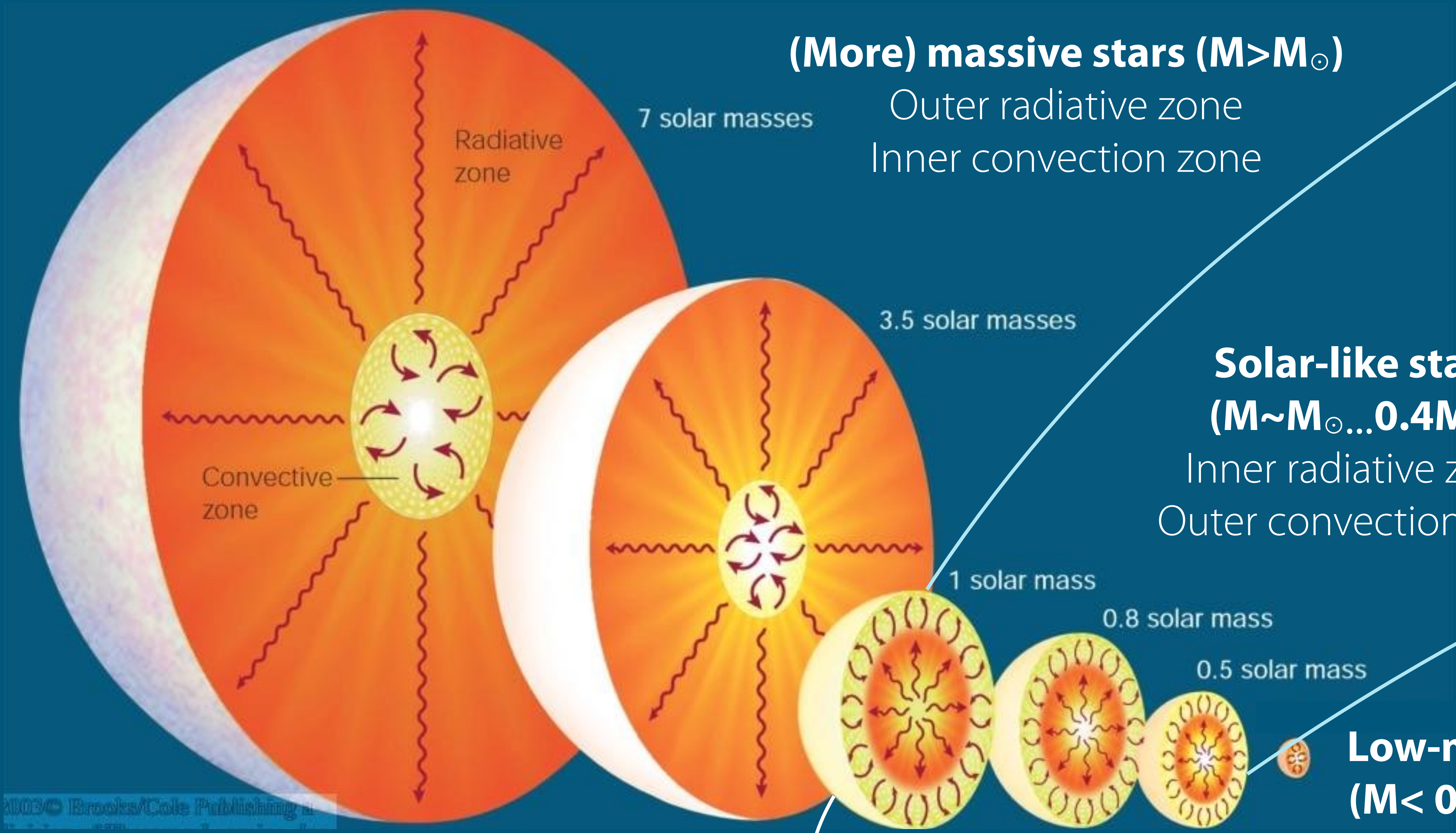
AST5770

Solar and stellar physics

Sven Wedemeyer, University of Oslo, 2023

Recap — Energy transport

Differences along the main sequence



(More) massive stars ($M>M_\odot$)
Outer radiative zone
Inner convection zone

Solar-like stars
($M \sim M_\odot ... 0.4 M_\odot$)
Inner radiative zone
Outer convection zone

Low-mass stars
($M < 0.3\text{-}0.4 M_\odot$)
Fully convective

Dominant fusion process	**CNO cycle**	**pp chain**

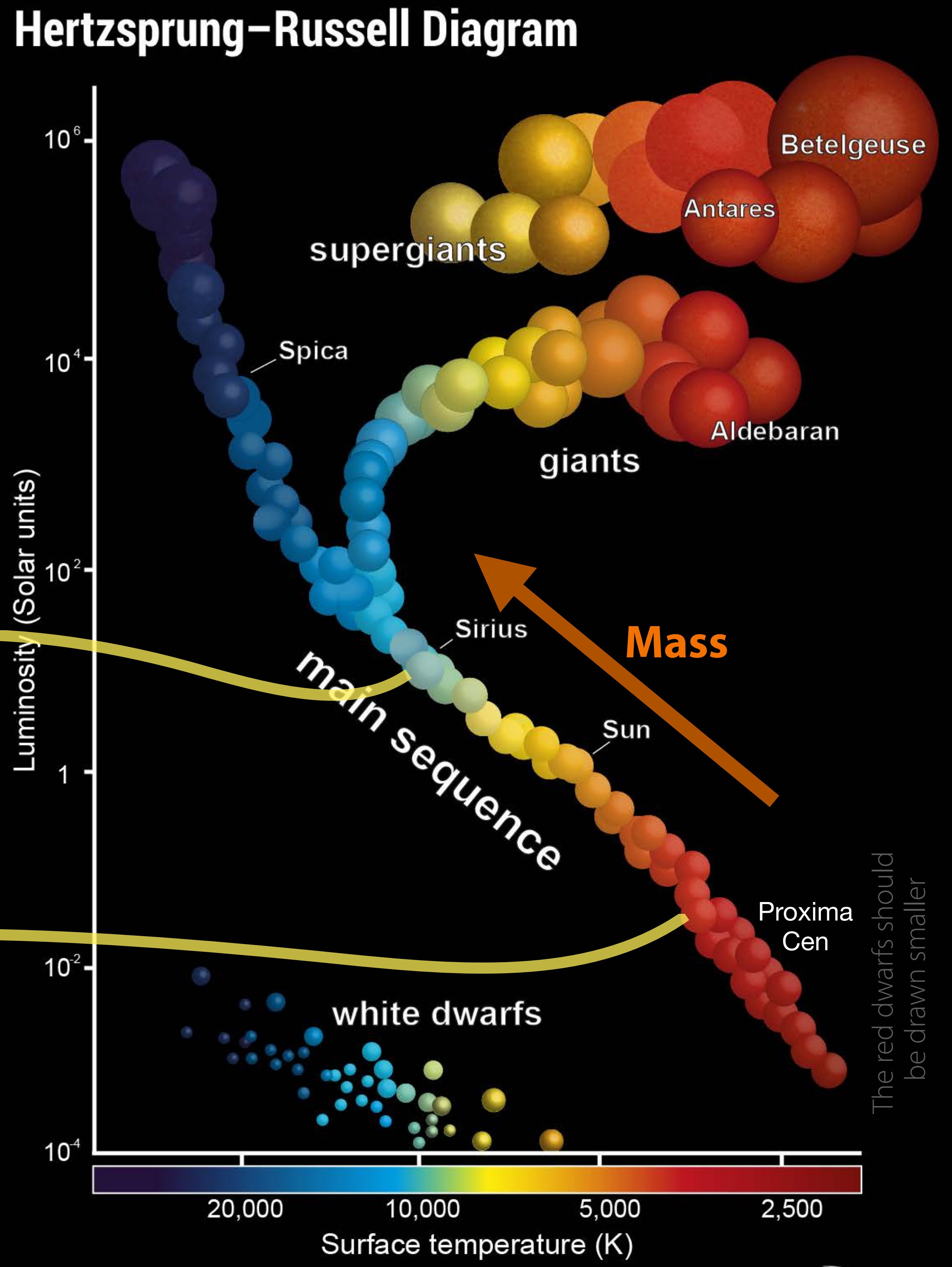
Recap

Differences along the main sequence

(More) massive stars (M>M⊙)
Outer radiative zone
Inner convection zone

Solar-like stars
(M~M⊙...0.4M⊙)
Inner radiative zone
Outer convection zone

Low-mass stars
(M< 0.3-0.4M⊙)
Fully convective



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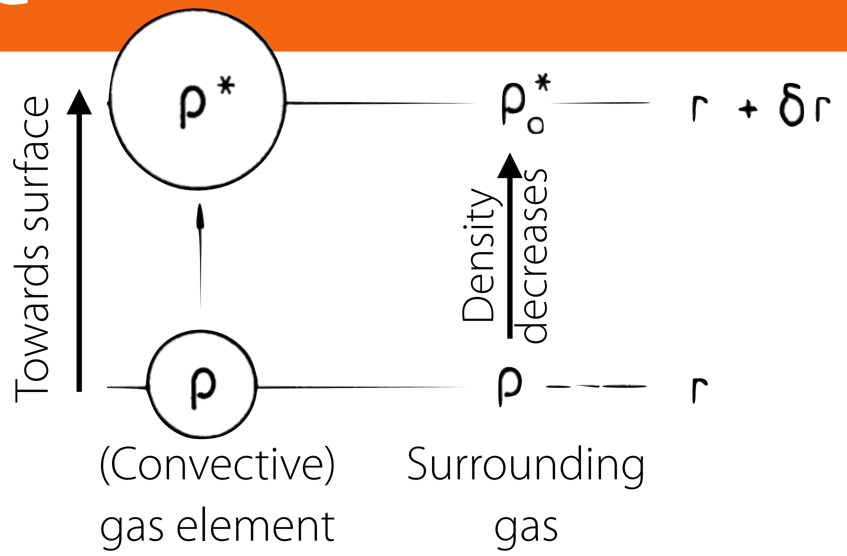
Recap — Energy transport

Convection — Stability criterion



- Compare gradient $\nabla_{\rm rad}$ for convectively stable stratification with adiabatic temperature gradient $\nabla_{\rm ad} \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)$

- **Ledoux criterion** of stability against convection

$$\boxed{\nabla_{\rm rad} < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T} \nabla_\mu}$$

∇_{rad} : spatial gradient of temperature

∇_μ: spatial gradient of mean molecular weight

∇_{ad} : adiabatic temperature variation in a gas element undergoing a change in pressure.

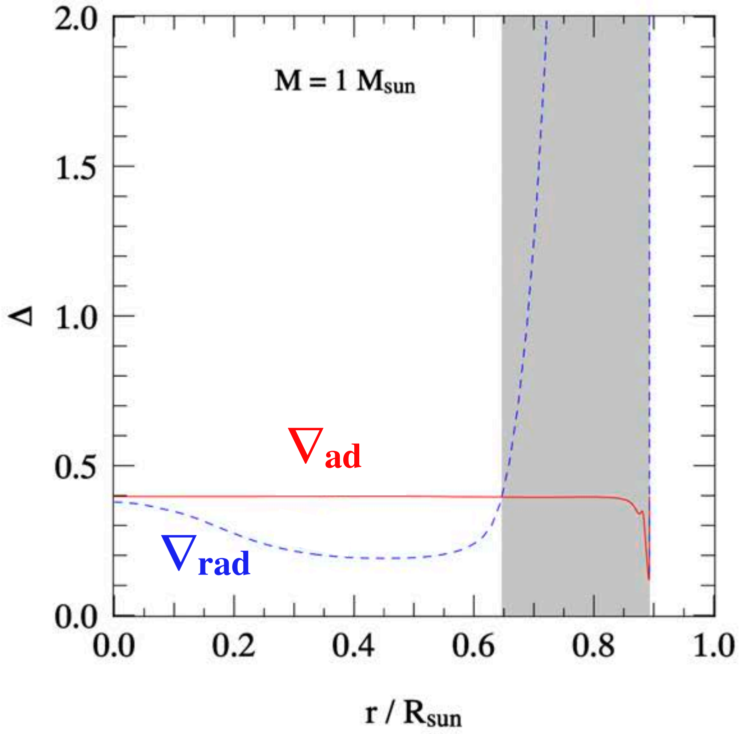
$\chi_T = \left(\frac{\partial \log P}{\partial \log T}\right)_{\rho, X_i}$ $\chi_\rho = \left(\frac{\partial \log P}{\partial \log \rho}\right)_{T, X_i}$: Indices: quantities held constant

- For chemically homogeneous gas: $\nabla_\mu = 0$:

➡ **Schwarzschild criterion of stability against convection**

$$\boxed{\nabla_{\rm rad} < \nabla_{\rm ad}}$$

- Note: In presence of fusion reactions: $\nabla_\mu \geq 0$
- Stabilising effect! (An upwards displaced element is heavier due to higher μ)

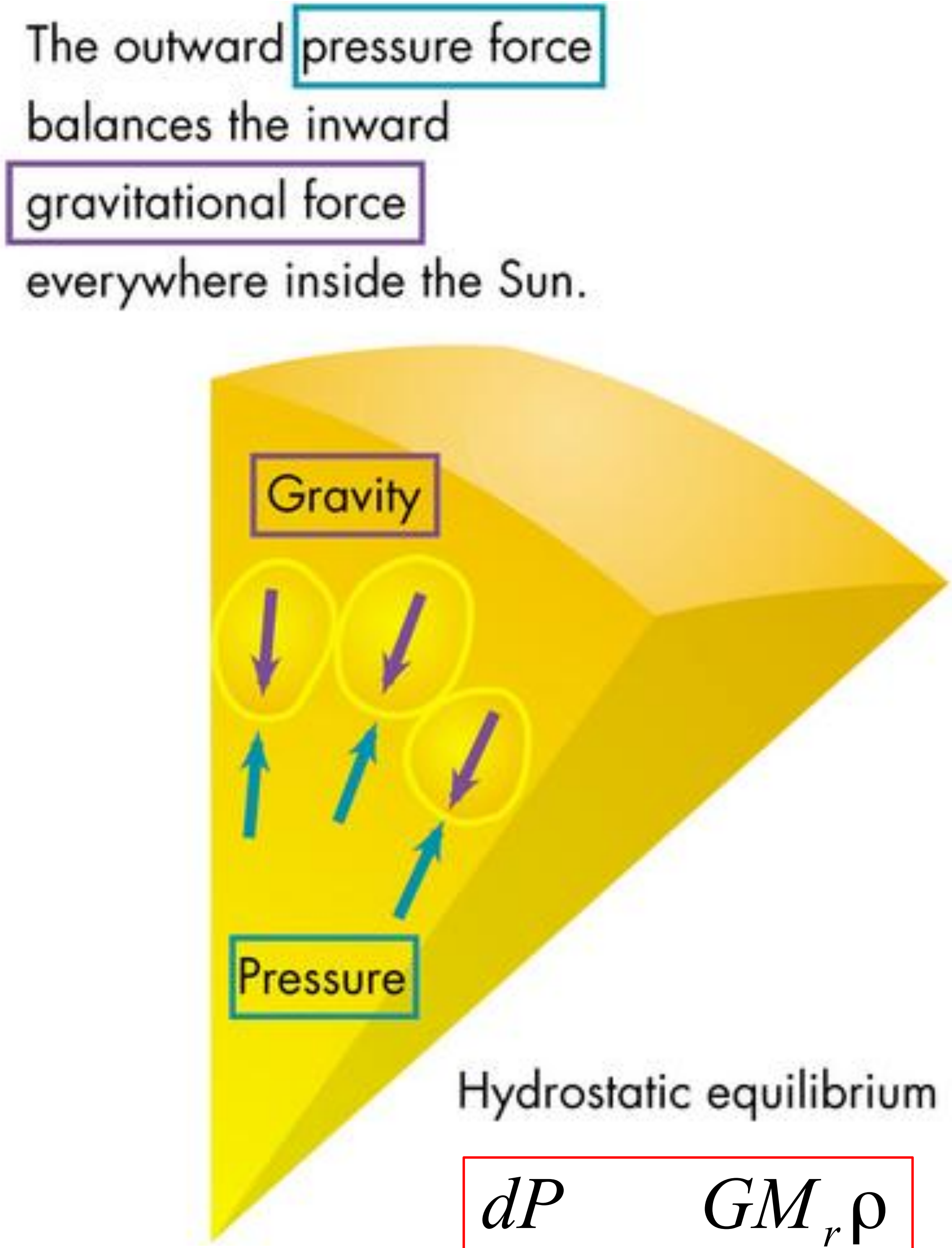


Stellar interiors

Equations of stellar structure

Stellar interior

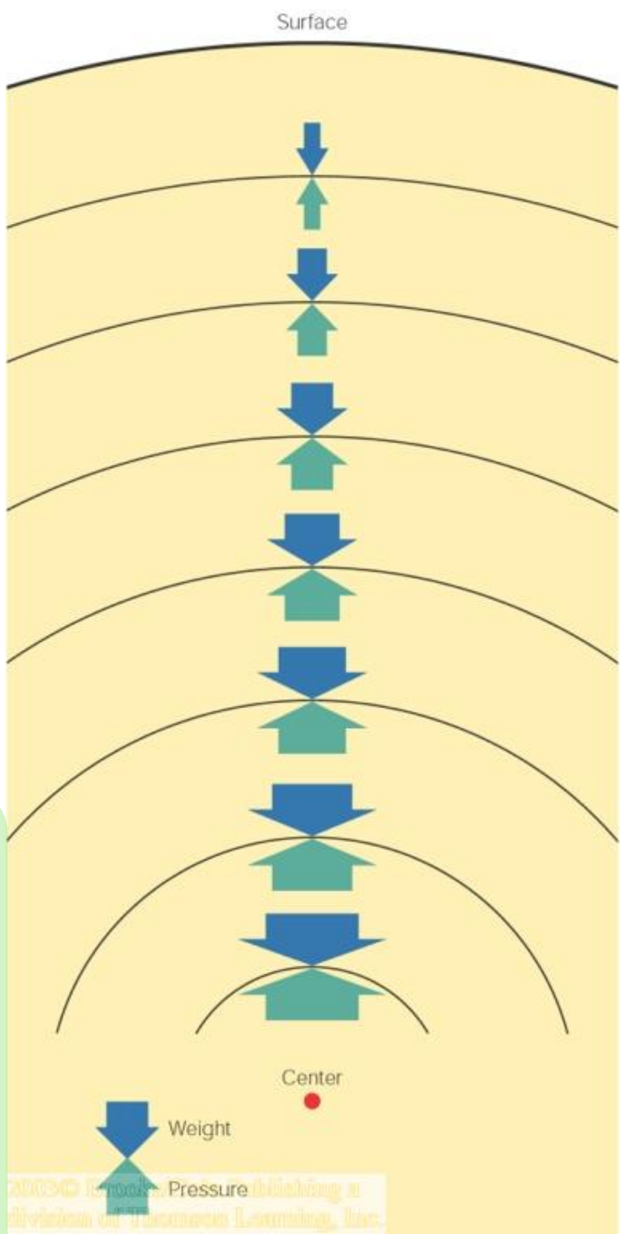
Hydrostatic Equilibrium



$$\boxed{\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}}$$

- Outward pressure of hot gas in the center balances the inward force due to gravity.
  - ➡ At any given radius balancing the weight of all layers above
  - ➡ Imbalance at some radius will result in corresponding adjustment of the stratification
  - ➡ Determines the interior structure (stratification)

- **Main Sequence in the Hertzsprung-Russell diagram is a narrow strip as it requires stability over long enough time**



Stellar interior

Equations of stellar structure

- The interior structure of a star in equilibrium can be described with the following “ingredients”
- Variables and their stratification (function of radius) P, ϱ, T, M_r, L_r
- The following equations :

