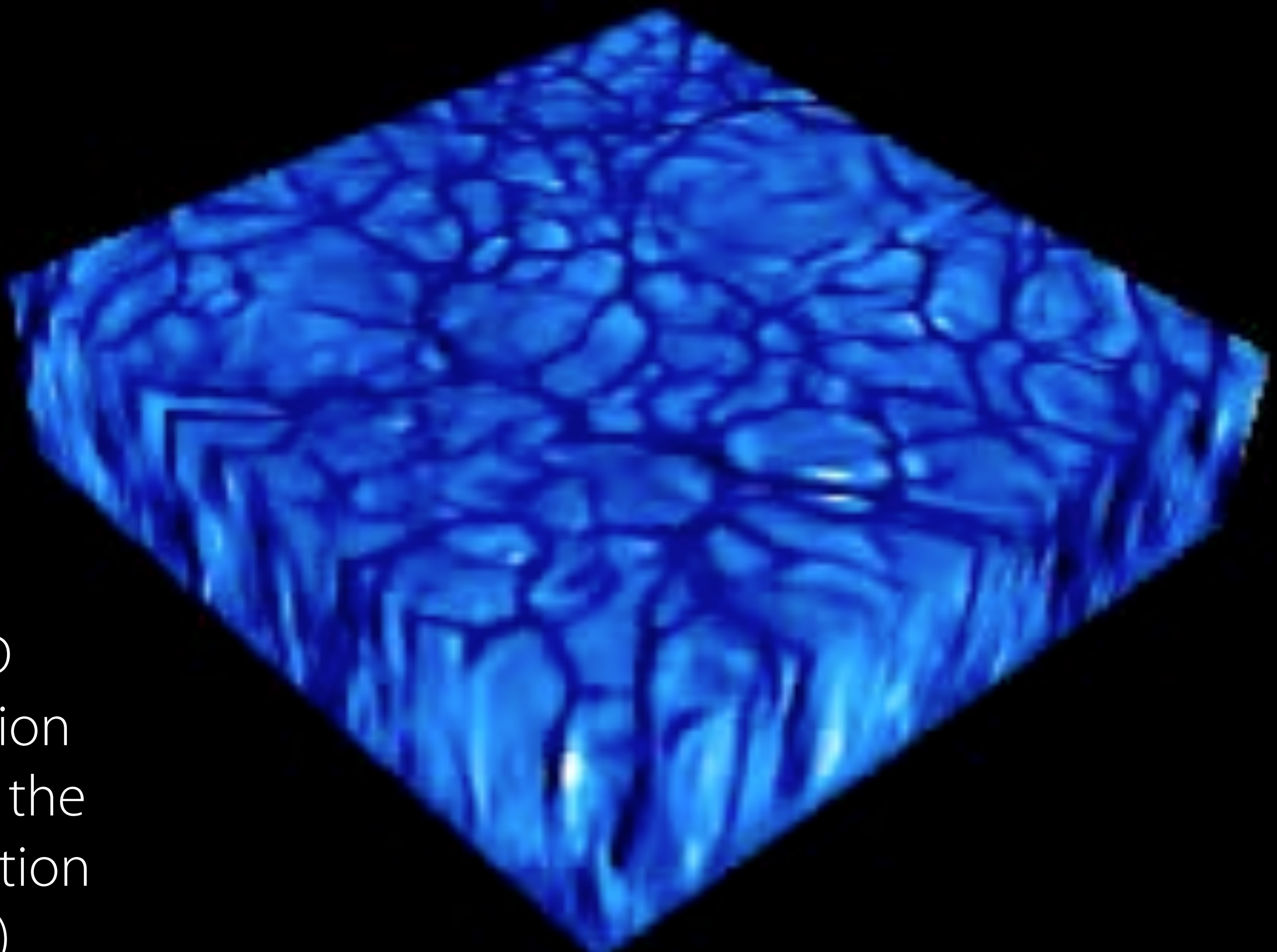


Top of convection zone — energy transport via convection (bulk motion)

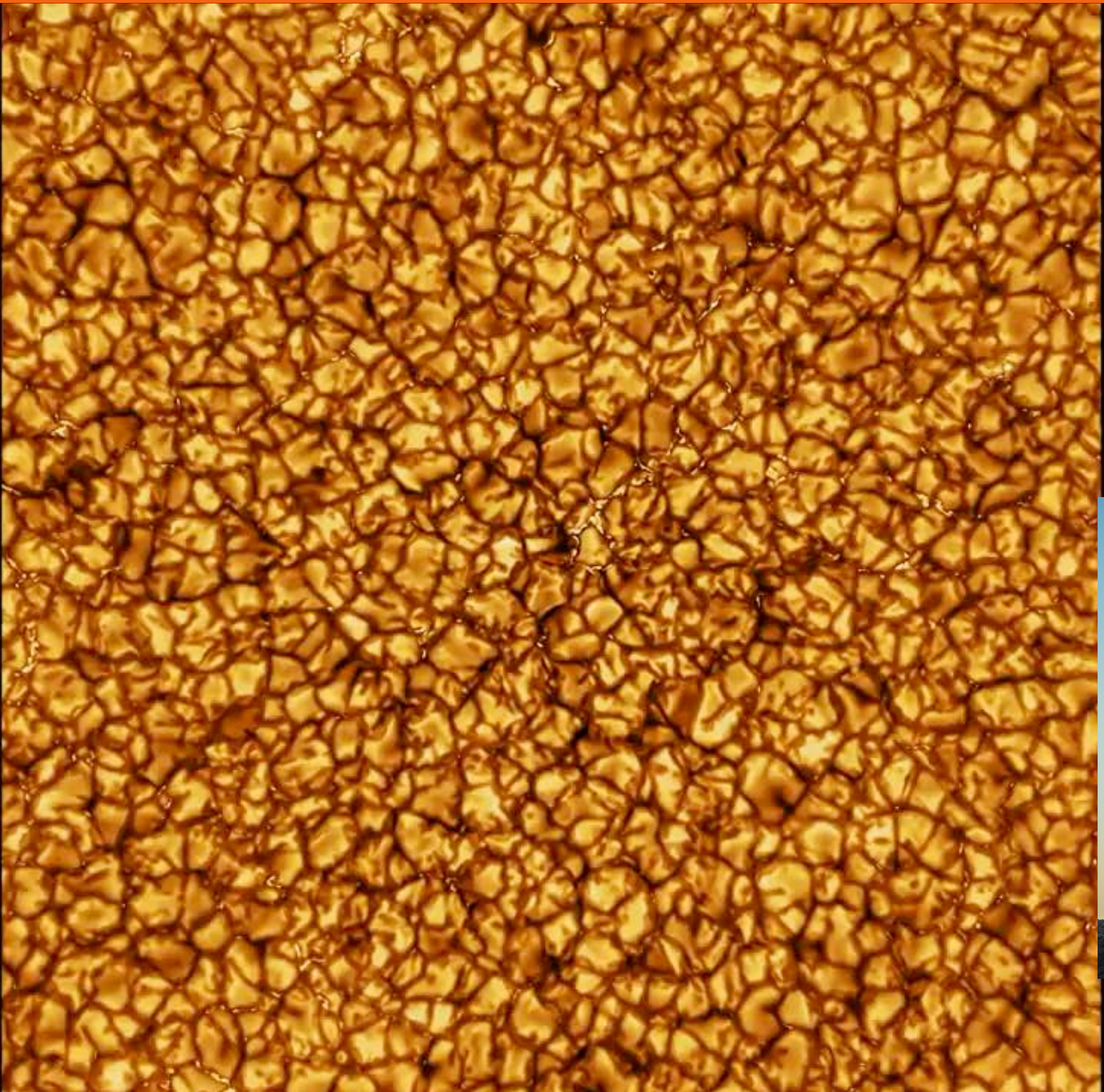
Energy transport

Convection — Solar surface convection



Time-dependent 3D hydrodynamic simulation showing convection at the top of the Sun's convection zone (at the surface)

