AST5770

Solar and stellar physics

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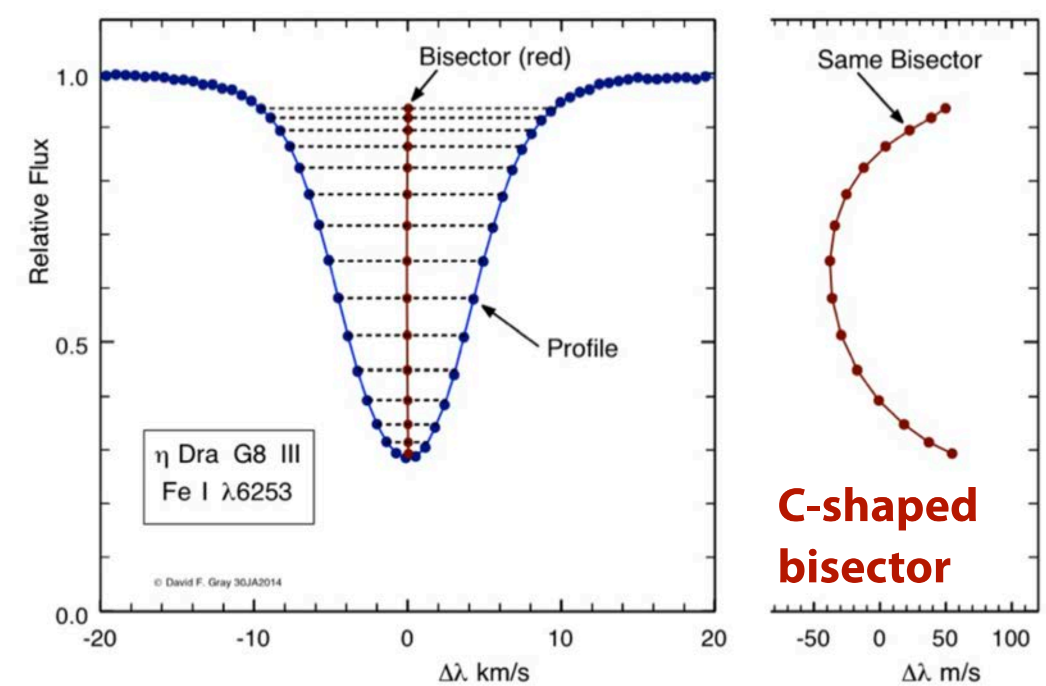
Observations of gas motion

- **Sun (spatially resolved observations):**
  - Granules visible in intensity but also in vertical velocity
  - Line-of-sight velocity from **Doppler shifts** of spectral line cores (viewing angle: vertical velocity at solar disk-centre)
  - Horizontal velocities via **local correlation tracking**
- **Other stars (spatially unresolved observations):**
  - Line broadening due to macro turbulence — mixing different velocity components due to viewing angle from (unresolved) disk-centre to limb
  - **Spectral line asymmetry** due to integrating up- and downward motion in granules and intergranular lanes with different area fractions
  - **Line bisectors**
    - Sample the velocity field across different formation heights (line core to continuum)
    - Different bisector shapes for different spectral types — differences in surface convection
- Combined information from weak and strong spectral lines, probing low and higher in the atmosphere



Helioseismology

k-ω diagram

- Observation $v\ (x,y,t)$

  ➡ "Spatiotemporal power spectrum ("2D power spectrum"): **k-ω diagram**

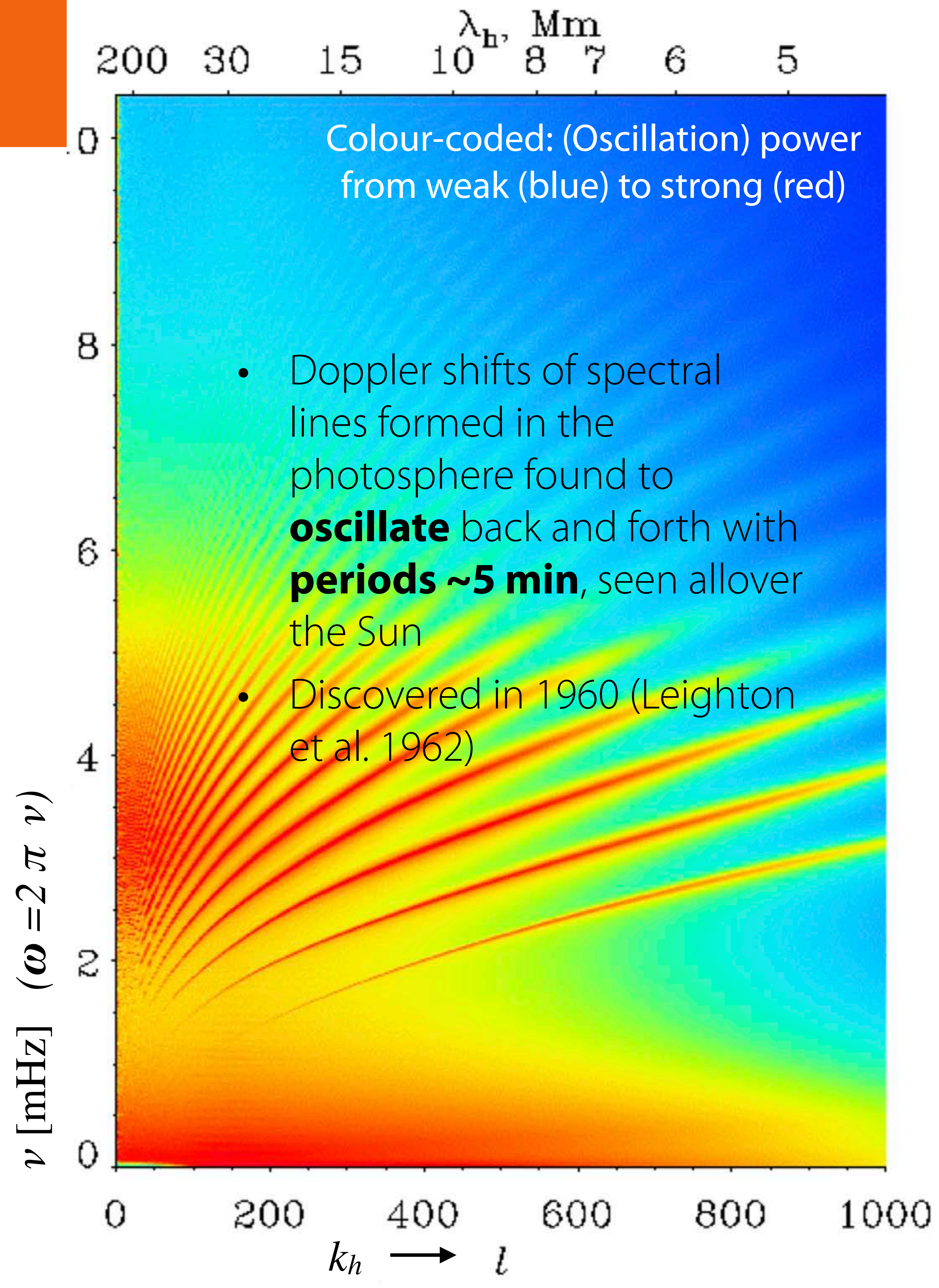
- The p-modes show a distinctive dispersion relation!