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$
Mass conservation/continuity	$\frac{dM_r}{dr} = 4\pi r^2 \rho$
Equation of state	$P = \frac{\rho k T}{\mu m_H}$
Energy “generation”	$\frac{dL_r}{dr} = 4\pi r^2 \rho\varepsilon$
Energy transport	$\frac{dT}{dr} = -\frac{3\kappa\rho}{16\pi ac}\frac{L_r}{r^2T^2}$

A “small” problem:
We cannot observe the interior of stars directly
but rely mostly on few measurable properties (M, L,…), assumptions, (tested) physical laws and material properties
(often with substantial uncertainties)

- And these boundary conditions:
  - In the centre (r=0): $M_r(0) = 0, \quad L_r(0) = 0$
  - At the surface (r=R): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$
- Note that the chemical composition affects the density and thus the stratification via the mean molecular weight μ as function of radius (see equation of state)

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Solution

Standard Model

assumes
[X:Y:Z]
=
[0.73:0.25:0.015]

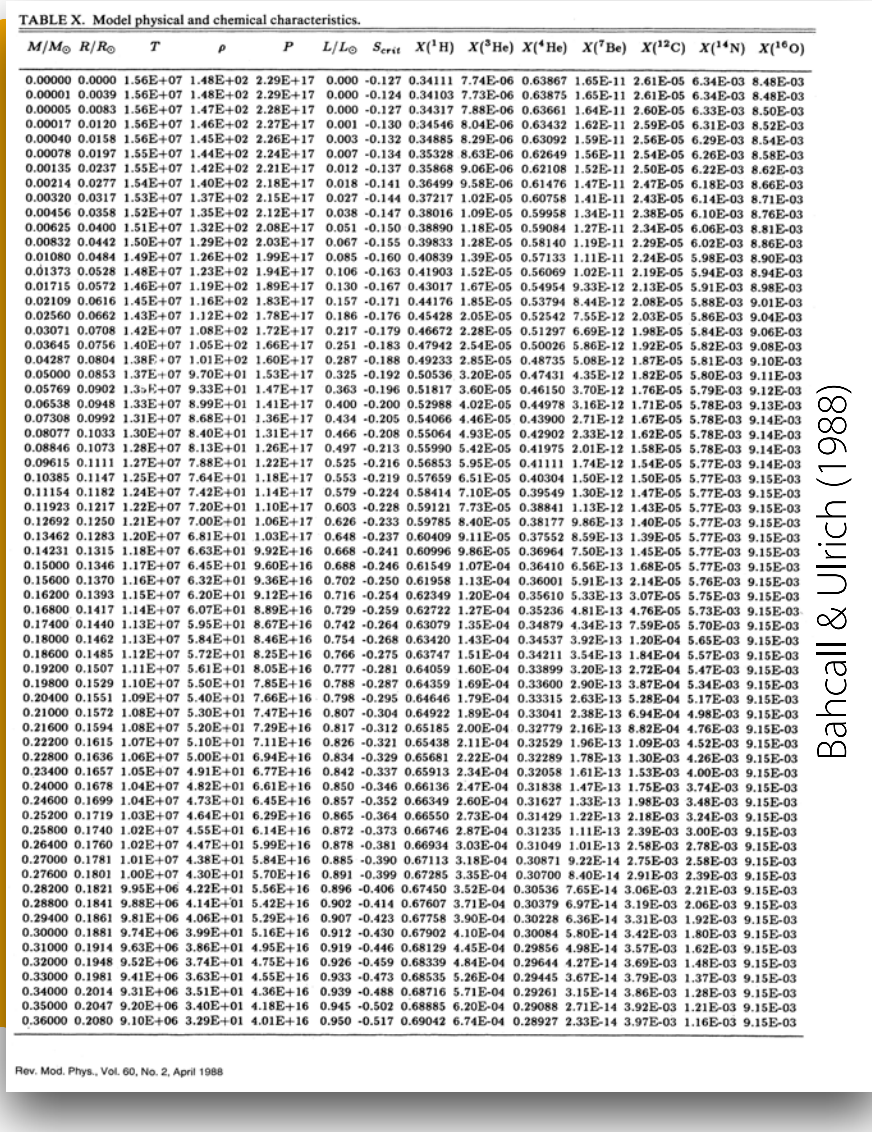

TABLE X. Model physical and chemical characteristics.

$M/M_\odot$	$R/R_\odot$	T	ρ	P	$L/L_\odot$	S_{crit}	$X(^1\mathrm{H})$	$X(^3\mathrm{He})$	$X(^4\mathrm{He})$	$X(^7\mathrm{Be})$	$X(^{12}\mathrm{C})$	$X(^{14}\mathrm{N})$	$X(^{16}\mathrm{O})$
0.00000	0.0000	1.56E+07	1.48E+02	2.29E+17	0.000	-0.127	0.34111	7.74E-06	0.63867	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00001	0.0039	1.56E+07	1.48E+02	2.29E+17	0.000	-0.124	0.34103	7.73E-06	0.63875	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00005	0.0083	1.56E+07	1.47E+02	2.28E+17	0.000	-0.127	0.34317	7.88E-06	0.63661	1.64E-11	2.60E-05	6.33E-03	8.50E-03
0.00017	0.0120	1.56E+07	1.46E+02	2.27E+17	0.001	-0.130	0.34546	8.04E-06	0.63432	1.62E-11	2.59E-05	6.31E-03	8.52E-03
0.00040	0.0158	1.56E+07	1.45E+02	2.26E+17	0.003	-0.132	0.34885	8.29E-06	0.63092	1.59E-11	2.56E-05	6.29E-03	8.54E-03
0.00078	0.0197	1.55E+07	1.44E+02	2.24E+17	0.007	-0.134	0.35328	8.63E-06	0.62649	1.56E-11	2.54E-05	6.26E-03	8.58E-03
0.00135	0.0237	1.55E+07	1.42E+02	2.21E+17	0.012	-0.137	0.35868	9.06E-06	0.62108	1.52E-11	2.50E-05	6.22E-03	8.62E-03
0.00214	0.0277	1.54E+07	1.40E+02	2.18E+17	0.018	-0.141	0.36499	9.58E-06	0.61476	1.47E-11	2.47E-05	6.18E-03	8.66E-03
0.00320	0.0317	1.53E+07	1.37E+02	2.15E+17	0.027	-0.144	0.37217	1.02E-05	0.60758	1.41E-11	2.43E-05	6.14E-03	8.71E-03
0.00456	0.0358	1.52E+07	1.35E+02	2.12E+17	0.038	-0.147	0.38016	1.09E-05	0.59958	1.34E-11	2.38E-05	6.10E-03	8.76E-03
0.00625	0.0400	1.51E+07	1.32E+02	2.08E+17	0.051	-0.150	0.38890	1.18E-05	0.59084	1.27E-11	2.34E-05	6.06E-03	8.81E-03
0.00832	0.0442	1.50E+07	1.29E+02	2.03E+17	0.067	-0.155	0.39833	1.28E-05	0.58140	1.19E-11	2.29E-05	6.02E-03	8.86E-03
0.01080	0.0484	1.49E+07	1.26E+02	1.99E+17	0.085	-0.160	0.40839	1.39E-05	0.57133	1.11E-11	2.24E-05	5.98E-03	8.90E-03
0.01373	0.0528	1.48E+07	1.23E+02	1.94E+17	0.106	-0.163	0.41903	1.52E-05	0.56069	1.02E-11	2.19E-05	5.94E-03	8.94E-03
0.01715	0.0572	1.46E+07	1.19E+02	1.89E+17	0.130	-0.167	0.43017	1.67E-05	0.54954	9.33E-12	2.13E-05	5.91E-03	8.98E-03
0.02109	0.0616	1.45E+07	1.16E+02	1.83E+17	0.157	-0.171	0.44176	1.85E-05	0.53794	8.44E-12	2.08E-05	5.88E-03	9.01E-03
0.02560	0.0662	1.43E+07	1.12E+02	1.78E+17	0.186	-0.176	0.45428	2.05E-05	0.52542	7.55E-12	2.03E-05	5.86E-03	9.04E-03
0.03071	0.0708	1.42E+07	1.08E+02	1.72E+17	0.217	-0.179	0.46672	2.28E-05	0.51297	6.69E-12	1.98E-05	5.84E-03	9.06E-03
0.03645	0.0756	1.40E+07	1.05E+02	1.66E+17	0.251	-0.183	0.47942	2.54E-05	0.50026	5.86E-12	1.92E-05	5.82E-03	9.08E-03
0.04287	0.0804	1.38E+07	1.01E+02	1.60E+17	0.287	-0.188	0.49233	2.85E-05	0.48735	5.08E-12	1.87E-05	5.81E-03	9.10E-03
0.05000	0.0853	1.37E+07	9.70E+01	1.53E+17	0.325	-0.192	0.50536	3.20E-05	0.47431	4.35E-12	1.82E-05	5.80E-03	9.11E-03
0.05769	0.0902	1.35E+07	9.33E+01	1.47E+17	0.363	-0.196	0.51817	3.60E-05	0.46150	3.70E-12	1.76E-05	5.79E-03	9.12E-03
0.06538	0.0948	1.33E+07	8.99E+01	1.41E+17	0.400	-0.200	0.52988	4.02E-05	0.44978	3.16E-12	1.71E-05	5.78E-03	9.13E-03
0.07308	0.0992	1.31E+07	8.68E+01	1.36E+17	0.434	-0.205	0.54066	4.46E-05	0.43900	2.71E-12	1.67E-05	5.78E-03	9.14E-03
0.08077	0.1033	1.30E+07	8.40E+01	1.31E+17	0.466	-0.208	0.55064	4.93E-05	0.42902	2.33E-12	1.62E-05	5.78E-03	9.14E-03
0.08846	0.1073	1.28E+07	8.13E+01	1.26E+17	0.497	-0.213	0.55990	5.42E-05	0.41975	2.01E-12	1.58E-05	5.78E-03	9.14E-03
0.09615	0.1111	1.27E+07	7.88E+01	1.22E+17	0.525	-0.216	0.56853	5.95E-05	0.41111	1.74E-12	1.54E-05	5.77E-03	9.14E-03
0.10385	0.1147	1.25E+07	7.64E+01	1.18E+17	0.553	-0.219	0.57659	6.51E-05	0.40304	1.50E-12	1.50E-05	5.77E-03	9.15E-03
0.11154	0.1182	1.24E+07	7.42E+01	1.14E+17	0.579	-0.224	0.58414	7.10E-05	0.39549	1.30E-12	1.47E-05	5.77E-03	9.15E-03
0.11923	0.1217	1.22E+07	7.20E+01	1.10E+17	0.603	-0.228	0.59121	7.73E-05	0.38841	1.13E-12	1.43E-05	5.77E-03	9.15E-03
0.12692	0.1250	1.21E+07	7.00E+01	1.06E+17	0.626	-0.233	0.59785	8.40E-05	0.38177	9.86E-13	1.40E-05	5.77E-03	9.15E-03
0.13462	0.1283	1.20E+07	6.81E+01	1.03E+17	0.648	-0.237	0.60409	9.11E-05	0.37552	8.59E-13	1.39E-05	5.77E-03	9.15E-03
0.14231	0.1315	1.18E+07	6.63E+01	9.92E+16	0.668	-0.241	0.60996	9.86E-05	0.36964	7.50E-13	1.45E-05	5.77E-03	9.15E-03
0.15000	0.1346	1.17E+07	6.45E+01	9.60E+16	0.688	-0.246	0.61549	1.07E-04	0.36410	6.56E-13	1.68E-05	5.77E-03	9.15E-03
0.15600	0.1370	1.16E+07	6.32E+01	9.36E+16	0.702	-0.250	0.61958	1.13E-04	0.36001	5.91E-13	2.14E-05	5.76E-03	9.15E-03
0.16200	0.1393	1.15E+07	6.20E+01	9.12E+16	0.716	-0.254	0.62349	1.20E-04	0.35610	5.33E-13	3.07E-05	5.75E-03	9.15E-03
0.16800	0.1417	1.14E+07	6.07E+01	8.89E+16	0.729	-0.259	0.62722	1.27E-04	0.35236	4.81E-13	4.76E-05	5.73E-03	9.15E-03
0.17400	0.1440	1.13E+07	5.95E+01	8.67E+16	0.742	-0.264	0.63079	1.35E-04	0.34879	4.34E-13	7.59E-05	5.70E-03	9.15E-03
0.18000	0.1462	1.13E+07	5.84E+01	8.46E+16	0.754	-0.268	0.63420	1.43E-04	0.34537	3.92E-13	1.20E-04	5.65E-03	9.15E-03
0.18600	0.1485	1.12E+07	5.72E+01	8.25E+16	0.766	-0.275	0.63747	1.51E-04	0.34211	3.54E-13	1.84E-04	5.57E-03	9.15E-03
0.19200	0.1507	1.11E+07	5.61E+01	8.05E+16	0.777	-0.281	0.64059	1.60E-04	0.33899	3.20E-13	2.72E-04	5.47E-03	9.15E-03
0.19800	0.1529	1.10E+07	5.50E+01	7.85E+16	0.788	-0.287	0.64359	1.69E-04	0.33600	2.90E-13	3.87E-04	5.34E-03	9.15E-03
0.20400	0.1551	1.09E+07	5.40E+01	7.66E+16	0.798	-0.295	0.64646	1.79E-04	0.33315	2.63E-13	5.28E-04	5.17E-03	9.15E-03
0.21000	0.1572	1.08E+07	5.30E+01	7.47E+16	0.807	-0.304	0.64922	1.89E-04	0.33041	2.38E-13	6.94E-04	4.98E-03	9.15E-03
0.21600	0.1594	1.08E+07	5.20E+01	7.29E+16	0.817	-0.312	0.65185	2.00E-04	0.32779	2.16E-13	8.82E-04	4.76E-03	9.15E-03
0.22200	0.1615	1.07E+07	5.10E+01	7.11E+16	0.826	-0.321	0.65438	2.11E-04	0.32529	1.96E-13	1.09E-03	4.52E-03	9.15E-03
0.22800	0.1636	1.06E+07	5.00E+01	6.94E+16	0.834	-0.329	0.65681	2.22E-04	0.32289	1.78E-13	1.30E-03	4.26E-03	9.15E-03
0.23400	0.1657	1.05E+07	4.91E+01	6.77E+16	0.842	-0.337	0.65913	2.34E-04	0.32058	1.61E-13	1.53E-03	4.00E-03	9.15E-03
0.24000	0.1678	1.04E+07	4.82E+01	6.61E+16	0.850	-0.346	0.66136	2.47E-04	0.31838	1.47E-13	1.75E-03	3.74E-03	9.15E-03
0.24600	0.1699	1.04E+07	4.73E+01	6.45E+16	0.857	-0.352	0.66349	2.60E-04	0.31627	1.33E-13	1.98E-03	3.48E-03	9.15E-03
0.25200	0.1719	1.03E+07	4.64E+01	6.29E+16	0.865	-0.364	0.66550	2.73E-04	0.31429	1.22E-13	2.18E-03	3.24E-03	9.15E-03
0.25800	0.1740	1.02E+07	4.55E+01	6.14E+16	0.872	-0.373	0.66746	2.87E-04	0.31235	1.11E-13	2.39E-03	3.00E-03	9.15E-03
0.26400	0.1760	1.02E+07	4.47E+01	5.99E+16	0.878	-0.381	0.66934	3.03E-04	0.31049	1.01E-13	2.58E-03	2.78E-03	9.15E-03
0.27000	0.1781	1.01E+07	4.38E+01	5.84E+16	0.885	-0.390	0.67113	3.18E-04	0.30871	9.22E-14	2.75E-03	2.58E-03	9.15E-03
0.27600	0.1801	1.00E+07	4.30E+01	5.70E+16	0.891	-0.399	0.67285	3.35E-04	0.30700	8.40E-14	2.91E-03	2.39E-03	9.15E-03
0.28200	0.1821	9.95E+06	4.22E+01	5.56E+16	0.896	-0.406	0.67450	3.52E-04	0.30536	7.65E-14	3.06E-03	2.21E-03	9.15E-03
0.28800	0.1841	9.88E+06	4.14E+01	5.42E+16	0.902	-0.414	0.67607	3.71E-04	0.30379	6.97E-14	3.19E-03	2.06E-03	9.15E-03
0.29400	0.1861	9.81E+06	4.06E+01	5.29E+16	0.907	-0.423	0.67758	3.90E-04	0.30228	6.36E-14	3.31E-03	1.92E-03	9.15E-03
0.30000	0.1881	9.74E+06	3.99E+01	5.16E+16	0.912	-0.430	0.67902	4.10E-04	0.30084	5.80E-14	3.42E-03	1.80E-03	9.15E-03
0.31000	0.1914	9.63E+06	3.86E+01	4.95E+16	0.919	-0.446	0.68129	4.45E-04	0.29856	4.98E-14	3.57E-03	1.62E-03	9.15E-03
0.32000	0.1948	9.52E+06	3.74E+01	4.75E+16	0.926	-0.459	0.68339	4.84E-04	0.29644	4.27E-14	3.69E-03	1.48E-03	9.15E-03
0.33000	0.1981	9.41E+06	3.63E+01	4.55E+16	0.933	-0.473	0.68535	5.26E-04	0.29445	3.67E-14	3.79E-03	1.37E-03	9.15E-03
0.34000	0.2014	9.31E+06	3.51E+01	4.36E+16	0.939	-0.488	0.68716	5.71E-04	0.29261	3.15E-14	3.86E-03	1.28E-03	9.15E-03
0.35000	0.2047	9.20E+06	3.40E+01	4.18E+16	0.945	-0.502	0.68885	6.20E-04	0.29088	2.71E-14	3.92E-03	1.21E-03	9.15E-03
0.36000	0.2080	9.10E+06	3.29E+01	4.01E+16	0.950	-0.517	0.69042	6.74E-04	0.28927	2.33E-14	3.97E-03	1.16E-03	9.15E-03

Rev. Mod. Phys., Vol. 60, No. 2, April 1988

Bahcall & Ulrich (1988)

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- Note that the **chemical composition** affects the density and thus the stratification via the mean molecular weight μ as function of radius (see equation of state)

