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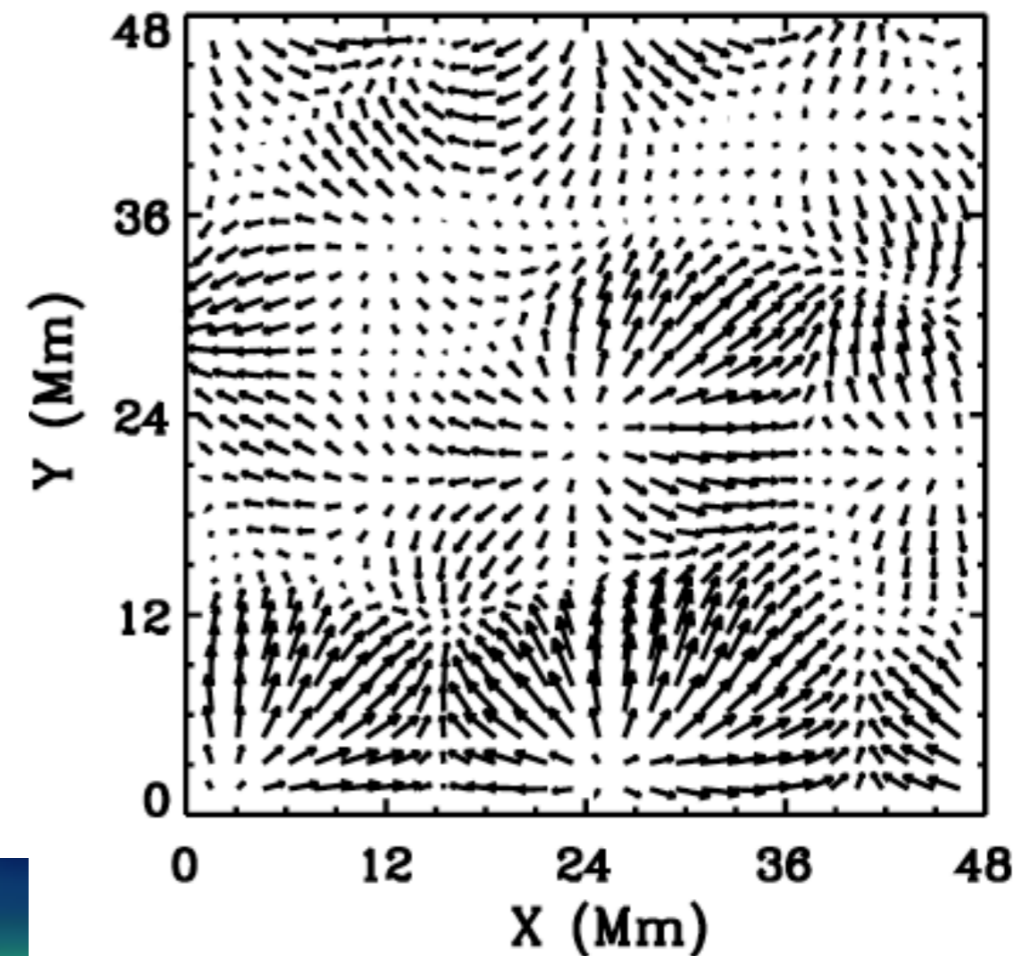
Solar and stellar physics

Sven Wedemeyer, University of Oslo, 2023

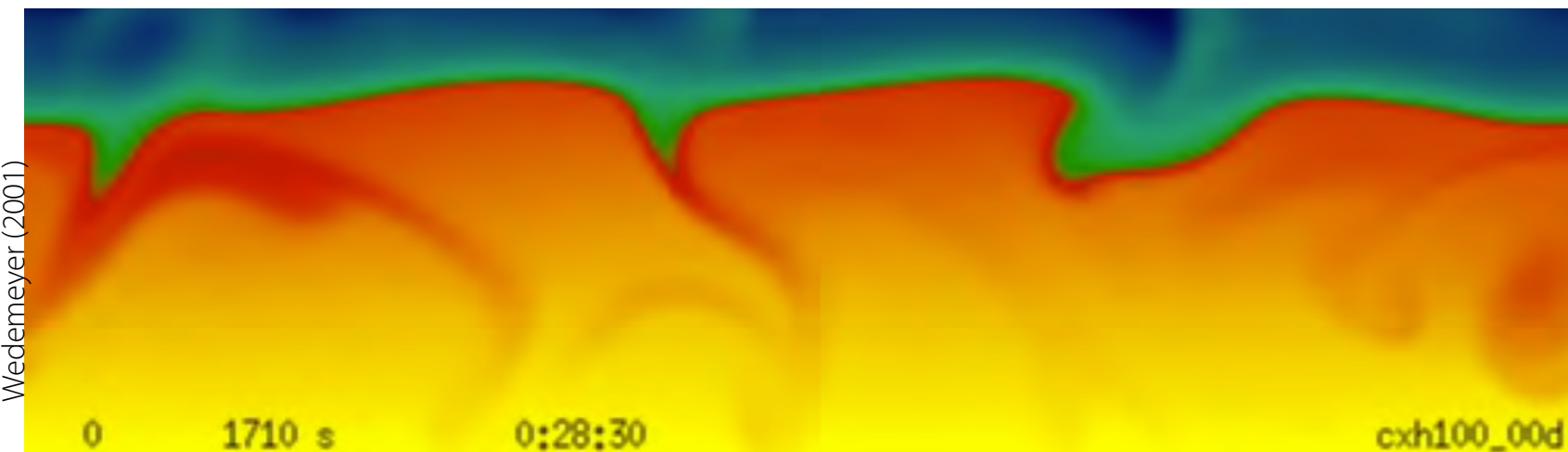
Convection

Flow field

- Hot gas elements rise upwards and diverge, typically within 1-2 pressure scale heights
- Downflowing material denser, can form turbulent plumes
 - Velocity field in a horizontal cross-section through a 3D simulation
 - Note the flow divergence!
- Vertical cross-section at the top of the convection zone — “surface” (photosphere)



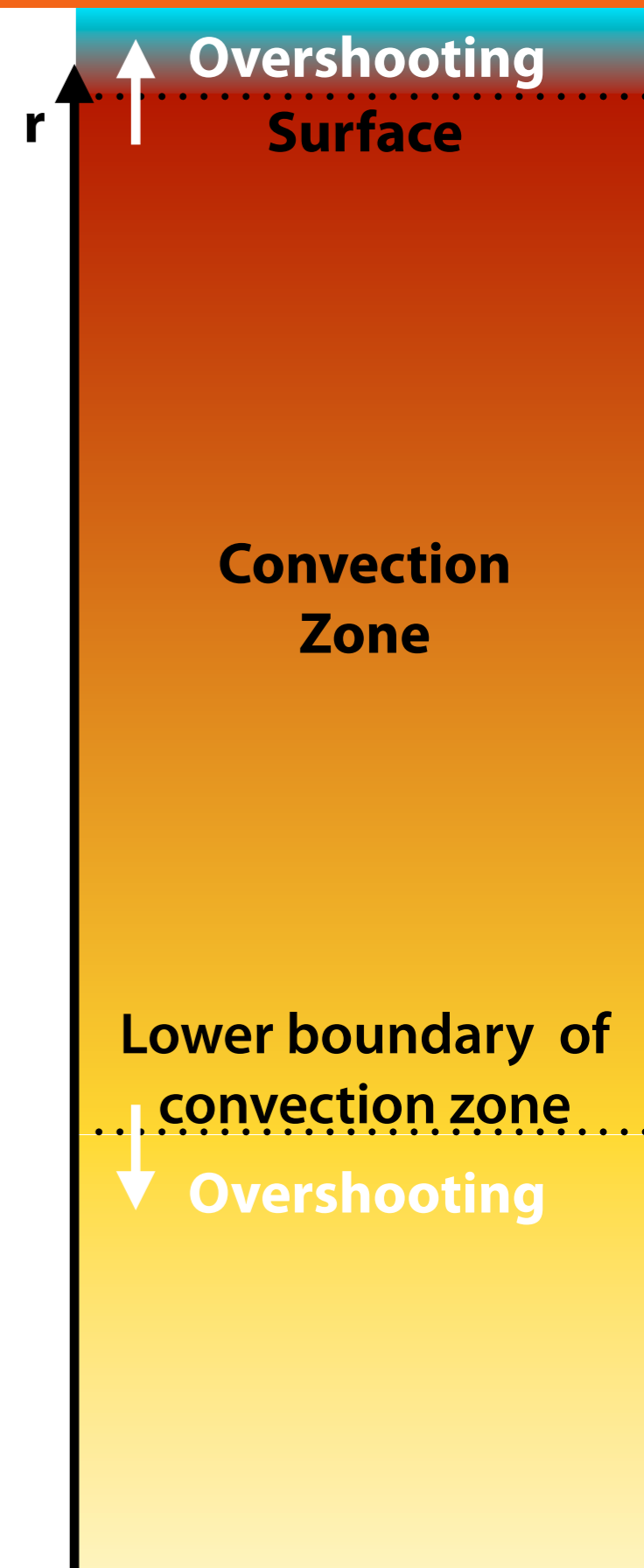
Nordlund et al (2009)



Convection

Convective overshooting

- Chemically homogeneous layer
(Schwarzschild criterion: $\nabla_{rad} < \nabla_{ad}$):
 - Convection zone stops where $\nabla_{rad} = \nabla_{ad}$
 - There, acceleration due to buoyancy force goes to zero:
 $a \approx g (\nabla - \nabla_{ad}) \rightarrow a=0$
 - Beyond at point, any gas element braked, resulting into convectively stable layer
 - BUT: Gas elements reaching that location have on average still **inertia** and will (on average) **overshoot the boundary by some distance**
 - Overshooting by less than a pressure scale height
- ➔ Schwarzschild criterion determines boundary of convection zone quite accurately
- ⦿ **IMPORTANT:** No net mass transport but convection **mixes** gas in neighbouring layers!



Convection

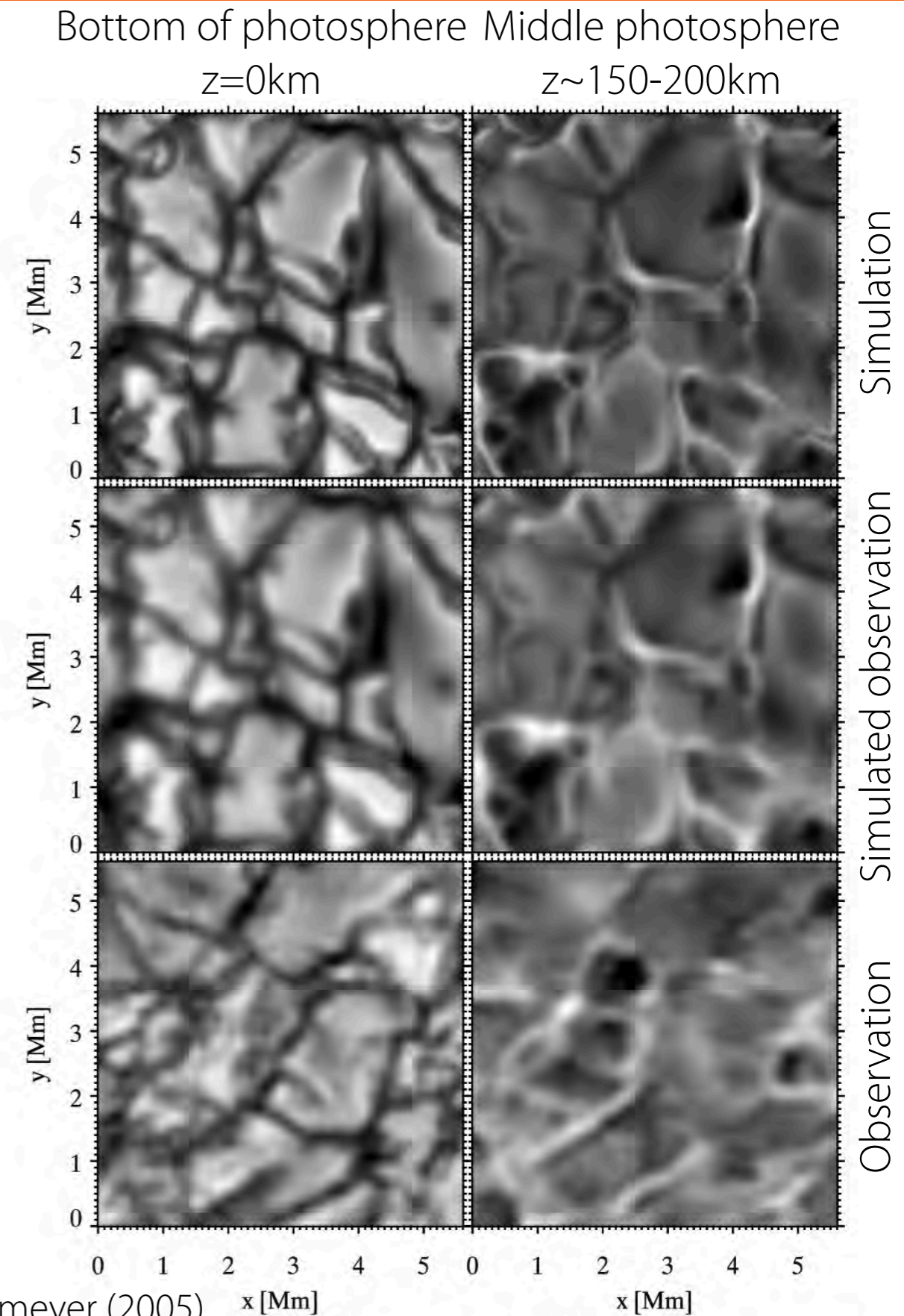
Convective overshooting

- Chemically inhomogeneous layer with $\nabla_{ad} \neq 0$ (Ledoux criterion: $\nabla_{rad} < \nabla_{ad} - \frac{\chi_{\mu}}{\chi_T} \nabla_{\mu}$)
 - Convective mixing decreases $|\nabla - \nabla_{ad}|$ and ∇_{μ} .
 - ➔ Effective buoyancy force increases
 - ➔ Positive feedback loop can develop with highly non-linear increase
 - ➔ Overshooting gas elements penetrate further and further.
 - Resulting overshoot distance is very uncertain and could be substantial.
- **Mixing due to convective overshooting important for stellar evolution.**
- In the core:
 - Convectively mixed core has “access” to a larger fuel supply for nuclear burning
 - ➔ Affects hydrogen-burning lifetime
 - ➔ Affects further evolution of that star
- Overshooting only taken into account in parameterised form, needs calibration for more detailed modelling

Convection

Convective overshooting

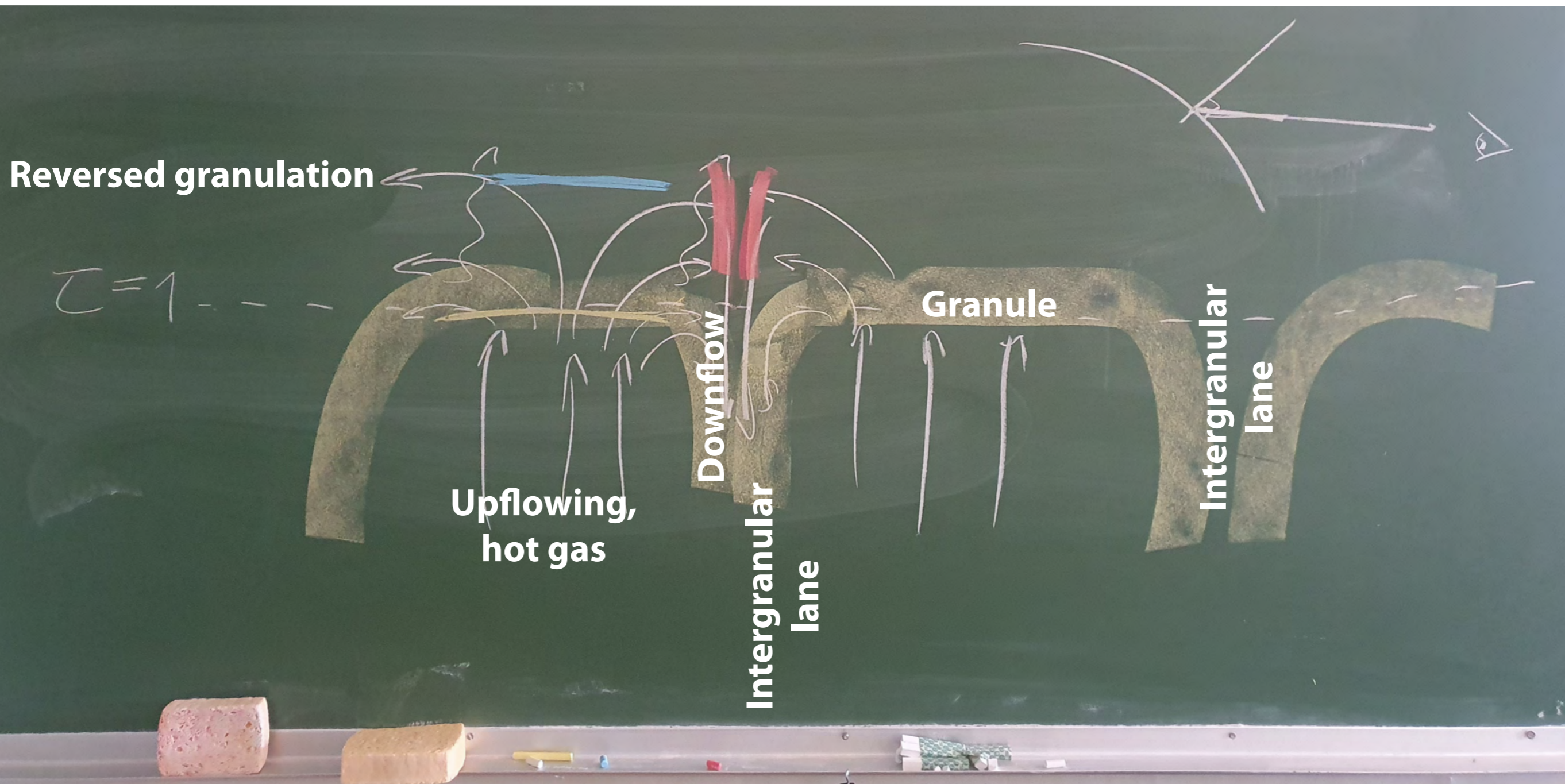
- In the Sun: Convective overshooting at the surface
- Overshooting into the stably (subadiabatically) stratified photosphere.
- Radiative losses make the overshooting material relatively cold and dense.
- Cooling by adiabatic expansion versus radiative heating (towards radiative equilibrium from the surrounding)



Convection

Convective overshooting

- Granulation pattern in the low photosphere (where optical depth $\tau \sim 1$)
- Reversed granulation pattern in the middle photosphere



Convection

Convective overshooting

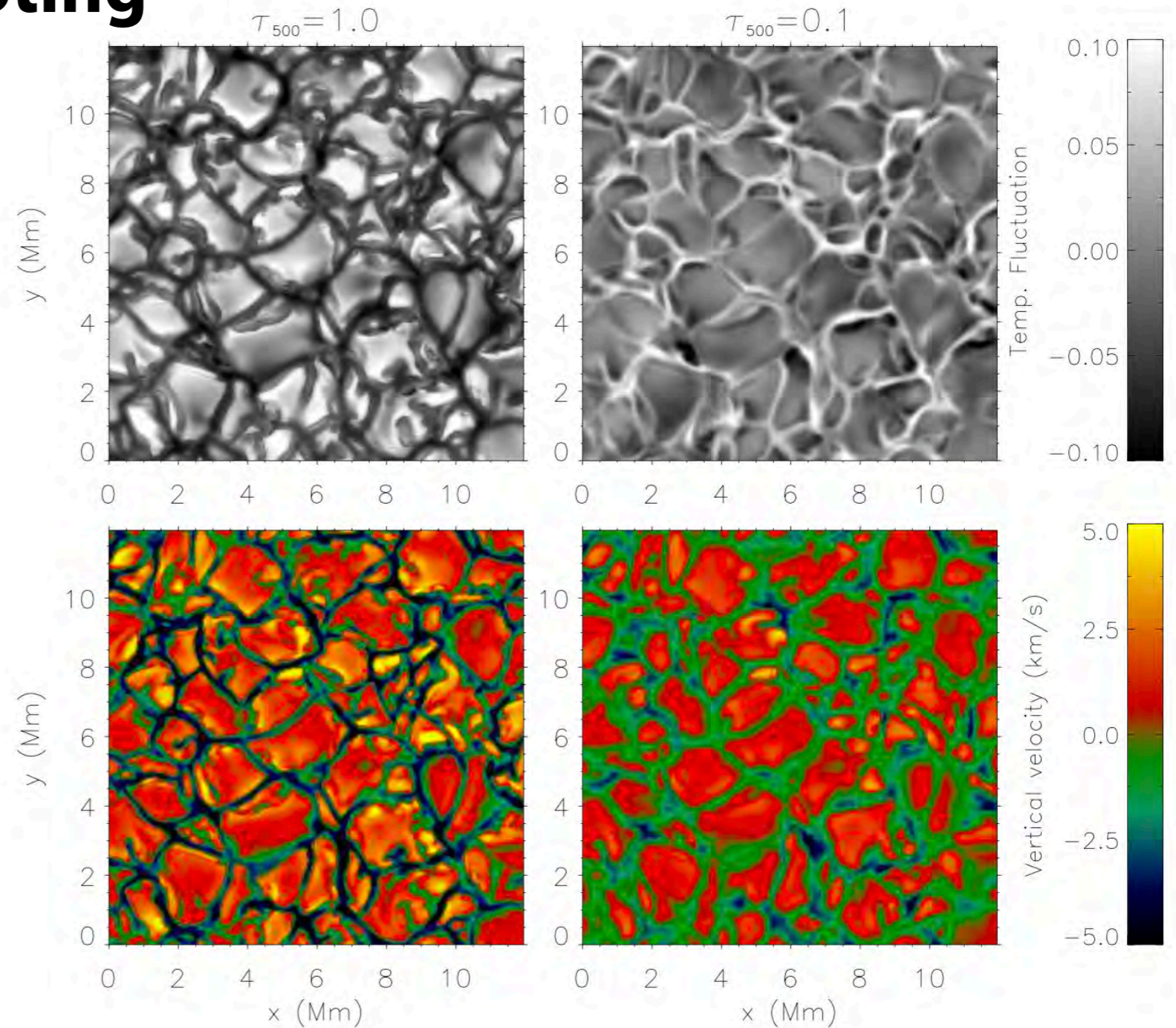


Figure 3: Temperature fluctuations ($\Delta T/\bar{T}$, upper row) and vertical velocity (lower row) at two surfaces of constant optical depth. The left column shows the patterns at $\tau_{500} = 1$, the right $\tau_{500} = 0.1$. The reversed granulation corresponds to a reversal of the temperature fluctuations while the velocity pattern remains qualitatively unchanged.