- Important: power only in distinct **ridges:** for a given k^2 only power at certain frequencies

➡ Discrete spectrum suggests the oscillations are trapped, eigenmodes of the Sun

➡ Set by the interior structure of the Sun

$$k_\mathrm{h} = (k_x^2 + k_y^2)^{1/2}$$

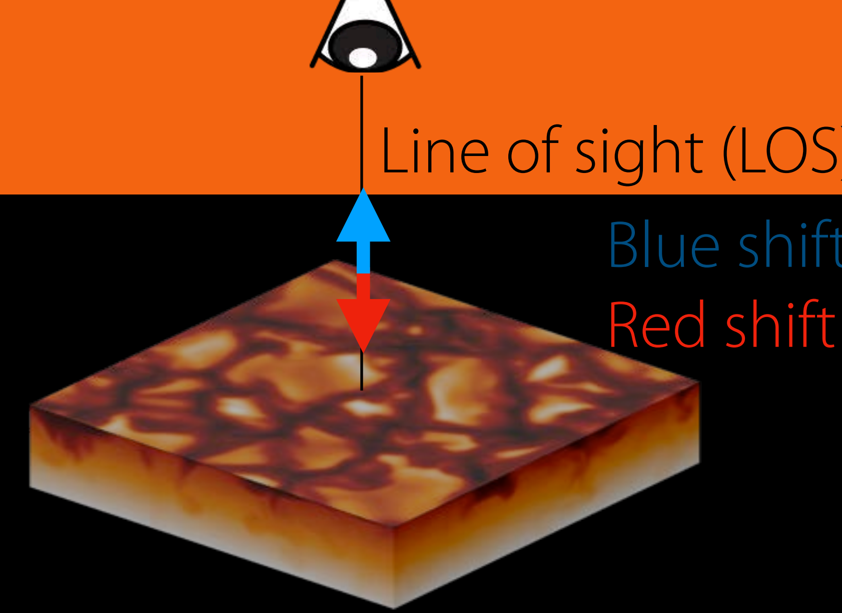


Colour-coded: (Oscillation) power from weak (blue) to strong (red)

- Doppler shifts of spectral lines formed in the photosphere found to **oscillate** back and forth with **periods ~5 min**, seen allover the Sun
- Discovered in 1960 (Leighton et al. 1962)

Helioseismology

Refraction & Reflection

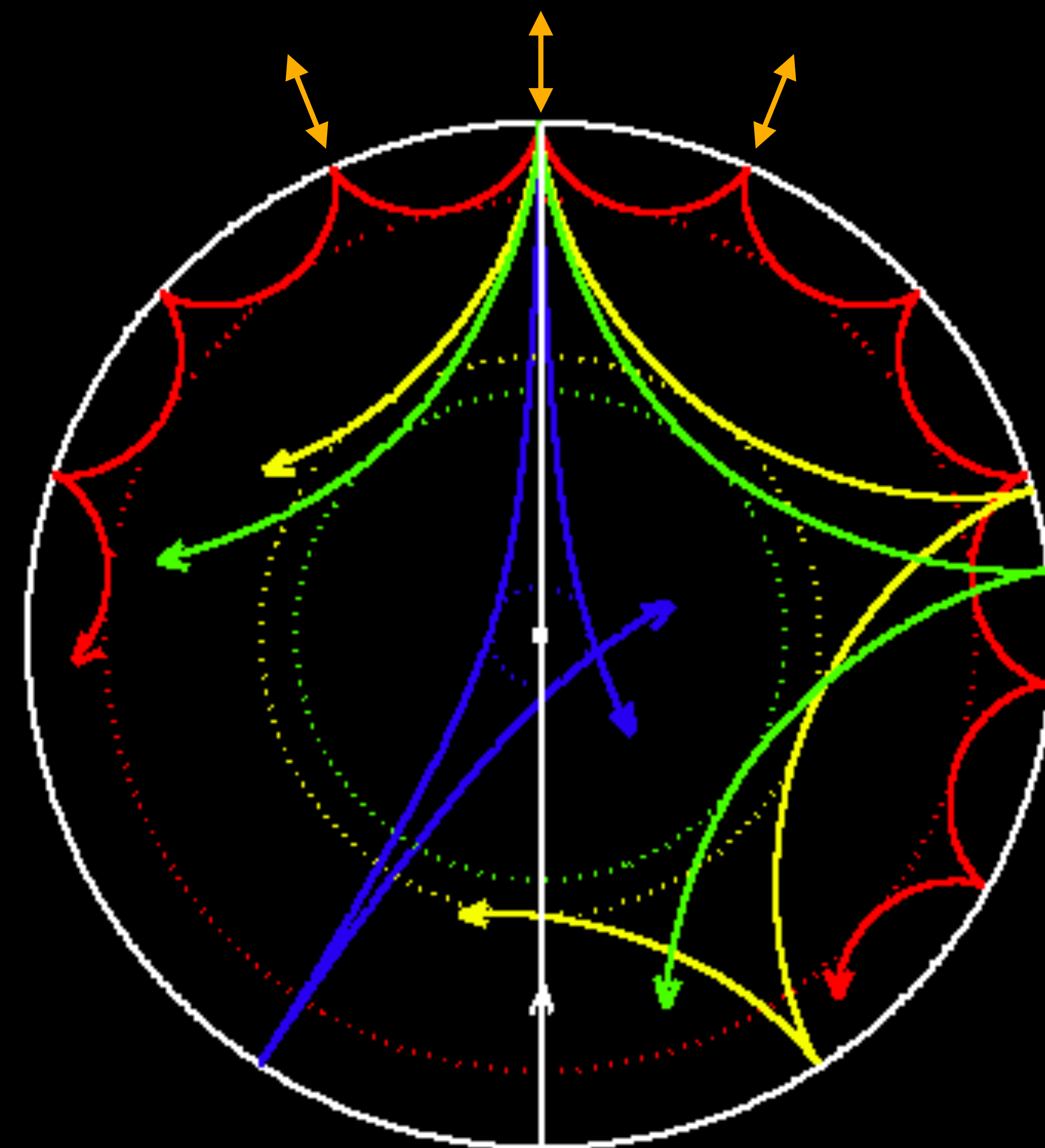


- Origin of oscillations identified as acoustic waves, called **p-modes**
- **Sound speed $c_s \sim T^{1/2}$ changes strongly as function of radius**