Stellar interior

Equations of stellar structure

- The interior structure of a star in equilibrium can be described with the following "ingredients"
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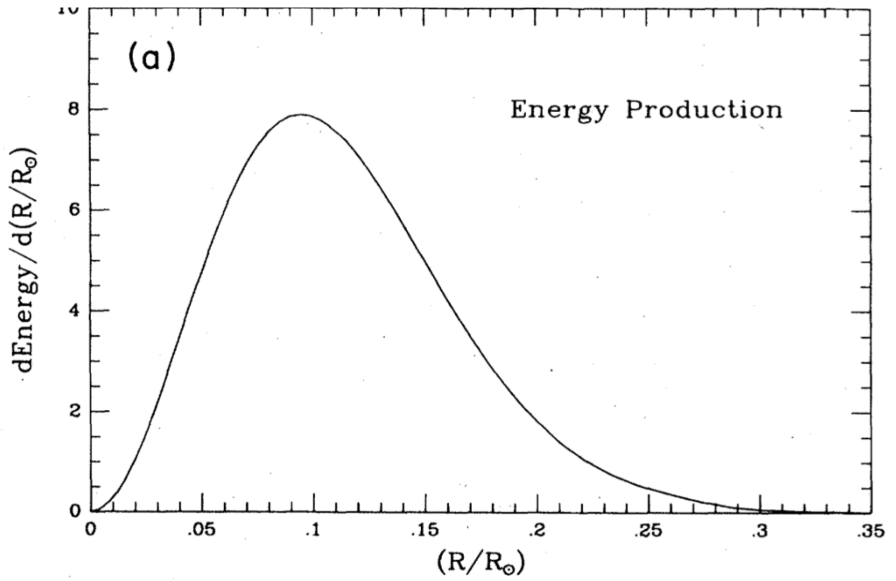
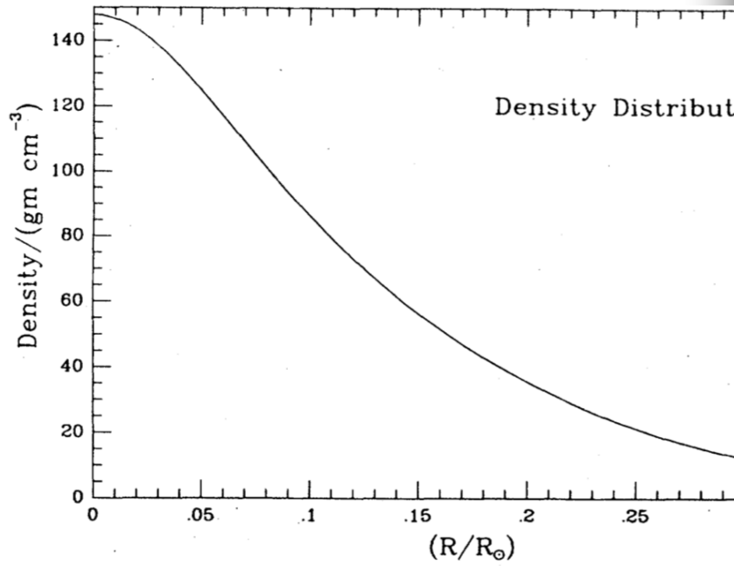
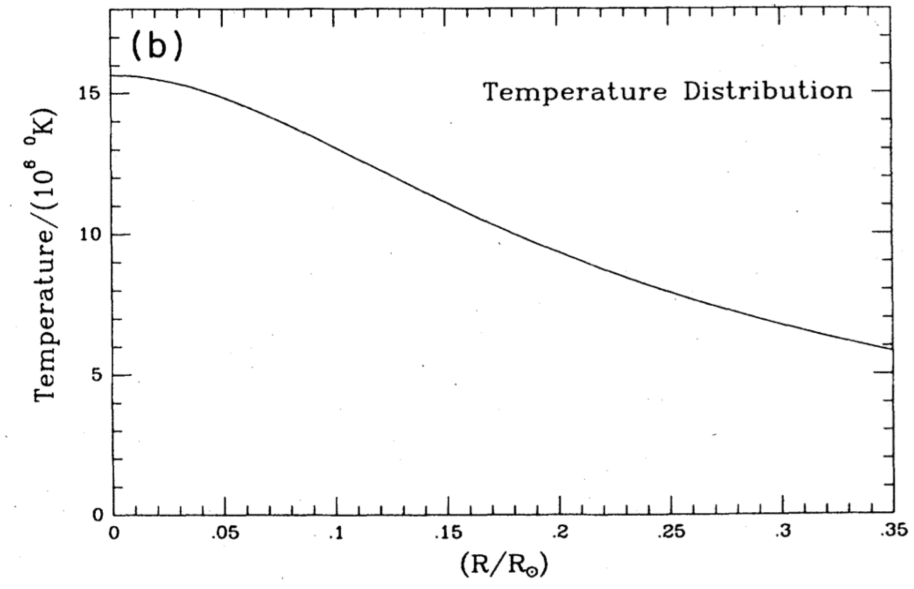
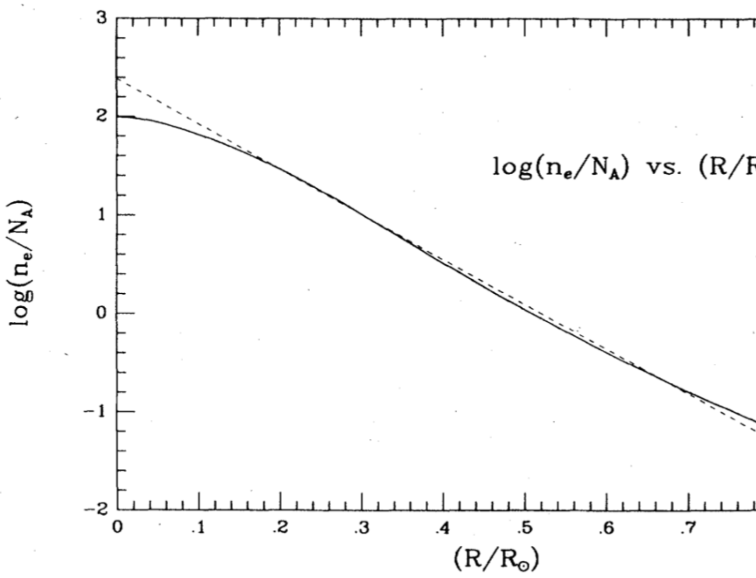
- **Solution: Standard Model** (for the Sun)

  assumes chemical composition [X:Y:Z] =[0.73:0.25:0.015]









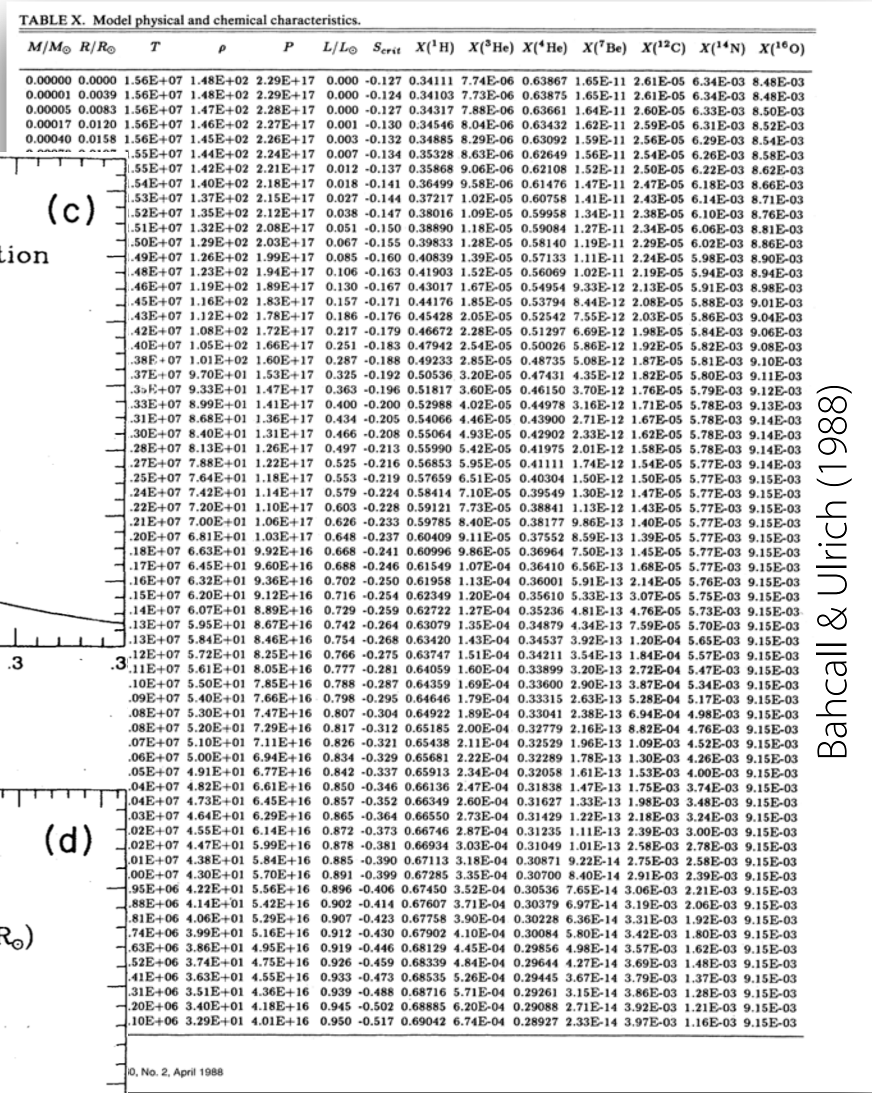


TABLE X. Model physical and chemical characteristics.

$M/M_\odot$	$R/R_\odot$	T	ρ	P	$L/L_\odot$	S_{crit}	$X(^1H)$	$X(^3He)$	$X(^4He)$	$X(^7Be)$	$X(^{12}C)$	$X(^{14}N)$	$X(^{16}O)$
0.00000	0.0000	1.56E+07	1.48E+02	2.29E+17	0.000	-0.127	0.34111	7.74E-06	0.63867	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00001	0.0039	1.56E+07	1.48E+02	2.29E+17	0.000	-0.124	0.34103	7.73E-06	0.63875	1.65E-11	2.61E-05	6.34E-03	8.48E-03
0.00005	0.0083	1.56E+07	1.47E+02	2.28E+17	0.000	-0.127	0.34317	7.88E-06	0.63661	1.64E-11	2.60E-05	6.33E-03	8.50E-03
0.00017	0.0120	1.56E+07	1.46E+02	2.27E+17	0.001	-0.130	0.34546	8.04E-06	0.63432	1.62E-11	2.59E-05	6.31E-03	8.52E-03
0.00040	0.0158	1.56E+07	1.45E+02	2.26E+17	0.003	-0.132	0.34885	8.29E-06	0.63092	1.59E-11	2.56E-05	6.29E-03	8.54E-03
		.55E+07	1.44E+02	2.24E+17	0.007	-0.134	0.35328	8.63E-06	0.62649	1.56E-11	2.54E-05	6.26E-03	8.58E-03
		.55E+07	1.42E+02	2.21E+17	0.012	-0.137	0.35868	9.06E-06	0.62108	1.52E-11	2.50E-05	6.22E-03	8.62E-03
		.54E+07	1.40E+02	2.18E+17	0.018	-0.141	0.36499	9.58E-06	0.61476	1.47E-11	2.47E-05	6.18E-03	8.66E-03
		.53E+07	1.37E+02	2.15E+17	0.027	-0.144	0.37217	1.02E-05	0.60758	1.41E-11	2.43E-05	6.14E-03	8.71E-03
		.52E+07	1.35E+02	2.12E+17	0.038	-0.147	0.38016	1.09E-05	0.59958	1.34E-11	2.38E-05	6.10E-03	8.76E-03
		.51E+07	1.32E+02	2.08E+17	0.051	-0.150	0.38890	1.18E-05	0.59084	1.27E-11	2.34E-05	6.06E-03	8.81E-03
		.50E+07	1.29E+02	2.03E+17	0.067	-0.155	0.39833	1.28E-05	0.58140	1.19E-11	2.29E-05	6.02E-03	8.86E-03
		.49E+07	1.26E+02	1.99E+17	0.085	-0.160	0.40839	1.39E-05	0.57133	1.11E-11	2.24E-05	5.98E-03	8.90E-03
		.48E+07	1.23E+02	1.94E+17	0.106	-0.163	0.41903	1.52E-05	0.56069	1.02E-11	2.19E-05	5.94E-03	8.94E-03
		.46E+07	1.19E+02	1.89E+17	0.130	-0.167	0.43017	1.67E-05	0.54954	9.33E-12	2.13E-05	5.91E-03	8.98E-03
		.45E+07	1.16E+02	1.83E+17	0.157	-0.171	0.44176	1.85E-05	0.53794	8.44E-12	2.08E-05	5.88E-03	9.01E-03
		.43E+07	1.12E+02	1.78E+17	0.186	-0.176	0.45428	2.05E-05	0.52542	7.55E-12	2.03E-05	5.86E-03	9.04E-03
		.42E+07	1.08E+02	1.72E+17	0.217	-0.179	0.46672	2.28E-05	0.51297	6.69E-12	1.98E-05	5.84E-03	9.06E-03
		.40E+07	1.05E+02	1.66E+17	0.251	-0.183	0.47942	2.54E-05	0.50026	5.86E-12	1.92E-05	5.82E-03	9.08E-03
		.38E+07	1.01E+02	1.60E+17	0.287	-0.188	0.49233	2.85E-05	0.48735	5.08E-12	1.87E-05	5.81E-03	9.10E-03
		.37E+07	9.70E+01	1.53E+17	0.325	-0.192	0.50536	3.20E-05	0.47431	4.35E-12	1.82E-05	5.80E-03	9.11E-03
		.35E+07	9.33E+01	1.47E+17	0.363	-0.196	0.51817	3.60E-05	0.46150	3.70E-12	1.76E-05	5.79E-03	9.12E-03
		.33E+07	8.99E+01	1.41E+17	0.400	-0.200	0.52988	4.02E-05	0.44978	3.16E-12	1.71E-05	5.78E-03	9.13E-03
		.31E+07	8.68E+01	1.36E+17	0.434	-0.205	0.54066	4.46E-05	0.43900	2.71E-12	1.67E-05	5.78E-03	9.14E-03
		.30E+07	8.40E+01	1.31E+17	0.466	-0.208	0.55064	4.93E-05	0.42902	2.33E-12	1.62E-05	5.78E-03	9.14E-03
		.28E+07	8.13E+01	1.26E+17	0.497	-0.213	0.55990	5.42E-05	0.41975	2.01E-12	1.58E-05	5.78E-03	9.14E-03
		.27E+07	7.88E+01	1.22E+17	0.525	-0.216	0.56853	5.95E-05	0.41111	1.74E-12	1.54E-05	5.77E-03	9.14E-03
		.25E+07	7.64E+01	1.18E+17	0.553	-0.219	0.57659	6.51E-05	0.40304	1.50E-12	1.50E-05	5.77E-03	9.15E-03
		.24E+07	7.42E+01	1.14E+17	0.579	-0.224	0.58414	7.10E-05	0.39549	1.30E-12	1.47E-05	5.77E-03	9.15E-03
		.22E+07	7.20E+01	1.10E+17	0.603	-0.228	0.59121	7.73E-05	0.38841	1.13E-12	1.43E-05	5.77E-03	9.15E-03
		.21E+07	7.00E+01	1.06E+17	0.626	-0.233	0.59785	8.40E-05	0.38177	9.86E-13	1.40E-05	5.77E-03	9.15E-03
		.20E+07	6.81E+01	1.03E+17	0.648	-0.237	0.60409	9.11E-05	0.37552	8.59E-13	1.39E-05	5.77E-03	9.15E-03
		.18E+07	6.63E+01	9.92E+16	0.668	-0.241	0.60996	9.86E-05	0.36964	7.50E-13	1.45E-05	5.77E-03	9.15E-03
		.17E+07	6.45E+01	9.60E+16	0.688	-0.246	0.61549	1.07E-04	0.36410	6.56E-13	1.68E-05	5.77E-03	9.15E-03
		.16E+07	6.32E+01	9.36E+16	0.702	-0.250	0.61958	1.13E-04	0.36001	5.91E-13	2.14E-05	5.76E-03	9.15E-03
		.15E+07	6.20E+01	9.12E+16	0.716	-0.254	0.62349	1.20E-04	0.35610	5.33E-13	3.07E-05	5.75E-03	9.15E-03
		.14E+07	6.07E+01	8.89E+16	0.729	-0.259	0.62722	1.27E-04	0.35236	4.81E-13	4.76E-05	5.73E-03	9.15E-03
		.13E+07	5.95E+01	8.67E+16	0.742	-0.264	0.63079	1.35E-04	0.34879	4.34E-13	7.59E-05	5.70E-03	9.15E-03
		.13E+07	5.84E+01	8.46E+16	0.754	-0.268	0.63420	1.43E-04	0.34537	3.92E-13	1.20E-04	5.65E-03	9.15E-03
		.12E+07	5.72E+01	8.25E+16	0.766	-0.275	0.63747	1.51E-04	0.34211	3.54E-13	1.84E-04	5.57E-03	9.15E-03
		.11E+07	5.61E+01	8.05E+16	0.777	-0.281	0.64059	1.60E-04	0.33899	3.20E-13	2.72E-04	5.47E-03	9.15E-03
		.10E+07	5.50E+01	7.85E+16	0.788	-0.287	0.64359	1.69E-04	0.33600	2.90E-13	3.87E-04	5.34E-03	9.15E-03
		.09E+07	5.40E+01	7.66E+16	0.798	-0.295	0.64646	1.79E-04	0.33315	2.63E-13	5.28E-04	5.17E-03	9.15E-03
		.08E+07	5.30E+01	7.47E+16	0.807	-0.304	0.64922	1.89E-04	0.33041	2.38E-13	6.94E-04	4.98E-03	9.15E-03
		.08E+07	5.20E+01	7.29E+16	0.817	-0.312	0.65185	2.00E-04	0.32779	2.16E-13	8.82E-04	4.76E-03	9.15E-03
		.07E+07	5.10E+01	7.11E+16	0.826	-0.321	0.65438	2.11E-04	0.32529	1.96E-13	1.09E-03	4.52E-03	9.15E-03
		.06E+07	5.00E+01	6.94E+16	0.834	-0.329	0.65681	2.22E-04	0.32289	1.78E-13	1.30E-03	4.26E-03	9.15E-03
		.05E+07	4.91E+01	6.77E+16	0.842	-0.337	0.65913	2.34E-04	0.32058	1.61E-13	1.53E-03	4.00E-03	9.15E-03
		.04E+07	4.82E+01	6.61E+16	0.850	-0.346	0.66136	2.47E-04	0.31838	1.47E-13	1.75E-03	3.74E-03	9.15E-03
		.04E+07	4.73E+01	6.45E+16	0.857	-0.352	0.66349	2.60E-04	0.31627	1.33E-13	1.98E-03	3.48E-03	9.15E-03
		.03E+07	4.64E+01	6.29E+16	0.865	-0.364	0.66550	2.73E-04	0.31429	1.22E-13	2.18E-03	3.24E-03	9.15E-03
		.02E+07	4.55E+01	6.14E+16	0.872	-0.373	0.66746	2.87E-04	0.31235	1.11E-13	2.39E-03	3.00E-03	9.15E-03
		.02E+07	4.47E+01	5.99E+16	0.878	-0.381	0.66934	3.03E-04	0.31049	1.01E-13	2.58E-03	2.78E-03	9.15E-03
		.01E+07	4.38E+01	5.84E+16	0.885	-0.390	0.67113	3.18E-04	0.30871	9.22E-14	2.75E-03	2.58E-03	9.15E-03
		.00E+07	4.30E+01	5.70E+16	0.891	-0.399	0.67285	3.35E-04	0.30700	8.40E-14	2.91E-03	2.39E-03	9.15E-03
		.95E+06	4.22E+01	5.56E+16	0.896	-0.406	0.67450	3.52E-04	0.30536	7.65E-14	3.06E-03	2.21E-03	9.15E-03
		.88E+06	4.14E+01	5.42E+16	0.902	-0.414	0.67607	3.71E-04	0.30379	6.97E-14	3.19E-03	2.06E-03	9.15E-03
		.81E+06	4.06E+01	5.29E+16	0.907	-0.423	0.67758	3.90E-04	0.30228	6.36E-14	3.31E-03	1.92E-03	9.15E-03
		.74E+06	3.99E+01	5.16E+16	0.912	-0.430	0.67902	4.10E-04	0.30084	5.80E-14	3.42E-03	1.80E-03	9.15E-03
		.63E+06	3.86E+01	4.95E+16	0.919	-0.446	0.68129	4.45E-04	0.29856	4.98E-14	3.57E-03	1.62E-03	9.15E-03
		.52E+06	3.74E+01	4.75E+16	0.926	-0.459	0.68339	4.84E-04	0.29644	4.27E-14	3.69E-03	1.48E-03	9.15E-03
		.41E+06	3.63E+01	4.55E+16	0.933	-0.473	0.68535	5.26E-04	0.29445	3.67E-14	3.79E-03	1.37E-03	9.15E-03
		.31E+06	3.51E+01	4.36E+16	0.939	-0.488	0.68716	5.71E-04	0.29261	3.15E-14	3.86E-03	1.28E-03	9.15E-03
		.20E+06	3.40E+01	4.18E+16	0.945	-0.502	0.68885	6.20E-04	0.29088	2.71E-14	3.92E-03	1.21E-03	9.15E-03
		.10E+06	3.29E+01	4.01E+16	0.950	-0.517	0.69042	6.74E-04	0.28927	2.33E-14	3.97E-03	1.16E-03	9.15E-03

0, No. 2, April 1988

Bahcall & Ulrich (1988)

Stellar interior

Equations of stellar structure

- The interior structure of a star in equilibrium can be described with the following "ingredients"
- Variables and their stratification (function of radius) P, ϱ, T, M_r, L_r
- The following equations :

Hydro**dynamic** equilibrium	$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} - \rho\frac{d^2r}{dt^2}$
Mass conservation/continuity	$\frac{dM_r}{dr} = 4\pi r^2 \rho$
Equation of state	$P = \frac{\rho k T}{\mu m_H}$
Energy "generation"	$\frac{dL_r}{dr} = 4\pi r^2 \rho \left[\varepsilon - T\frac{dS}{dt}\right]$
Energy transport	$\frac{dT}{dr} = -\frac{3\kappa\rho}{16\pi a c}\frac{L_r}{r^2 T^2}$

More general, **time-dependent form**

Stellar structure is time-dependent and will adjust

➡ S: entropy

➡ T (dS/dt) is the energy of collapse expressed in terms of the entropy change

- And these boundary conditions:
  - In the centre (r=0): $M_r(0) = 0, \quad L_r(0) = 0$
  - At the surface (r=R): $M_r(R) = M, \quad L_r(R) = L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$

- Note that the mean molecular weight μ and the opacity $\varkappa$ are now functions of radius and time (due to fusion in the core)

Energy transport

Mixing length theory

- In the context of stellar (interior) structure and evolution, we want to know about convection zones:
    - **How much energy** can be transported by convection?
    - What is the **temperature gradient** ?