Cheung et al. (2006)

Surface convection

Granulation on other stars

(More) massive stars ($M > M_{\odot}$)

Outer radiative zone, no surface convection

Spectral types A, F

- exhibit inverse bisectors
- granulation has different geometry

Solar-like and low-mass stars

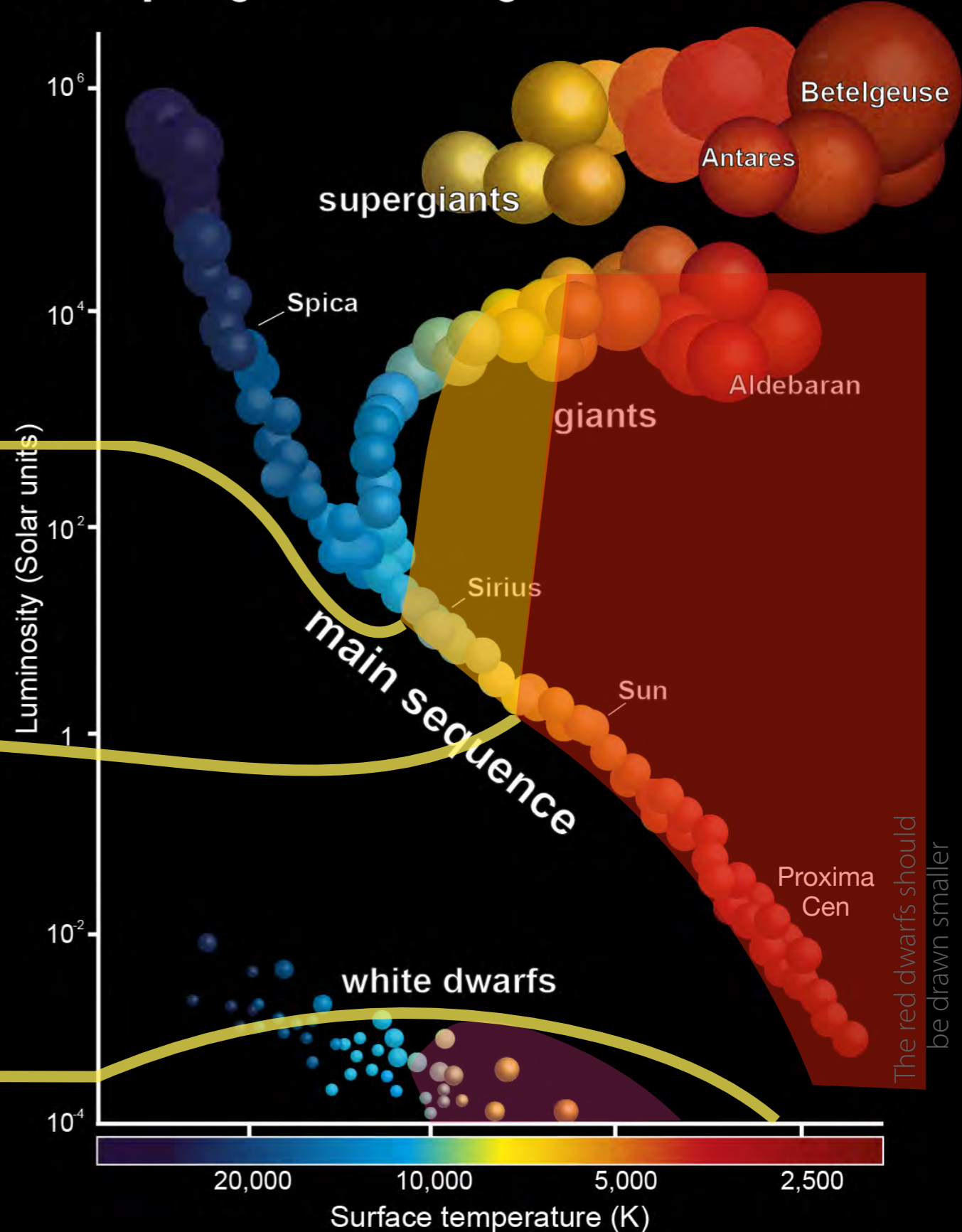
Spectral types F, G, K & M

- outer convection zones and
- exhibit observable signs of convection

(Cooler) White Dwarfs

- exhibit observable signs of convection

Hertzsprung–Russell Diagram



Surface convection

Granule sizes on other stars

- Size of granules (at the surface!) seems to scale with pressure scale height for different stellar types
- Typical granulation size determined from 3D simulations for different stellar types (Freytag et al. 2002):

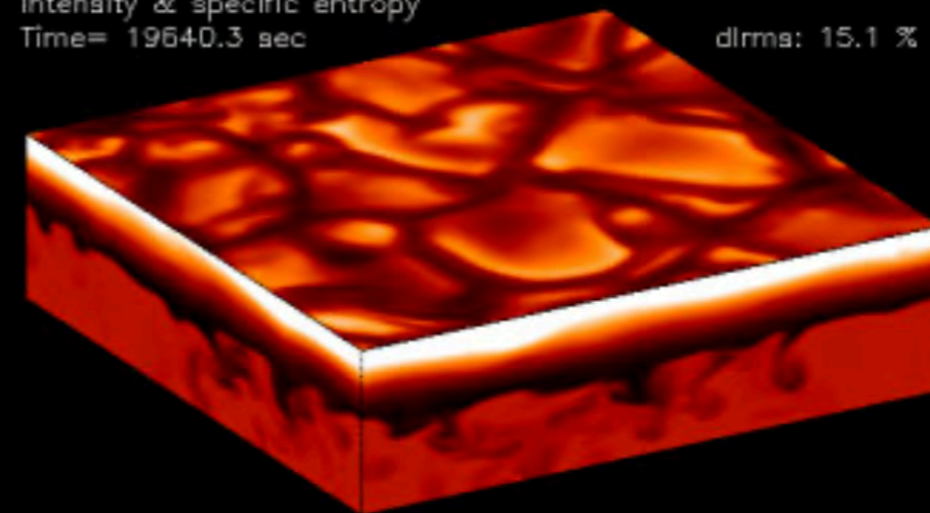
$$\frac{x_{\text{gran}}}{R_*} \approx 0.0025 * \frac{R_*}{R_{\odot}} \frac{T_{\text{eff},*}}{T_{\text{eff},\odot}} \frac{M_{\odot}}{M_*}$$

- Sun: $x_{\text{gran}} = 0.0025 R_{\odot} \approx 2000 \text{ km}$

- **Note:** $x_{\text{gran}} \ll R_{\odot}$ and turnover timescales change by 4 orders of magnitude
- Simulations of the whole convection computationally very challenging

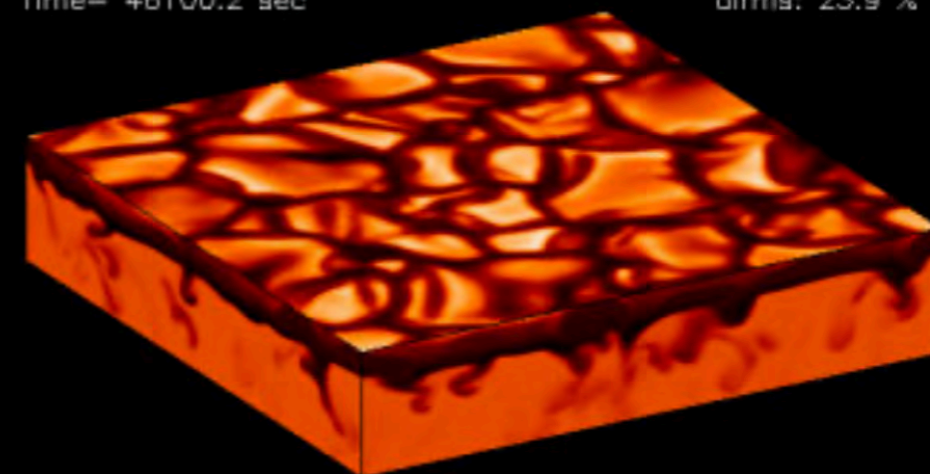
Solar Granulation:
Intensity & specific entropy
Time= 19640.3 sec

dIrms: 15.1 %



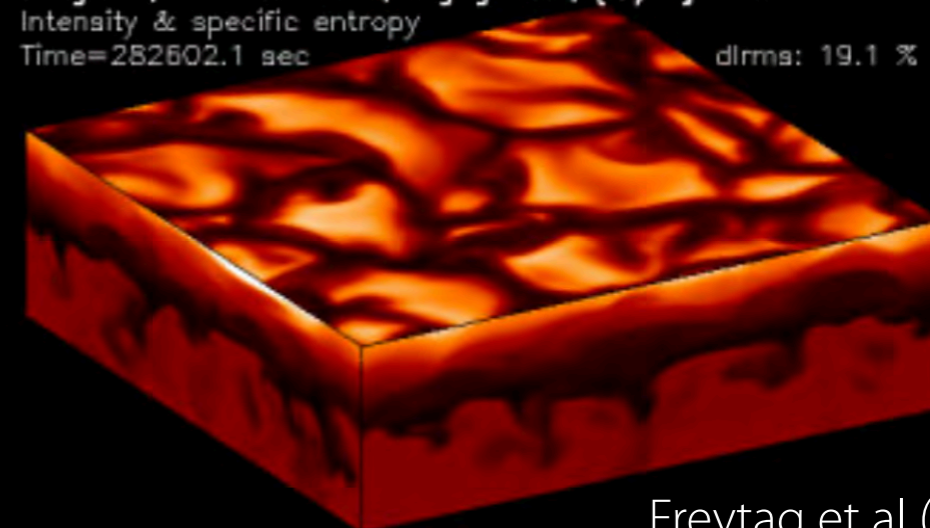
F-dwarf, $T_{\text{eff}}=6300 \text{ K}$, $\log g=4.0$, $[M/H]=-2$
Intensity & specific entropy
Time= 46100.2 sec

dIrms: 23.9 %



G-giant, $T_{\text{eff}}=5000 \text{ K}$, $\log g=2.9$, $[M/H]=-2$
Intensity & specific entropy
Time=282602.1 sec

dIrms: 19.1 %



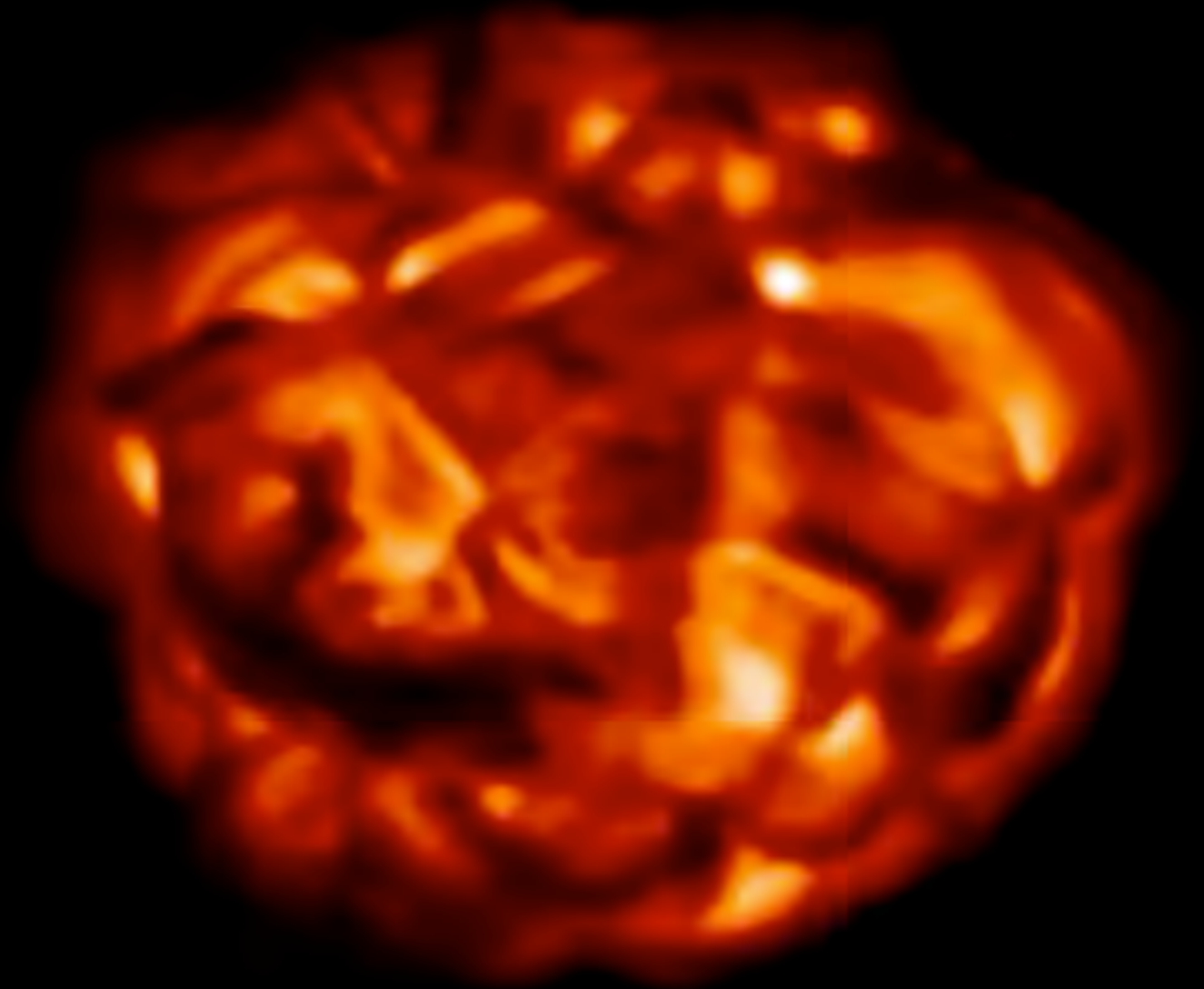
Surface convection

Convection

st35gm04n26: Surface Intensity(3I), time(0.0)=30.263 yrs

Time-dependent 3D hydrodynamic simulation of **Betelgeuse**, here **intensity**

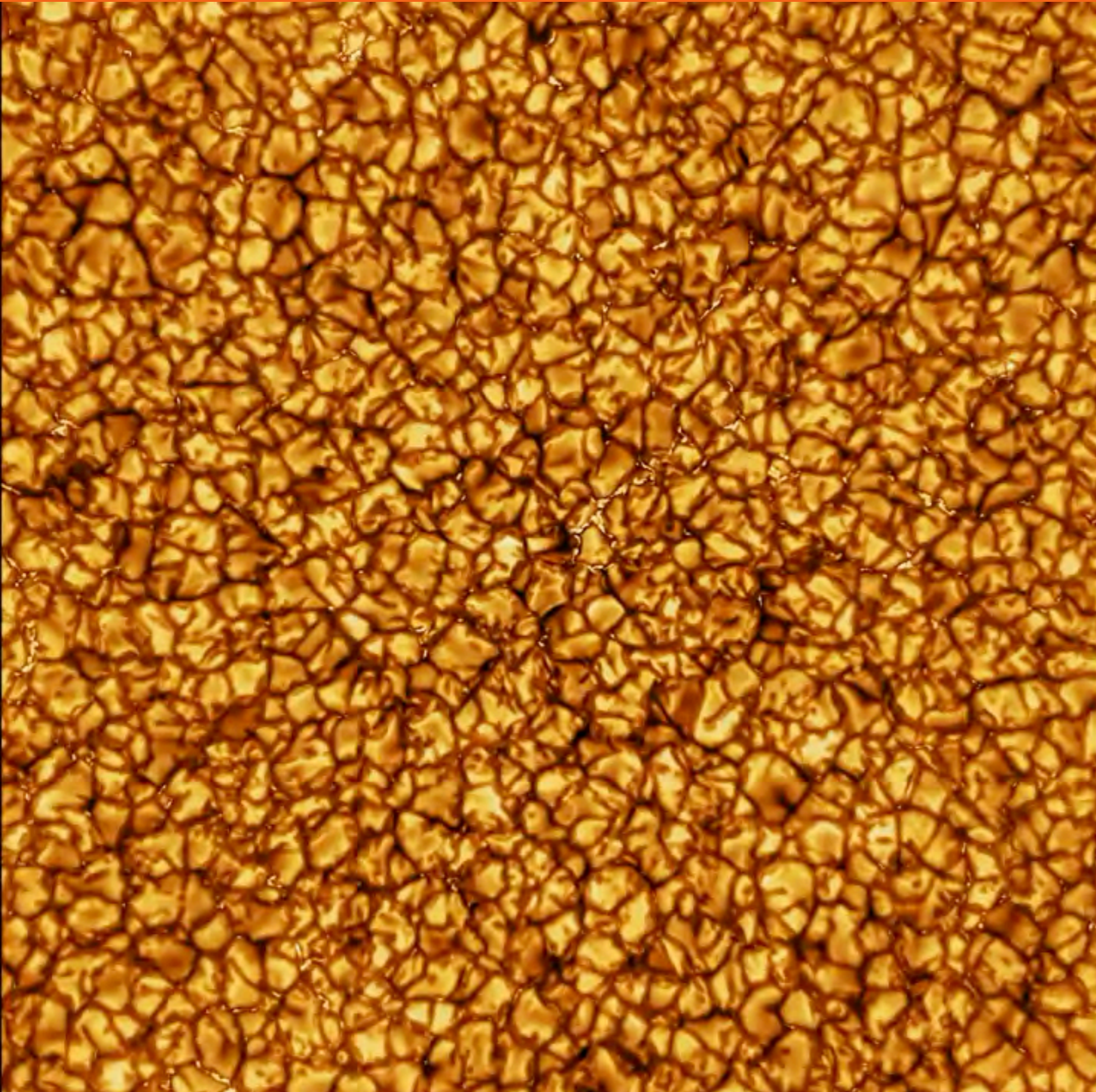
- Large convection cells
- Slower temporal evolution (Note time at top right)
- Observations of Betelgeuse: variations of radial velocity imply existence of large convection cells.



Freytag

Simulation produced with the same code as the models for the project assignment (Freytag et al. 2012)

Granulation



Highest-resolution observations of the Sun's granulation ever taken.