Energy transport



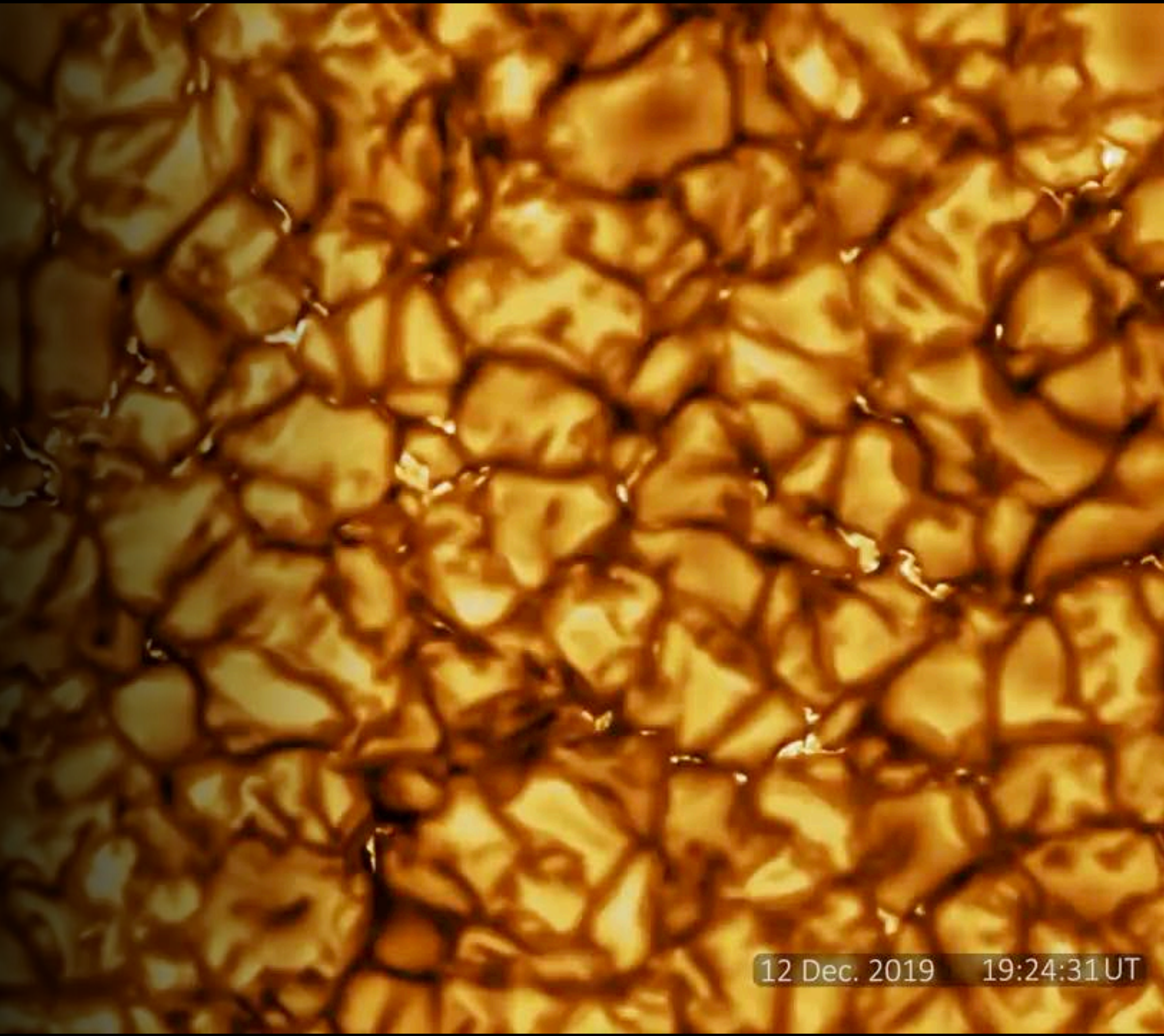
Highest-resolution
observations of the Sun's
granulation ever taken.
DKIST (4m)
(NSO/AURA/NSF)

10 Dec. 2019 19:24:31 UT

Energy transport

Surface convection — solar granulation

Highest-resolution observations of the Sun's granulation ever taken (DKIST, NSO/AURA/NSF)



Granules

- Diameters ~1000-2000km
- Lifetime ~8min

Bright features in inter-granular lanes

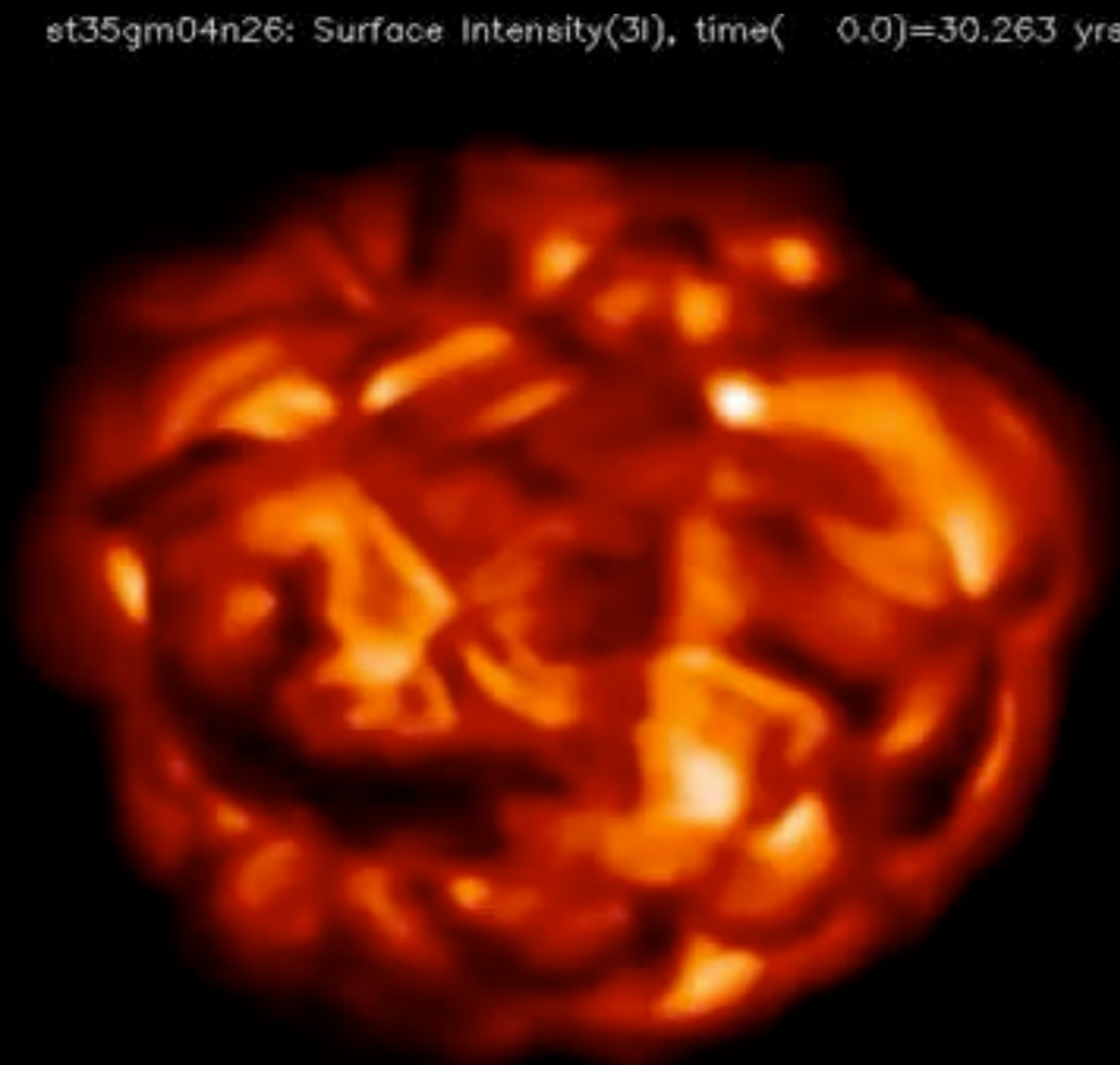
- (Mostly) due to magnetic field concentrations