➡ **Sound waves get refracted in the solar interior.**

➡ **Penetration depth of sound waves depends on their wavelength.**

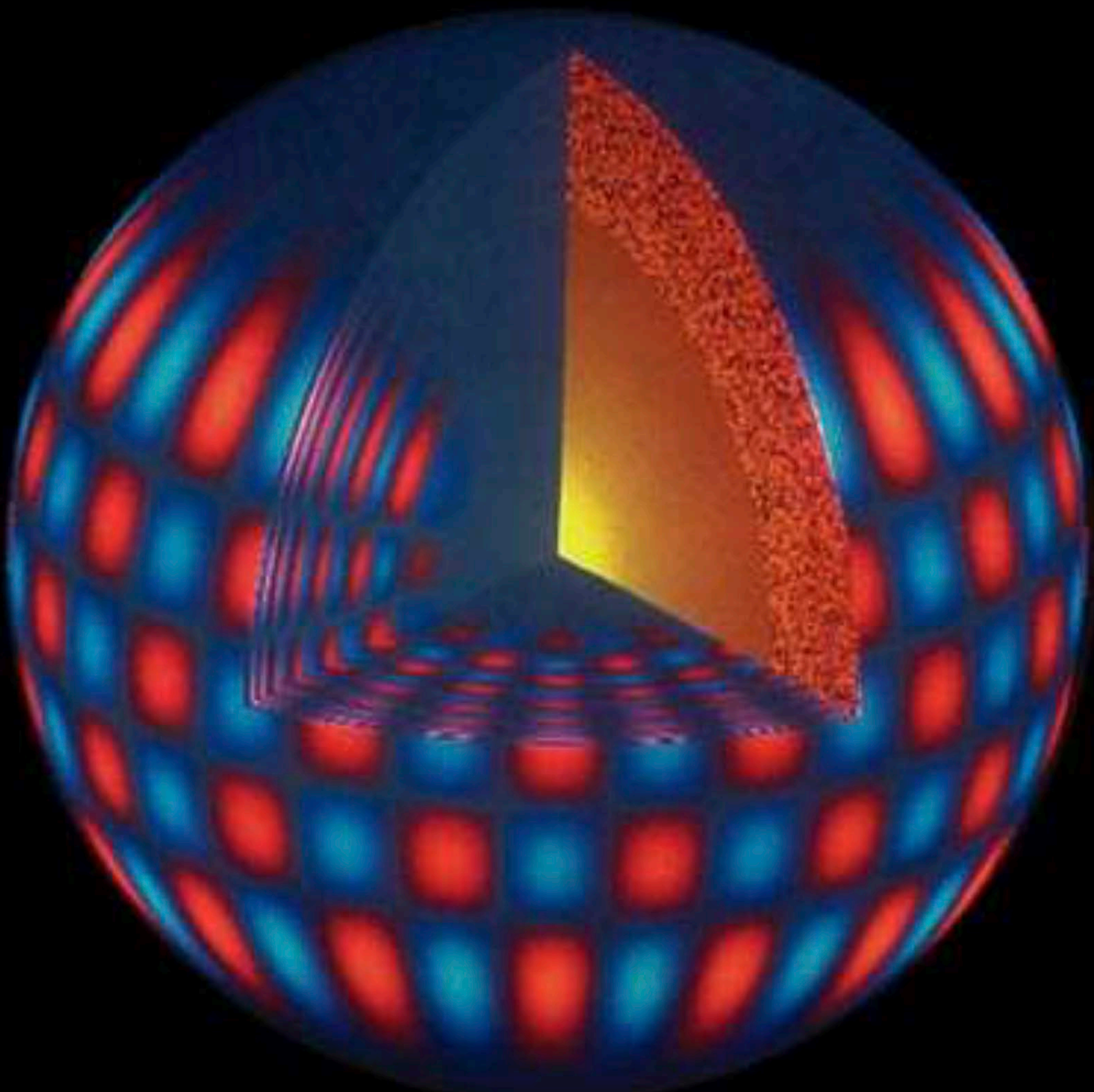
- Different wavelengths probe different depths
- All wavelengths together probe the stratification of the solar interior!
- Sound waves reflected at surface results in surface (patch) to oscillate up and down accordingly
- Observation (Doppler shifts, intensity variation) and interpretation of these oscillations at the surface provides information about the interior structure of the Sun!



Helioseismology

Description of solar eigenmodes

- Eigen-oscillations of a sphere are described by spherical harmonics
- Each oscillation mode is identified by a set of three parameters:
  - n = number or radial nodes
  - l = number of nodes on the solar surface
  - m = number of nodes passing through the poles



NSO/GONG

Helioseismology

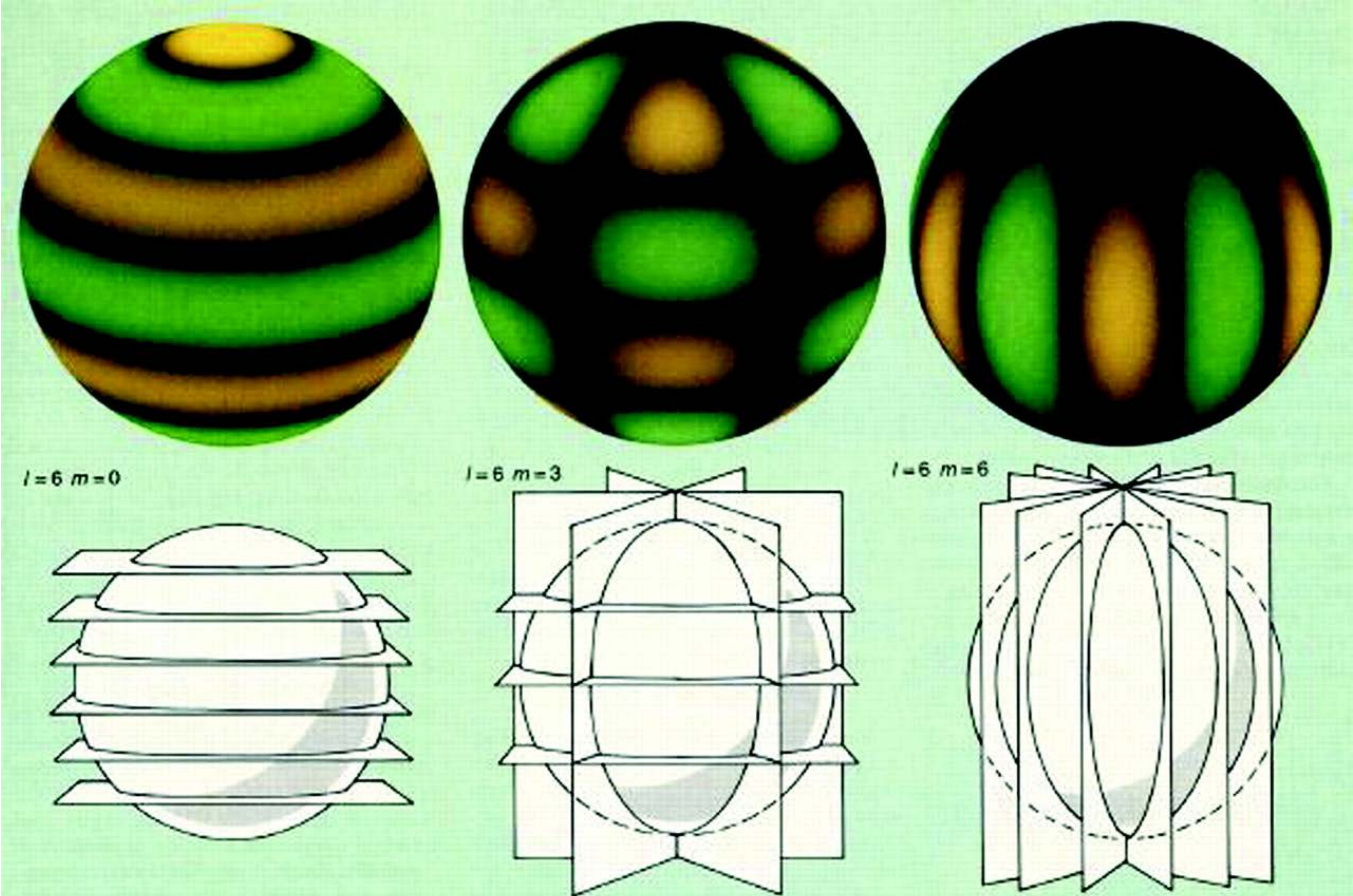
Spherical harmonics

Input signal:
Measured velocity signal at solar surface, no radial dependence

- So far cartesian coordinates (ok for distance « radius of the Sun)
- Better, more general: spherical polar coordinates (r, θ, ϕ): $v\ (x,y,t) \longrightarrow v\ (\theta,\phi,t)$
- Express velocity signal $v\ (\theta,\phi,t)$ now as spherical surface harmonics:

$P_l^{|m|}\ (\theta)$ = associated Legendre Polynomial

$$v(\theta,\phi,t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm}(t) Y_l^m(\theta,\phi) \quad \text{with} \quad Y_l^m(\theta,\phi) = P_l^{|m|}(\theta) e^{im\phi}$$



- Temporal dependence in amplitude a_{lm}
- Spatial dependence in spherical harmonic Y_l^m
- Fourier transform of amplitude a_{lm}: $F(a)$

➡ Fourier power = $F(a)F(a)^*$ (due normalisation of spherical harmonic)

- l = total number of nodes (=degree)
- m = number of nodes connecting the "poles" (=order)

Helioseismology

Spherical harmonics

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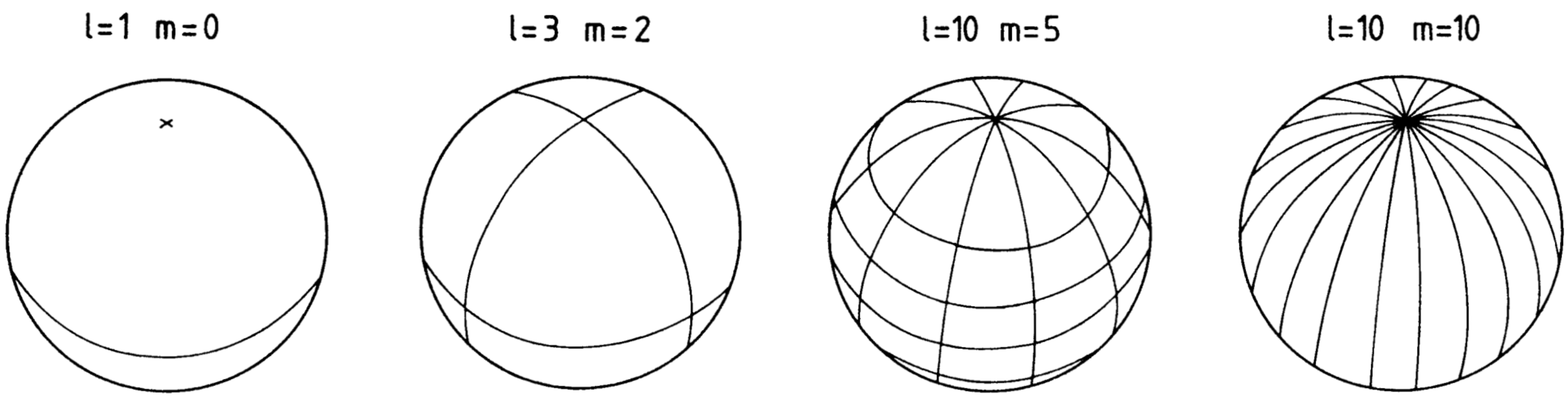

Node circles of spherical harmonics. After Noyes and Rhodes (1984)

- l = total number of nodes (=degree)
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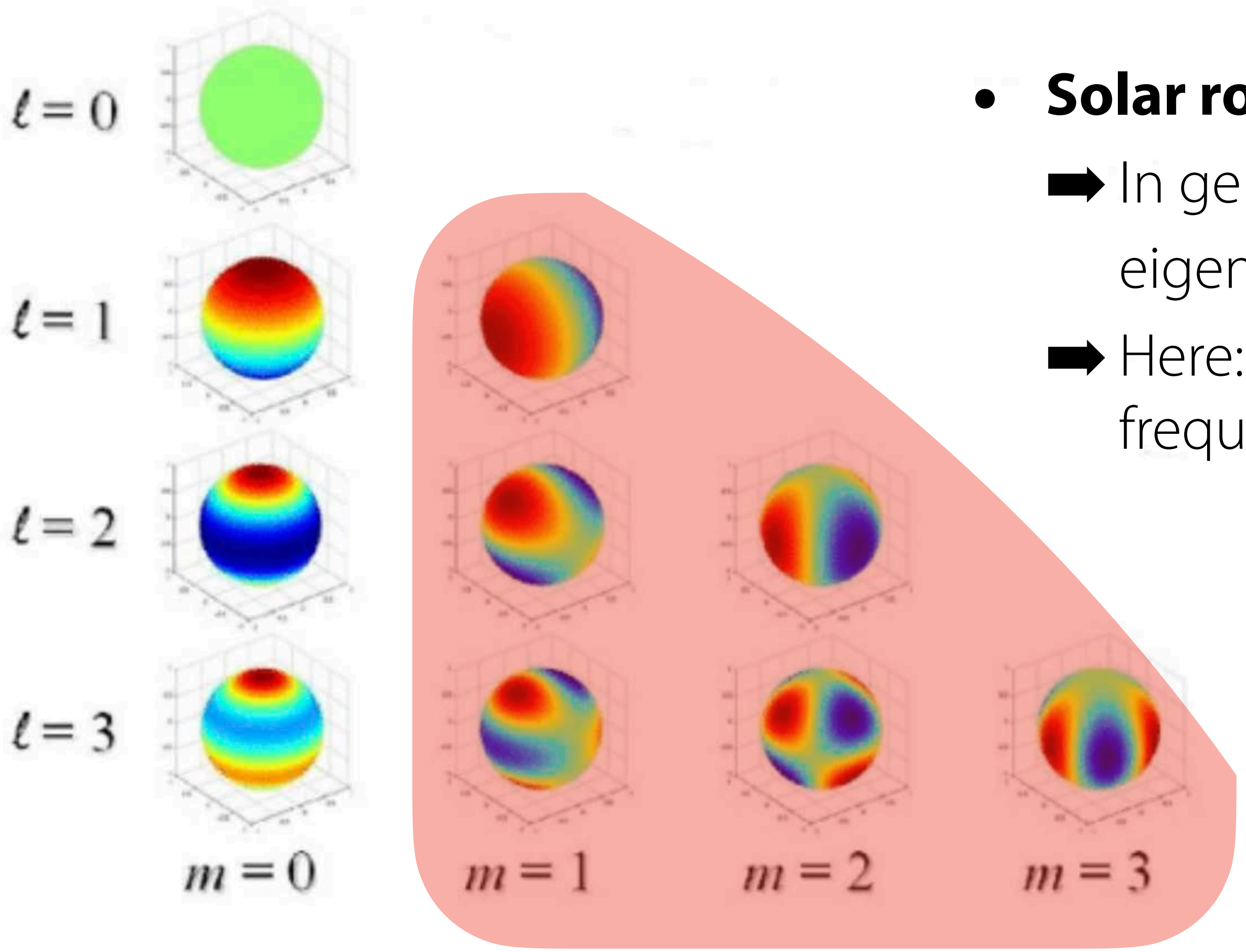
Helioseismology