DKIST (4m)
(NSO/AURA/NSF)

- Prominent scale:
Granulation with 1-2Mm cell diameters



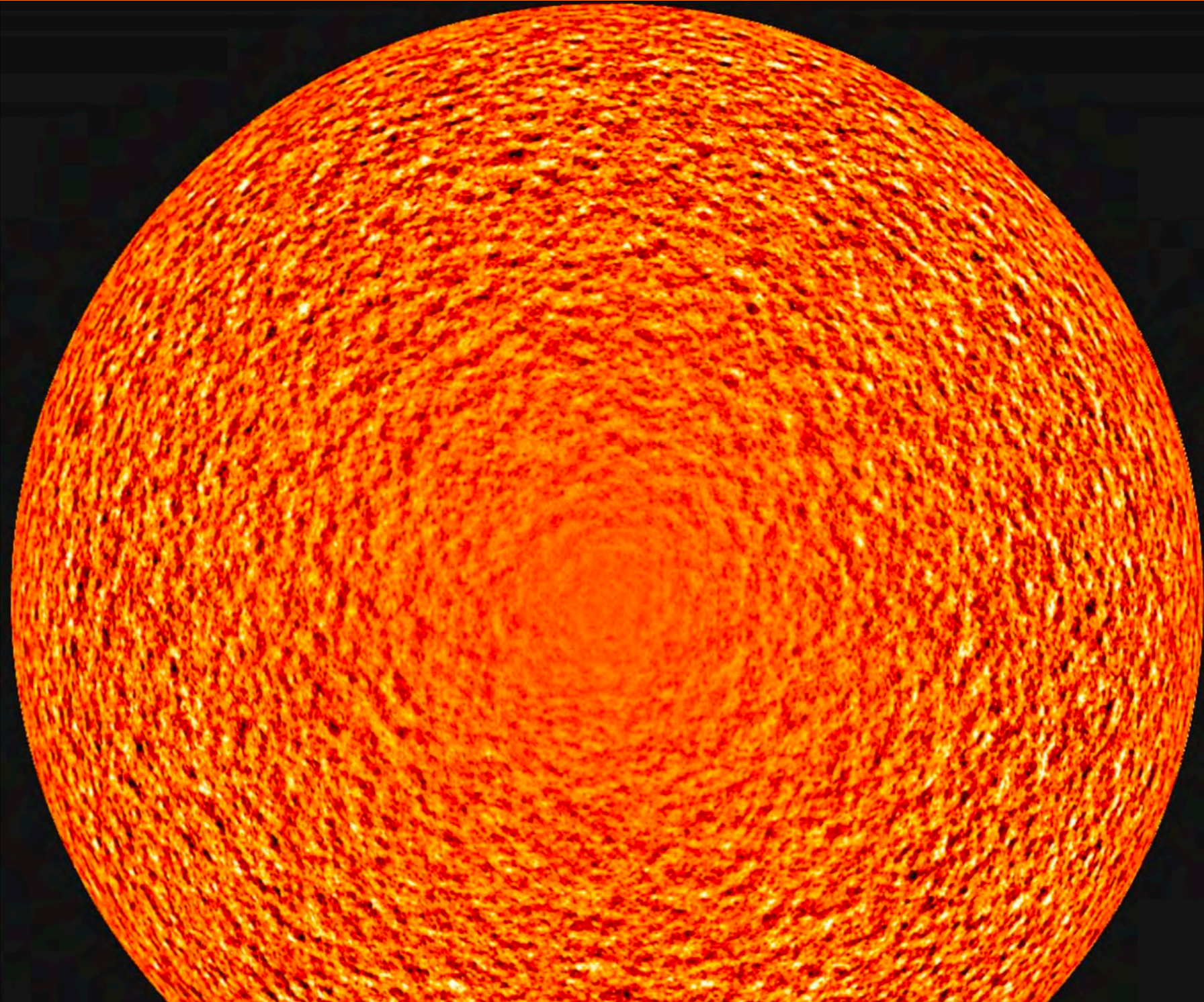
KRajala ©

10 Dec. 2019 19:24:31 UT

Granulation

Supergranulation

- Dopplergram revealing the supergranulation pattern (credits SOHO/MDI/ESA).
- Prominent scale here: Supergranulation with typical cell diameters of $\sim 30\text{-}40\text{ Mm}$ (wider range $10\text{-}70\text{ Mm}$)
- Cell lifetime $\sim 40\text{ h}$
- Typical horizontal velocities $\sim 0.2\text{ km/s}$
- Convection on larger scales
- But: may only extend 5 Mm into convection zone (tbc)



- Supergranulation can be subtle, depending on the way the Sun is observed (wavelength etc.)
- Can be revealed by tracing the horizontal motion of granules / feature (local correlation tracking)
- Observation over such long uninterrupted periods are only possible from space

Granulation

Supergranulation

09/Oct/14 12:05:48
Spectroheliogram K3

P: 26.2 B0: 5.9

- A bit higher in the atmosphere:
map in the Ca II K line core at (393.37 nm), showing the chromospheric network (from Meudon Observatory)
- Also outlines supergranulation scales



Granulation

Spatial scales

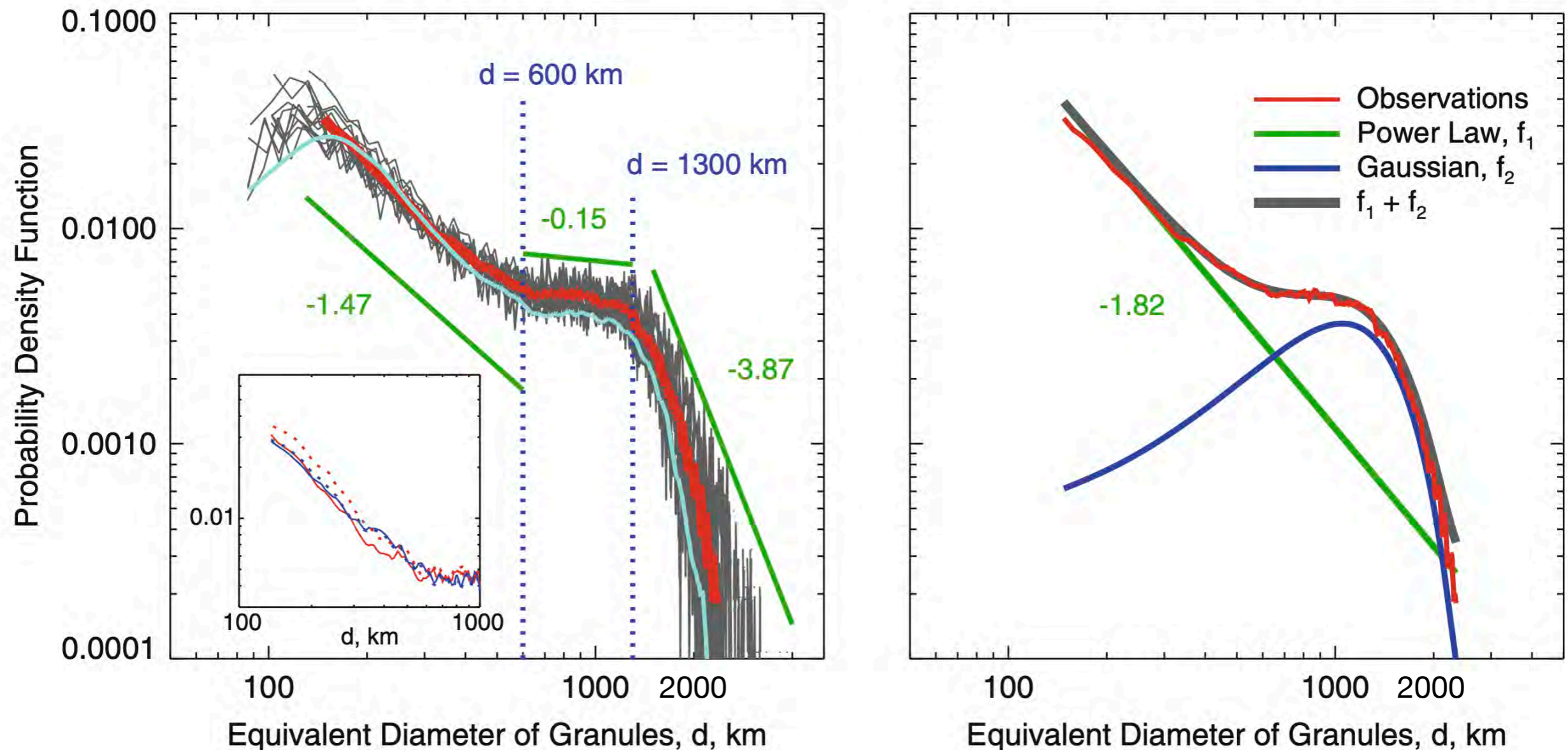


Fig. 4.11 The probability density function of the equivalent diameter of granules (in units of km) is shown, observed in Quiet Sun regions with the *New Solar Telescope (NST)*. The regular granules have a size of $w \approx 500\text{--}2000$ km, while the range of $w \approx 100\text{--}500$ km exhibits the new phenomenon of “mini-granules” (Abramenko et al. [2012](#))

Granulation

Spatial scales

- 'Velocity spectrum'
- Fourier transformation of velocity
- ➔ velocity amplitudes over a large range of spatial scales

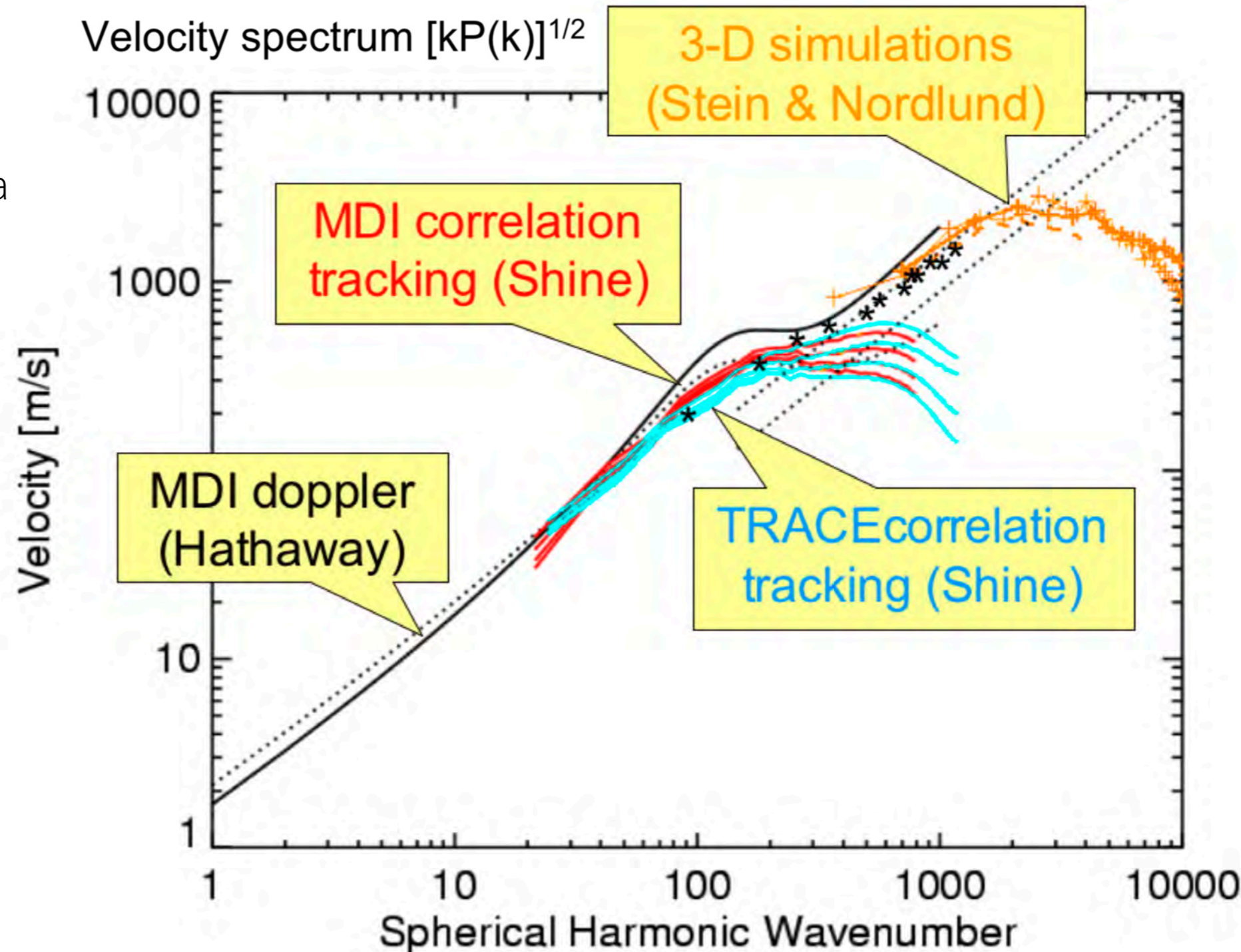
$$V(k) = \sqrt{kP(k)}$$

- $P(k)$: power spectrum (velocity power per unit linear wave number)
- k : wavenumber ($k = 2\pi/\Delta x$)

Giant cells

Supergranulation

Granulation



Granulation

Spatial scales of solar granulation

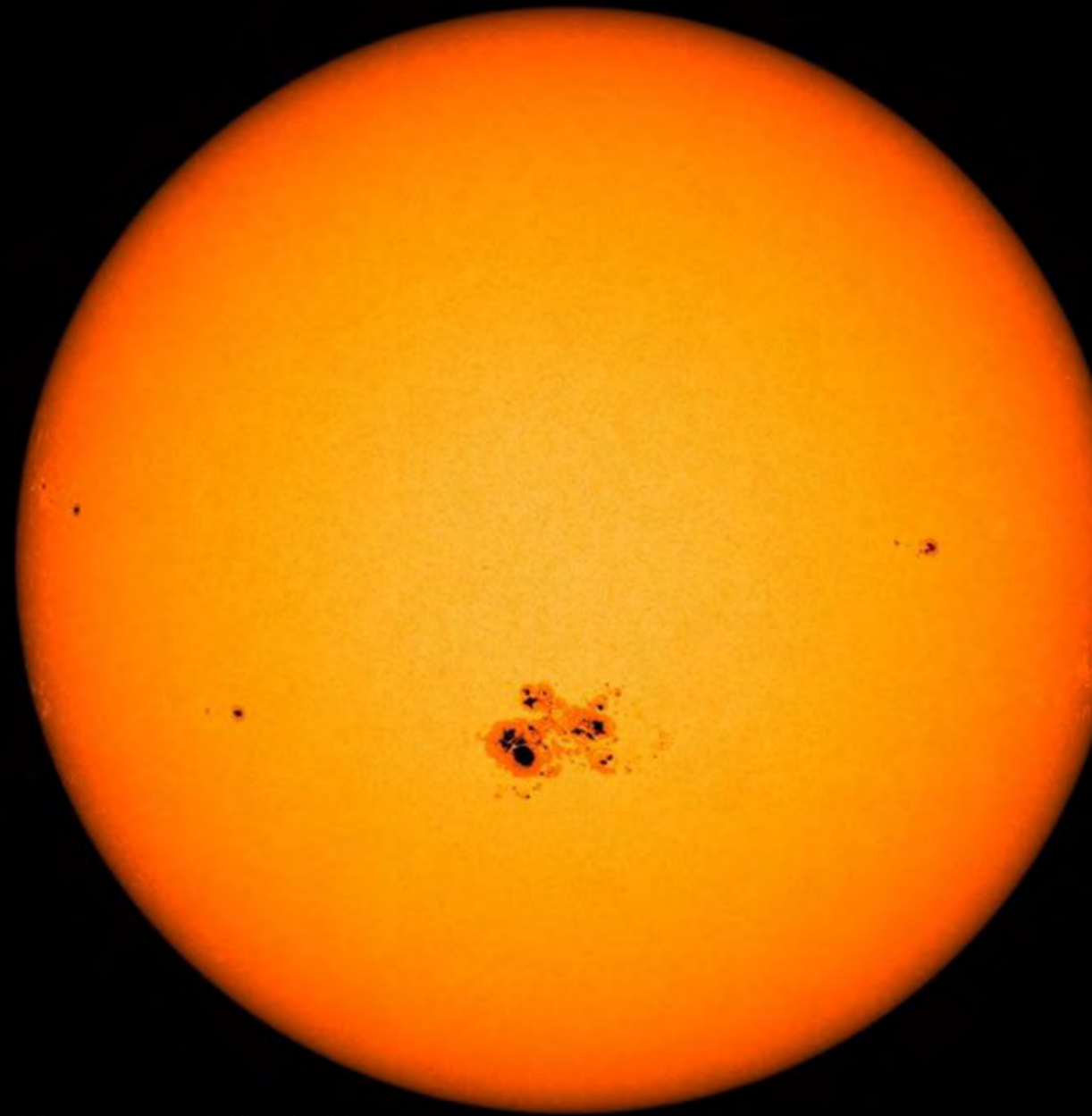
- **Mesogranulation:**
 - between granulation and supergranulation, first observational indication in 1980
 - Still debated if a true scale exhibited by convection or not
- **Giant cells:**
 - Larger than supergranulation on scales > 100 Mm
 - Observations need to cover large areas on the Sun over very long time; possible with space-borne telescopes since ~ 2000 (e.g., SOHO/MDI)
- **Summary:**
 - Continuous distribution of spatial scales and corresponding timescales (lifetimes) for surface convection cells
 - Granulation and supergranulation clearly present
 - Indications for giant cells, while existence of mesogranulation still debated

Rotation

Solar rotation

Introduction

- Solar rotation becomes obvious from observing over several days as sunspots move across the solar disk
- Already reported long ago by Galilei and Scheiner
- Standard value:
Carrington rotation period = 27.28 days
 - = time taken for the solar coordinate system to rotate once).
- The Sun rotates faster at the equator than at poles — both depending on latitude and depth
 - **Differential rotation**
- Rotation axis inclined by 7.1 deg (Sun's equator is inclined by 7.1 deg relative to the ecliptic).

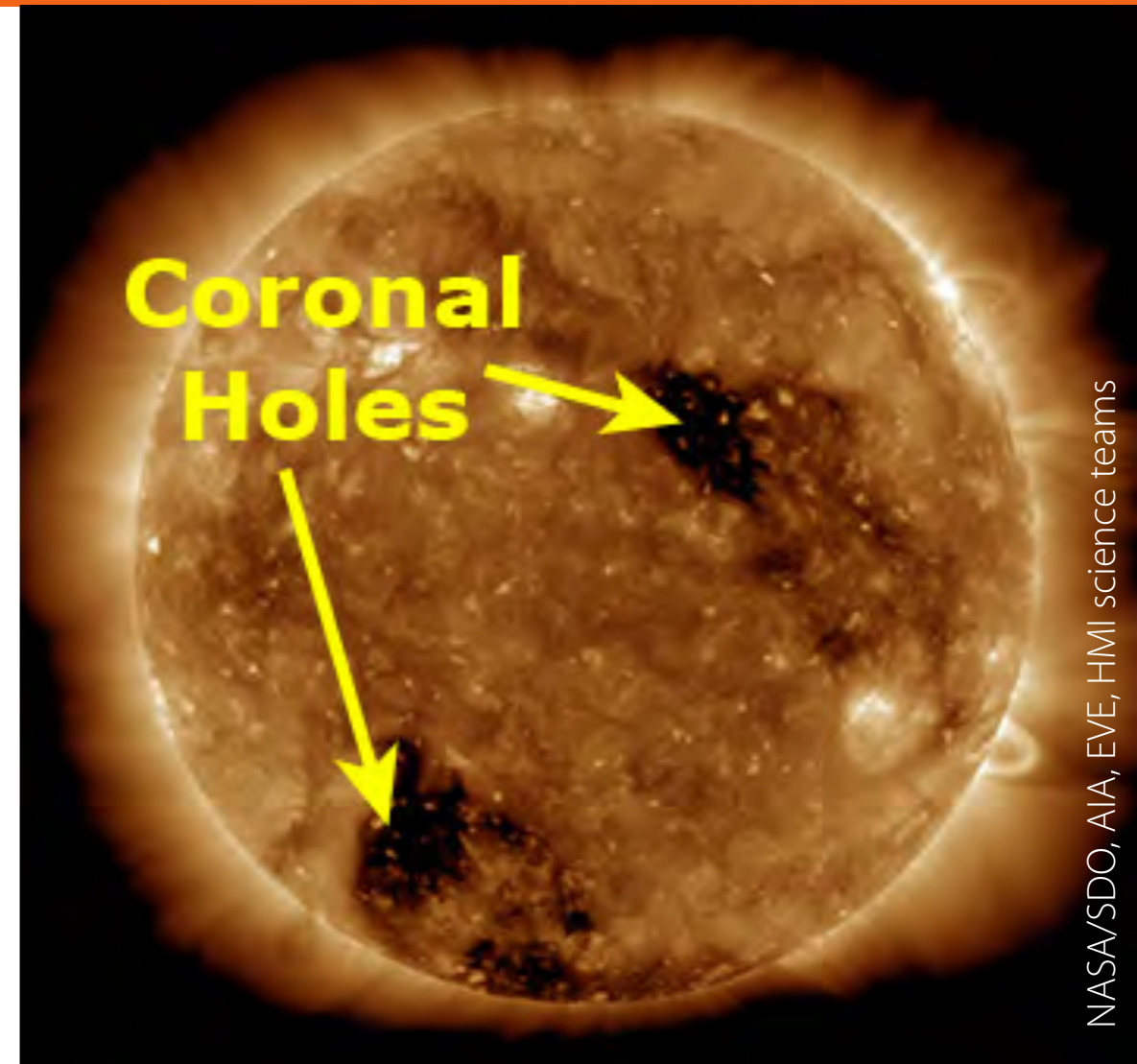


SDO/HMI Quick-Look Continuum: 20141023_131500

Solar rotation

Differential rotation — surface

- Surface differential rotation from measurements of:
 - Tracking the movement of magnetic features (sunspots, magnetic field elements, coronal holes)
 - Doppler shifts (of the gas at the surface)