Energy transport

Convection

Time-dependent 3D hydrodynamic simulation of **Betelgeuse**, here **intensity**

- Large convection cells
- Slower temporal evolution (Note time at top right)



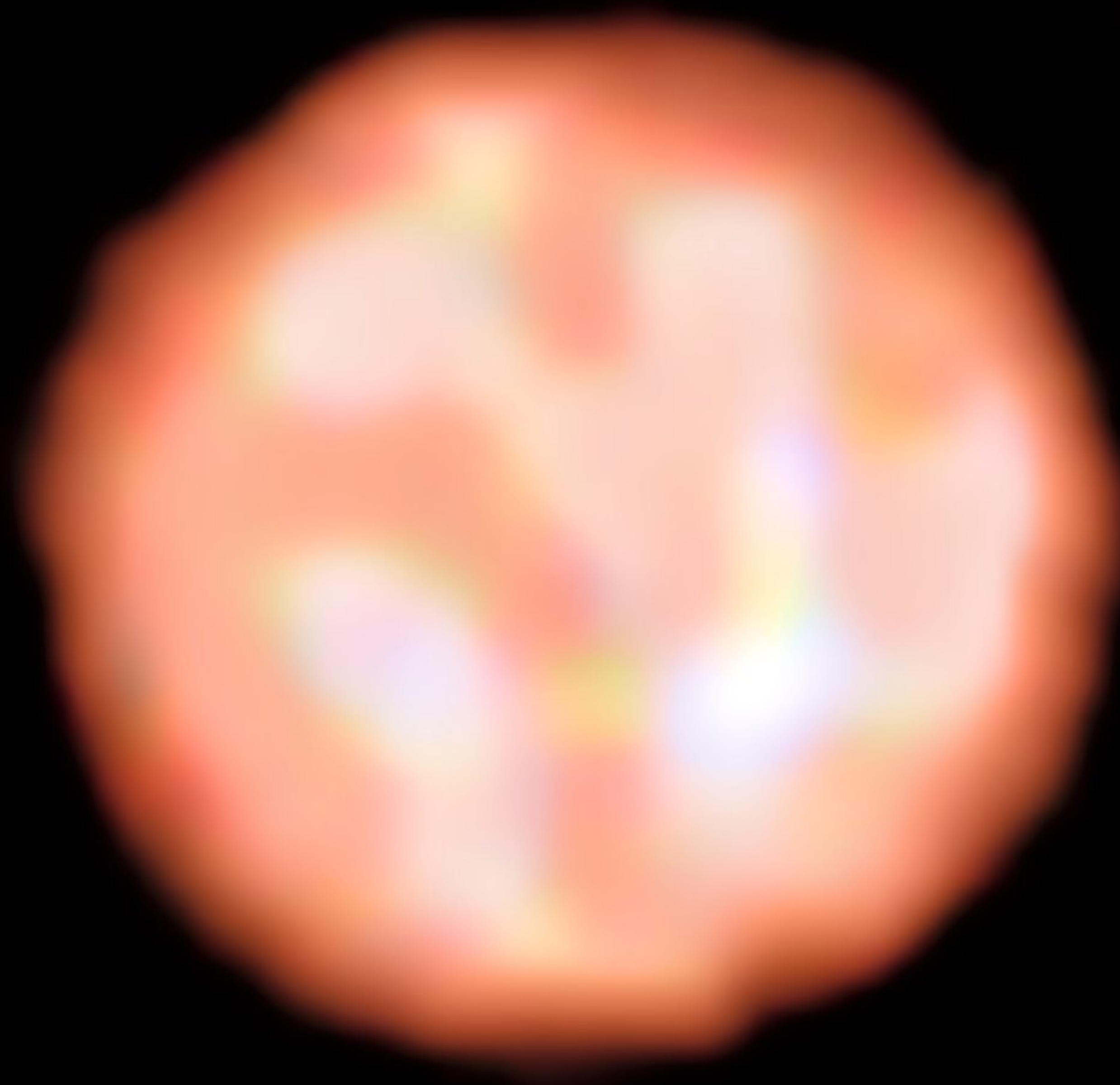
Simulation produced with the same code as the models for the project assignment (Freytag et al. 2012)

Freytag

Energy transport

Surface convection on other stars

- Observations of the cool red giant π^1 Gruis (with PIONIER/VLT)
- Spectral type M1I
- 1.5 solar masses
- 350 times the diameter of the Sun
- Few giant convection cells