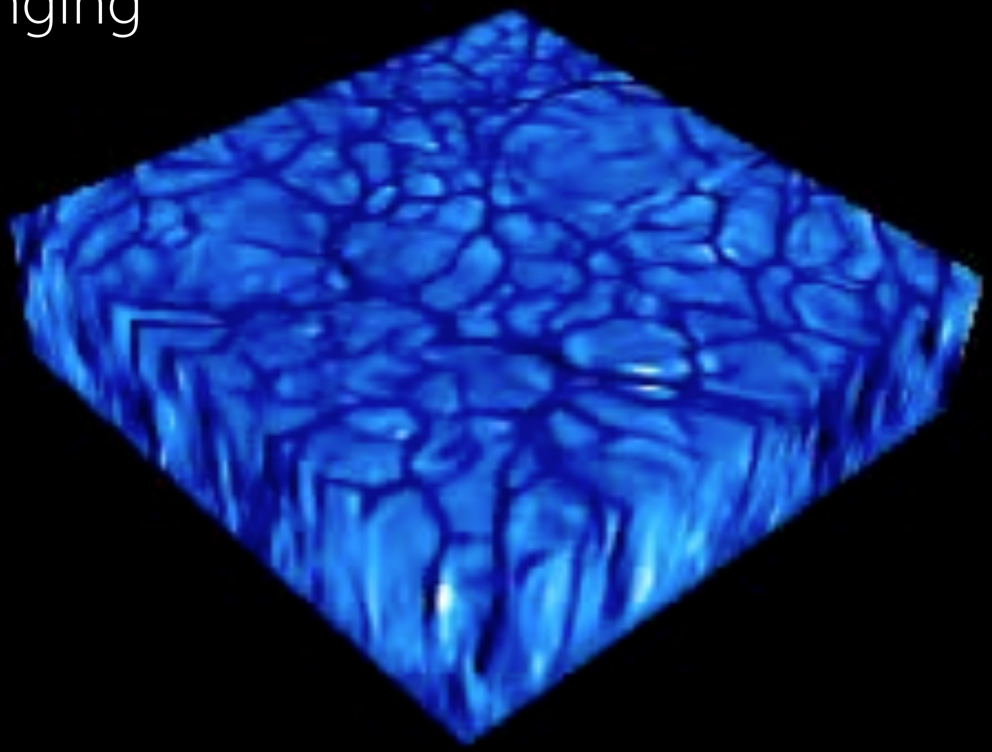
- But: Detailed theory of convection and its practical application still challenging today

- Addressed with numerical simulations but very challenging and computationally expensive

➡ Prohibitive to use as part of stellar evolution calculations.

➡ Simpler approach needed

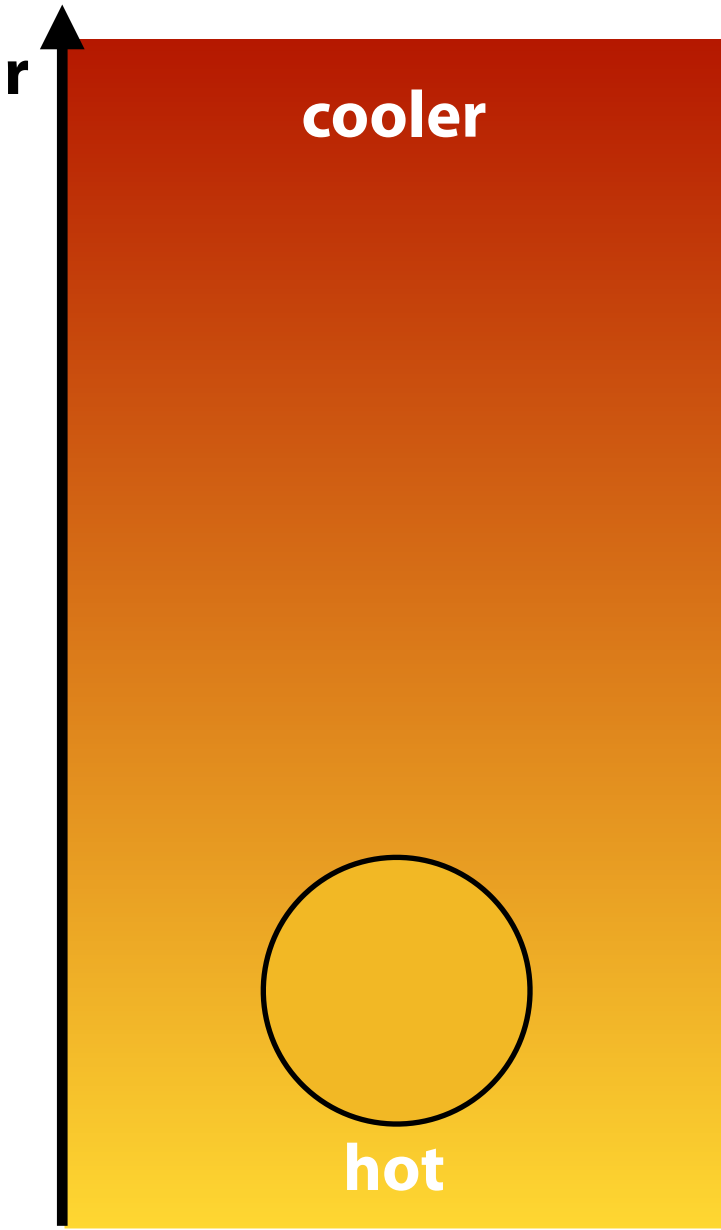
➡ **Mixing Length Theory (MLT)**



Energy transport

Mixing length theory

- Simplified picture of convective energy transport:
    1. Gas element rises (or sinks) over a radial distance
    2. Gas element dissolves (becomes part of the new environment) and releases excess heat
  - Mixing length l_m
  - If gas element sinks, it absorbs deficit energy from environment
  - Mixing length l_m is an unknown free parameter!
  - Assumption: l_m on the order of local pressure scale height H_P



Energy transport

Mixing length theory

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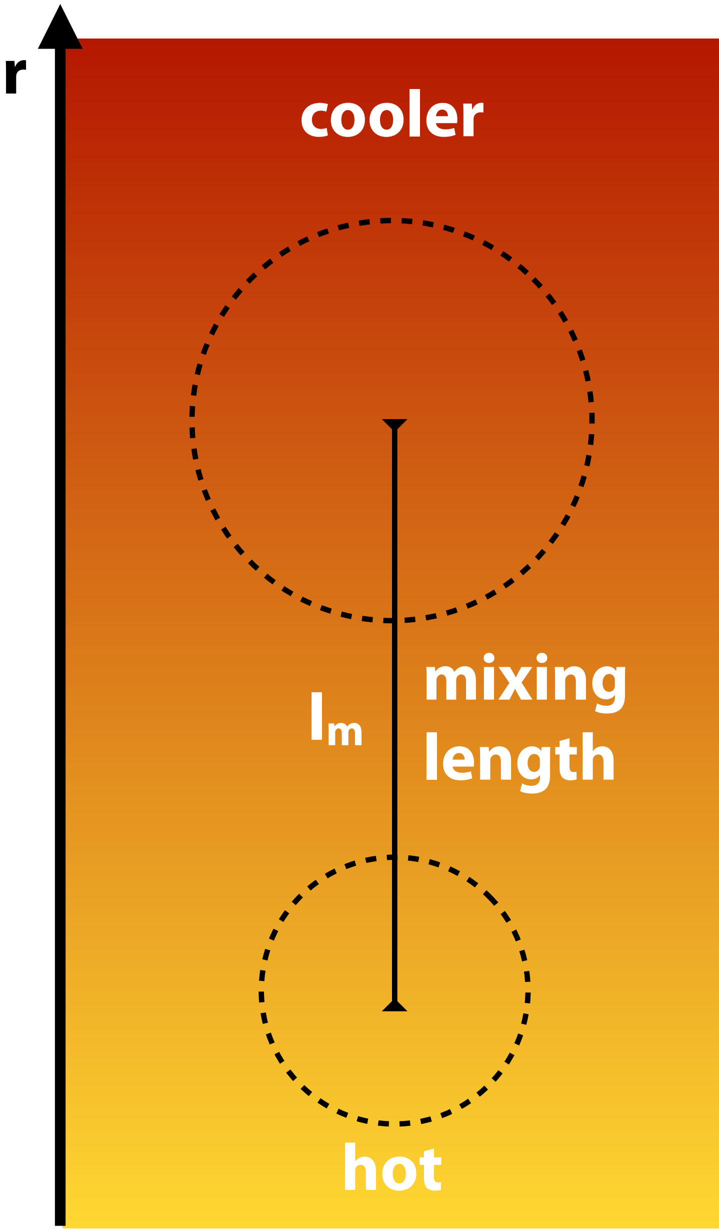


- **Pressure scale height:** (in a stratified medium) the radial distance over which the pressure changes by a factor 1/e

$$H_P = \left| \frac{\mathrm{d}r}{\mathrm{d}\ln P} \right| = \frac{P}{\rho g}.$$

Energy transport

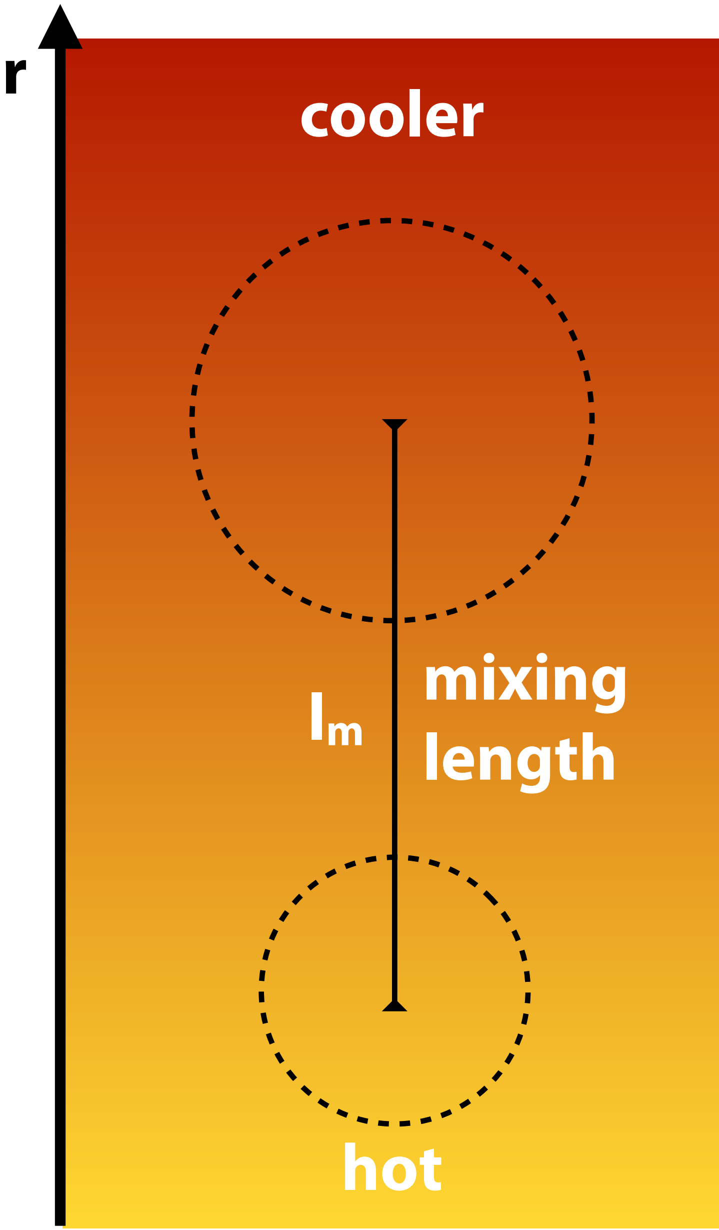
Mixing length theory

- Assumption: $l_m = \alpha H_P$
  - Valid in hydrostatic equilibrium.
  - Reasonable as gas element expands while rising

- Assume spherical surface inside the convection zone
  - 1/2 covered by rising blobs
  - 1/2 covered by sinking blobs

➡ Expanding rising blobs would cover most of the area after rising 1-2 pressure scale heights.

➡ **Net energy transport upwards** (down the gradient)



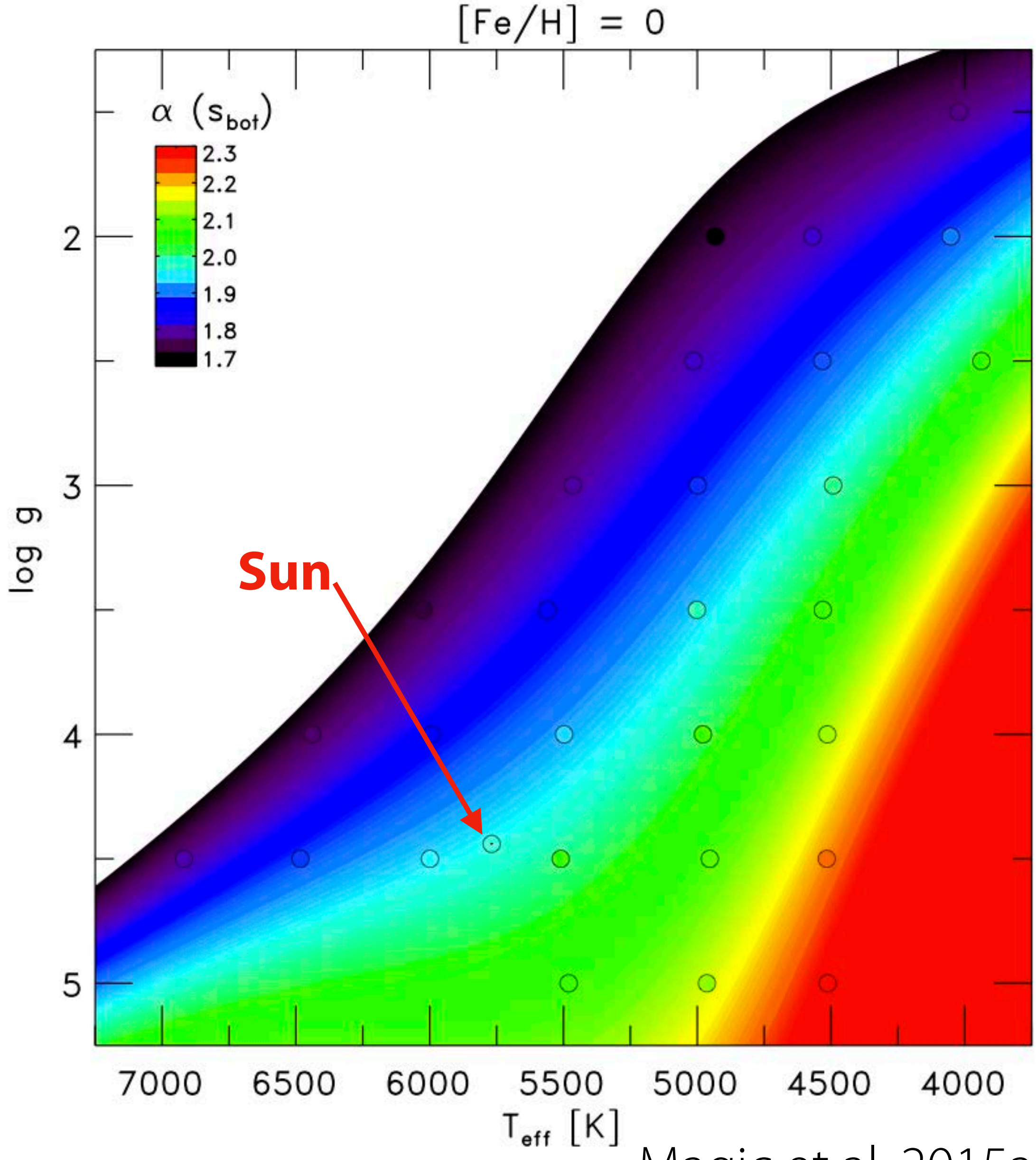
- **Pressure scale height:** (in a stratified medium) the radial distance over which the pressure changes by a factor 1/e

$$P(r) = P_0 e^{-(r/H_P)} \Longrightarrow H_P = \left|\frac{dr}{d\ln P}\right| = \frac{P}{\rho g}$$

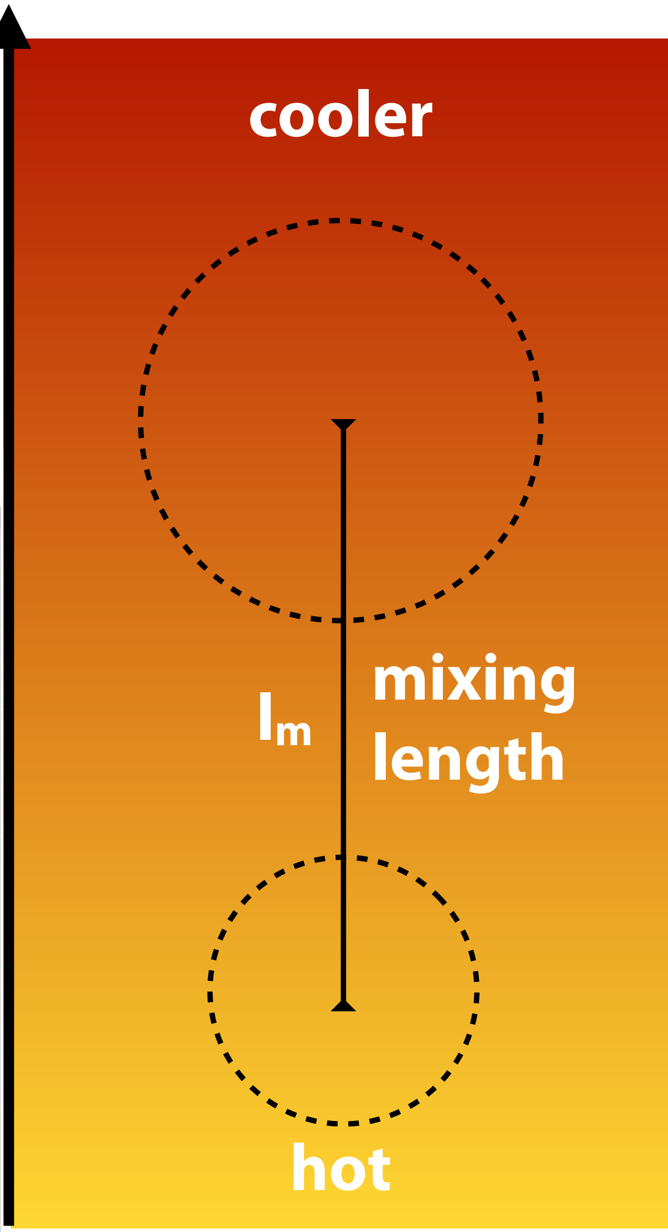
Energy transport

Mixing length theory

- Detailed numerical model calculations
  (in comparison to observation)
  to derive/calibrate corresponding mixing length
- Mixing length (via parameter depends on
  - Effective temperature T_{eff}
  - Grav. Acceleration $log\ g$
  - Metallicity $[Fe/M]$
- Values for α typically ~2