Solar rotation

Differential rotation — surface

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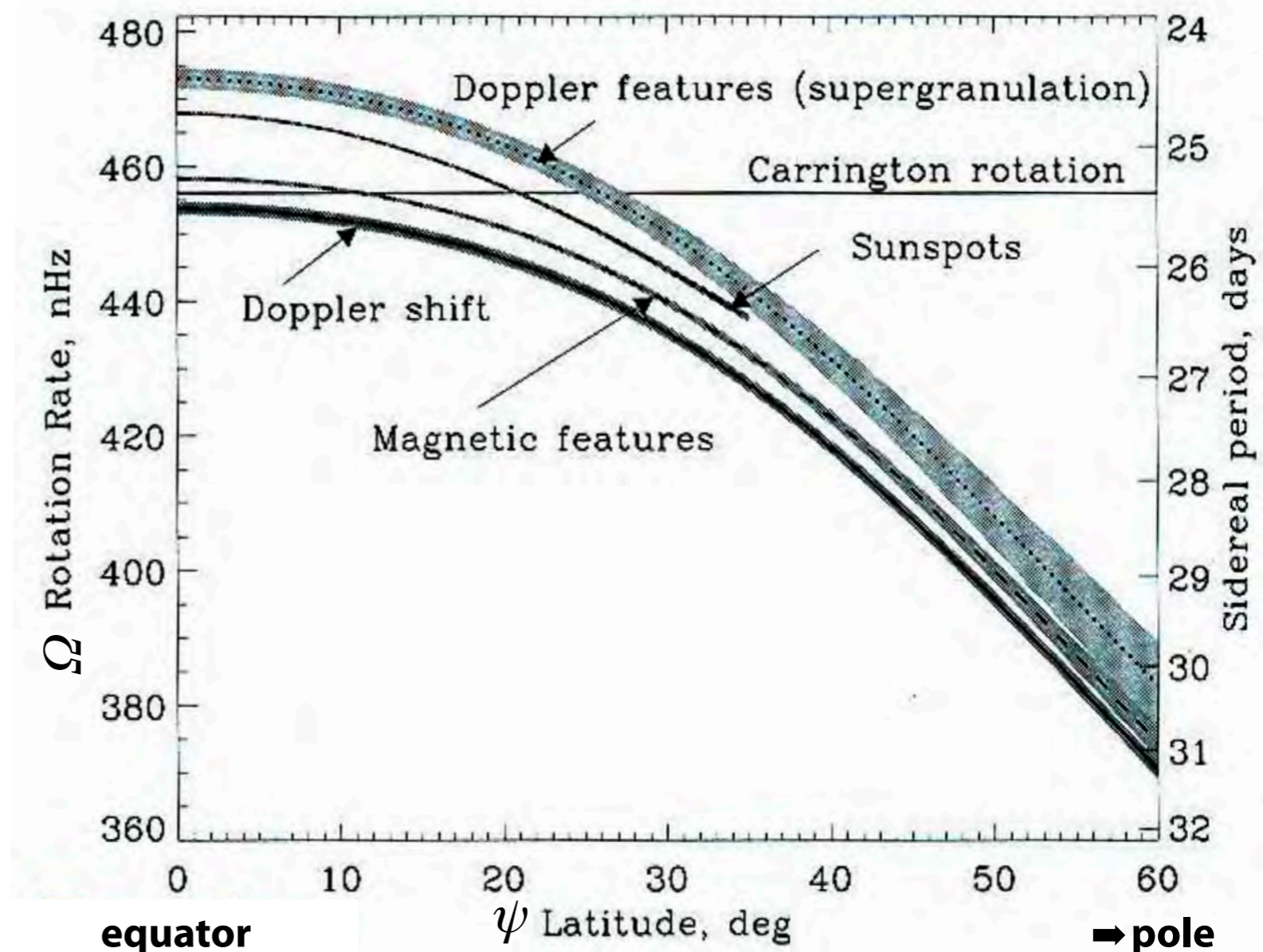
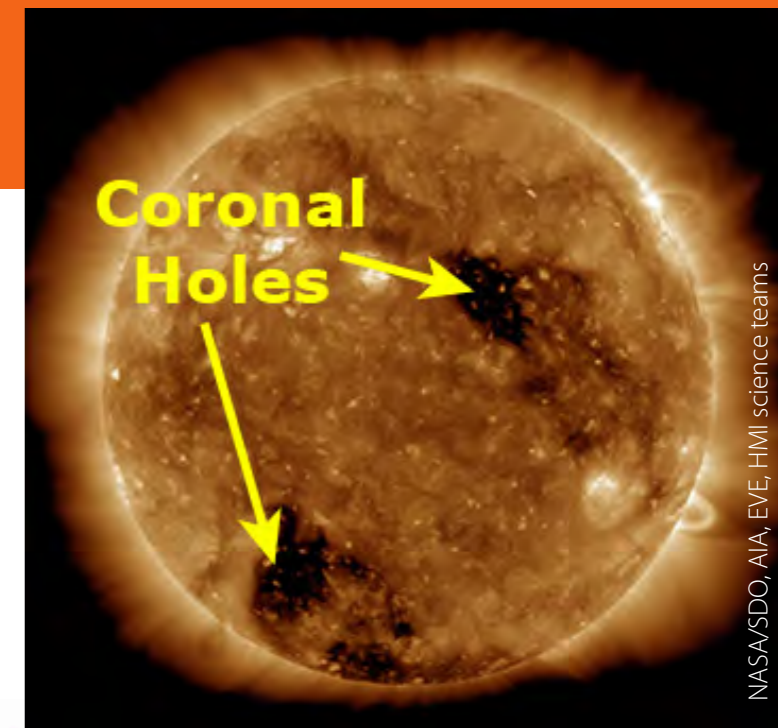
- **Empirical rotation law:** Rotation rate Ω as function of latitude ψ :

$$\Omega = A + B \sin \psi^2 + C \sin \psi^4$$

➔ Equator: $\Omega = A$

➔ Poles: $\Omega = A + B + C$

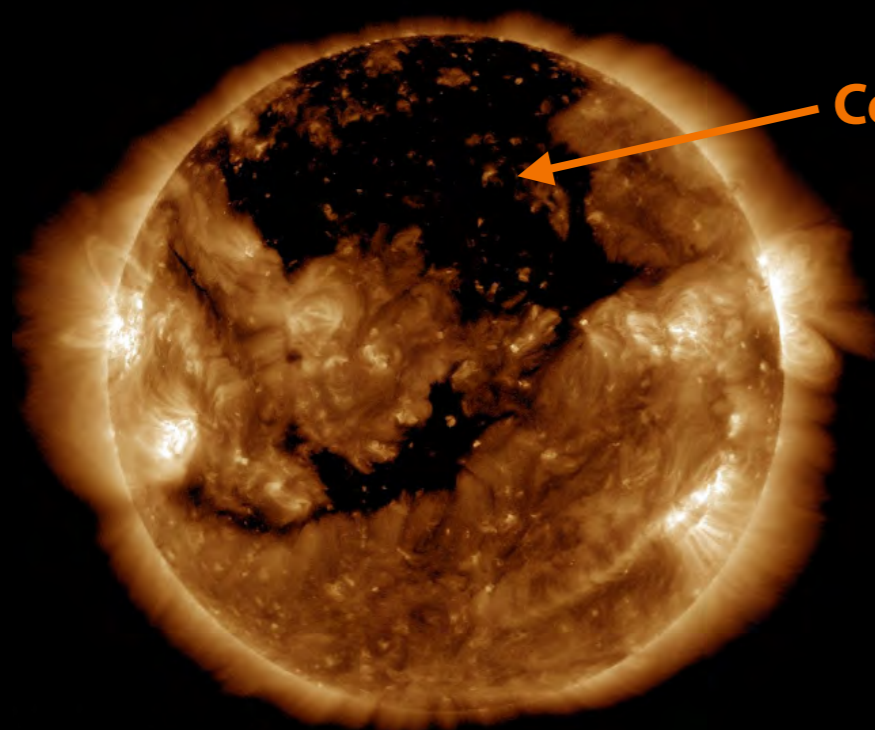
- Different tracers give different values of A, B, C!
(e.g. spots rotate faster than the gas at the surface!)



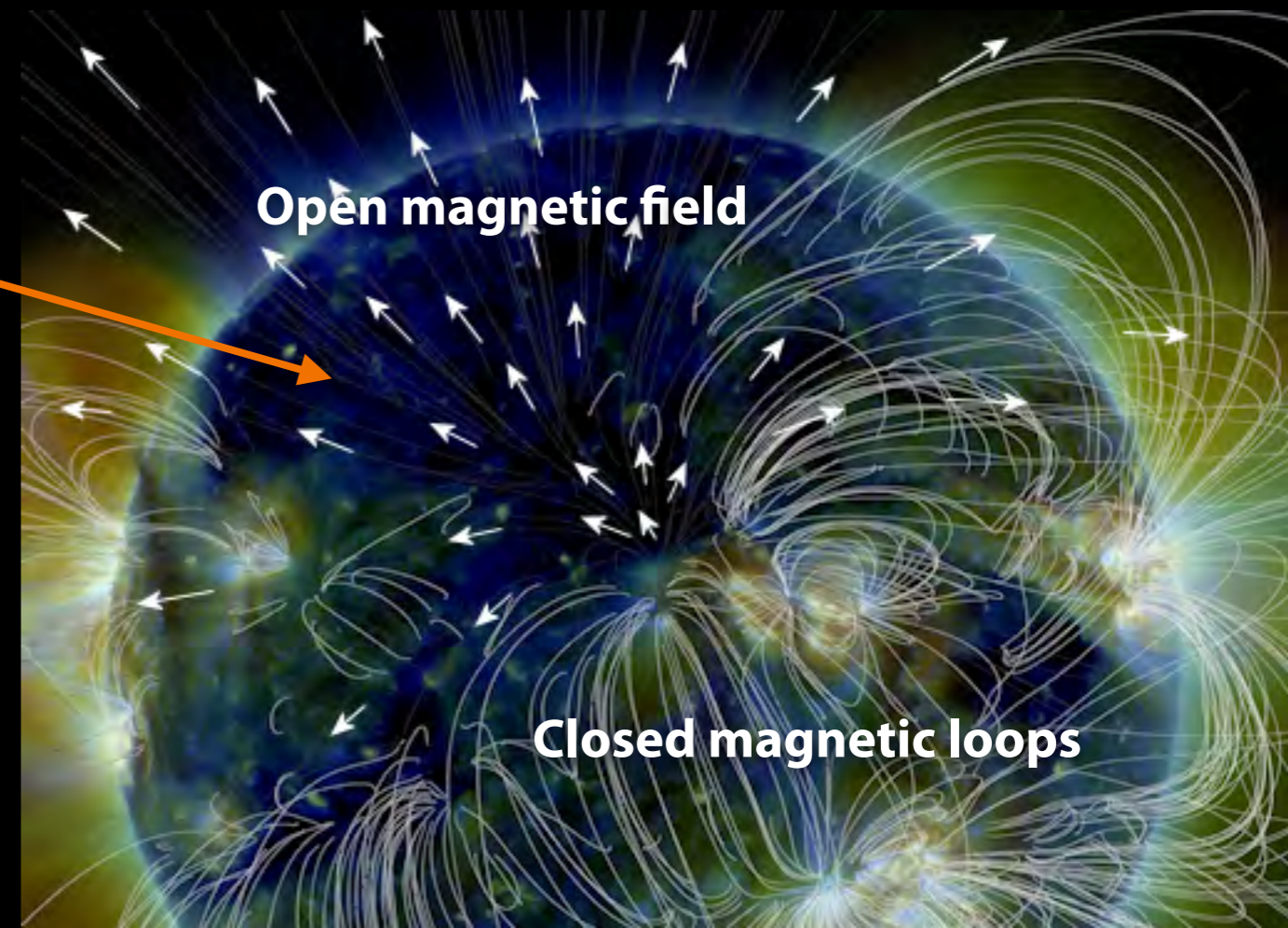
Solar rotation

Differential rotation — surface

- **Different tracers give different values of A, B, C!**
 - Sunspots rotate faster than the gas at the surface
 - Young sunspots rotate faster than older spots (older spots seem to be slowed down by the surrounding gas)
 - Coronal holes rigidly while photospheric magnetic field exhibits differential rotation
 - ➔ Magnetic features move in and out of coronal holes and would need to interact with the surrounding magnetic field (magnetic reconnection)
 - ➔ Observational confirmed.



Coronal hole



Open magnetic field

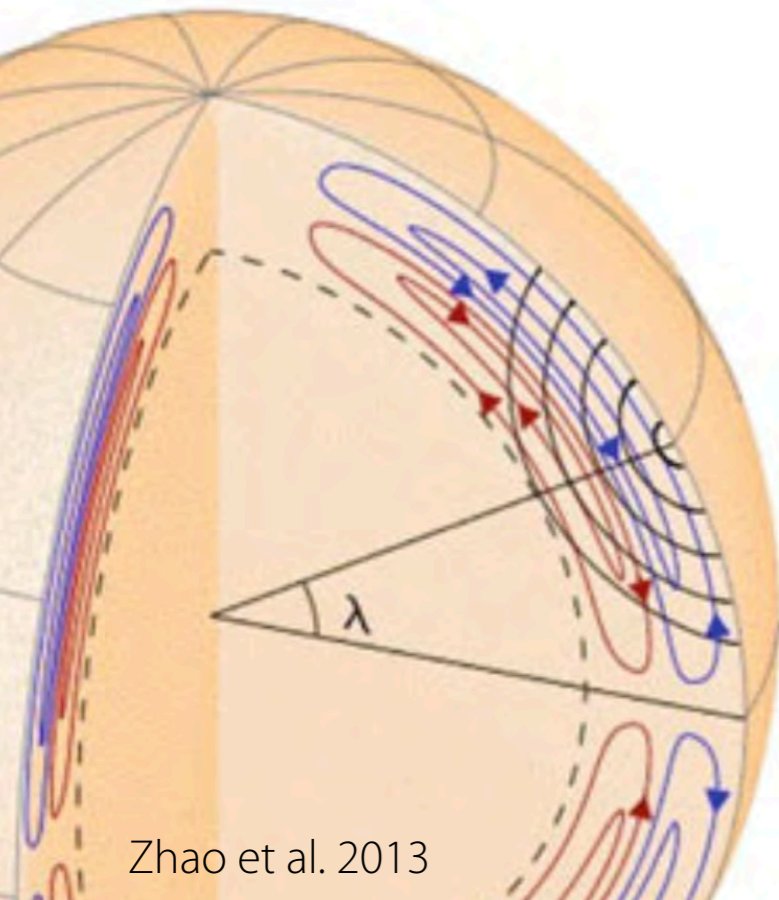
Closed magnetic loops

Solar rotation

Differential rotation — surface

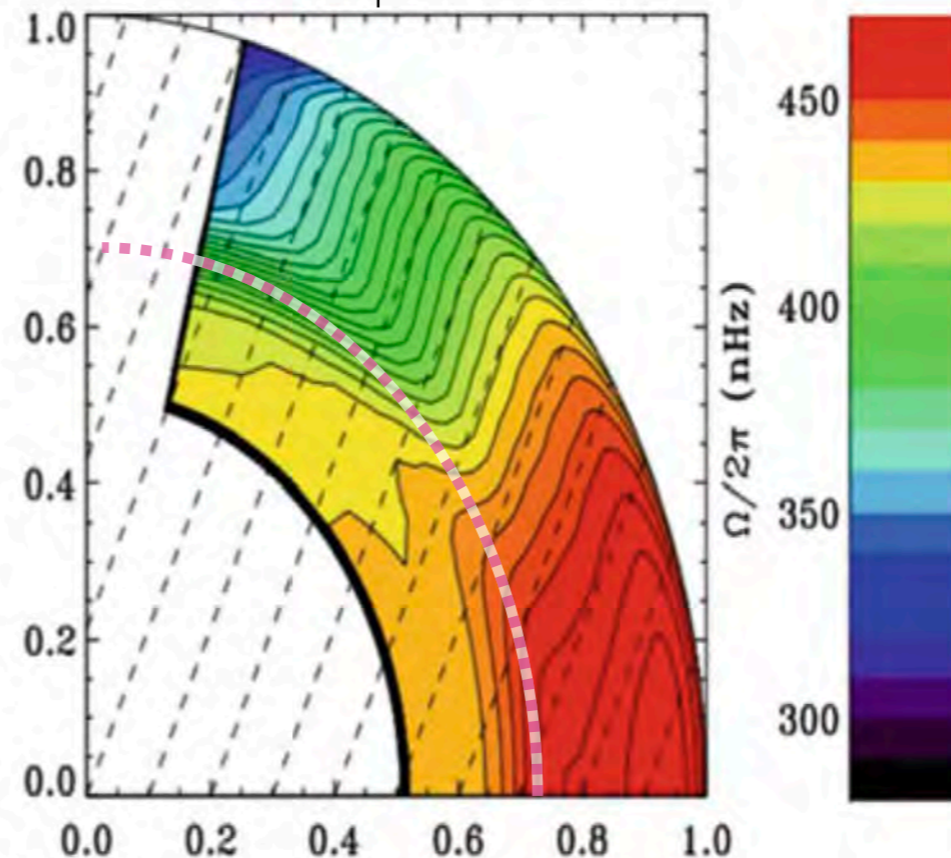
- **Different tracers imply different rotation laws $\Omega(\psi)$ — Why?**
 - Are different tracers anchored at different depths in the convection zone?
 - How does the magnetic field structure change as function of radius?
- Remember the internal rotation speeds and meridional flows as probed with helioseismological methods!

➔ Strong variation of rotation speed as function of radius, depending on latitude

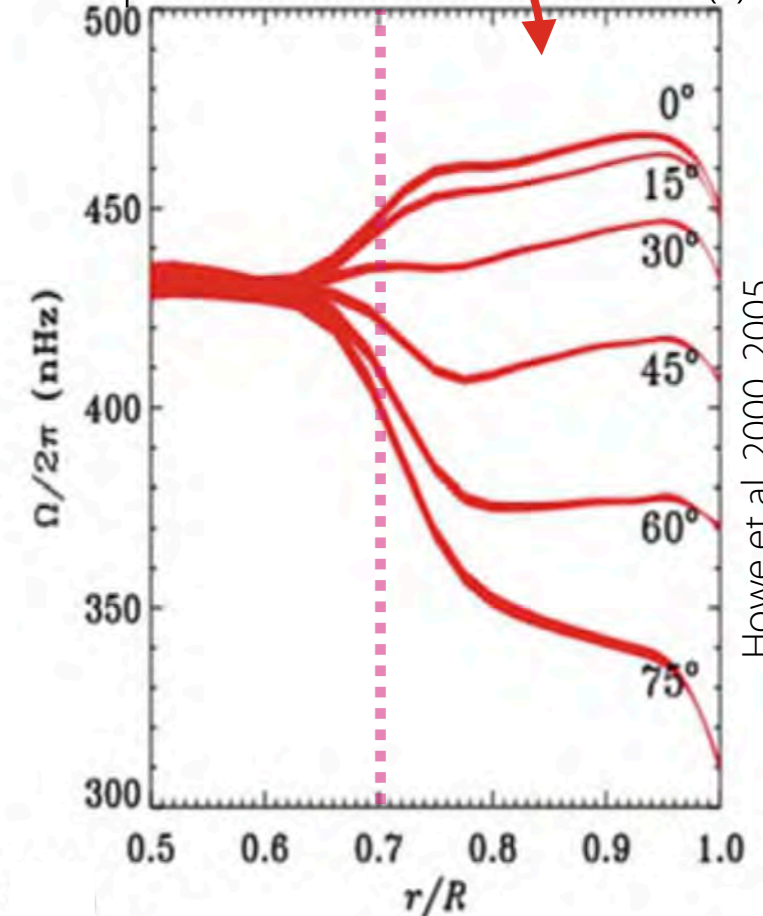


Zhao et al. 2013

Mean rotation profile from GONG data



Radial profiles at constant latitude(s)



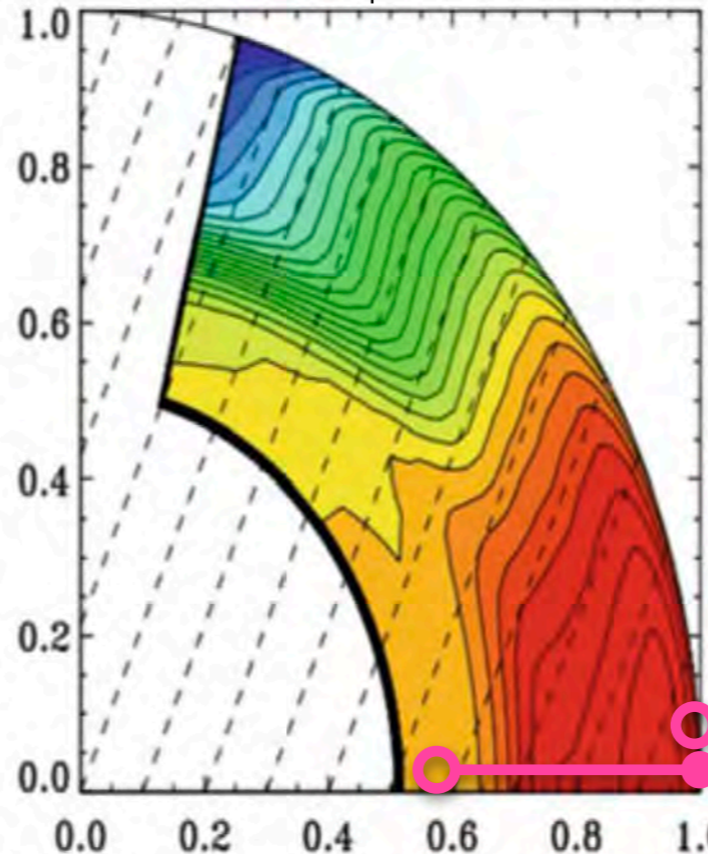
Solar rotation

Differential rotation — surface

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Mean rotation profile from GONG data



➔ What happens if a feature is anchored below the surface in gas that rotates slow compared to a feature at the surface that rotates faster?