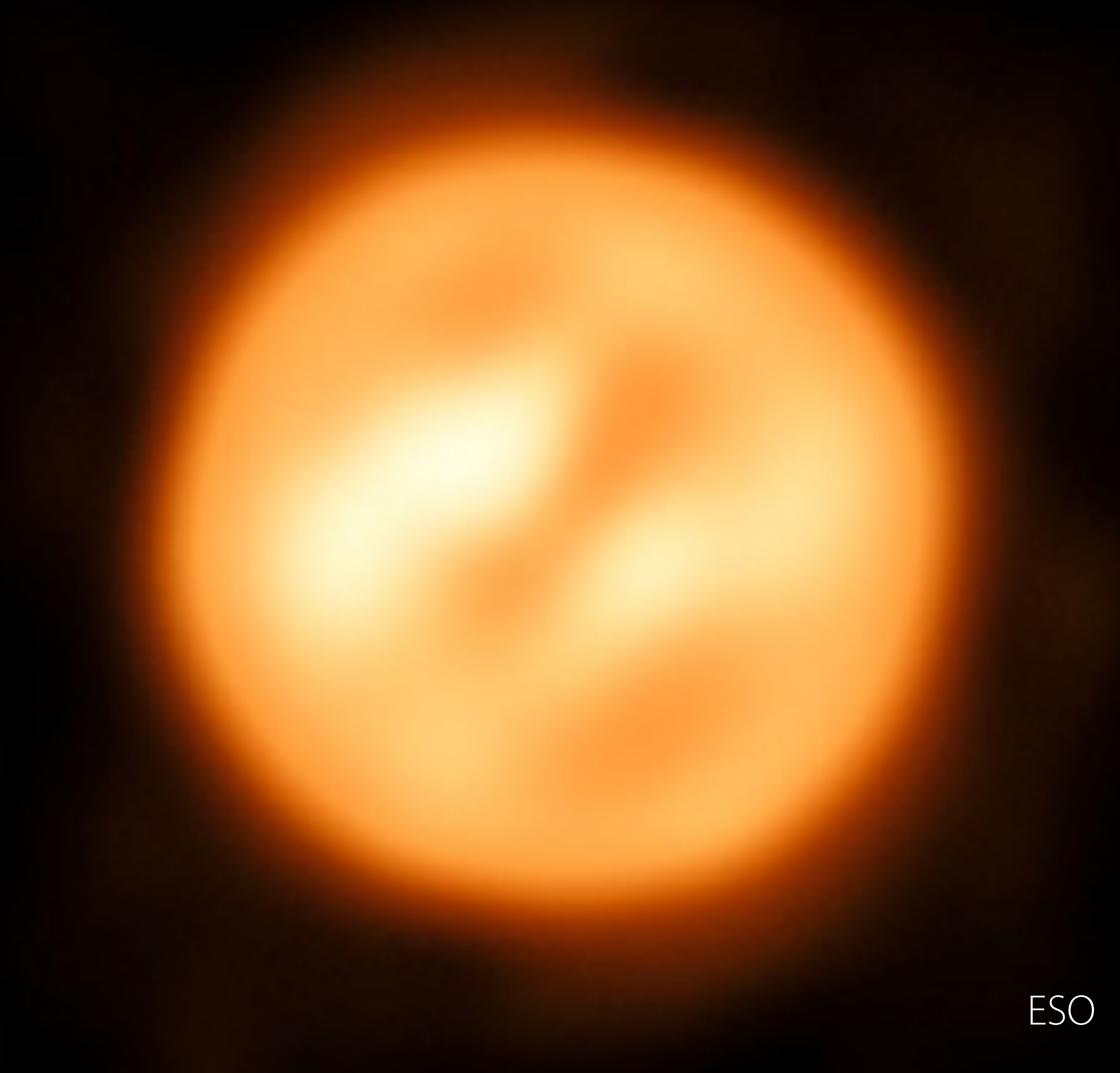


ESO

Energy transport

Surface convection on other stars

- Observations of the cool red supergiant Antares (with AMBER/VLTI)
- Spectral type M1.5I
- ~11-14 solar masses
- ~700 times the diameter of the Sun

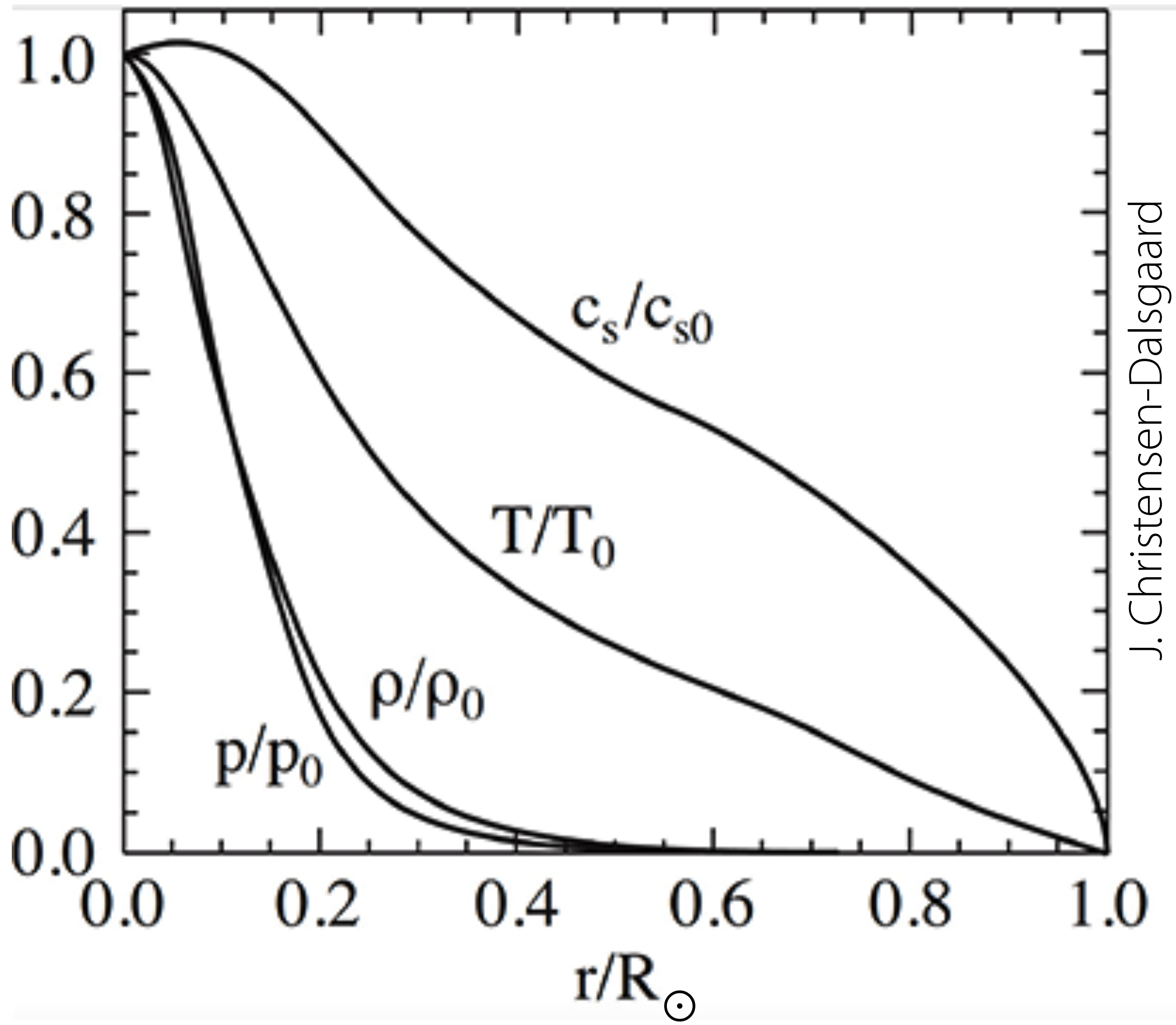


ESO

Stellar interior

Standard model of the solar interior

- Variation of (average) quantities as function of radius in the solar interior ($r/R_\odot$)
- Scaled to value at solar centre
- Temperature T_0=1.57 10^7 K
- Mass density ρ_0=1.54 10^5 kg m^{-3}
- Pressure p_0=2.35 10^{16} Nm^{-2}
- Sound speed $c_{s,0}$=5.05 10^5 ms^{-1}