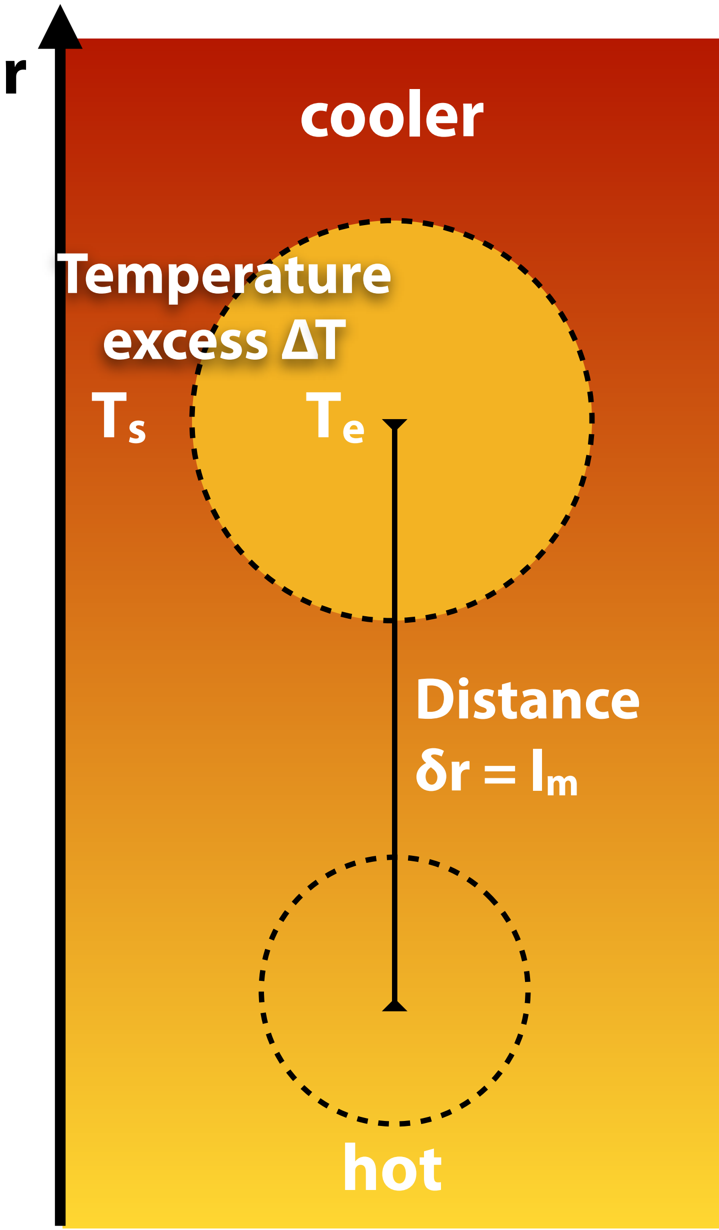


Magic et al. 2015a



Energy transport

The convective energy flux



- Gas element after rising a distance $\delta r = l_m$:

➡ Mean excess temperature ΔT between element and surrounding:

$$\Delta T = T_e - T_s = \left[\left(\frac{dT}{dr}\right)_e - \frac{dT}{dr}\right]\ell_m = \Delta\left(\frac{dT}{dr}\right)\ell_m$$

  - dT/dr : temperature gradient in surrounding
  - $(dT/dr)_e$: variation of temperature with radius r for the gas element while rising and expanding adiabatically
  - $\Delta(dT/dr)$: difference between the two gradients.

- Rewrite the equation above with gradients $\nabla \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)$ and $H_P = \left|\frac{dr}{d\ln P}\right| = \frac{P}{\rho g}$, deriving and using the following equation

$$\frac{dT}{dr} = T\frac{d\ln T}{dr} = T\frac{d\ln T}{d\ln P}\frac{d\ln P}{dr} = -\frac{T}{H_P}\nabla \quad \text{and} \quad \left(\frac{dT}{dr}\right)_e = -\frac{T}{H_P}\nabla_{ad}.$$

➡ $$\Delta T = T\frac{\ell_m}{H_P}(\nabla - \nabla_{ad})$$

Energy transport

The convective energy flux

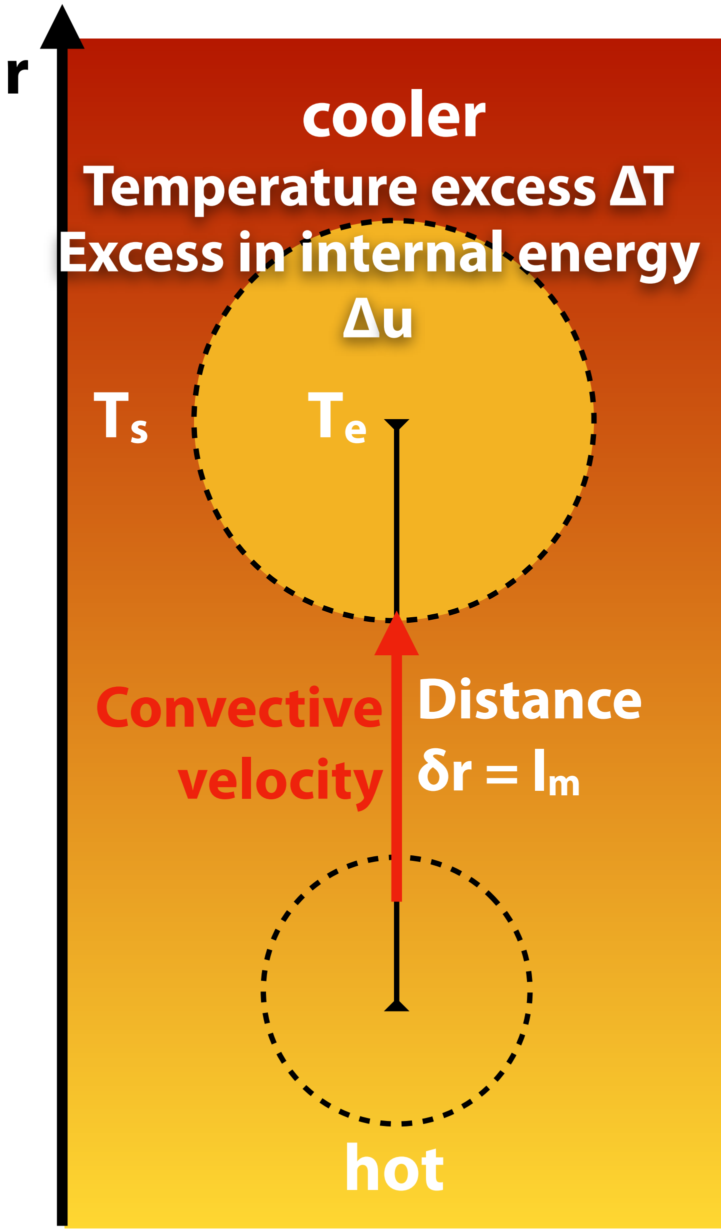
- Mean temperature excess ΔT is related to an excess in internal energy between the gas element and the surrounding
  $\Delta u = c_P \, \Delta T \, \varrho$
- Energy flux carried by gas elements at (average) velocity v_c (the convective velocity)
  $F_{\mathrm{conv}} = v_c \, \varrho \, \Delta u = v_c \, \varrho \, c_P \, \Delta T$

➡ What is the **convective velocity**?

➡ Gas element moves over distance l_m in the time t starting from resting position at constant acceleration: $l_m = {}^1/_2 \, a \, t^2$

  ➡ Average velocity $v_c \approx l_m / t = \sqrt{\frac{1}{2}\ell_{\mathrm{m}} a}$

➡ Buoyancy force provides the acceleration $a = -g \dfrac{\Delta\rho}{\rho} \approx g \dfrac{\Delta T}{T}$



$$\Delta T = T \, \frac{\ell_{\mathrm{m}}}{H_P} \, (\nabla - \nabla_{\mathrm{ad}})$$

Energy transport

The convective energy flux

➡ Convective velocity $v_c \approx \sqrt{\frac{1}{2}\ell_m g \frac{\Delta T}{T}} \approx \sqrt{\frac{{\ell_m}^2 g}{2H_P}(\nabla - \nabla_{ad})}$

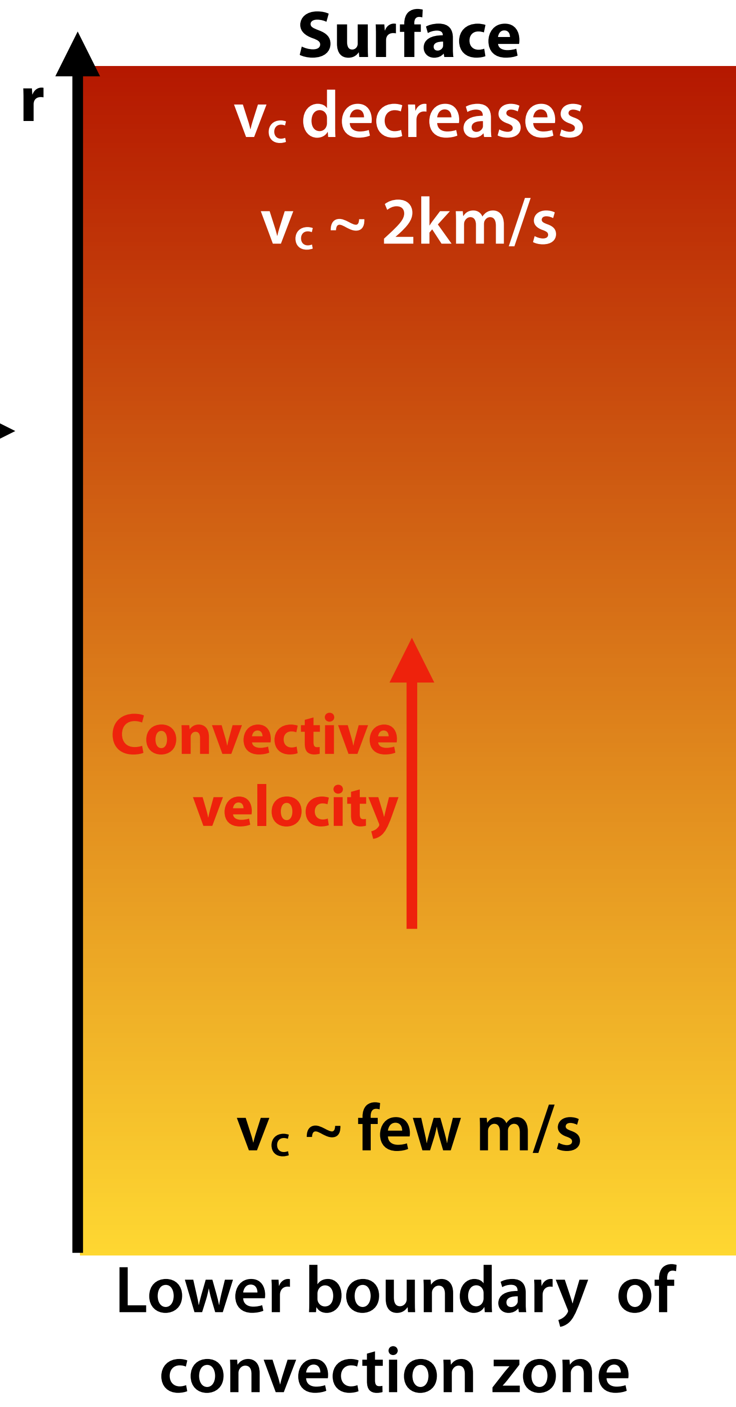
- Increases with radius
- For the Sun up to ~2 km/s (average velocity) ⟶

➡ **Convective energy flux** $\boxed{F_{conv} = \rho c_P T \left(\frac{\ell_m}{H_P}\right)^2 \sqrt{\tfrac{1}{2} g H_P}\,(\nabla - \nabla_{ad})^{3/2}}$

- **Superadiabaticity $\nabla - \nabla_{ad}$**: degree to which the actual temperature gradient ∇ exceeds the adiabatic value ∇_{ad}.

- **What $\nabla - \nabla_{ad}$ is needed to carry the whole energy flux by convection?**
  - With typical values for the whole star (using the virial theorem):

➡ $F_{conv} \sim \frac{M}{R^3}\left(\frac{GM}{R}\right)^{3/2}(\nabla - \nabla_{ad})^{3/2}$



Surface

r

v_c decreases

v_c ~ 2km/s

Convective velocity

v_c ~ few m/s

Lower boundary of convection zone

Energy transport

The convective energy flux and temperature gradient

- Combine $F_{\rm conv} = \rho c_P T \left(\frac{\ell_{\rm m}}{H_P}\right)^2 \sqrt{\frac{1}{2} g H_P} \, (\nabla - \nabla_{\rm ad})^{3/2}$ and $F_{\rm conv} \sim \frac{M}{R^3} \left(\frac{GM}{R}\right)^{3/2} (\nabla - \nabla_{\rm ad})^{3/2}$

➡ **Superadiabaticity** $\nabla - \nabla_{\rm ad} \sim \left(\frac{LR}{M}\right)^{2/3} \frac{R}{GM}$

- Typical values in the interior of the Sun $\nabla - \nabla_{\rm ad} \sim 10^{-5} - 10^{-7}$

➡ Only very small superadiabaticity needed!

➡ Convective energy transport is very efficient with $F_{\rm conv} \gg F_{\rm rad}$

- **Temperature gradient** in a convective region can be derived by simply using $\nabla \approx \nabla_{\rm ad}$

➡ $$\frac{\mathrm{d}T}{\mathrm{d}m} = -\frac{Gm}{4\pi r^4} \frac{T}{P} \nabla$$

Convection near the surface

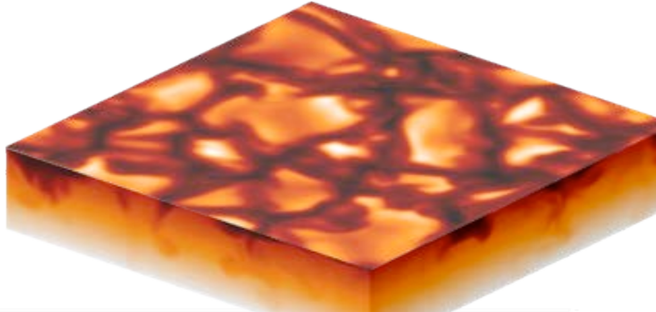
- Density and temperature much smaller
- Superadiabaticity much larger
- Temperature gradient depends on detailed properties of the convective motions