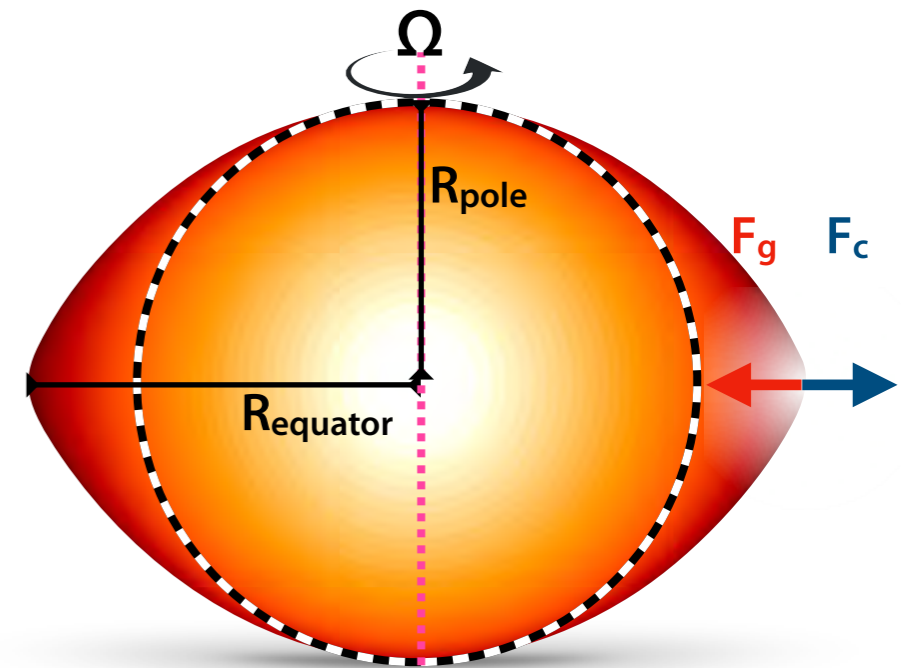
Feature A: anchored at surface, rotates fast

Feature B: anchored in deep convection zone, rotates slower

Solar rotation

Solar oblateness

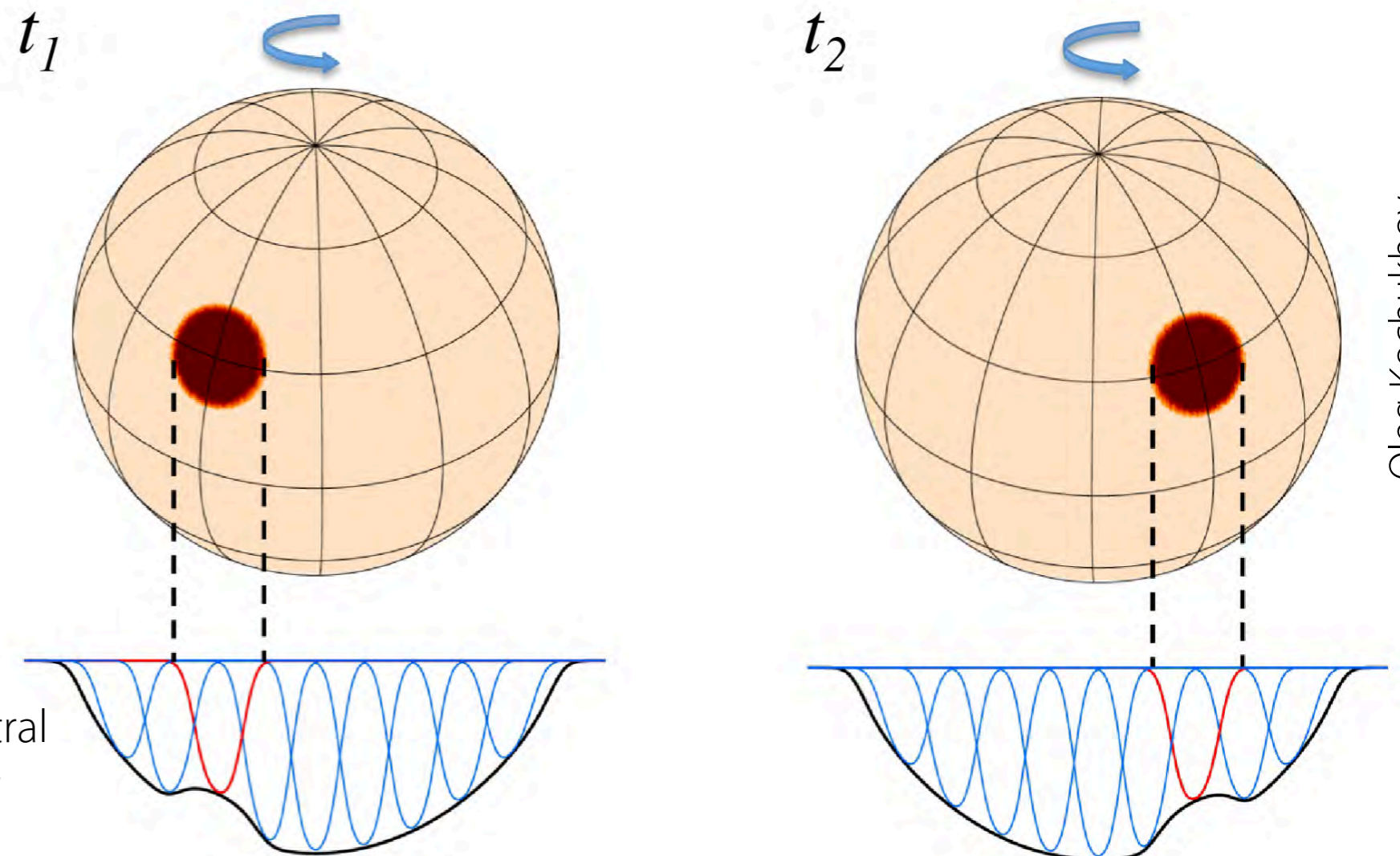
- Centrifugal force due to solar rotation leads larger solar diameter at equator than between the poles
- Oblateness := $(R_{\text{equator}} - R_{\text{pole}}) / \langle R \rangle = \Delta R / \langle R \rangle$
 - Note: If core rotates more rapidly than surface, then oblateness will be larger than expected due to surface rotation rate
- **Direct measurements:** $(\Delta R/R)_{\odot} \approx 10^{-5}$ (which thus means $\Delta R \approx 14$ km)
 - Best spatial resolution of images still ~ 50 km
 - Systematic errors due to concentration of magnetic activity to low latitudes affects measurements of solar diameter since shape of limb is distorted.
- **Helioseismic measurements** also: $(\Delta R/R)_{\odot} \approx 10^{-5}$ (Redouane Mecheri)



Stellar rotation

Measuring rotation periods

- **Spectroscopically**: rotational broadening of spectral lines.
- From **asteroseismic** measurements
- Detection of **rotational modulation** (over time) caused by a non-uniform surface on the star, .e.g. varying imprints of **starspots** in stellar spectra/light curves as they move across the unresolved stellar disk over time.



Rotational modulation of a spectral line caused by a large starspot

Stellar rotation

Spectral line broadening due to rotation

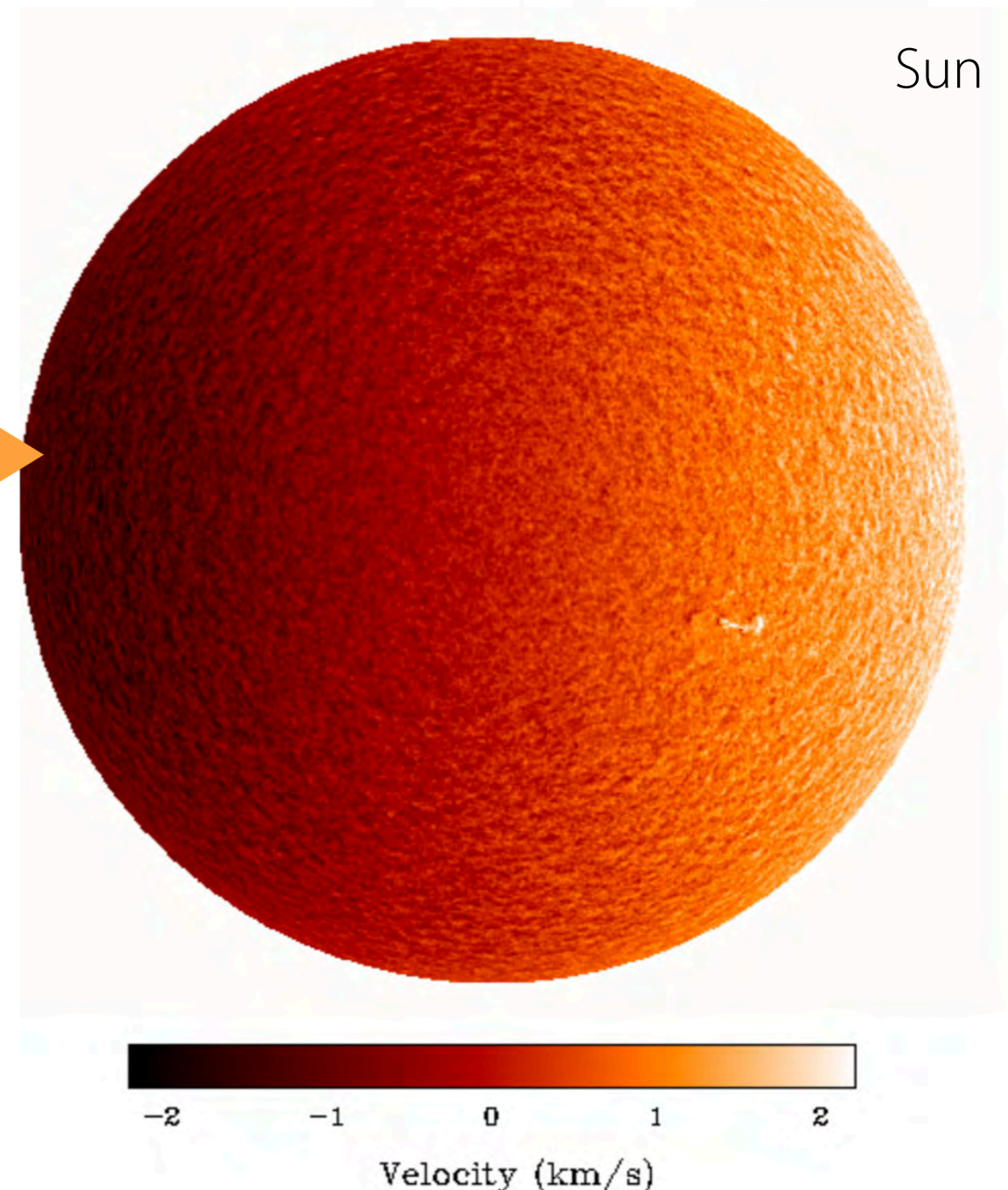
- Doppler effect — velocity component along line of sight (LOS)
- Stars: LOS velocity due to rotation of the different parts of the stellar disk integrated into a spatially averaged line profile
- Combining spectral line profiles that are Doppler-shifted between +/- LOS rotation velocity at the limb

➔ Broadening of spectral lines allows to derive only the **projected** rotation rate / apparent rotational velocity:

$$v' = v \sin i$$

- i : angle between LOS and rotation axis of the star

Full-disk Dopplergram
9 July 1996, 9:00:00



Stellar rotation

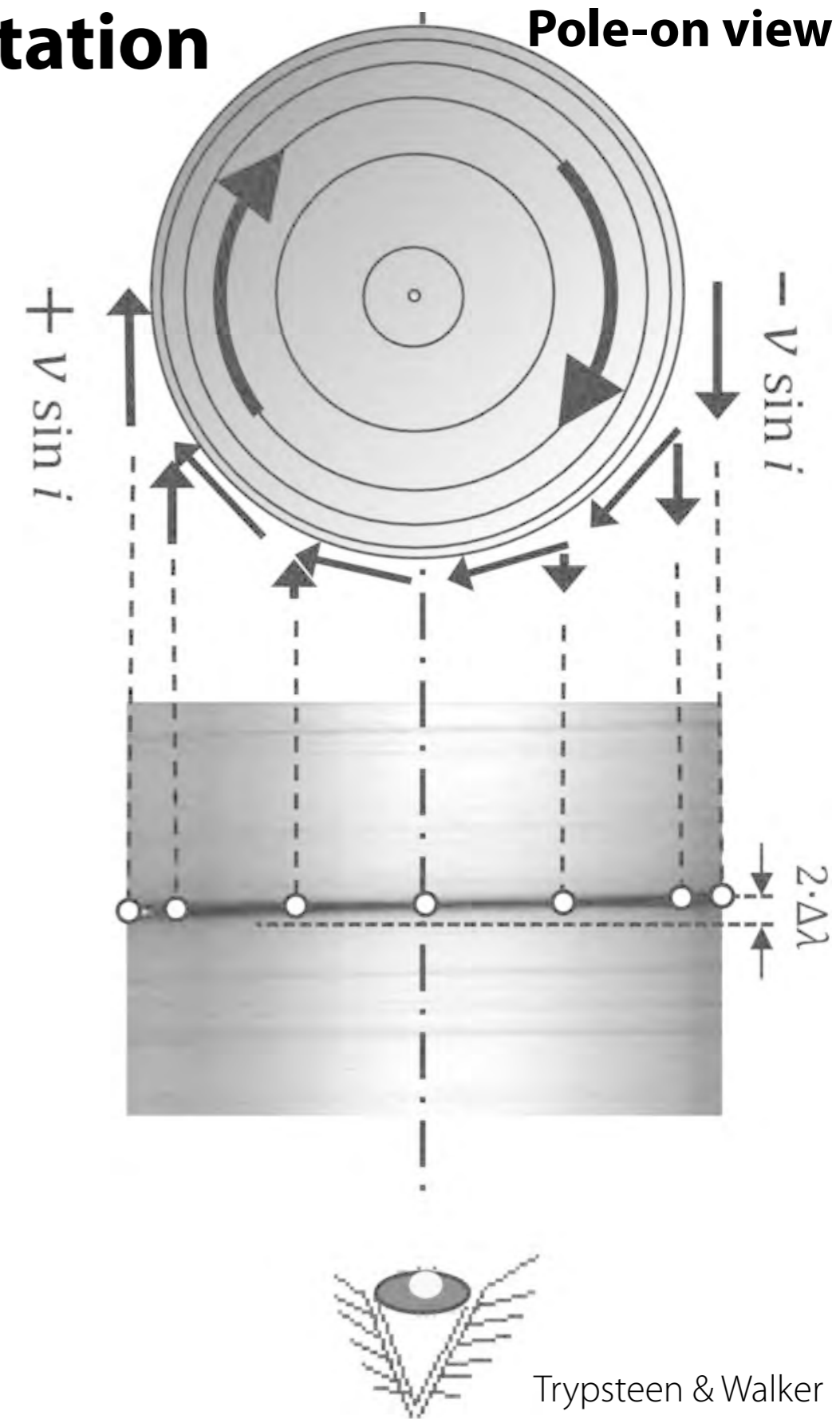
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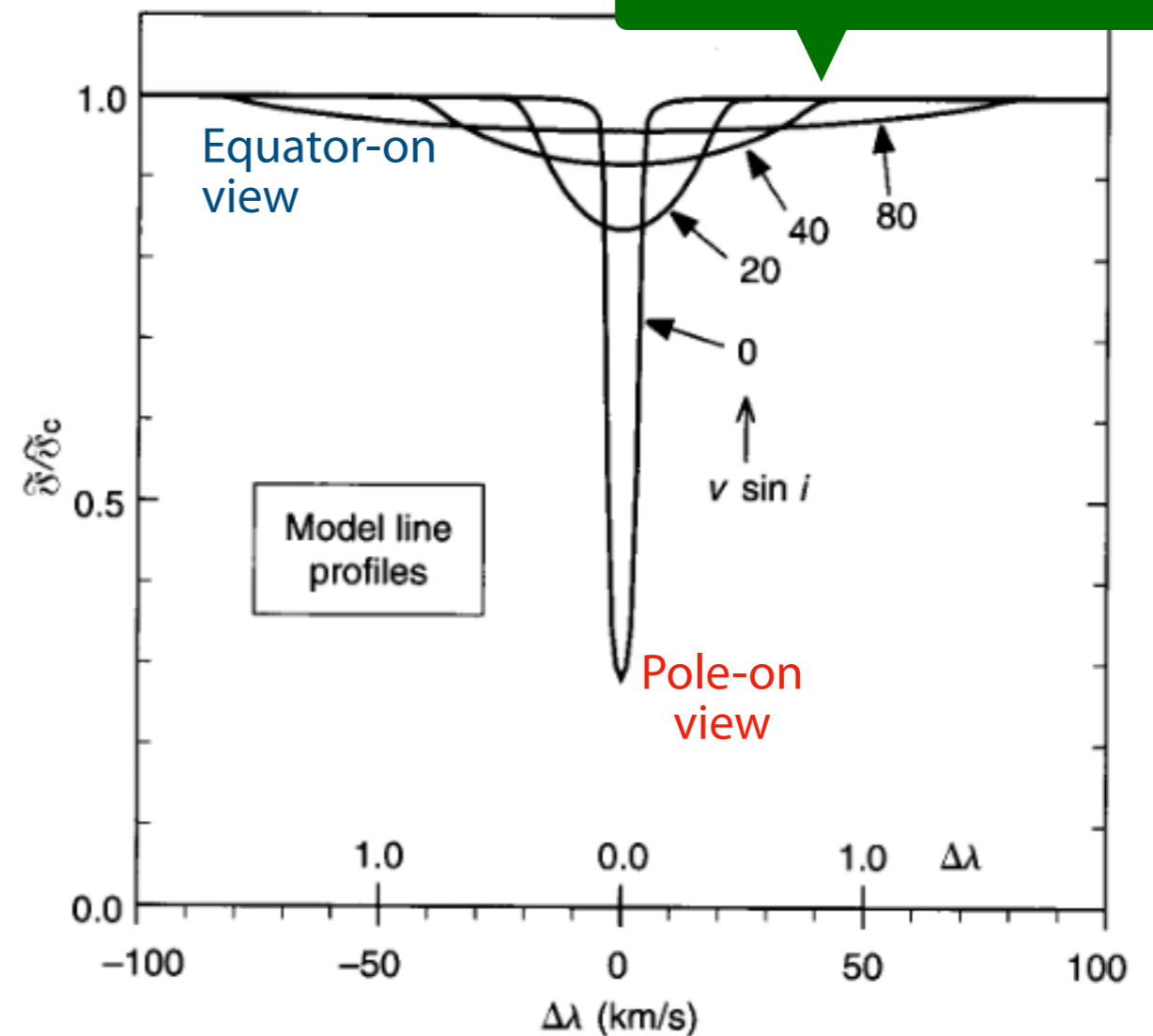
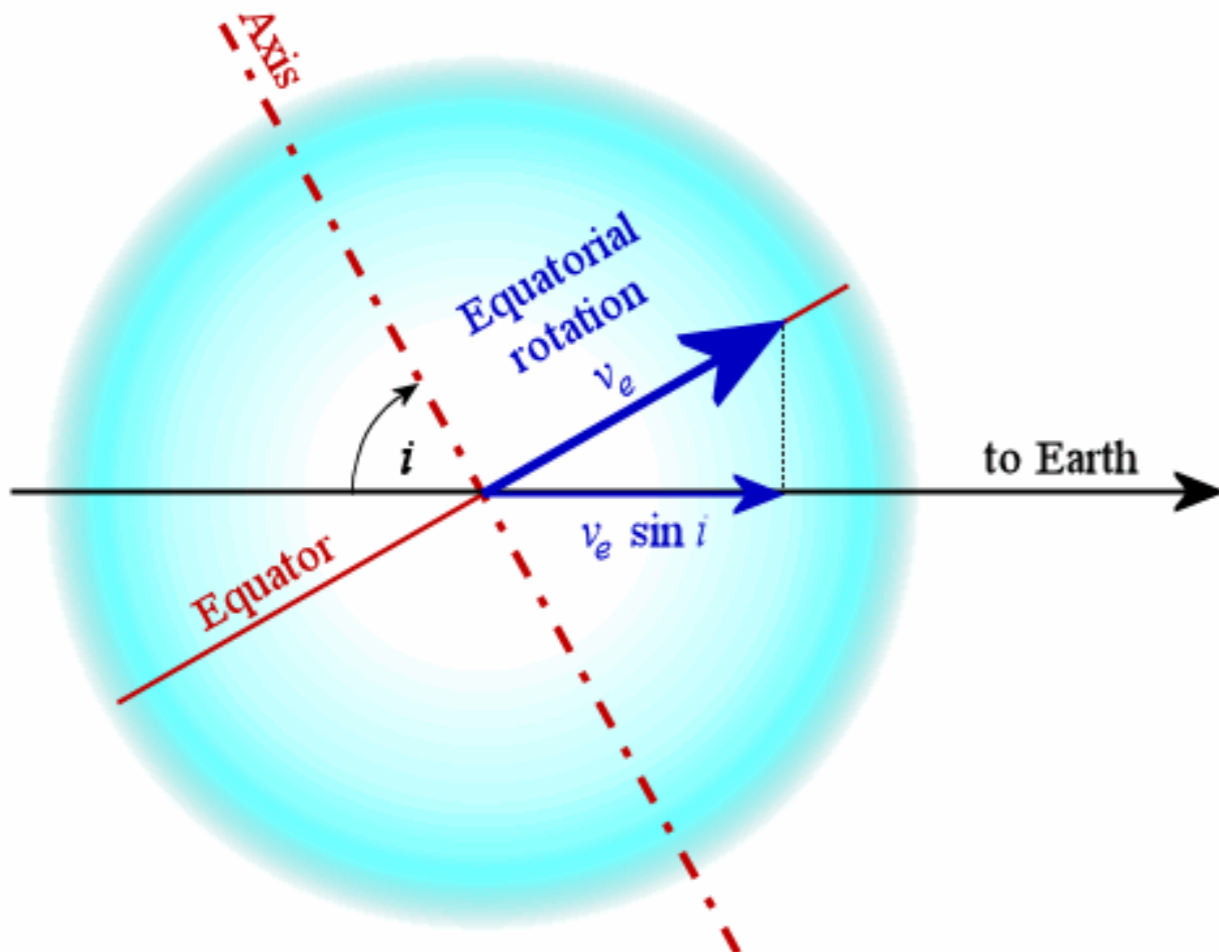
Stellar rotation