Spherical harmonics



- **Solar rotation axis:**
  ➡ In general, oscillations and their eigenfrequencies would then depend on m
  ➡ Here: frequency of rotation ≪ oscillation frequencies
  ➡ Oscillation frequencies essentially independent of m.
  ➡ Simplifying assumption $m=0$

➡ The degree l of the spherical surface harmonic connected to horizontal wavenumber k_h

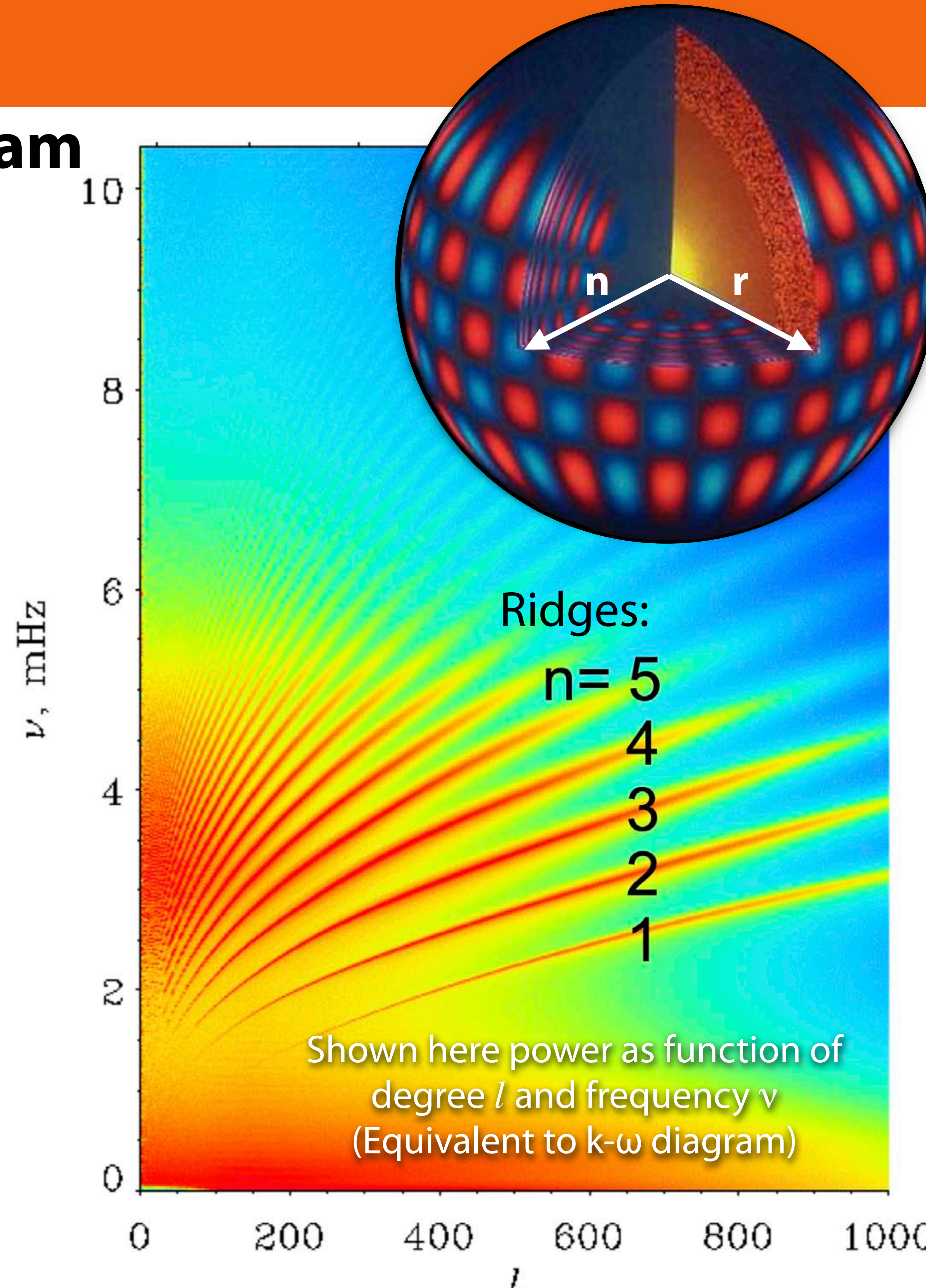
$$k_{\mathrm{h}} r_{\odot} = [l(l+1)]^{1/2}$$

➡ Now can evaluate the power as function of degree l and frequency ν equivalently to k-ω

Helioseismology

Interpretation of k-ω diagram

- Power ridges belong to different orders n (n = number of radial nodes)
- Power in ridge with increasing l
  ➡ Increase in frequency ν (or ω)
- Most prominent power along ridges for small n intermediate/large degree l