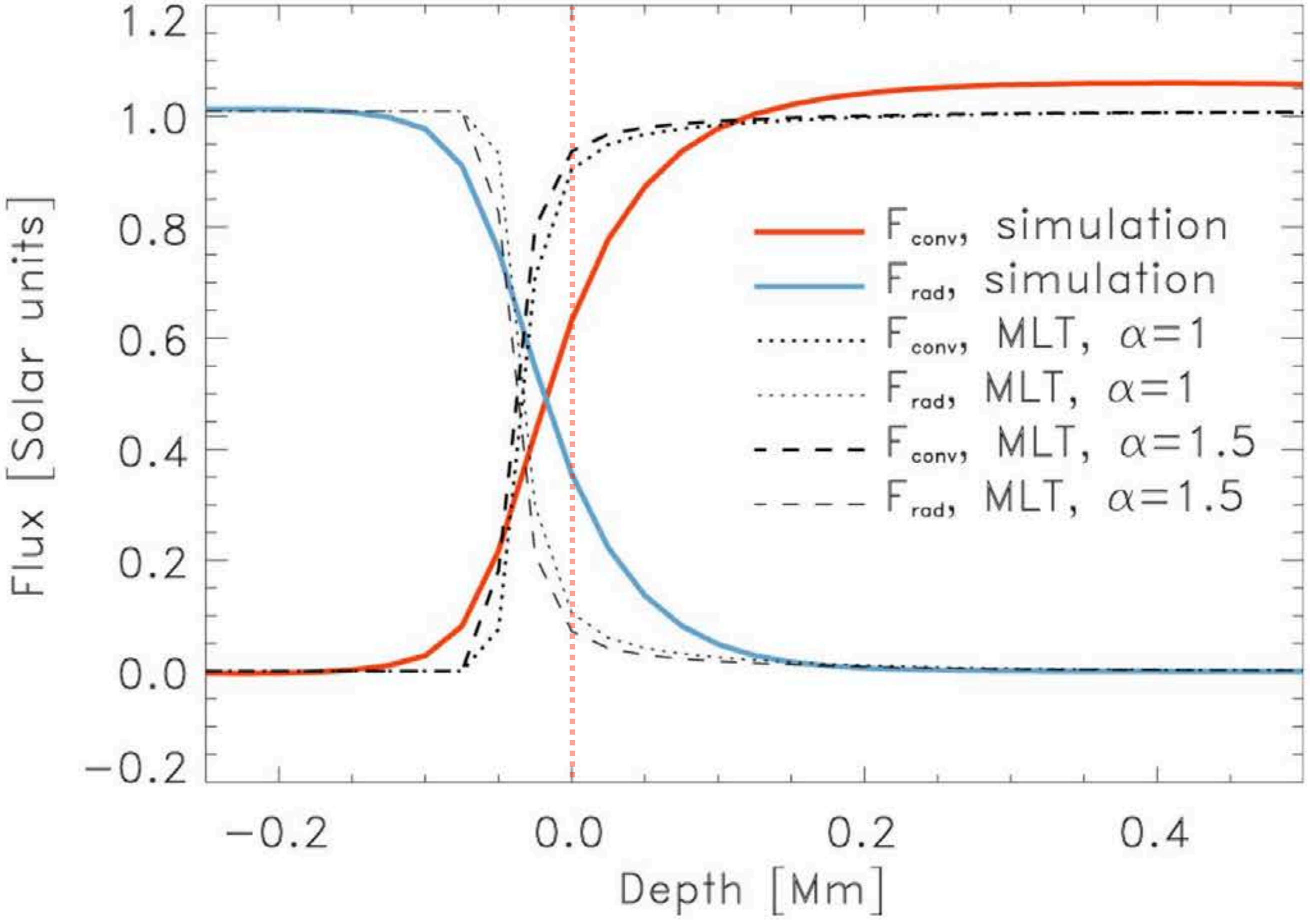
Energy transport

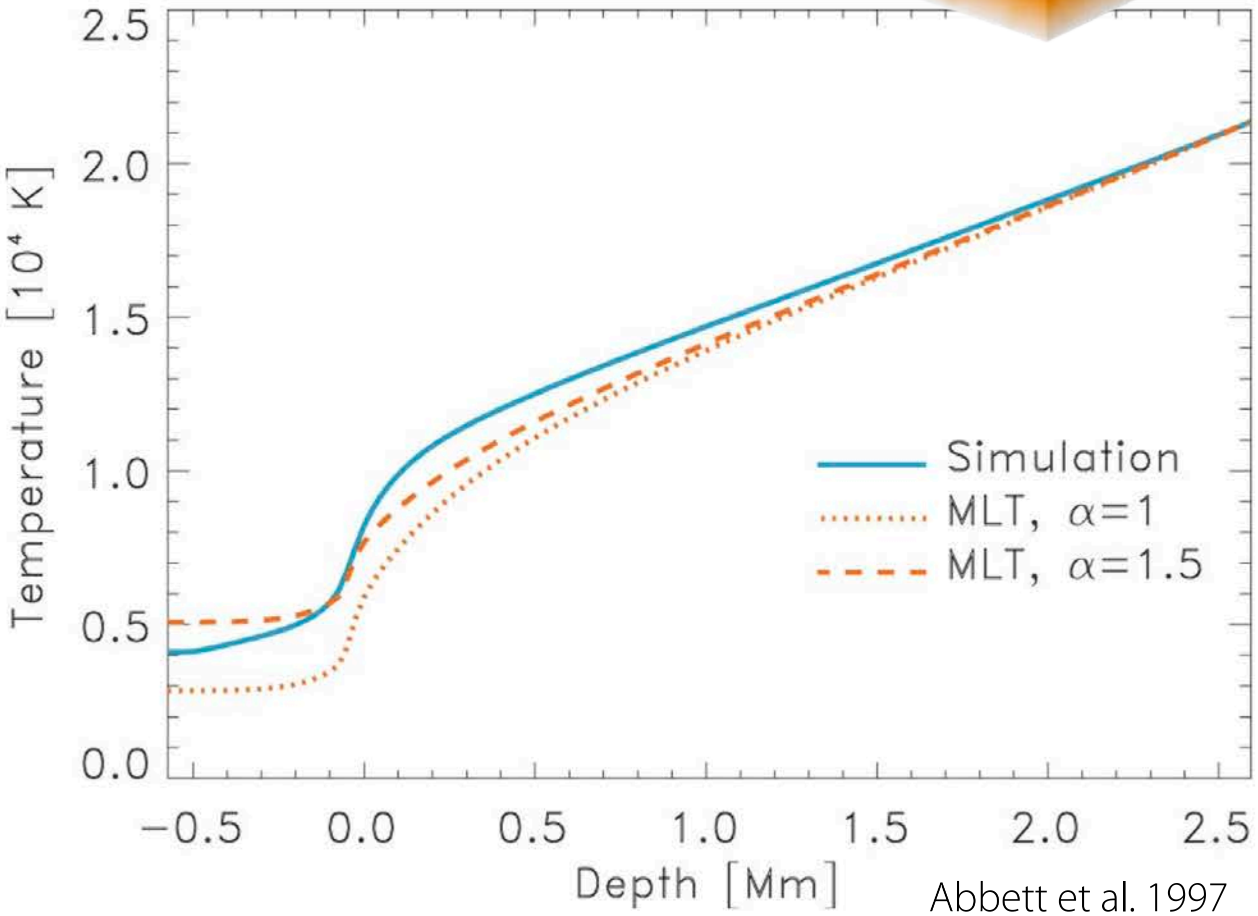
Mixing length - conclusion

$$\boxed{\nabla_{\rm rad} < \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T}\nabla_\mu}$$

- Radiative energy transport = default (in a stable layer)
- Convective energy transport very efficient, occurs if a layer is unstable against convection
- For stellar interior structure/evolution simplified mixing length theory valid in good approximation ($l_m = \alpha H_P$)
- Calibration of α from comparison of detailed 3D numerical models





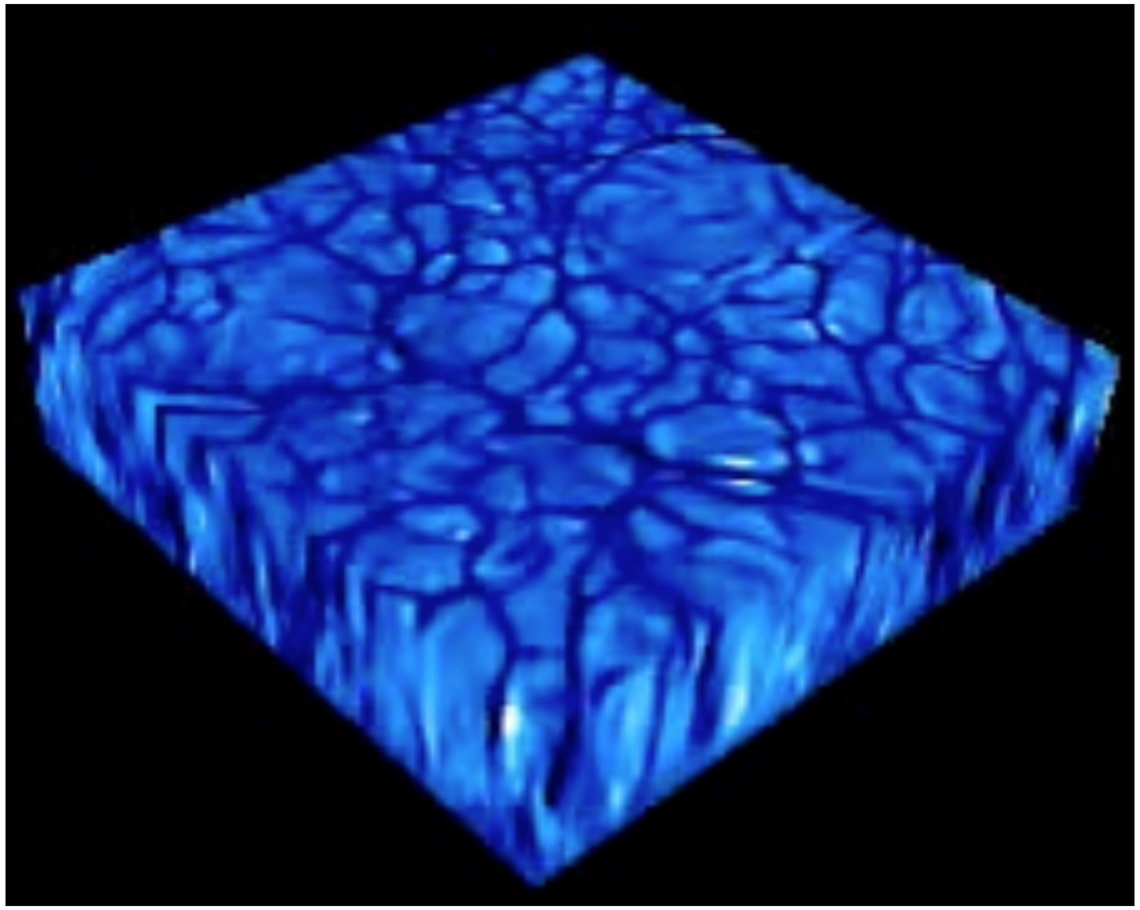


Abbett et al. 1997

Stellar interior

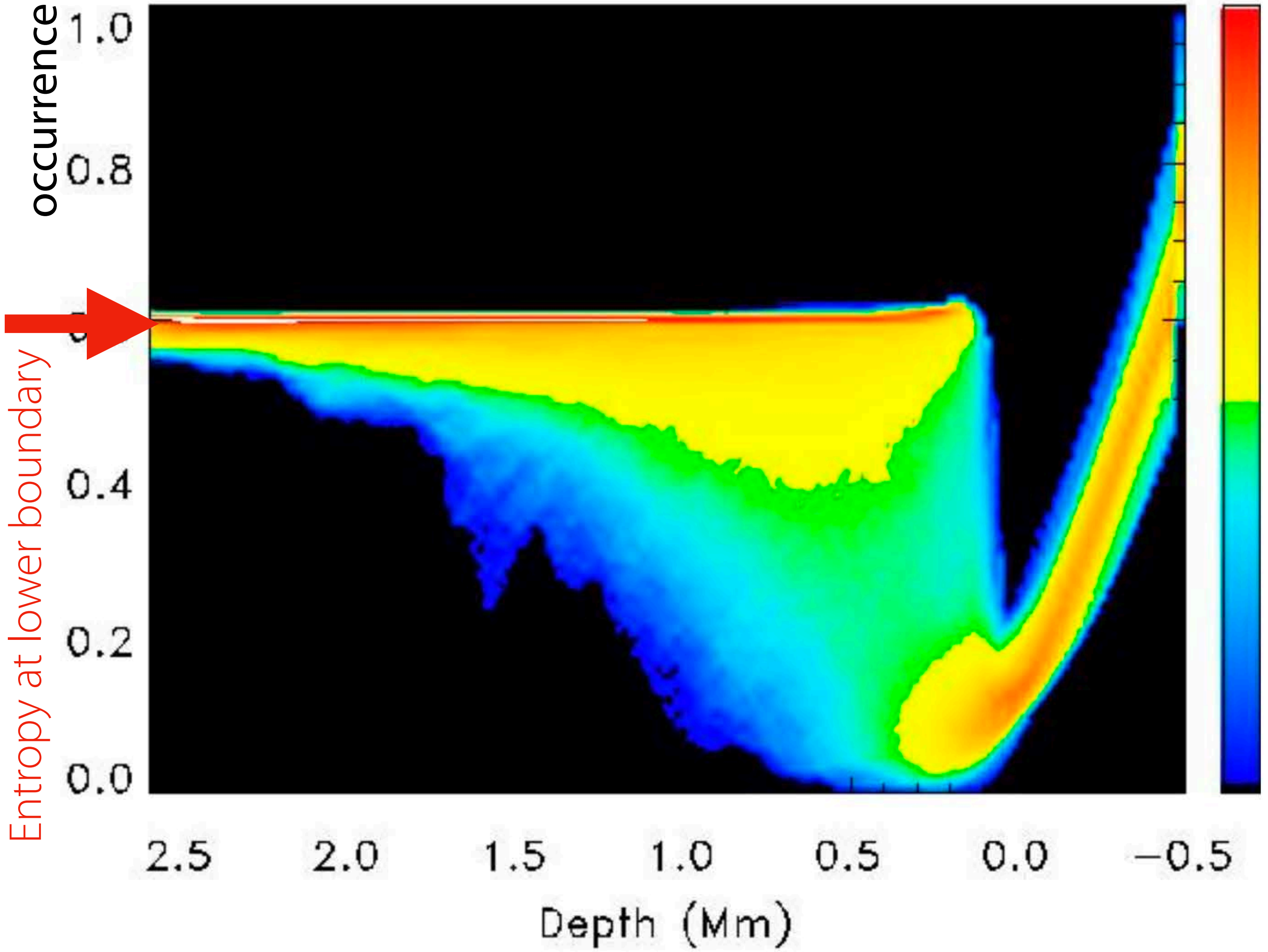
Entropy

- Entropy quite constant in the upper solar convection zone (in contrast to stratification in temperature, pressure etc.)
- Exploited by some simulation codes for the lower boundary (less sensitive to the exact depth, prescribed entropy of gas inflowing through the lower boundary)



Histogram of the entropy

(<u>logarithmic</u> color scale with arbitrary units) as a function of depth

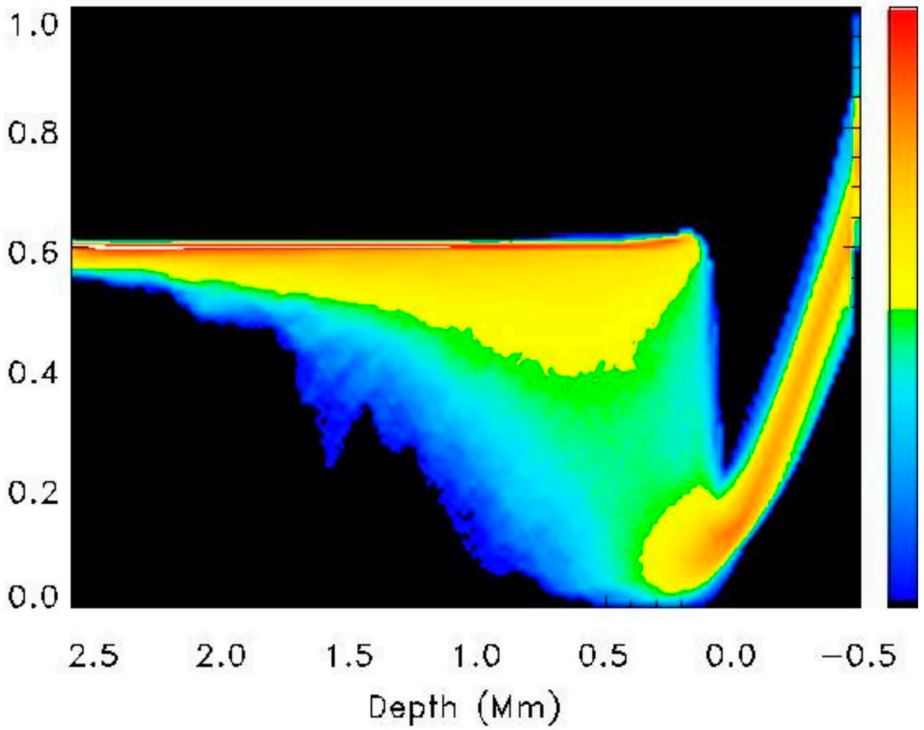


Stellar interior

Entropy

- At the surface: Plasma becomes (more) transparent (longer mean free path of photons)
  ➡ Radiative energy transport becomes efficient
  ➡ Plasma at surface looses thermal energy via radiative cooling
  ➡ Hydrogen ions **recombine** with free electrons to form neutral hydrogen atoms
  ➡ Large amount of ionisation energy set free, also radiated away
  ➡ Escaping photons remove energy and also entropy
  ➡ Resulting overdense fluid sinks back into convection zone (due to gravity)

- Rising plasma in the granules: hot, underdense, high entropy
- Sinking plasma in the intergranular lanes: cool, overdense, low entropy
- Strong impact on entropy at the surface (**entropy jump** + fluctuations) but not so much deeper in the convection zone as diverging upflowing plasma all has almost the same entropy



Stellar interior

Entropy

- Convection needs to transport the flux $F = L_\odot/4\,\pi\,r^2$
- Smaller **superadiabaticity*** $\nabla - \nabla_{ad} \Leftrightarrow$ More efficient transport of energy flux by convection

*difference between actual and adiabatic temperature gradient

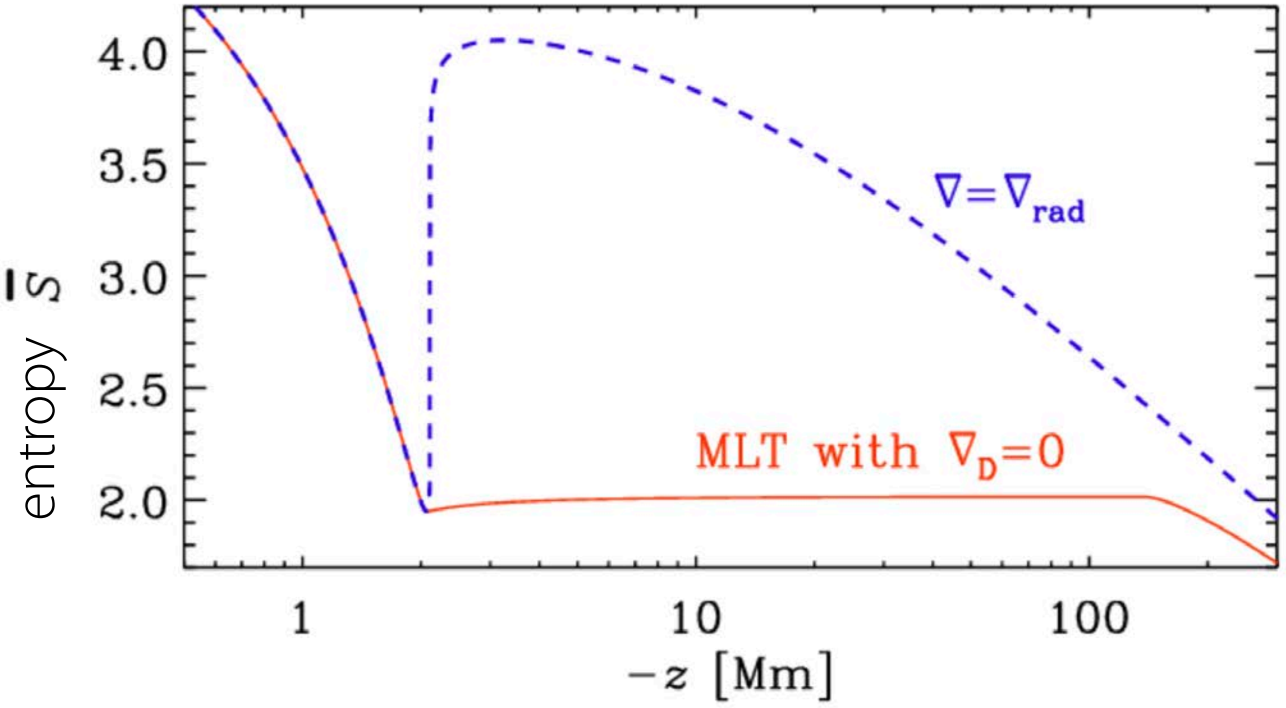
- Related to the excess of specific entropy over the entropy of the marginal state ($\nabla = \nabla_{ad}$)

$$\Delta S = \int c_P (\nabla - \nabla_a)\, d \ln P$$

- ΔS: entropy “jump” across the outermost layers of the convection zone (only there $\nabla = \nabla_{ad}$, significantly larger than zero)