Spectral line broadening due to rotation

- Only the **projected** rotation rate can be derived from broadening of spectral lines

$$v \sin i \quad i: \text{angle between LOS and rotation axis of the star}$$

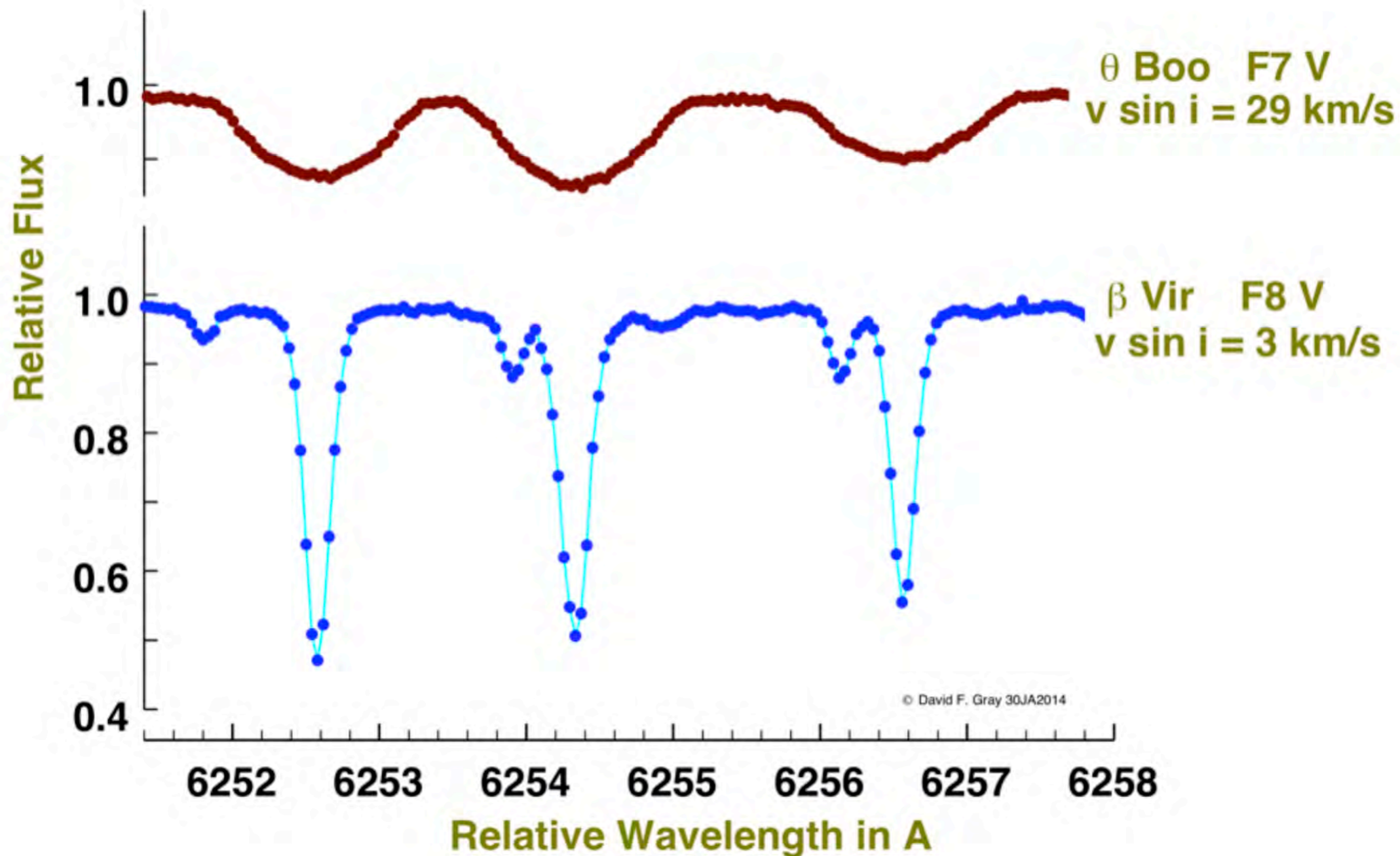
- View on pole: $i = 0$, no rotational broadening;
- View on equator: $i = 90^\circ$, maximum rotational broadening.



Stellar rotation

Spectral line broadening due to rotation

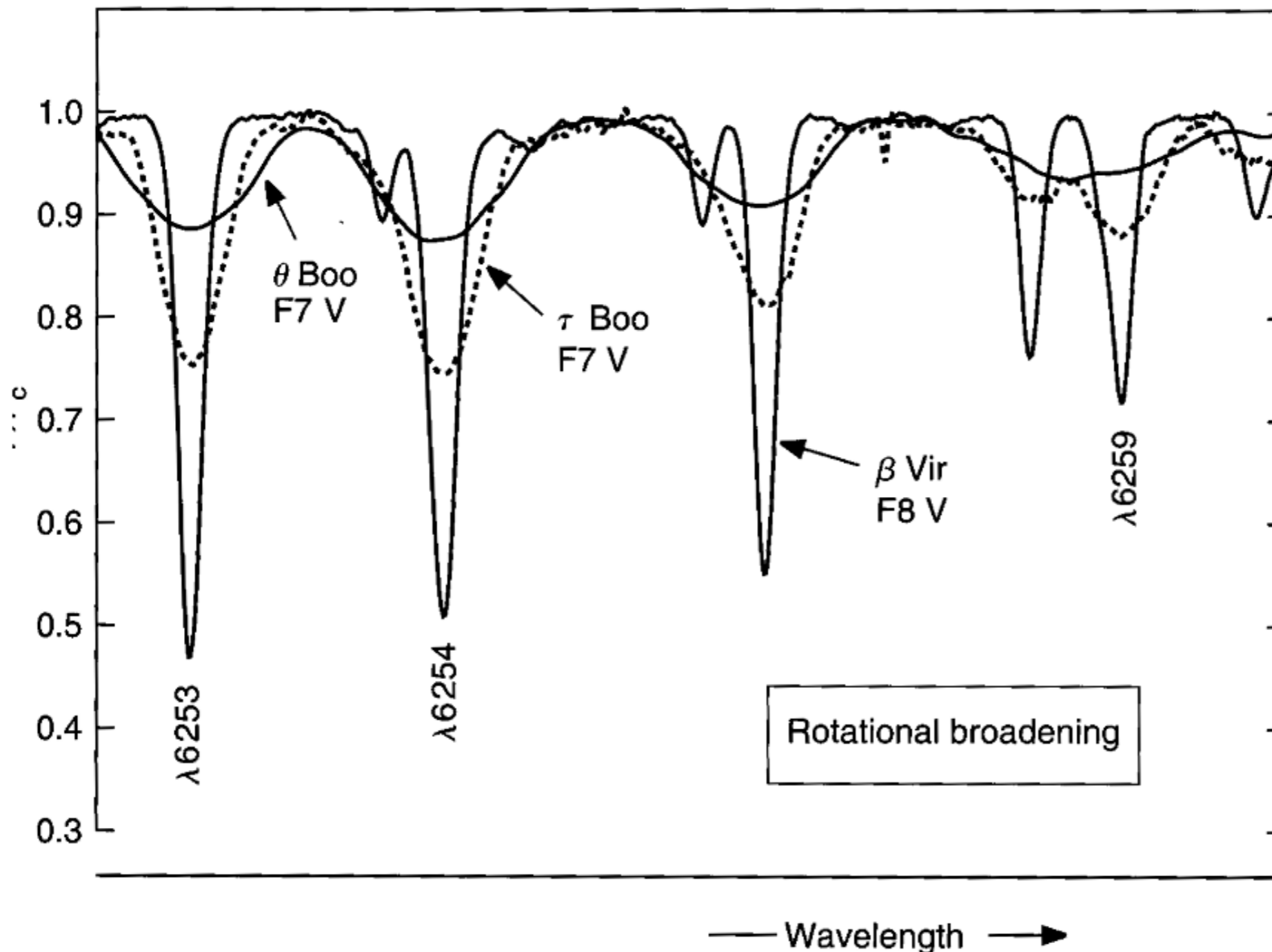
- Comparison of two (relatively) similar stars



Stellar rotation

Spectral line broadening due to rotation

- Comparison of three (relatively) similar stars



A small problem:

We measure only

$$v \sin i$$

→ The true rotational velocity v will remain uncertain unless the inclination i of the rotation axis is known.

Stellar rotation

Differential rotation

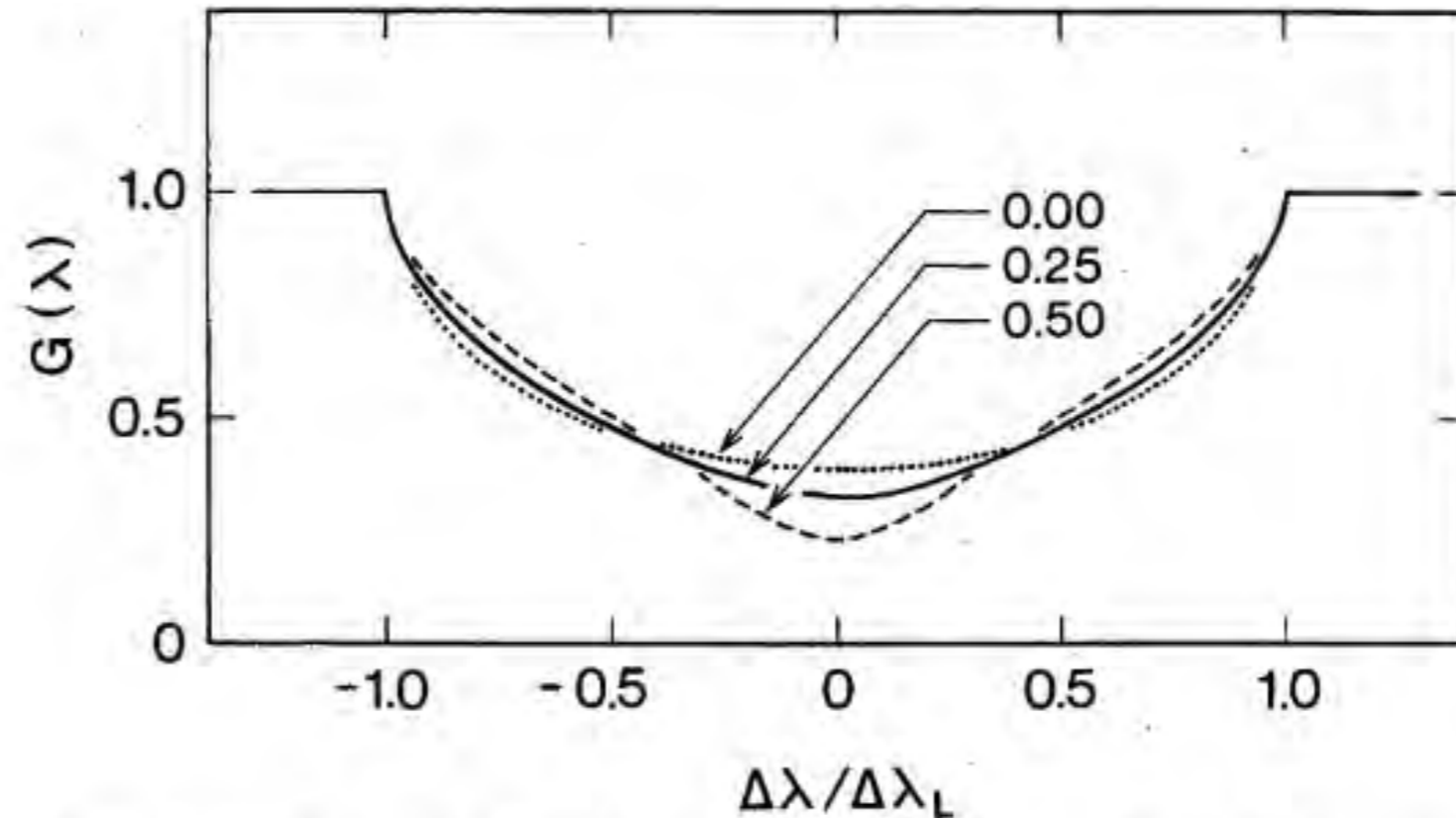


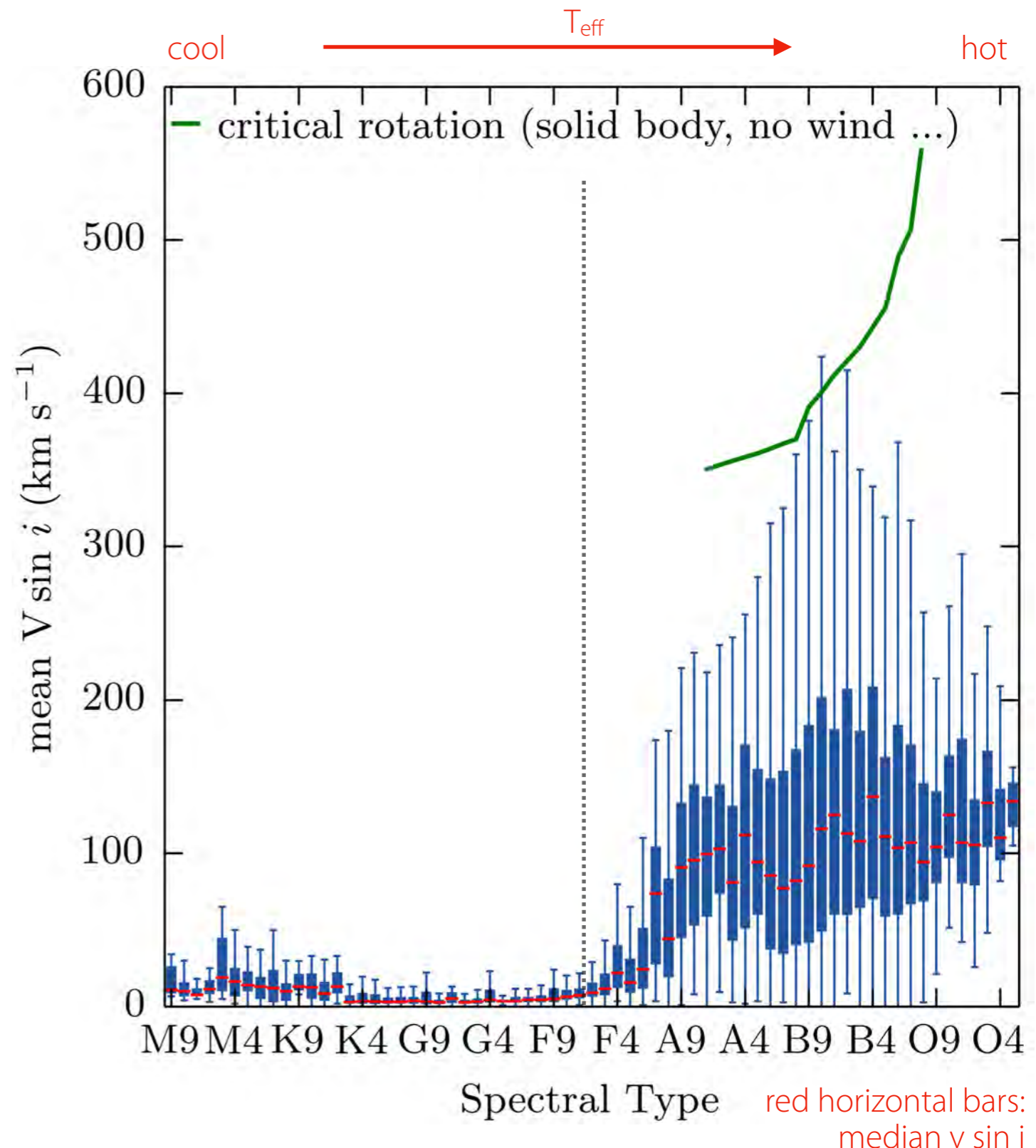
FIG. 3.—Rotation profiles for three values of the differential rotation parameter $\alpha = \omega_2/(\omega_0 + \omega_2)$.

- Equatorial acceleration \rightarrow lines narrower and more V-shaped
- Polar acceleration \rightarrow lines fatter and more U-shaped
- Evidence for differential rotation found in F-stars
- Significant rotational broadening of the line profile by several km/s needed to be detectable, solar-type stars rotate too slowly

Stellar rotation

Measured $v \sin i$

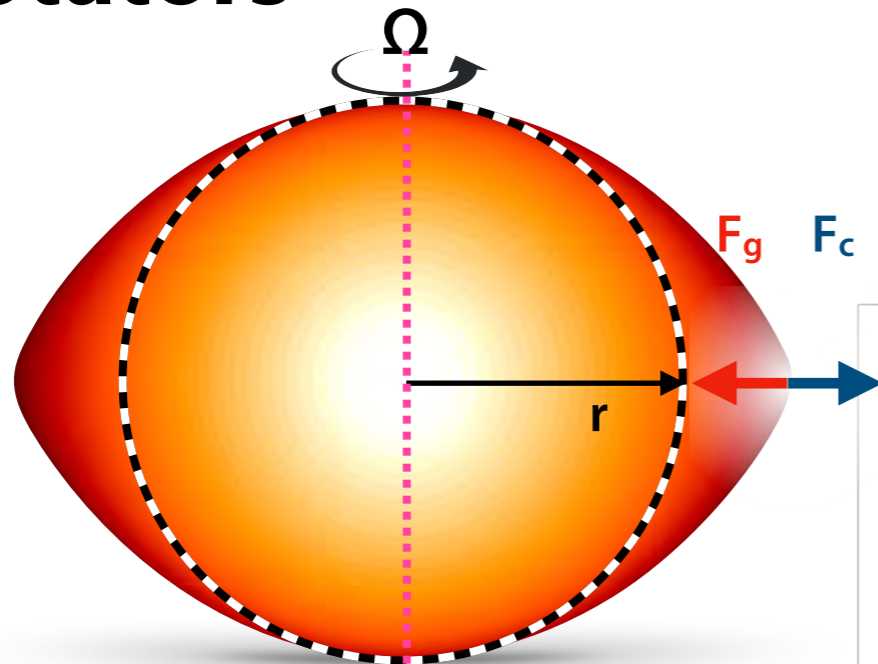
- Distribution of apparent rotational velocities ($v \sin i$) as a function of spectral type.
- Some hot stars rotate as fast as 450 km/s
- Two stellar populations:
 - **Slow rotators** — stars cooler than F7: typically $v \sin i < 50$ km/s; for many solar-like stars < 6 km/s.
 - **Fast rotators** — hotter stars: often $v \sin i > 100$ km/s.
- The Sun rotates comparatively slowly at ~ 2 km/s
- Wide range at earlier spectral types (hot stars) due to
 - Large spread in actual rotation rates
 - Large spread in inclination angles



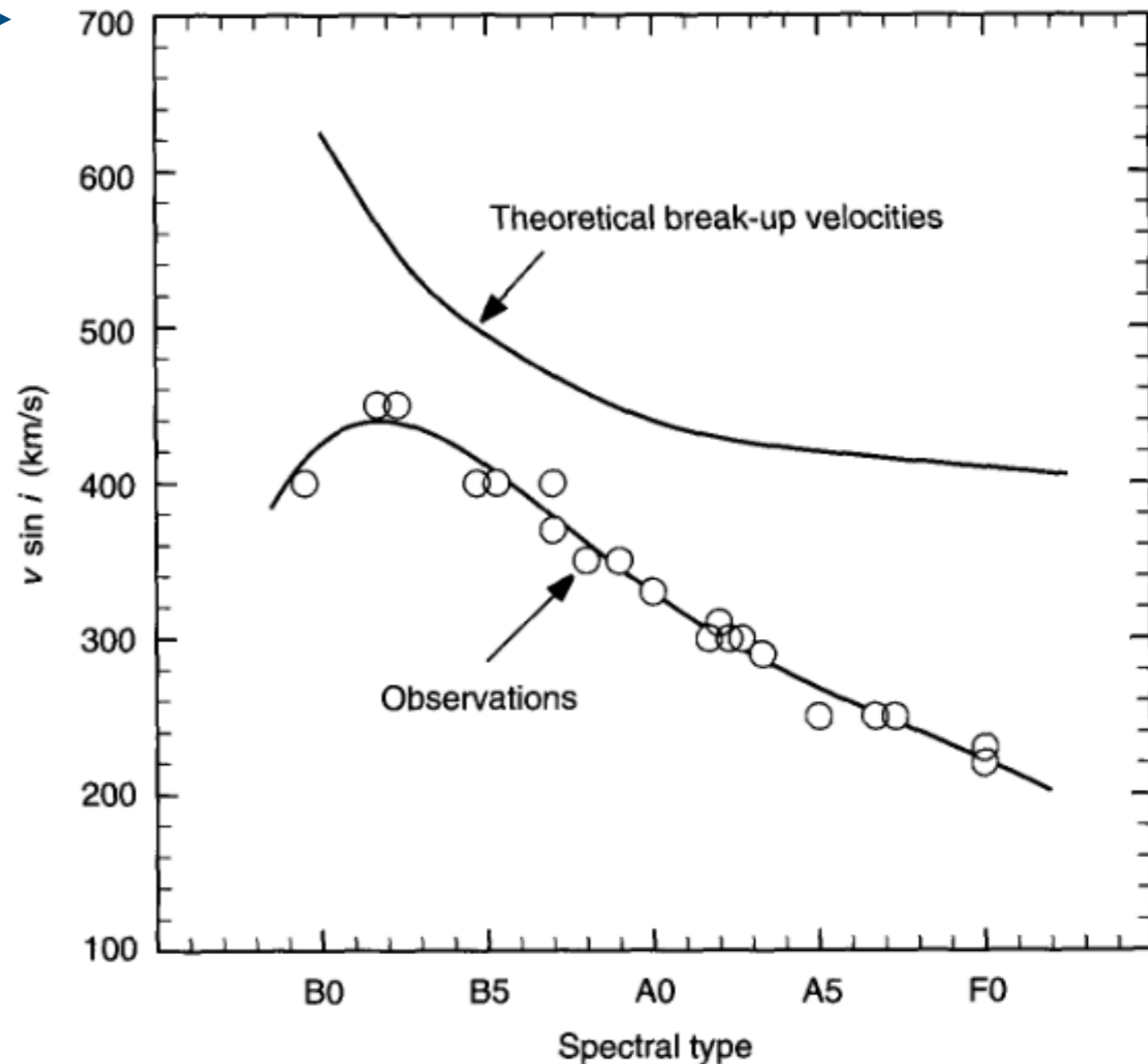
Y.Frémat

Values are taken from the catalog of Glebocki R. & Gnacinski P. 2005.