J. Christensen-Dalsgaard

Energy transport

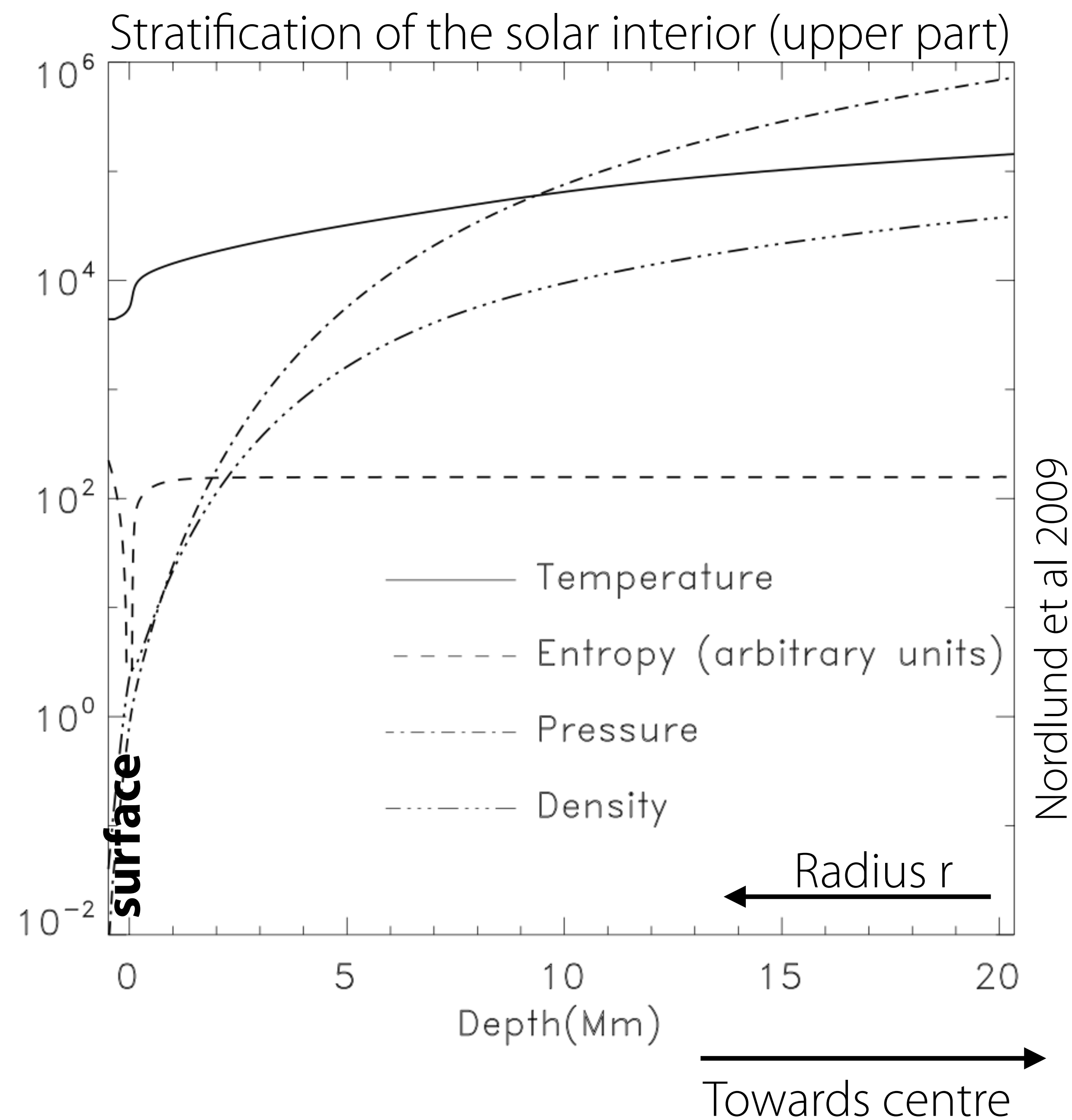
Convection — Stability criterion

- **Plasma/gas inside a star not a perfectly stratified but small perturbations occur**

➡ Is a layer stable against small perturbations?

➡ Or can initially small perturbations grow and result in significant deviations?

- **In the stratified stellar interior:** Pressure and density decreases with radius r towards surface

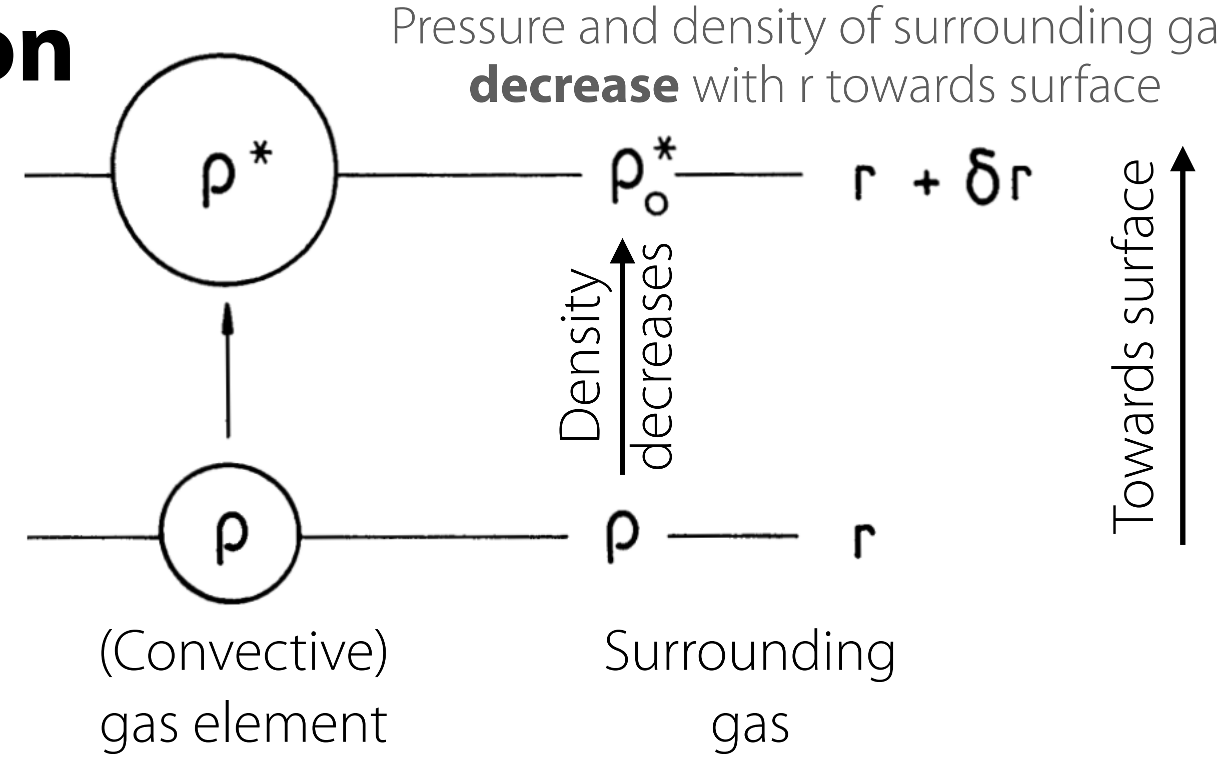


Energy transport

Convection — Stability criterion

- Gas element at distance r from the centre of the star
- Initially: Element in equilibrium with its surroundings at r:

➡ Pressure P and density ρ are the same as in its surroundings.