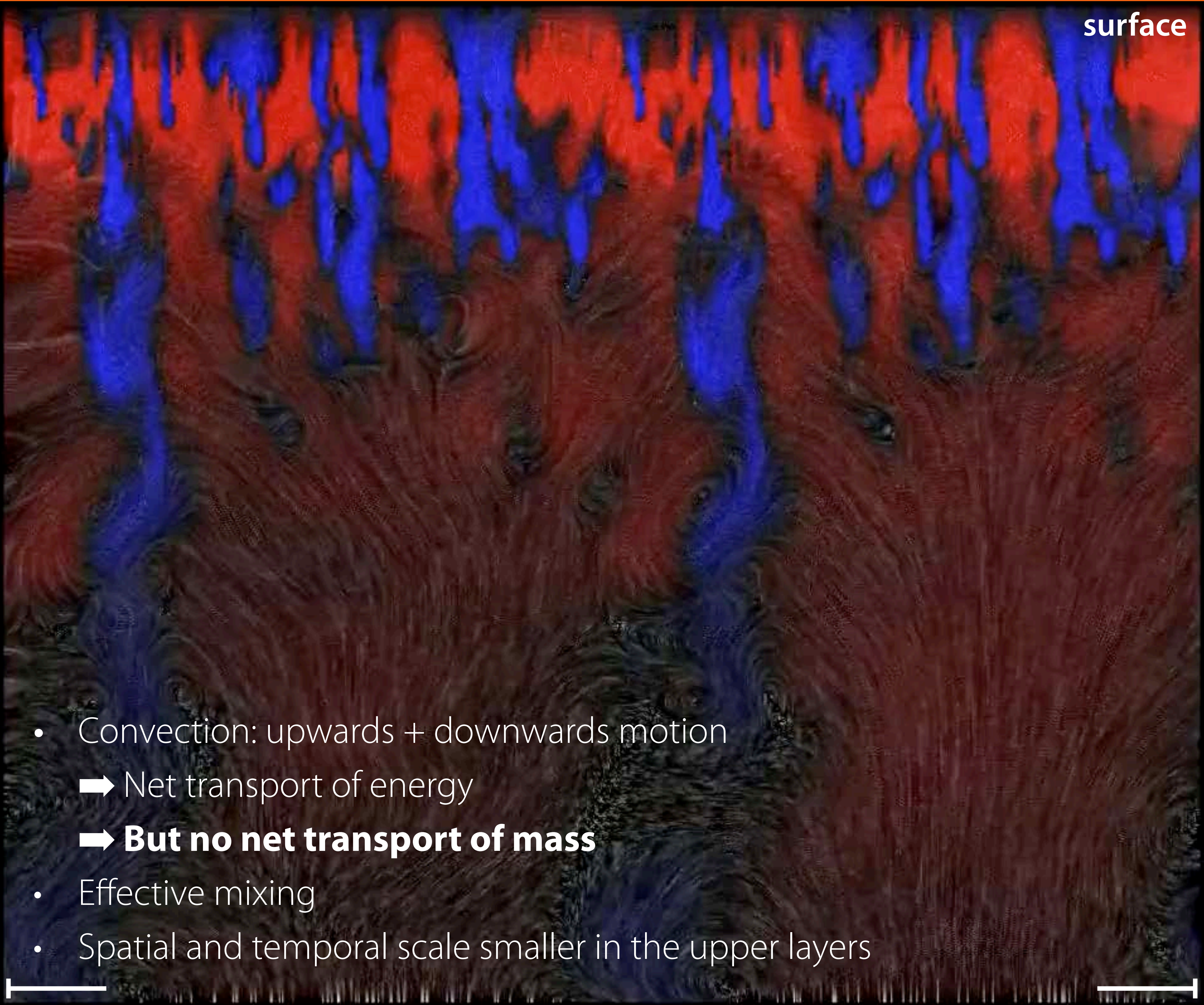
- Note: Efficiency of convection connected to mixing length parameter α
- Detailed properties of convection (including impact of downdraft on entropy) affect the mixing length parameter

Standard convective solution ($\nabla_D = 0$) vs.
nonconvective radiative solution ($\nabla = \nabla_{rad}$)



Brandenburg 2015

Convection

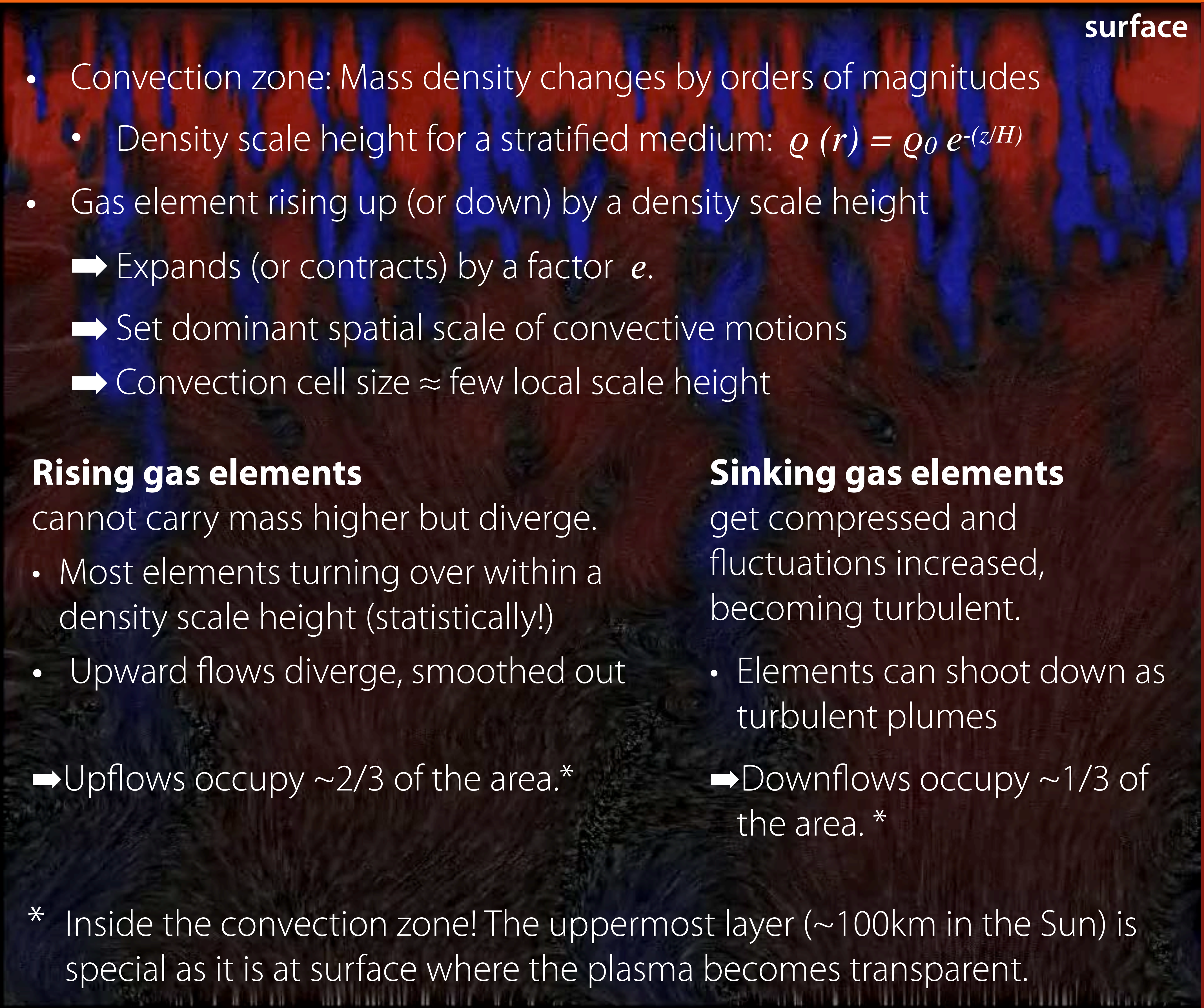


- Simulated vertical velocity in a vertical cross-section trough the Sun’s upper convection zone, played fast forward
- **red: upward**
- **blue: downward**
- streamlines

(C. Henze, NASA Advanced Supercomputing Division, Ames Research Center).

- Convection: upwards + downwards motion
  ➡ Net transport of energy
  ➡ **But no net transport of mass**
- Effective mixing
- Spatial and temporal scale smaller in the upper layers

Convection



- Convection zone: Mass density changes by orders of magnitudes
  - Density scale height for a stratified medium: $\varrho\,(r) = \varrho_0\, e^{-(z/H)}$
- Gas element rising up (or down) by a density scale height

➡ Expands (or contracts) by a factor e.

➡ Set dominant spatial scale of convective motions

➡ Convection cell size ≈ few local scale height

Rising gas elements cannot carry mass higher but diverge.

- Most elements turning over within a density scale height (statistically!)
- Upward flows diverge, smoothed out

➡Upflows occupy ~2/3 of the area.*

Sinking gas elements get compressed and fluctuations increased, becoming turbulent.

- Elements can shoot down as turbulent plumes

➡Downflows occupy ~1/3 of the area. *

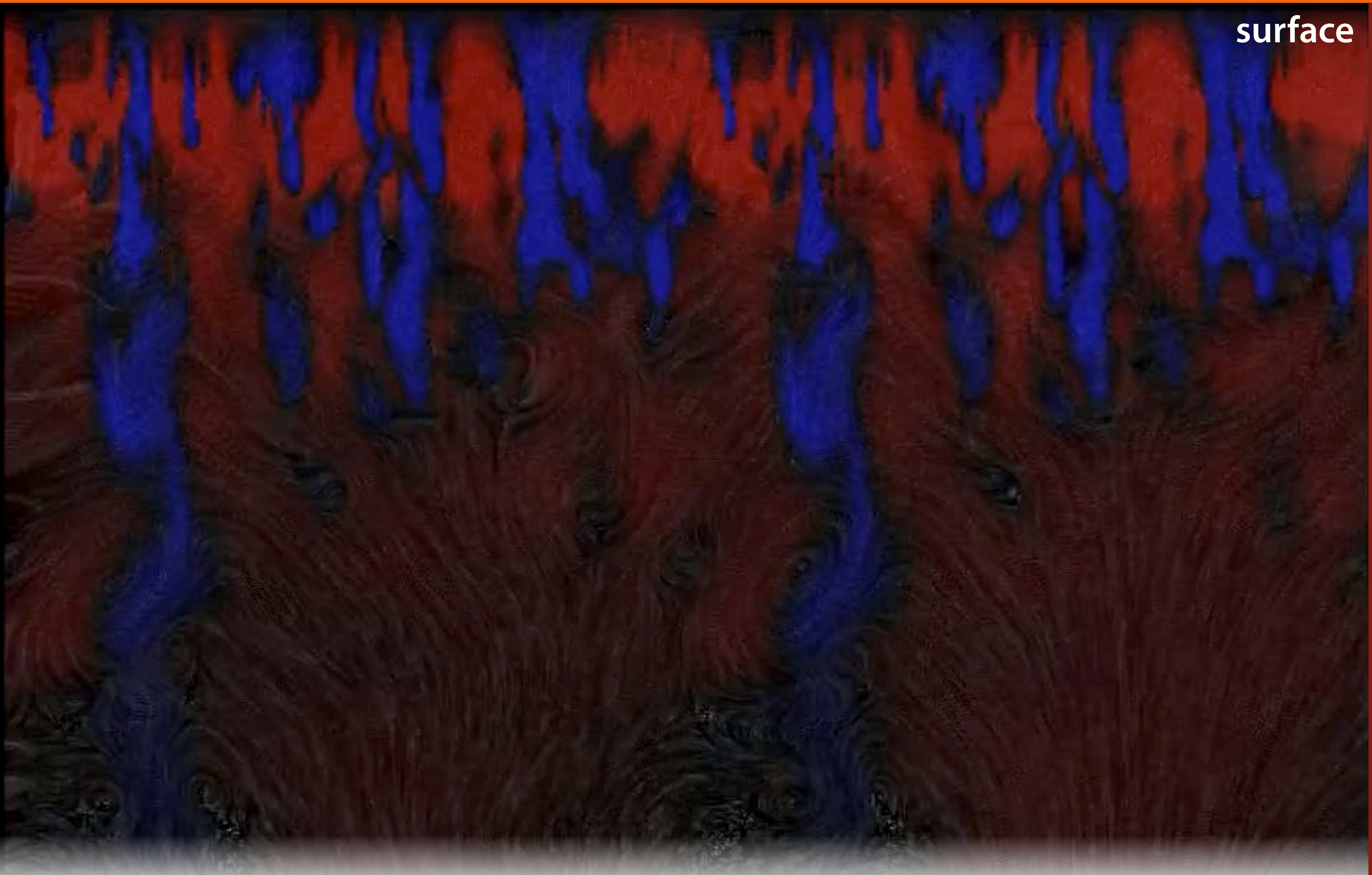
* Inside the convection zone! The uppermost layer (~100km in the Sun) is special as it is at surface where the plasma becomes transparent.

- **Asymmetry** up/downflows but **conservation of mass** must be obeyed!
- Temperature and density increase with depth

➡ Scale height $H_P = P/(\varrho g)$ increases with depth

➡ Typical size of convective cells set by mass conservation and local conditions, increase with depth

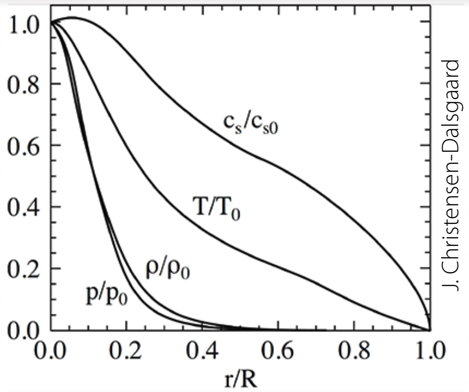
Convection



- **Sound speed** in the adiabatic case (no heat exchange with surrounding) increases with depth, too!

$$c_{\mathrm{s}} = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} = \sqrt{\frac{\gamma P_0}{\rho_0}} = \sqrt{\frac{\gamma k_{\mathrm{B}} T_0}{\mu m_{\mathrm{H}}}} = \sqrt{\gamma g H_p}$$

k_{B}: Boltzmann constant μ: mean molecular weight m_{H}: mass of hydrogen



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Convection

Spatial scales — Convection cell sizes

- Conservation of mass must be obeyed!
- **Rough estimate for typical radius of a convection cell:**
  - Assume a volume in the form of a cylinder with height H and radius r
  - Inflow of matter through bottom with area $\mathrm{A_{in}} = \pi\, \mathrm{r}^2$
  - Flow turns over within one scale height H

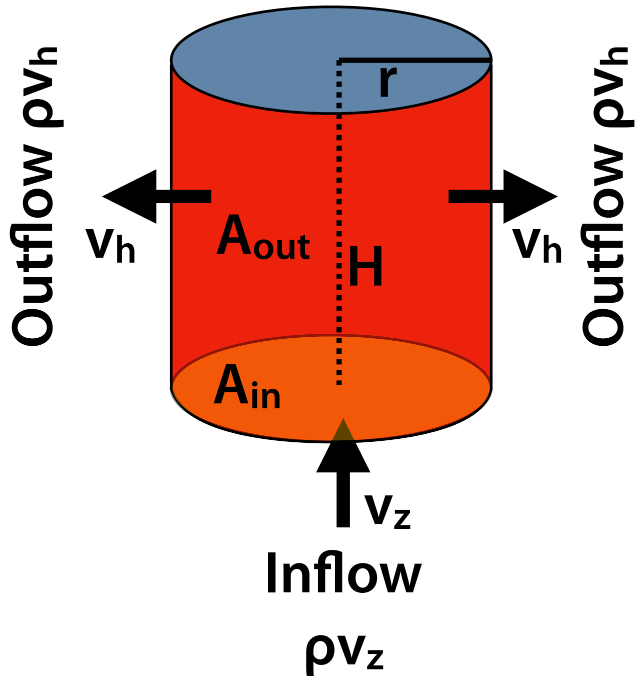
➡ Outflow through the sides of the cylinder with area $\mathrm{A_{out}} = 2\, \pi\, \mathrm{r}\, \mathrm{H}$

- Conservation: Outflow = inflow

➡ $A_{\mathrm{in}}\, \varrho v_z = A_{\mathrm{out}}\, \varrho v_h$

➡ $\pi\, \mathrm{r}^2\, \varrho v_z = 2\, \pi\, \mathrm{r}\, \mathrm{H}\, \varrho v_h$

➡ $\mathrm{r} = 2\, \mathrm{H}\, v_h / v_z$



v_z: vertical velocity
v_h: horizontal velocity

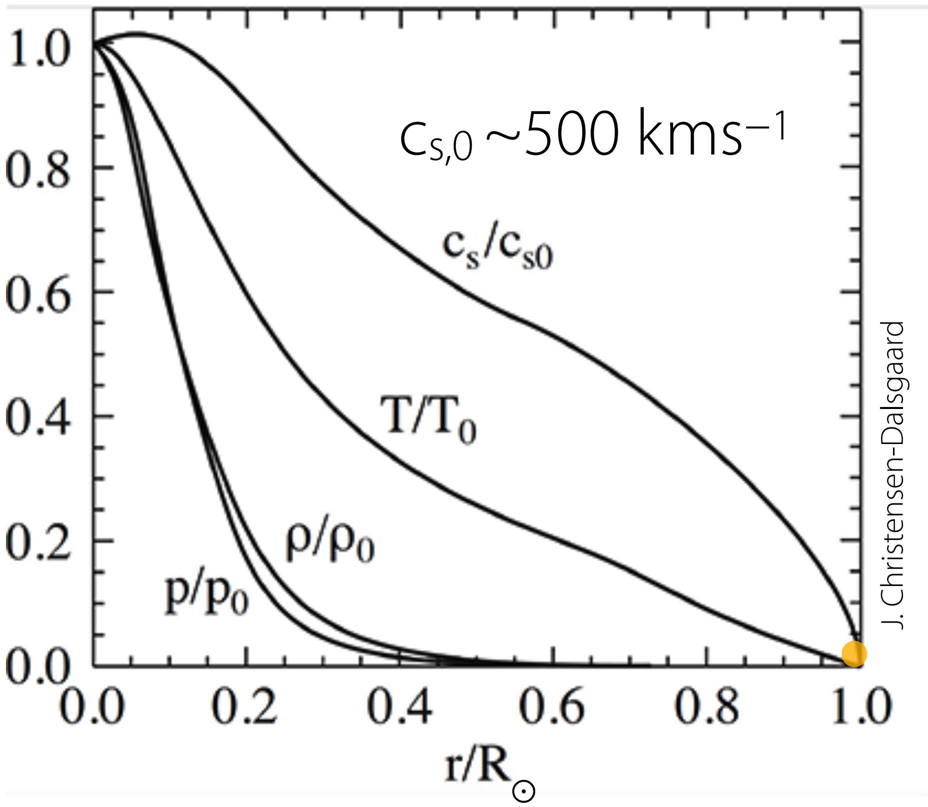
Convection

Spatial scales — Convection cell sizes

- **Typical radius of a convection cell** $r = 2\,H\,v_h / v_z$
- Vertical velocity v_z = convective velocity
- Horizontal velocity? **Upper limit** set by local sound speed

$$v_h < c_s$$

- Convective velocity v_z and sound speed c_s ($>v_h$) depend on the local thermodynamic conditions and are thus functions of radius



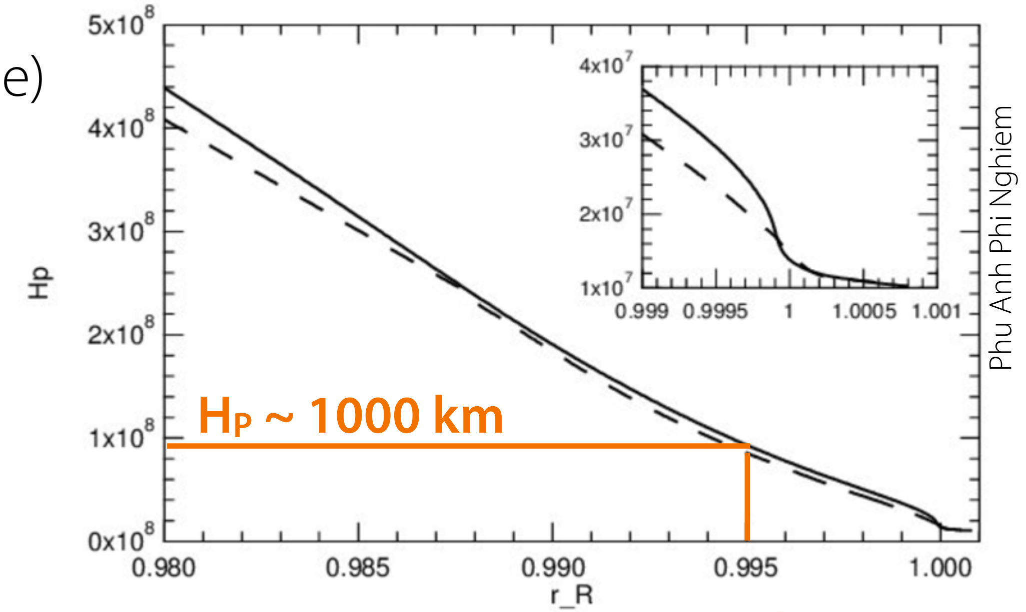
J. Christensen-Dalsgaard

- **Example 1:** Top of the convection zone (well below surface)
  - $v_z \sim 2\,km/s,\ \ H \sim 1000\ km,\ c_s = (\gamma\, H_P\, g)^{1/2} \approx 20$ km/s

➡ $r = 2\,H\,v_h / v_z < 2000\ km \times 20/2 = 20\,000\ km$

➡ Horizontal cell diameters are <u>smaller</u> than that

➡ Will change strongly as function of radius / depth!