Fast rotators



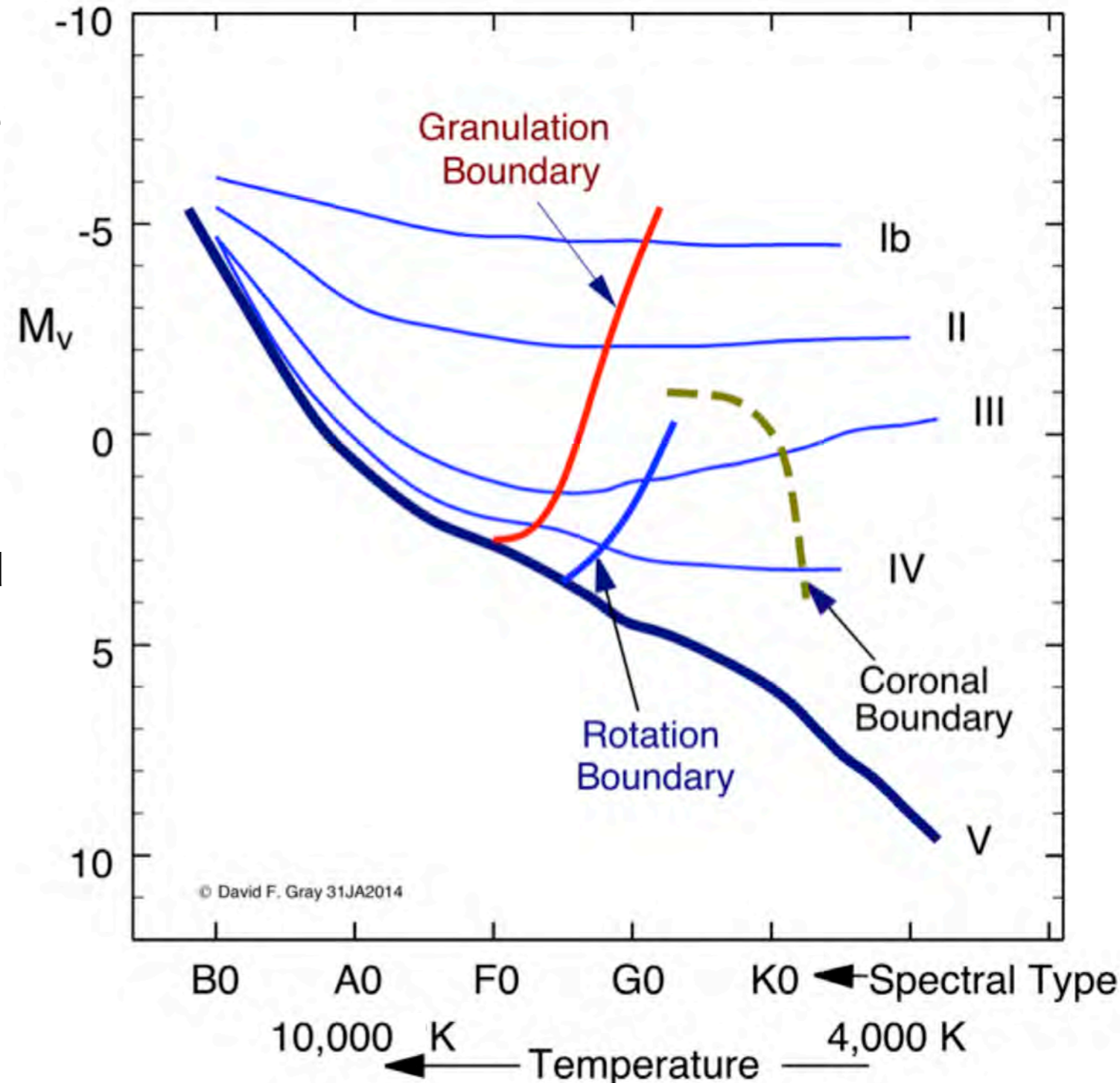
- Fast rotation causes centrifugal force, counteracts gravity, strongest effect at equator
 ➔ Oblateness
- Limiting **breakup velocity** = rotation speed at which the centrifugal force exceeds the gravitational force
 ➔ Stellar material in fastest rotating areas no longer gravitationally bound



Stellar rotation

Across the HRD

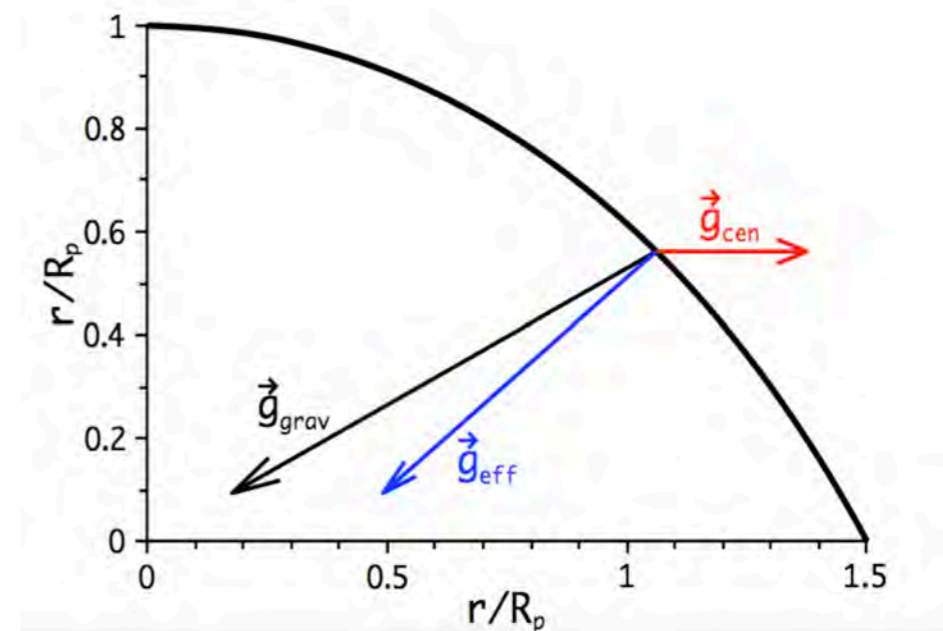
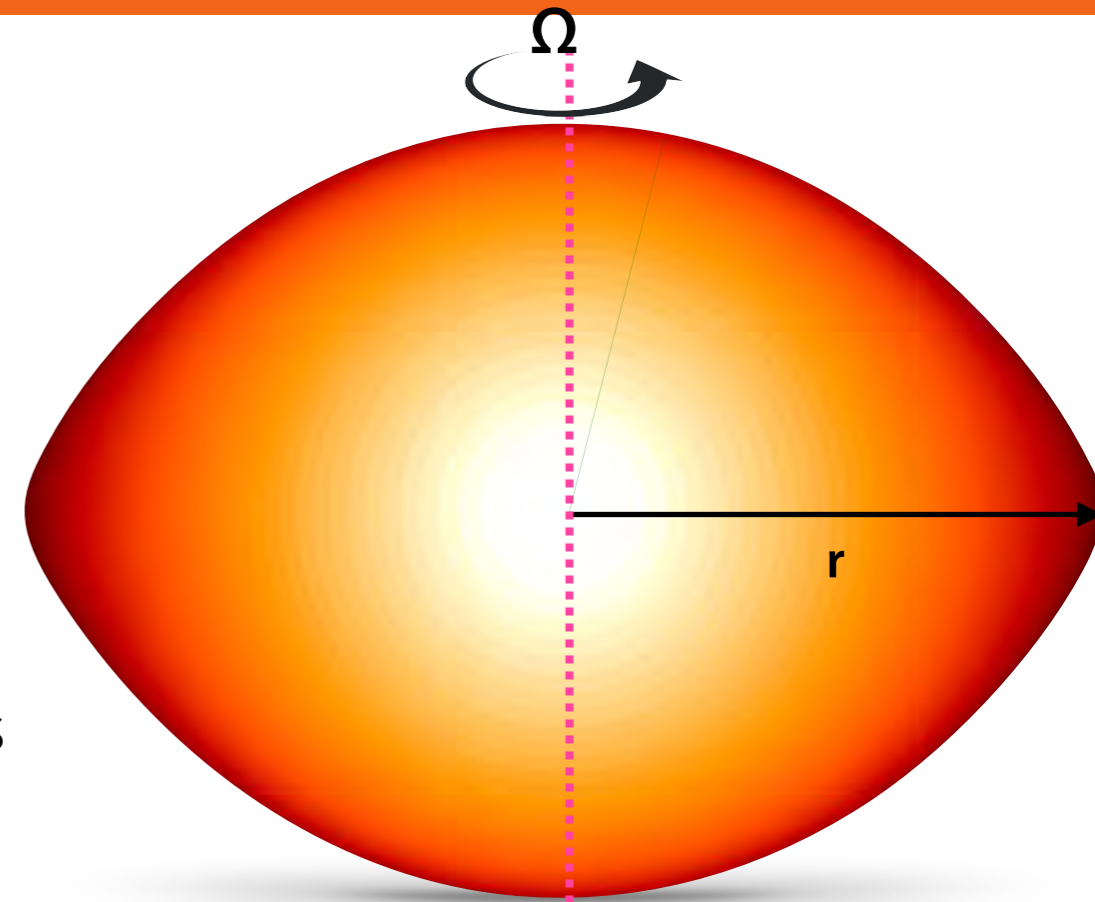
- “**rotation boundary**” — sharp transition between fast and slow rotators
- Stars on the cool side basically one value for any given effective temperature and evolutionary status.
- Possible explanation: magnetic braking of the star due to the impact of convection on rotation properties (rotostat mechanism)
- Note that the rotation boundary is found not far from the granulation boundary beyond which there is no more surface convection
- Exceptions: Close binary stars can transfer orbital angular momentum to companion star through gravitational coupling, resulting in anomalous rotation



Stellar rotation

Effects at fast rotation

- Oblateness due to fast rotation, star no longer spherical symmetric
- Surface further away from centre at the equator than at the pole
- ➔ Gravity acceleration at surface varies with latitude!
 - Effective gravity acceleration g_{eff} larger at the poles than at the equator
- At the equator: centrifugal force outwards “compensates” for parts of gravitational force inwards, provides “hydrostatic support”
- ➔ Effects the hydrostatic stratification:
 - ➔ Lower gas pressure required to maintain equilibrium
 - ➔ Lower temperature (equation of state! Ideal gas: $P \propto T$)
 - Lower temperature at the equator than at the poles
 - Von Zeipel law (1924): $T_{\text{eff}} \propto g_{\text{eff}}^{1/4}$

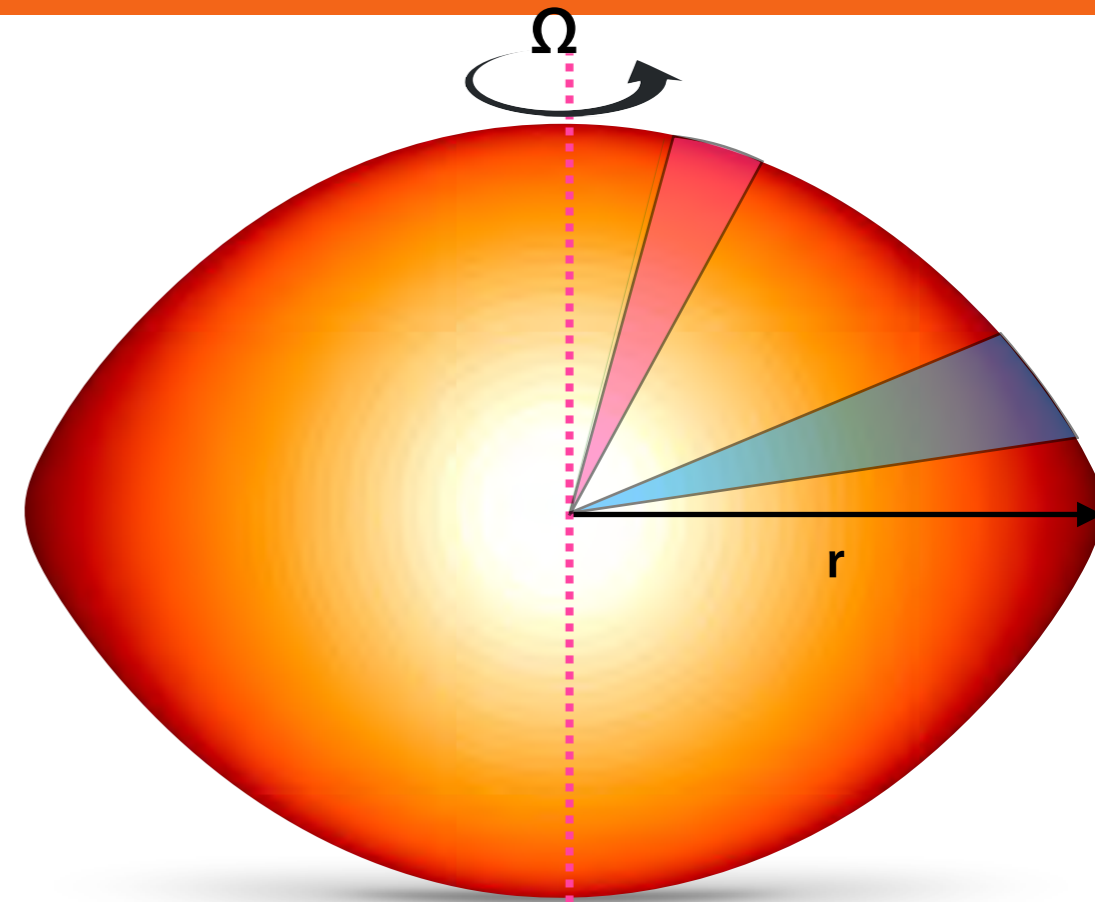


Stellar rotation

Effects at fast rotation

- Oblateness due to fast rotation, star no longer spherical symmetric
- Surface further away from centre at the equator than at the pole
- Energy flux in the interior radiated in all directions into solid angle
- Solid angle now corresponds to different area at the surface
 - Smaller area at poles and larger area at equator
 - Energy flux spread over different area at the surface
 - Stefan-Boltzmann law $T_{\text{eff}} = (F / \sigma)^{1/4}$
 - Smaller area, higher flux, higher effective temperature
 - T_{eff} is higher at the pole than at the equator

➔ Effect is called **gravity darkening**

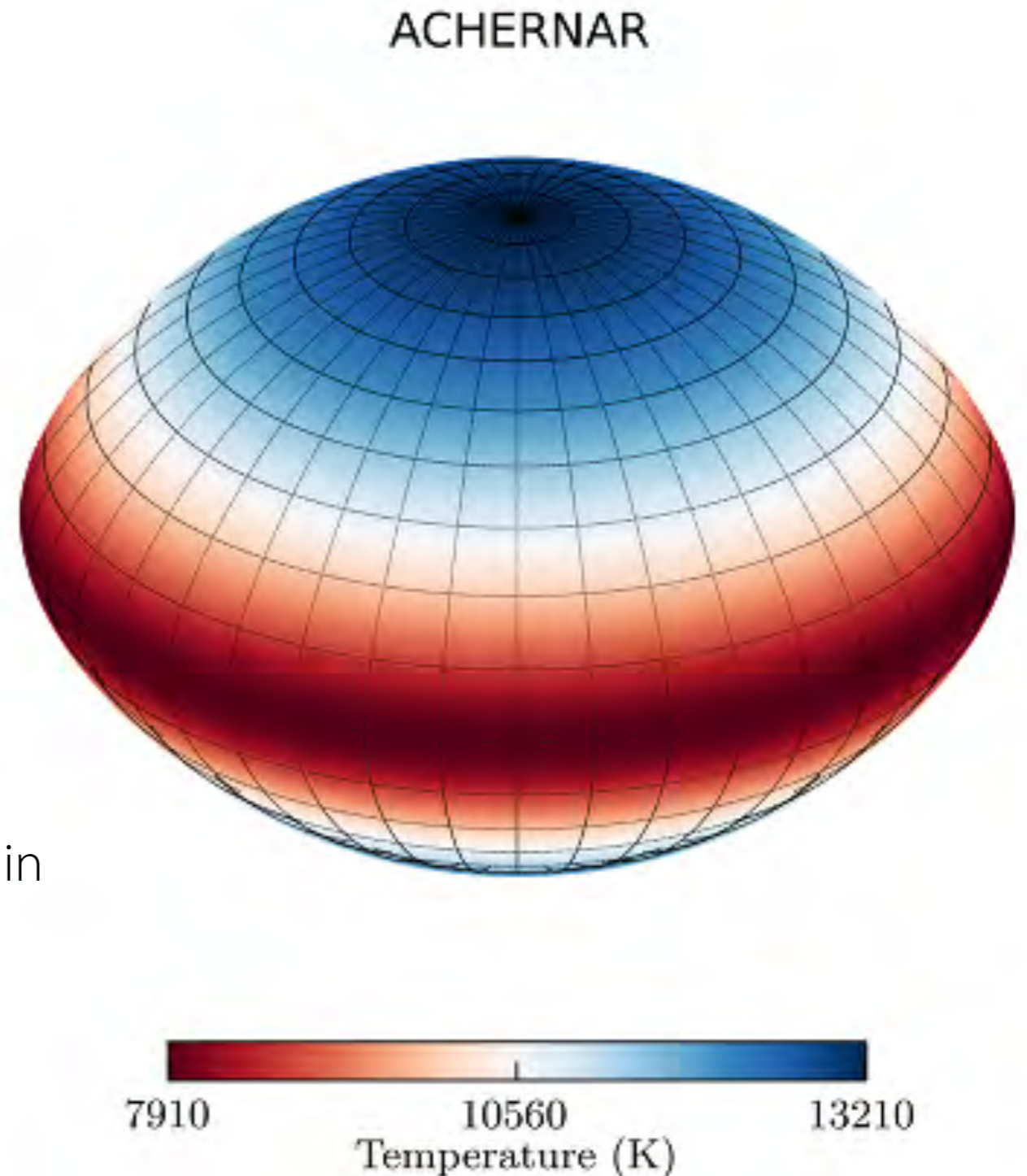


- Strong deviations from spherical symmetry
 - To be considered:
 - Conservation of angular momentum
 - Impact on energy balance
- ➔ Meridional circulation might develop

Stellar rotation

Effects at fast rotation — gravity darkening

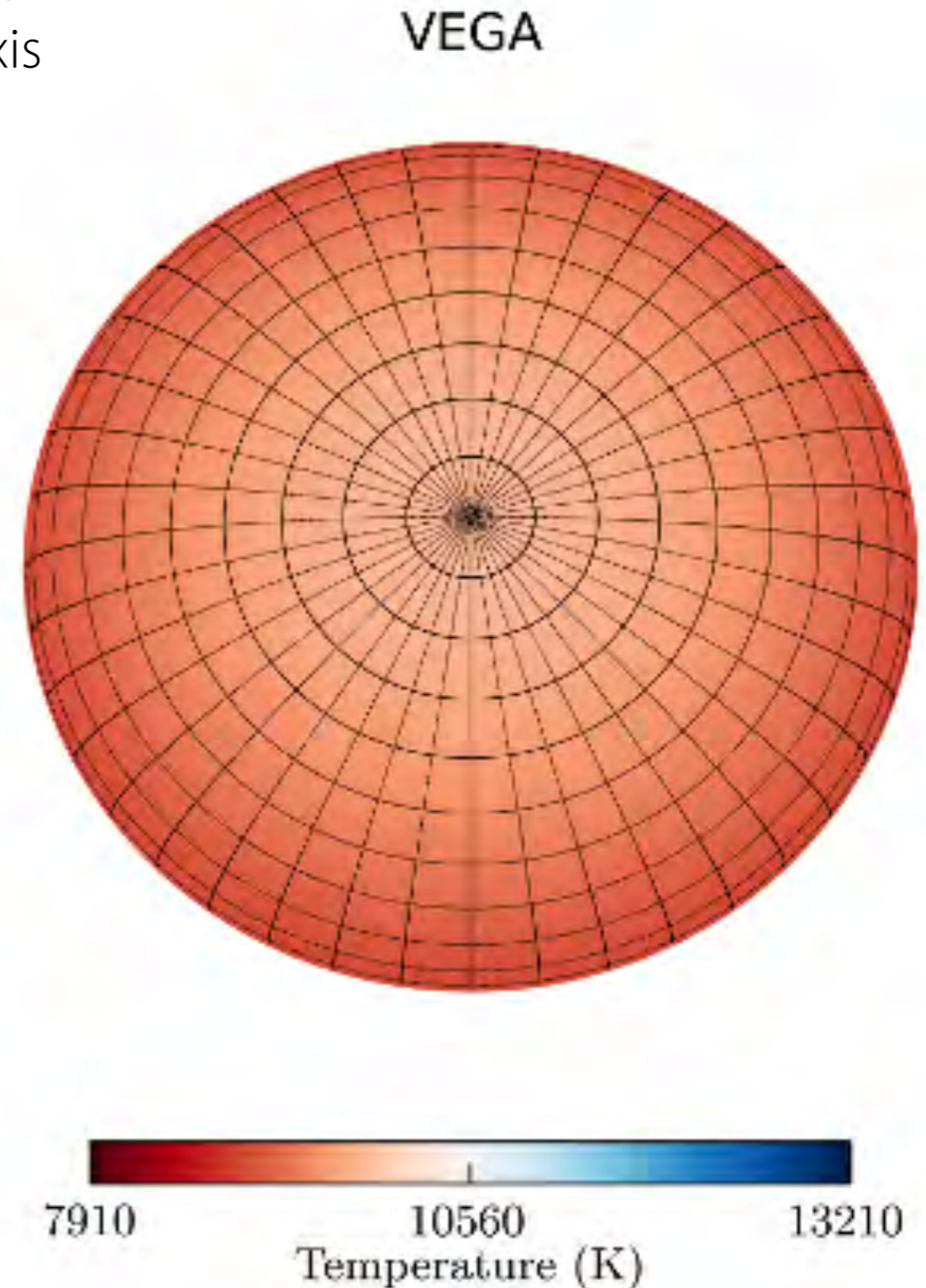
- **Example: α Eridani A — Achernar**
- 9th brightest star in the sky
- Distance ~ 140 ly (43 pc)
- Spectral type B6V
- Mass: $M = 6.7 M_{\odot}$
- **Rotational velocity ($v \sin i$) 250 km/s !**
- Radius:
 - Pole: $R = 7.3 R_{\odot}$
 - Equator: $R = 11.4 R_{\odot}$
 - ➔ Oblateness $\Delta R/R \sim 10$
- Gravity darkening produces large differences in surface temperature!