- Now — perturbation: **element** displaced (**rises**) a vertical distance δr **adiabatically** (no heat exchanged with environment) but slow enough that pressure is adjusted to new balance with outside pressure
  - Occurs when <u>time scale</u> **of heat exchange** is long compared to **time scale of expansion** of the element (the latter = local dynamical time scale, set by local sound speed); happens in the optically thick solar interior

➡ Element expands to restore pressure equilibrium with surrounding

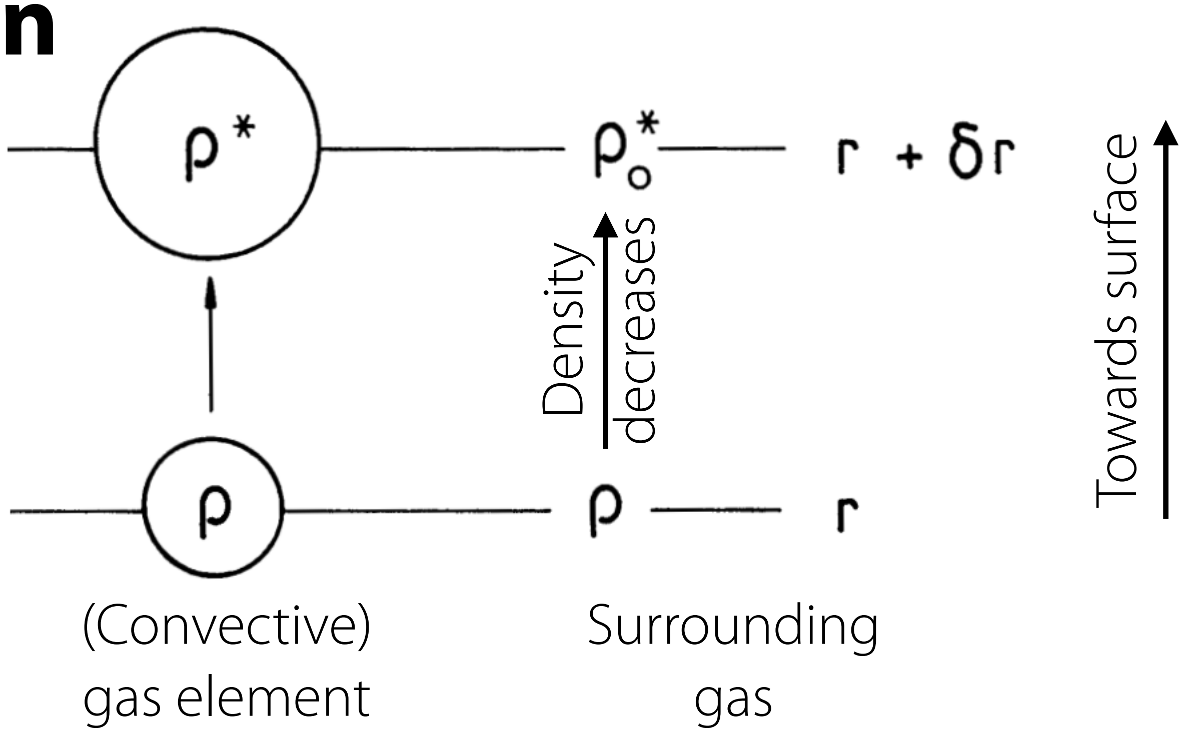
➡ Pressure in the element reduce as it rises

➡ Now at $r+\delta r$: Compare density of the element ρ^* with density of (new) surrounding ρ_0^*

Energy transport

Convection — Stability criterion

- Gas element at distance r from the centre of the star rises …
- Initially in equilibrium with its surroundings at r



- $\varrho^* > \varrho_0^*$: Element will fall back to initial height — stratification is **convectively stable**
- $\varrho^* < \varrho_0^*$: Element will keep rising up (net buoyancy!) — stratification is **convectively unstable**

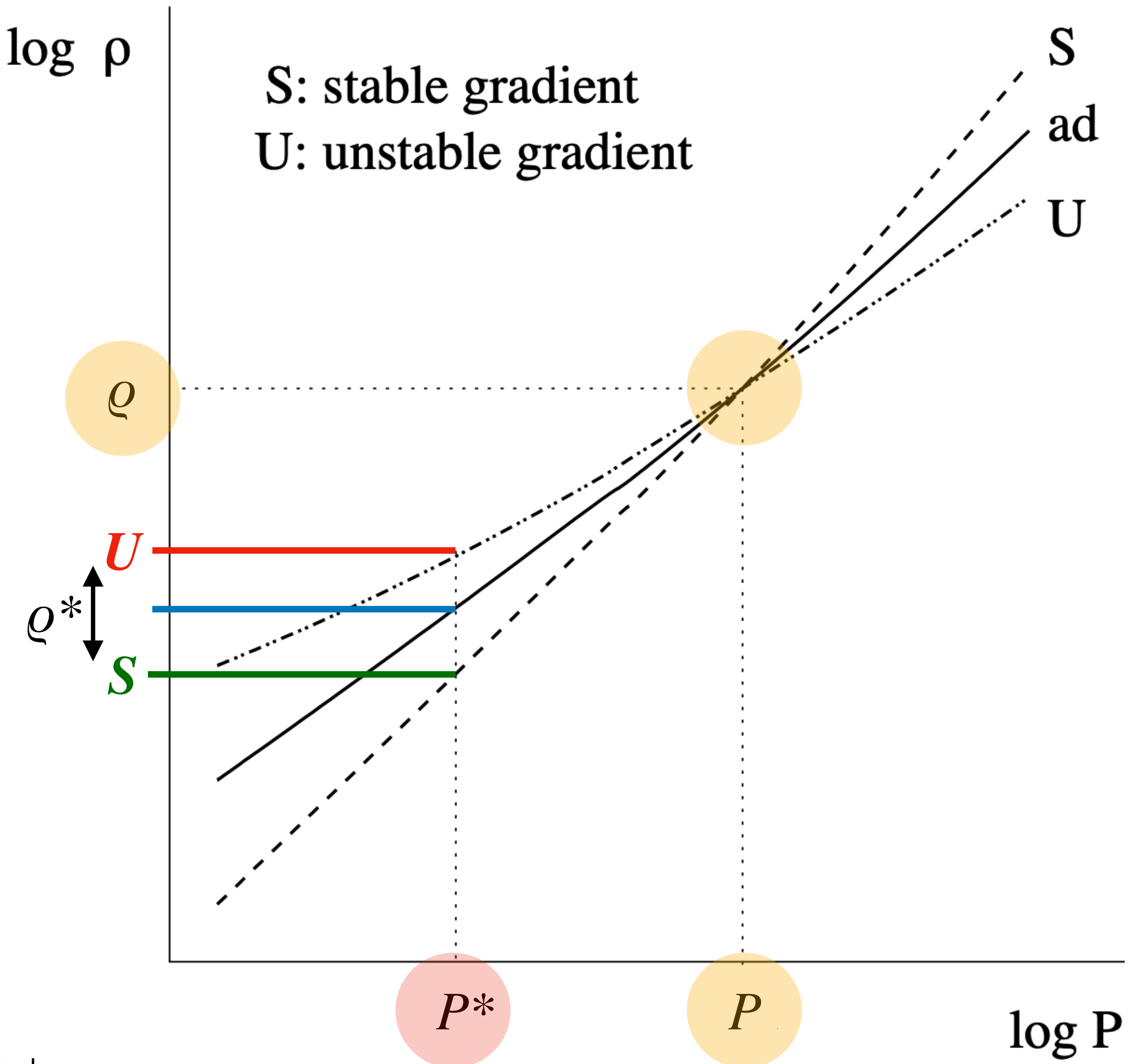
- At $r+\delta r$: Density difference ϱ^* - ϱ_0^* — gradients!
- For adiabatic expansion: $$\frac{\delta P^*}{P^*} = \gamma_{\rm ad} \frac{\delta \varrho^*}{\varrho^*}$$