- n = number or radial nodes
- l = number of nodes on the solar surface

Helioseismology

p-modes

- Note the grouping of modes
- Large separations

$$(n, l) \longleftrightarrow (n\text{-}1, l)$$

- Small separations

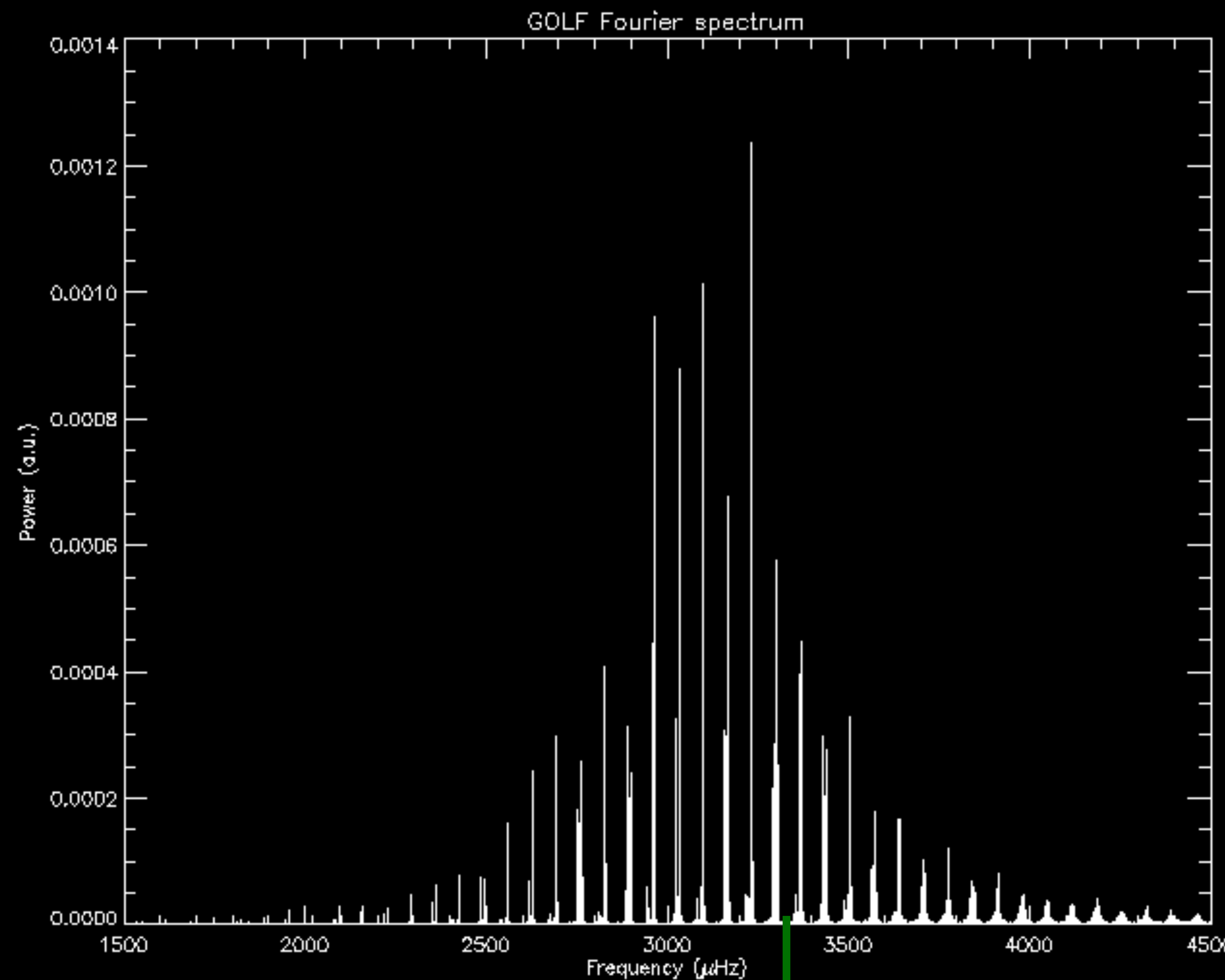
$$(n, l) \longleftrightarrow (n\text{-}1, l+2)$$

- n = number or radial nodes
- l = number of nodes on the solar surface

5min = 300 s

ν = 1/300 s = 3.333 mHz

GOLF (Global Oscillations at Low Frequencies)



SOHO (ESA & NASA)

Helioseismology

p-modes

- Close-up: Power spectrum for modes $l = 0$ and $l = 2$
- Noise due to random re-excitation of the oscillation mode by turbulence
-

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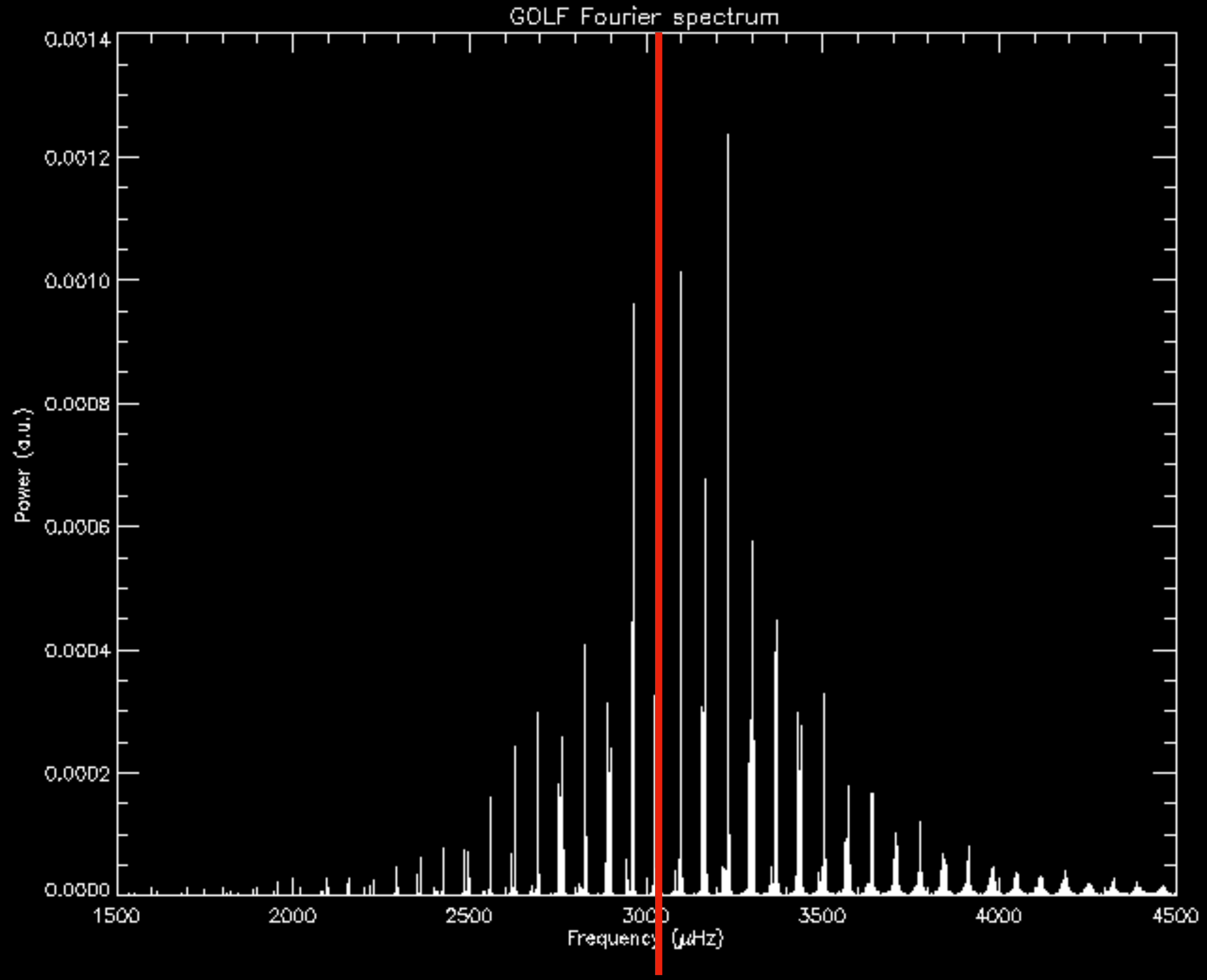

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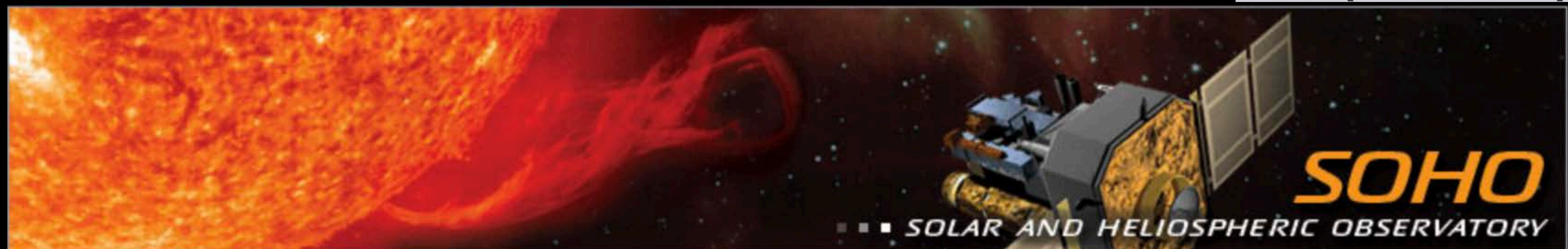