Stellar rotation

Effects at fast rotation — gravity darkening

- Note: Effect only measurable directly when seen at large enough angle with respect to the rotation axis (i.e. not pole-on)
- **Example: α Lyrae — Vega**
 - 5th brightest star in the sky
 - Distance ~ 25 ly (7.7 pc)
 - Spectral type A0V
 - Mass: $M = 2.1 M_{\odot}$
 - **Rotational velocity ($v \sin i$) 20.5 km/s !**
 - Radius:
 - Pole: $R = 2.3 R_{\odot}$
 - Equator: $R = 2.8 R_{\odot}$
 - **➔ Oblateness $\Delta R/R \sim 2.2$**
- We do not see gravity darkening as we observe Vega **pole-on**



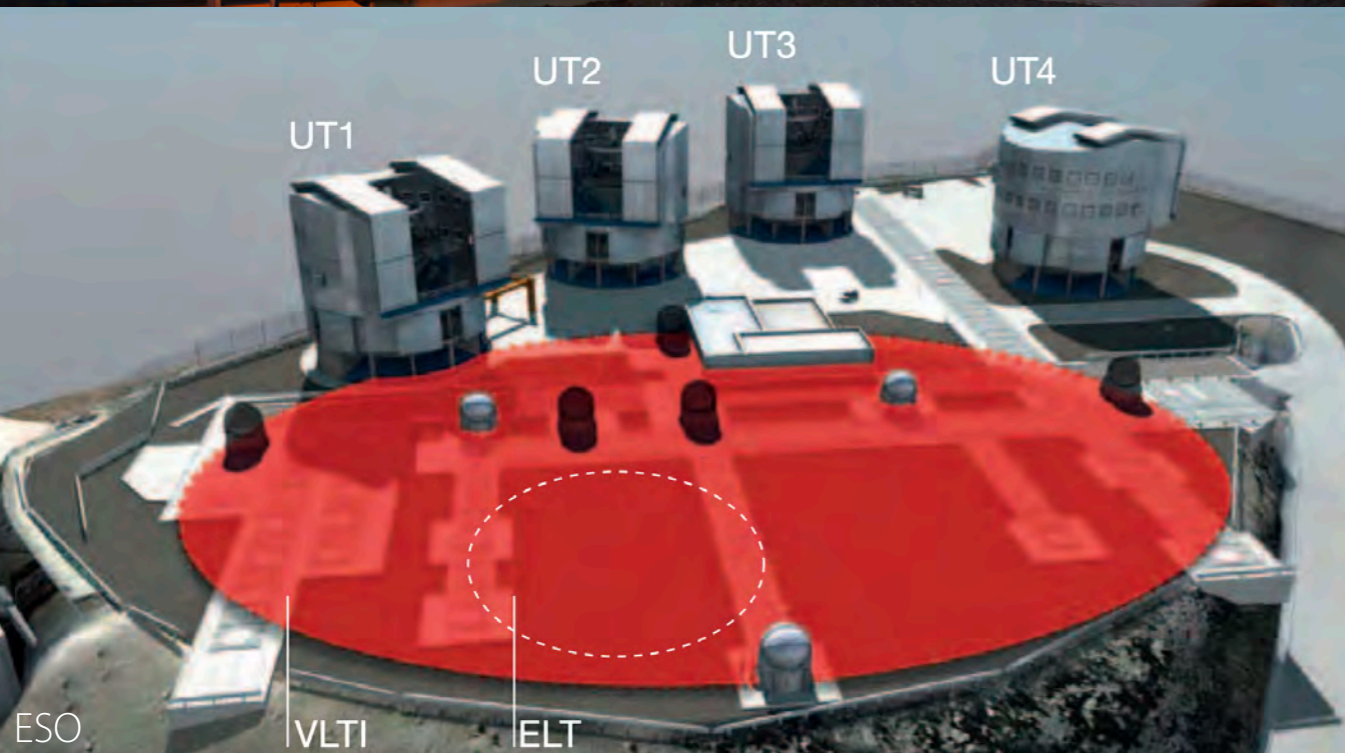
Stellar rotation

Effects at fast rotation — gravity darkening

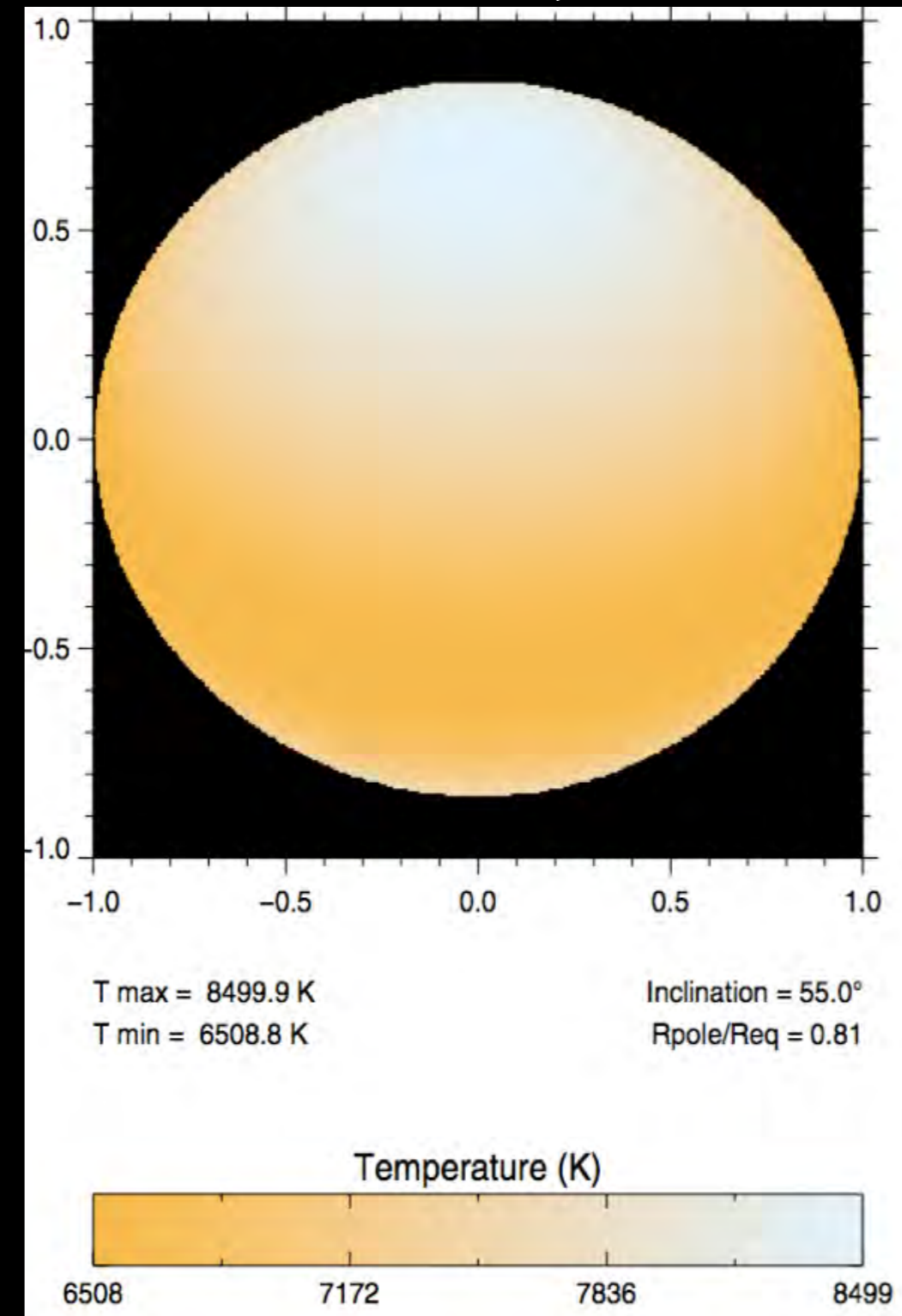
- Gravity darkening: Direct observational confirmation by optical interferometry with VLTi: Altair
- Rotational velocity ($v \sin i$) 240 km/s!



My own :-)



Altair (spectral type A7IV-V)



Stellar rotation

Coriolis force

- Not a “real” force but due to rotation of the frame of reference
- Impact on motion depends on involved temporal and spatial scales and velocities

➔ Rossby number: $Ro = \frac{U}{fL}$

U : velocity, U

L : length scale

f : Coriolis parameter

Ω : rotation rate

φ : latitude

- The Rossby number is the ratio of inertial to Coriolis forces.
- Coriolis parameter: $f = 2\Omega \sin \varphi$.
($f = 0$ at the equator, $\varphi=0$)
- Ratio between the rotation period of a star and the local convective turnover time
- Large Rossby number: inertial and centrifugal forces dominate (e.g. in tornadoes on Earth, bathtub flow)
- Small Rossby number: Coriolis forces important, stellar rotation impacts resulting flow pattern — notably over large spatial scales on a star

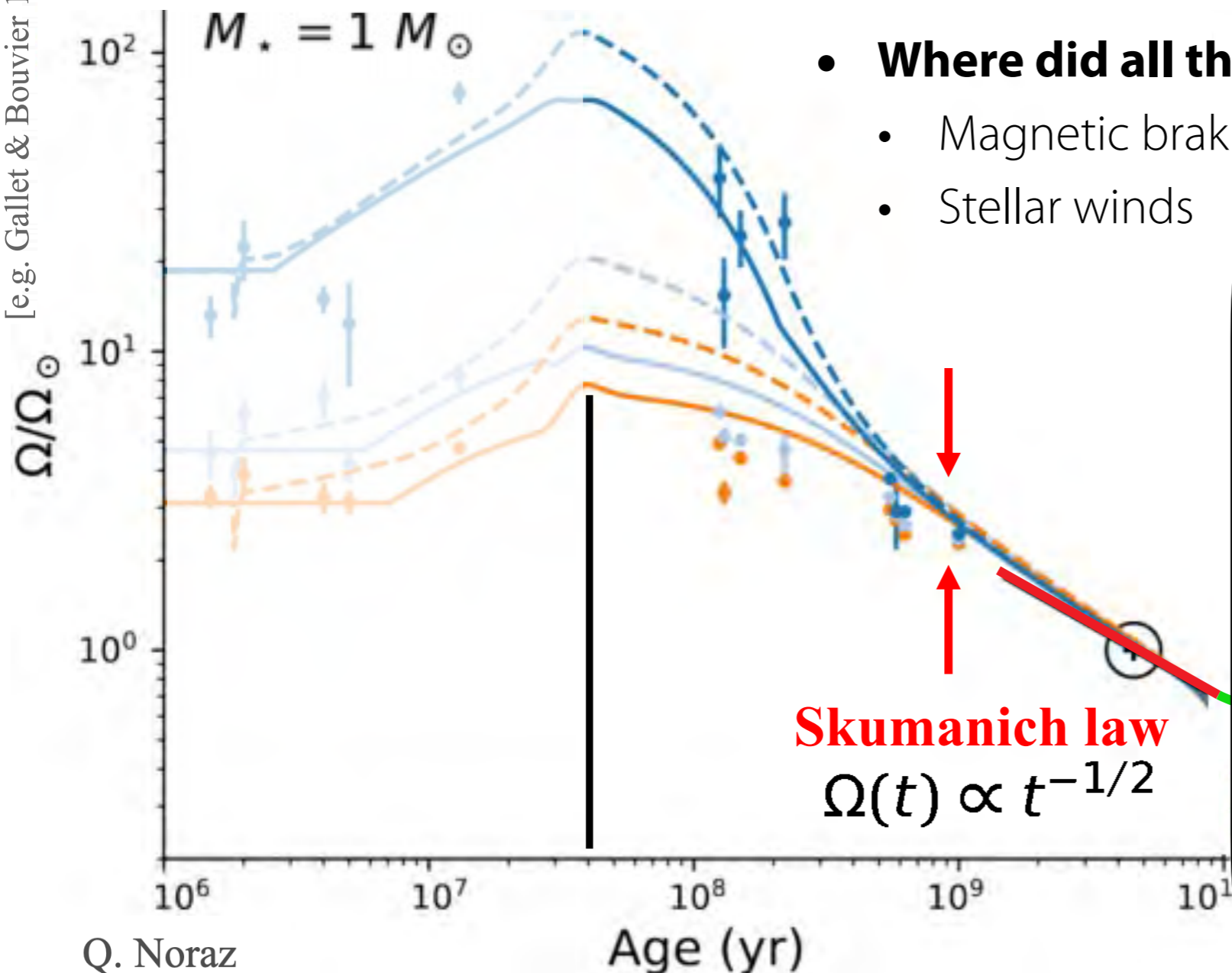
Stellar rotation

Evolution of rotation

- **Young stars are seen to rotate up to 100 times faster than the Sun.**

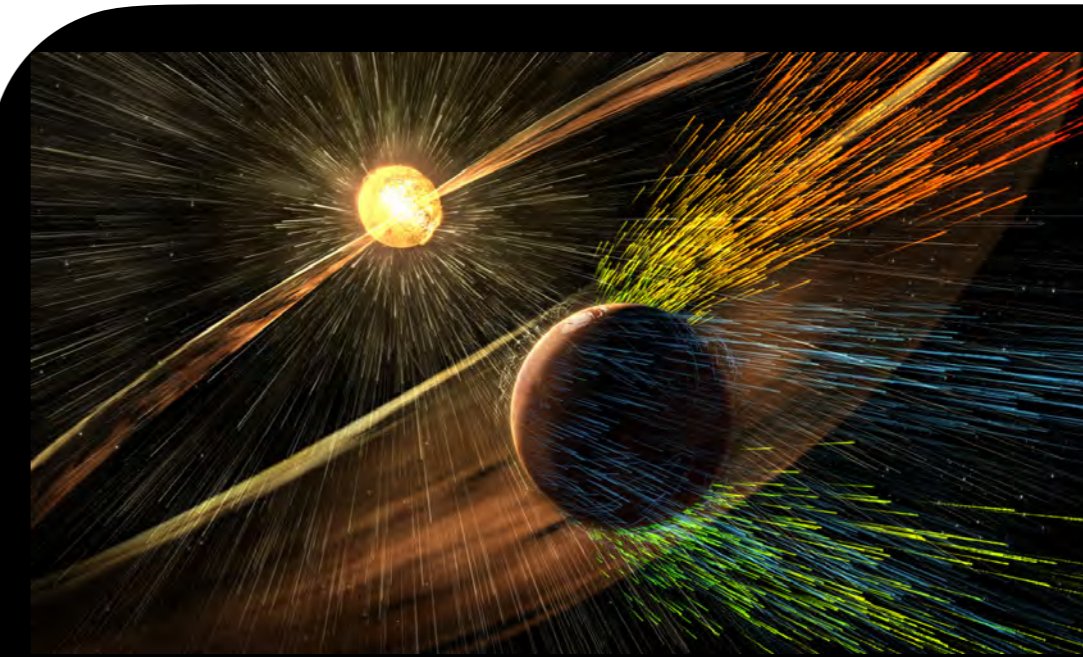
- Skumanich law: $\Omega \propto t^{-1/2}$ (t: age of the star)
- Sun also rotated faster as a young star.

[e.g. Gallet & Bouvier 15; Ahuir+ 21]



- **Where did all the angular momentum go?**

- Magnetic braking
- Stellar winds



NASA/GSFC

Solar/stellar wind:

Stream of charged particles released from the upper atmospheric layers of a star

Stellar rotation

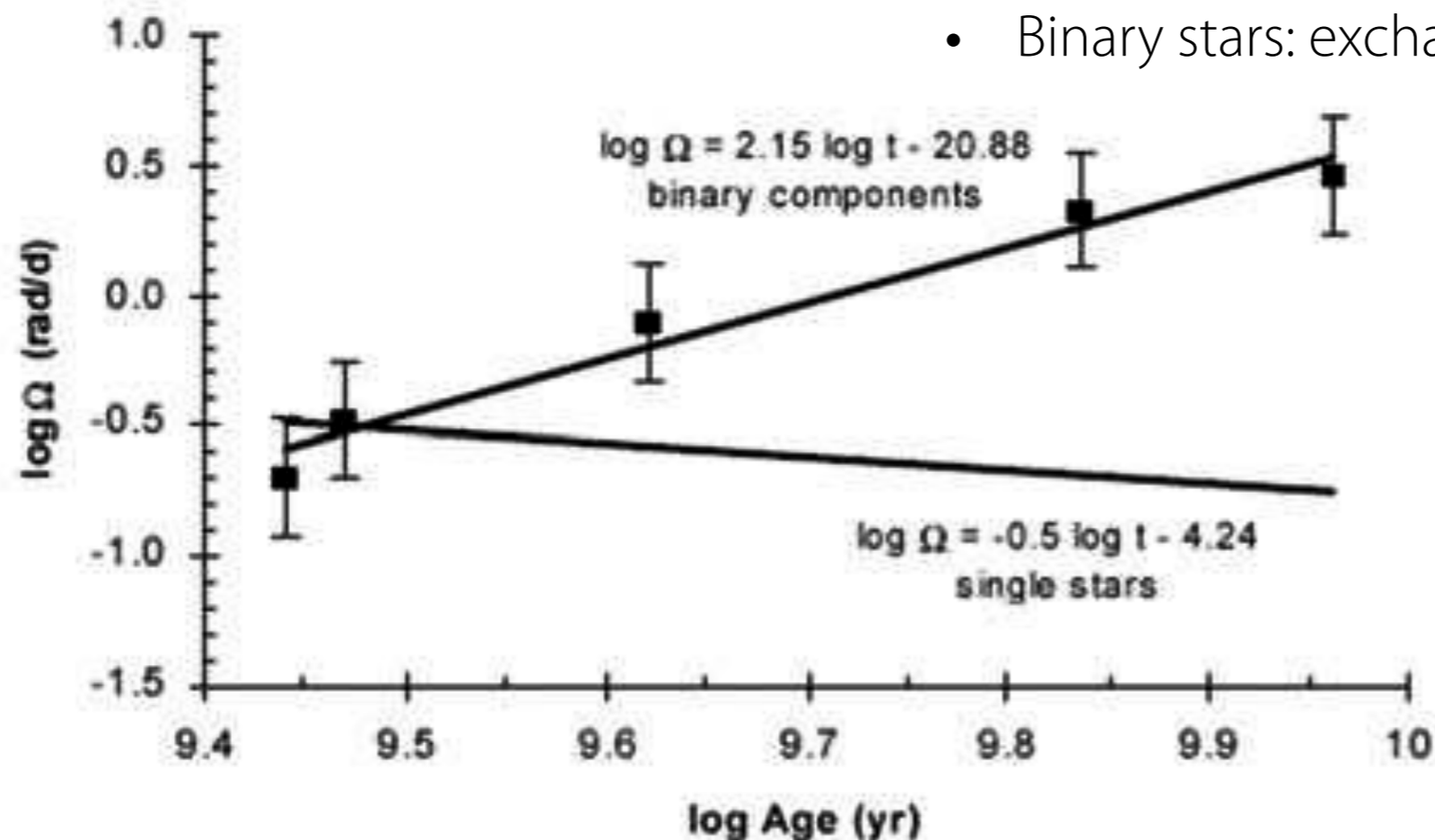
Evolution of rotation

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- Stellar winds
- Binary stars: exchange of orbital angular momentum



Stellar rotation

Summary

- Rotation velocities derived as $v \sin i$
 - Can be determined from rotational broadening of spectral lines
 - Inclination angle i must be known to determine actual rotation rate of the star
- Two distinct groups along main sequence
 - Fast rotators — hotter stars: often $v \sin i > 100$ km/s.
 - Some hot stars rotate as fast as 450 km/s
 - Slow rotators — stars cooler than F7: typically $v \sin i < 50$ km/s;
 - Sun ~ 2 km/s.
 - Surface convection leads to reduction of rotation rates
- Fast rotation results in oblateness (deviation from spherical symmetry), causing effects such as gravitational darkening
- Stellar rotation slows down (slowly) with time
 - Loss of angular momentum due to stellar wind