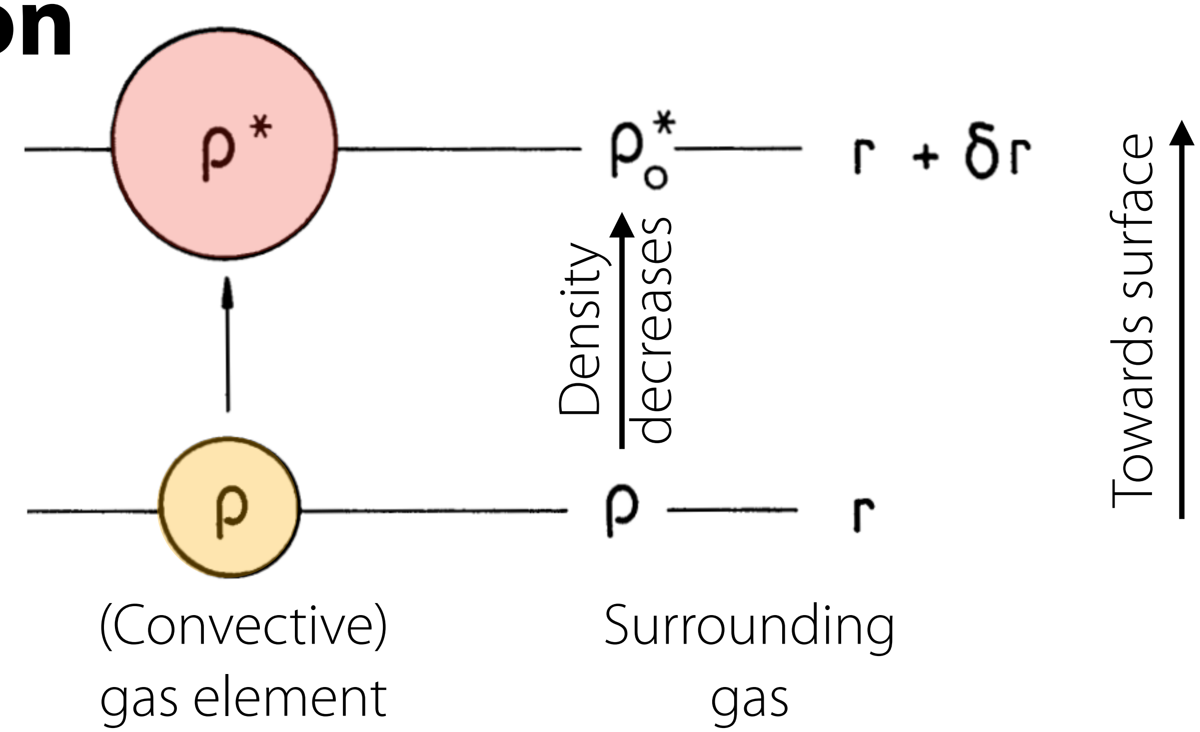
➡ For the situation above, we derive the following criterion for the gas remaining convectively stable:

$$\frac{d \log \rho}{d \log P} > \frac{1}{\gamma_{\rm ad}}$$

Energy transport

Convection — Stability criterion





$$\frac{d\log\rho}{d\log P} > \frac{1}{\gamma_{ad}}$$

Pols

Energy transport

Convection — Stability criterion

- Compare gradient $\nabla_{\rm rad}$ for convectively stable stratification with adiabatic temperature gradient $\nabla_{\rm ad} \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)$

- **Ledoux criterion** of stability against convection

$$\nabla_{\rm rad} < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T} \nabla_\mu$$

∇_{rad}: spatial gradient of temperature

∇_μ: spatial gradient of mean molecular weight

∇_{ad}: adiabatic temperature variation in a gas element undergoing a change in pressure.

$$\chi_T = \left(\frac{\partial \log P}{\partial \log T}\right)_{\rho, X_i} \quad \chi_\rho = \left(\frac{\partial \log P}{\partial \log \rho}\right)_{T, X_i}$$

: Indices: quantities held constant

- For chemically homogeneous gas: $\nabla_\mu = 0$:

➡ **Schwarzschild criterion** of stability against convection

$$\nabla_{\rm rad} < \nabla_{\rm ad}$$

- Note: In presence of fusion reactions: $\nabla_\mu \geq 0$
- Stabilising effect! (An upwards displaced element is heavier due to higher μ)

Energy transport

Convection — Stability criterion

- Compare gradient $\nabla_{\rm rad}$ for convectively stable stratification with adiabatic temperature gradient $\nabla_{\rm ad} \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)$