Phu Anh Phi Nghiem

3500 km below surface

✳ $\gamma = 5/3$ (ideal monatomic gas). $\log g_\odot = 4.4$ (cgs!)

Convection

Spatial scales — Convection cell sizes

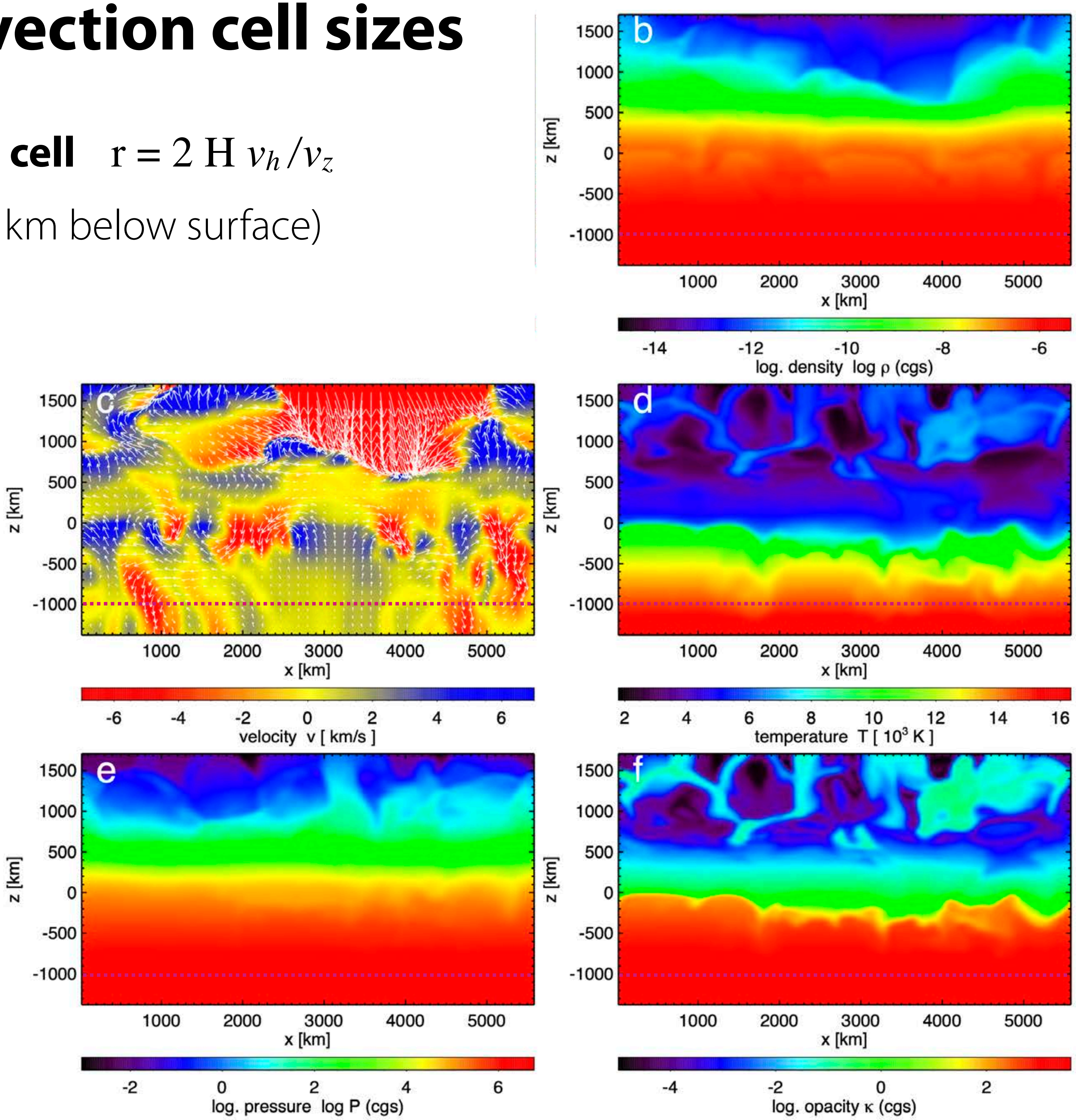
- **Typical radius of a convection cell** $r = 2\,H\,v_h/v_z$
- **Example 2:** 3D simulation, 1000 km below surface)
- All in cgs units!
  - $\log g = 4.4$
  - $v_z \sim 2\,\mathrm{km/s}$
  - $\log P \sim 6$
  - $\log \varrho \sim -6$

➡ $H_P = P/(\varrho g) = 400\ \mathrm{km}$

➡ $c_s = (\gamma\, H_P\, g)^{1/2} \approx 13\ \mathrm{km/s}$

➡ $r < 2\,H_P\, c_s/v_z \approx 5000\ \mathrm{km}$

➡ Convection cell diameter smaller than 10 Mm



Wedemeyer (2003)

Convection

Spatial scales — Convection cell sizes

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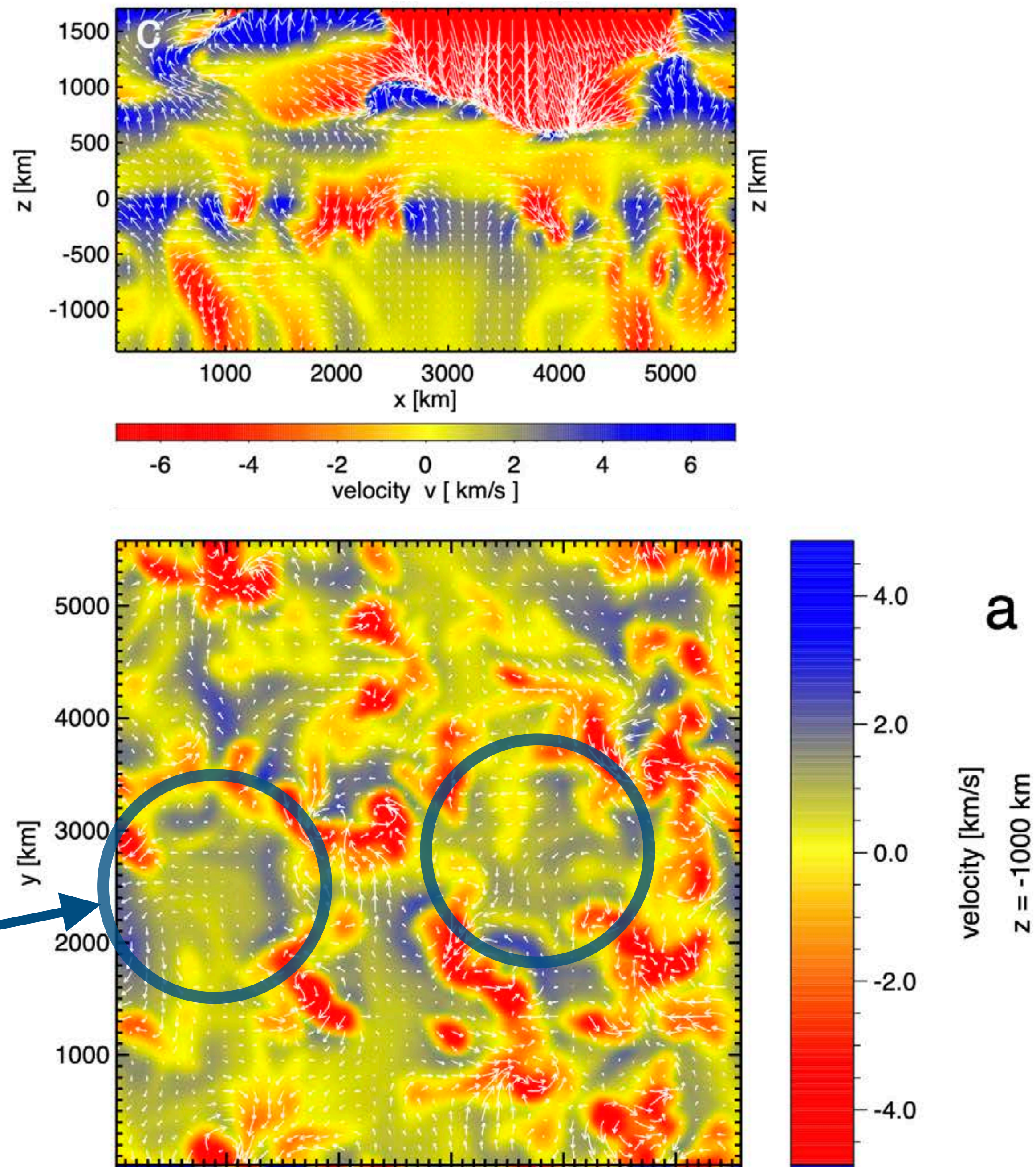
➡ $c_s = (\gamma\, H_P\, g)^{1/2} \approx 13\ \mathrm{km/s}$

➡ $r < 2\, H_P\, c_s / v_z \approx 5000\ km$

- **Very rough upper limit only!**

➡ From simulation diameter of convection cells at z=-1000km on the order of ~2000km

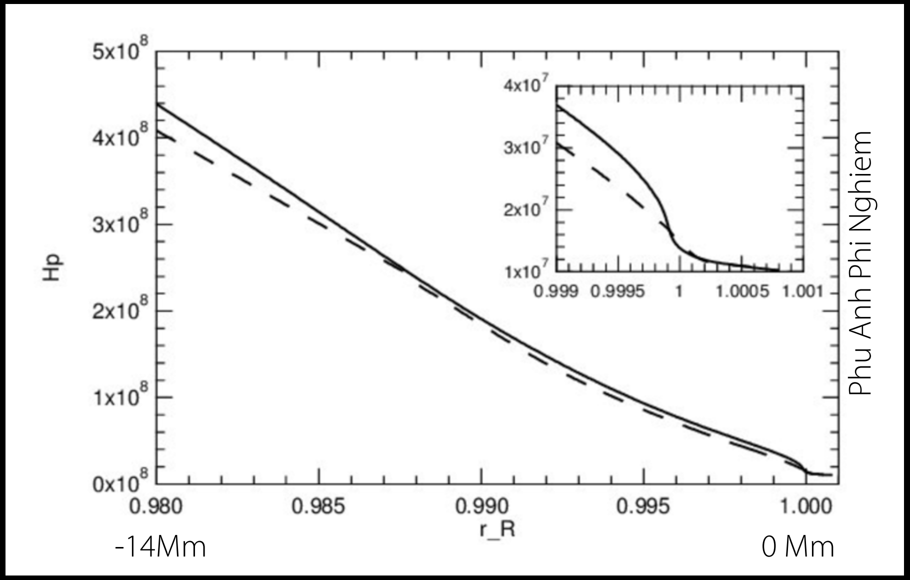


Wedemeyer (2003)

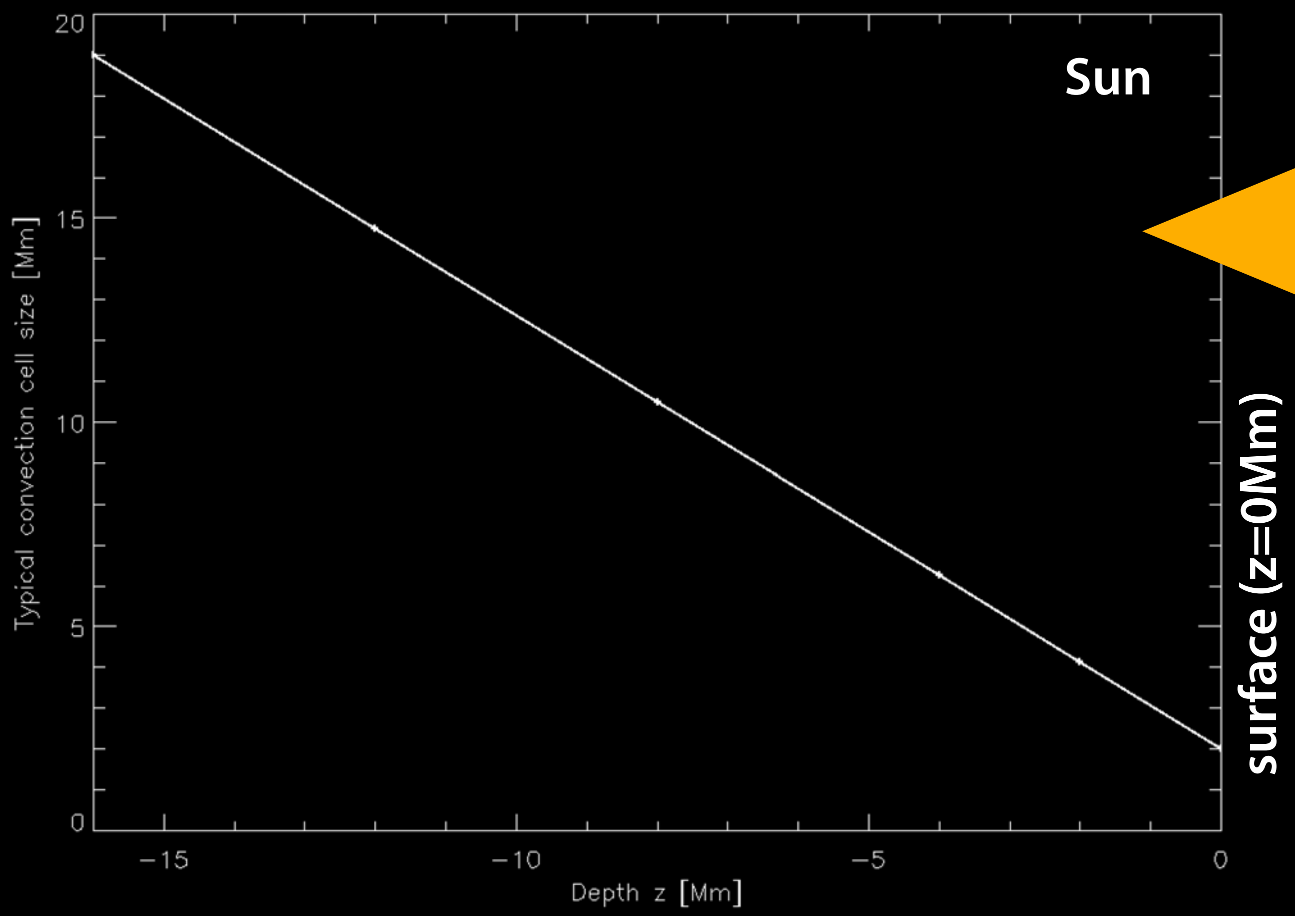
Convection

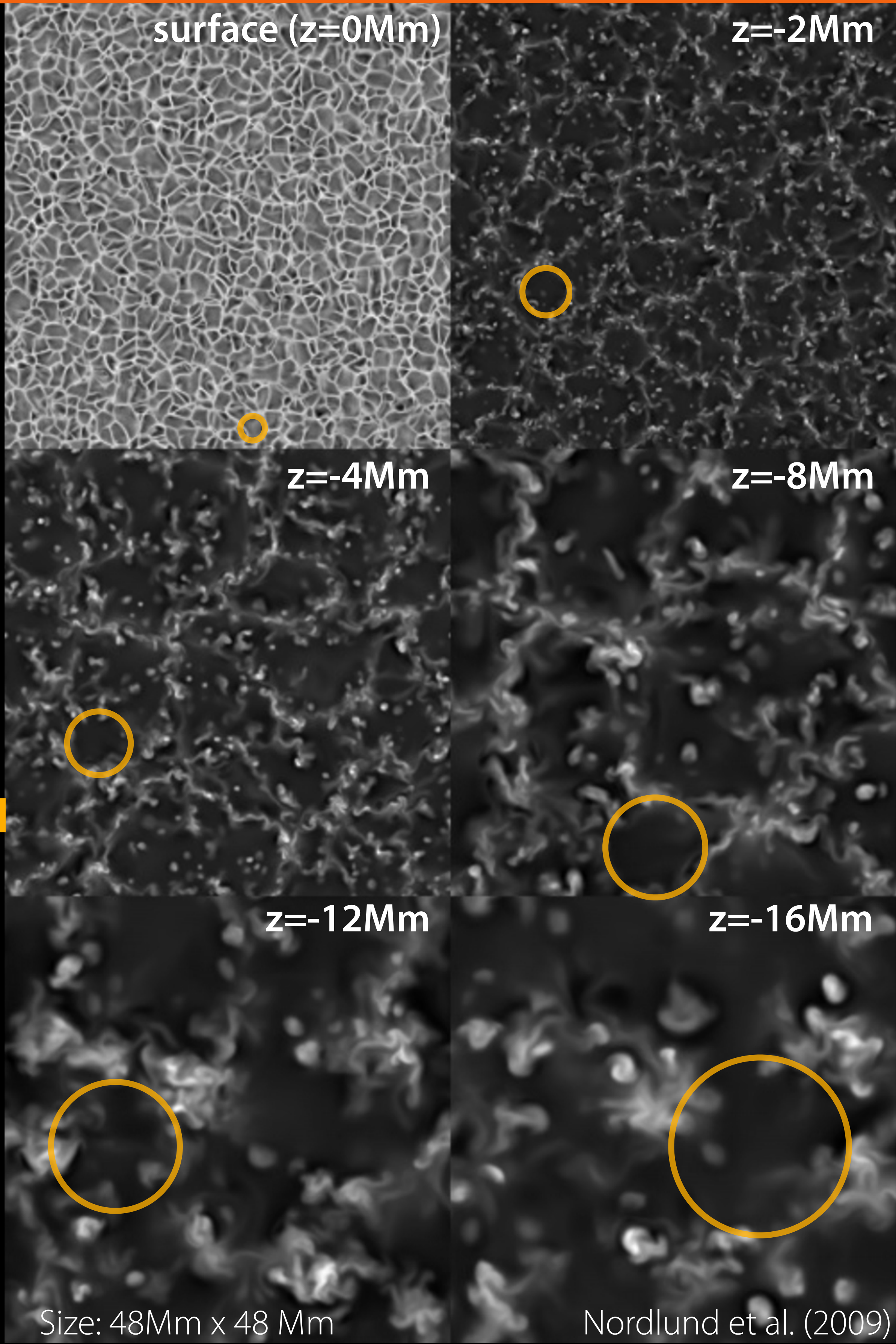
Spatial scales

- Pressure scale height $H_P = P/(\varrho g)$



Phu Anh Phi Nghiem





Size: 48Mm x 48 Mm

Nordlund et al. (2009)

Convection

Convective turnover time scale

- Turnover time scale $t_{\mathrm{to}} \propto H_P / v_c$

- Solar convection zone highly stratified!

➡ Convective turnover time scale varies by 4 orders of magnitude from surface bottom of the convection zone

- Surface: $t_{\mathrm{to}} \approx 200$ s
- Bottom of convection zone: $t_{\mathrm{to}} \approx 25$ d

Convection

Spatial scales — Convection cell sizes

- Pressure scale height $H_P = P/(\varrho g)$
- **Upshot — Convection cell size**
  - Scales with pressure scale height
  - Increases with depth due to increasing pressure, density, temperature
  - About 2Mm at surface but many Mm deeper in the interior
  - Rough estimates, more precise numbers require detailed numerical simulations (and/or a more detailed description of convection)
- Note the dependence on g^{-1}

➡ What does it mean when we consider a giant star with $\log g \sim 0$ instead of the Sun with $\log g = 4.4$?

Nordlund & Stein 1997

Convection

Spatial scales — Convection cell sizes

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➡ What does it mean when we consider a giant star with $\log g \sim 0$ instead of the Sun with $\log g = 4.4$?

- Expect convection cells several orders of magnitude larger!

Nordlund & Stein 1997