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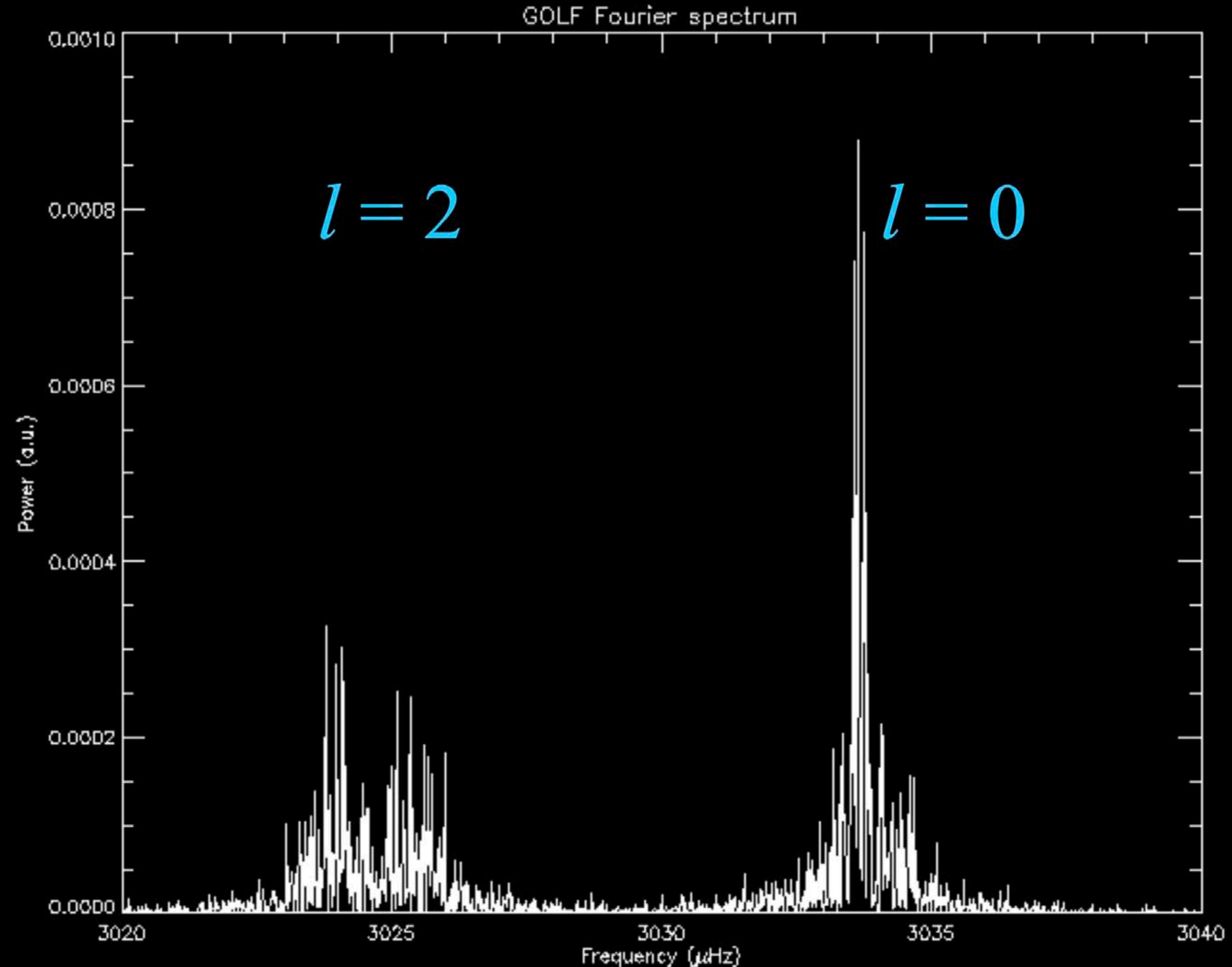

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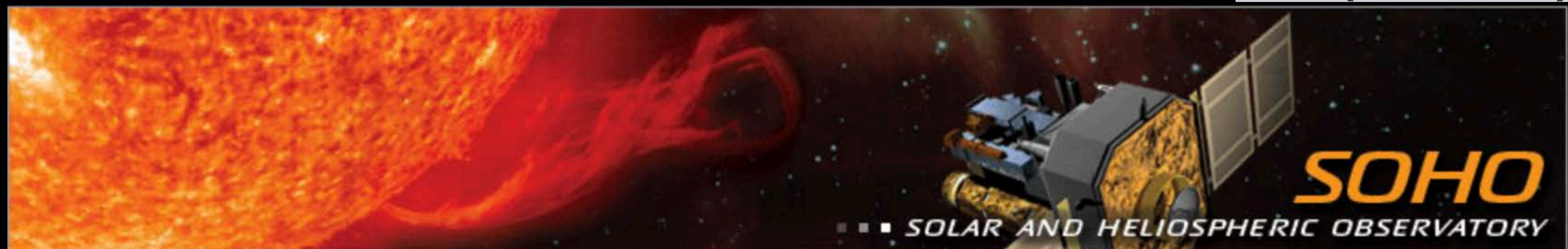

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Oscillation modes

- **Why discrete frequencies?**
  - Sun acts as a resonant cavity producing resonant oscillation modes set by the structure of the Sun's interior
  - p-modes are excited by turbulence, which excites all frequencies but eigenmodes develop only at the Sun's eigenfrequencies

- **Detection — frequency vs amplitude**
  - Amplitudes depend on the excitation, while the frequencies do not.
  - Frequencies carry the main information on the structure of the solar interior
  - Frequencies can be measured more precisely .

- **Detected modes:**
  - $\sim 10^7$ modes known, each oscillation mode samples different parts of the solar interior
  - periods: ~1.5 min — ~20 min
  - horizontal wavelengths: less than a few thousand kilometers — solar diameter

Helioseismology

Observational limitations

- Only **half of the Sun observed**

  ➡ Decomposition of the sum of all oscillations into spherical harmonics not unique!

  ➡ Uncertainty in the derived l and m

- Complexity: **10^7 modes** are present on the surface of the Sun at any given time (and interfering with each other).

- **Amplitudes:** single mode typically < 20 cm/s, all 10^7 modes combined a few 100 m/s
  - Accuracy of current instruments better than 1 cm/s

- Highest detectable frequency ~ **cadence** of obs.

- Frequency resolution ~ **length of time series** ~ lowest detectable frequency

  ➡ Longer time series are better but technically more challenging

  - Gaps in time series produce side lobes
    (i.e. spurious unwanted peaks in the power spectrum)

  ➡ Precise measurements require global long-term observational efforts!