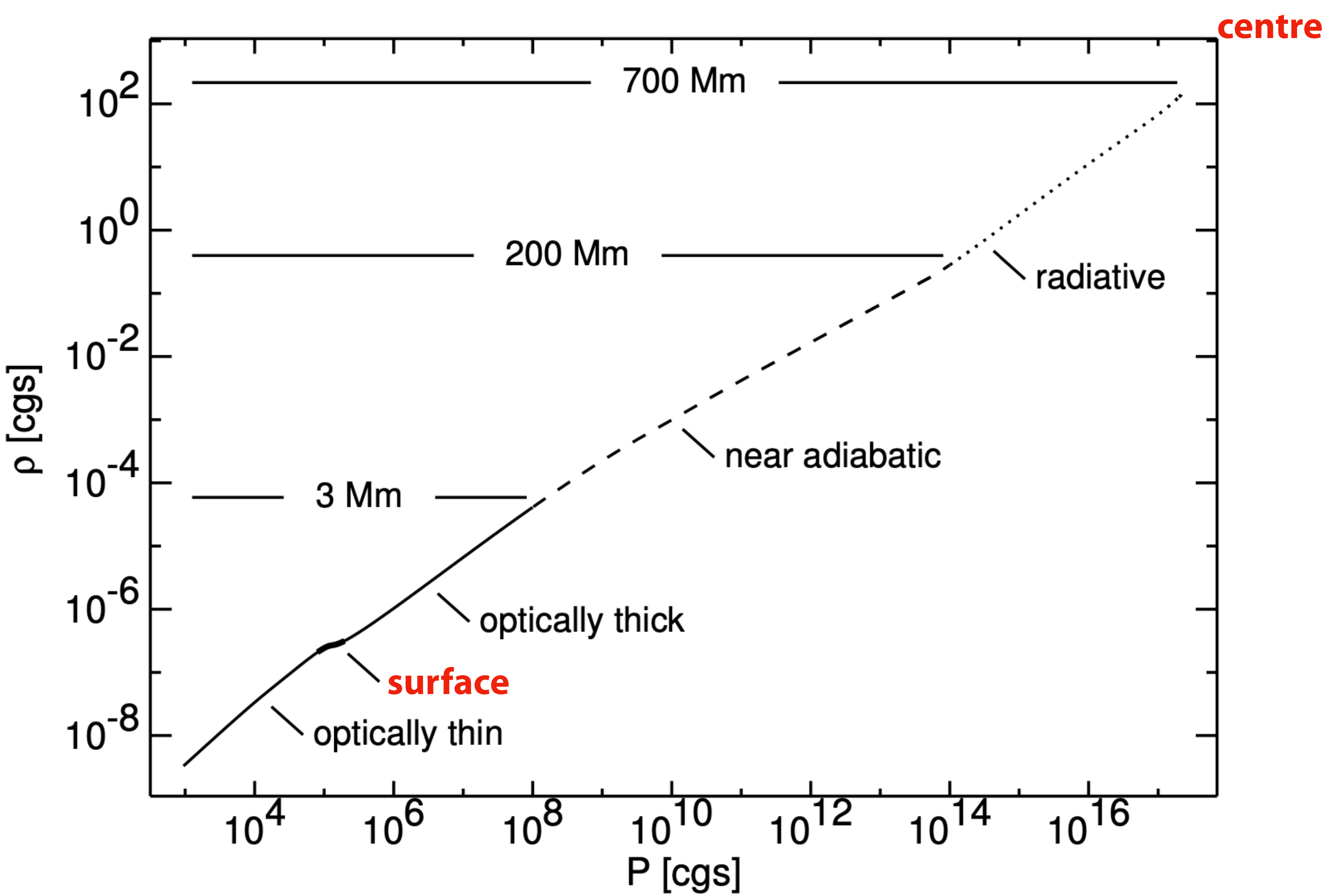
- **Ledoux criterion** of stability against convection

$$\nabla_{\rm rad} < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T} \nabla_\mu$$

- Stratification **convectively stable: Energy transport by radiation**
- Stratification **convectively unstable: Energy transport by convection**

Energy transport

Convection — Stability criterion in the solar interior



Energy transport

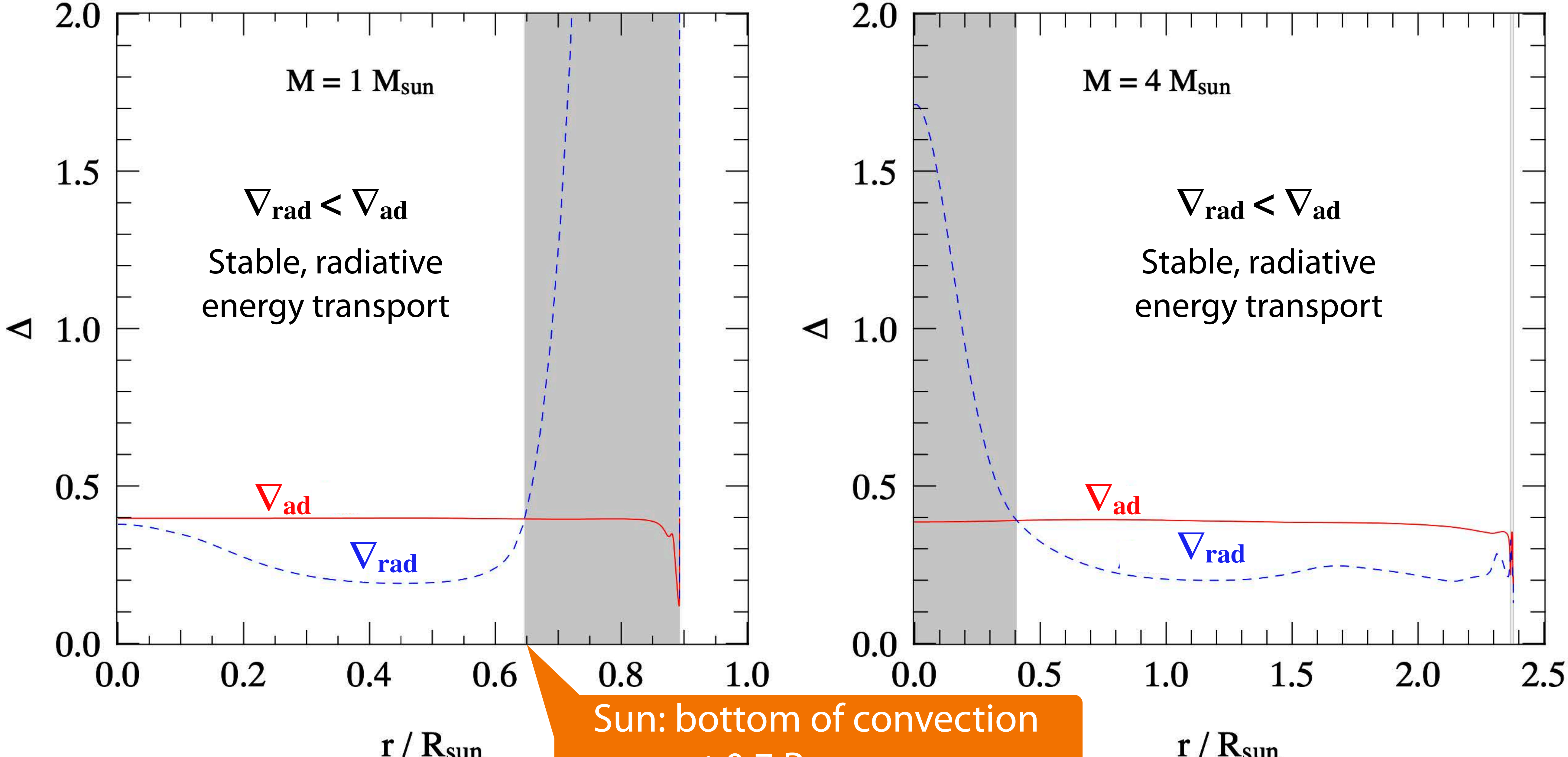
Convection — Stability criterion

M = 1 M_{sun}

Outer convection zone

Inner convection zone

M = 4 M_{sun}



Sun: bottom of convection zone at 0.7 R⊙