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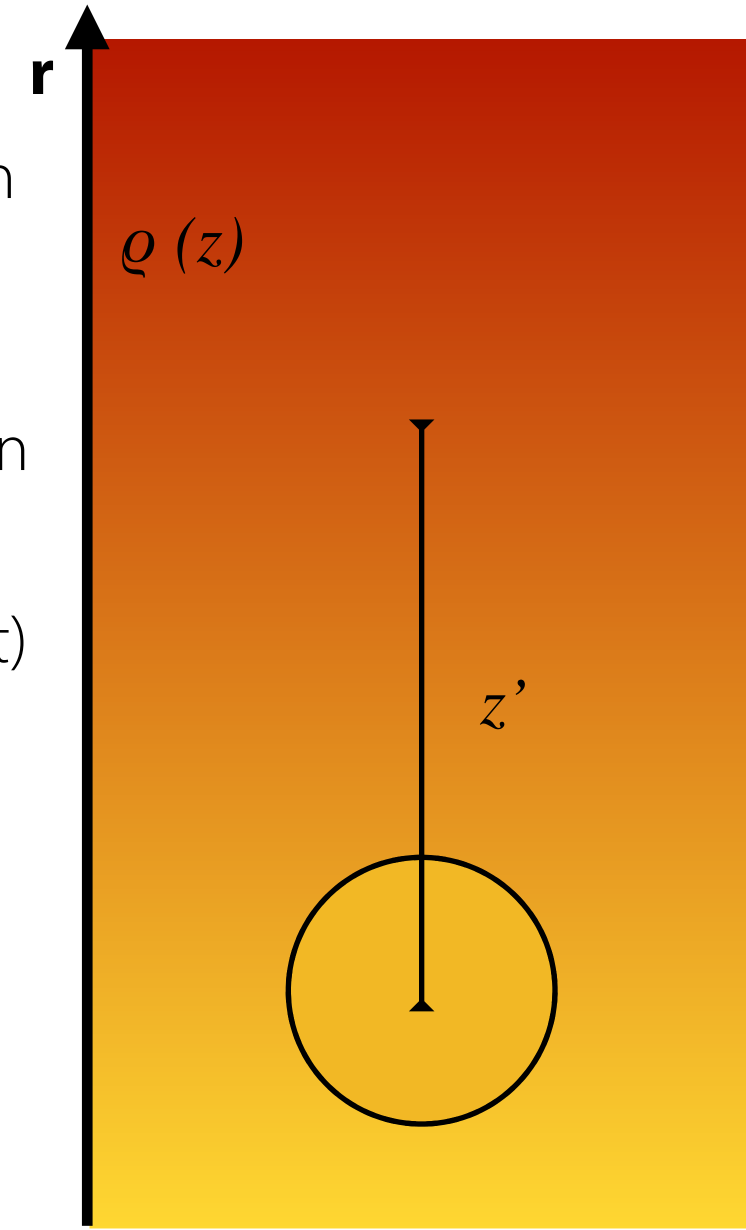
Stability against convection



- Stability of a vertically displaced gas element against convection can be evaluated in terms of the **Brunt–Väisälä frequency** (buoyancy frequency)
  - Frequency at which a vertically displaced gas element oscillates in a convectively stable layer
- **Gravity as restoring force** (working against upwards displacement)
  - gravitational acceleration:

$$g = -\rho_0^{-1} dP_0/dr.$$

- Now: vertical displacement by z'

➡ $$\rho_0 \frac{\partial^2 z'}{\partial t^2} = -g\left[\rho(z) - \rho(z+z')\right]$$

➡ $$\frac{\partial^2 z'}{\partial t^2} = \frac{g}{\rho_0}\frac{\partial \rho(z)}{\partial z} z'$$

- Solutions of the form $z' = z'_0 e^{i\sqrt{N^2}t}$ with **Brunt–Väisälä frequency** N

➡Oscillations may occur depending on N!

Helioseismology

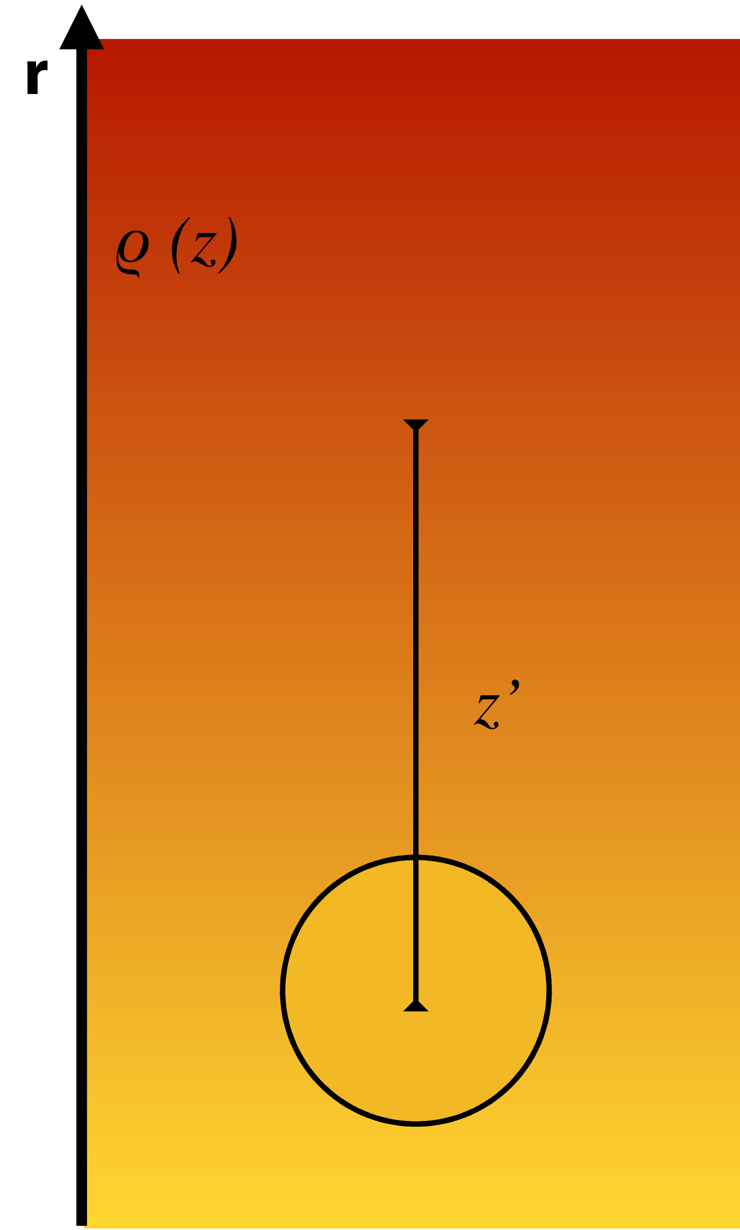
Stability against convection

- **Brunt–Väisälä frequency** N: $$N^2 = g\left(\frac{1}{\Gamma_1 P_0}\frac{dP_0}{dz} - \frac{1}{\rho_0}\frac{d\rho_0}{dz}\right)$$

- $N^2 > 0$: Oscillation around the height where density of surrounding matches density of gas element
- $N^2 = 0$: Gas element in rest after displacement
- $N^2 < 0$: (N imaginary) perturbation leads to instability, run away growth

- Buoyancy vs gravity as restoring force
- Gravity waves — g-modes (not to be confused with gravitational waves)

- Condition $N^2 > 0$ is equivalent to Schwarzschild / Ledoux criteria for stability against convection



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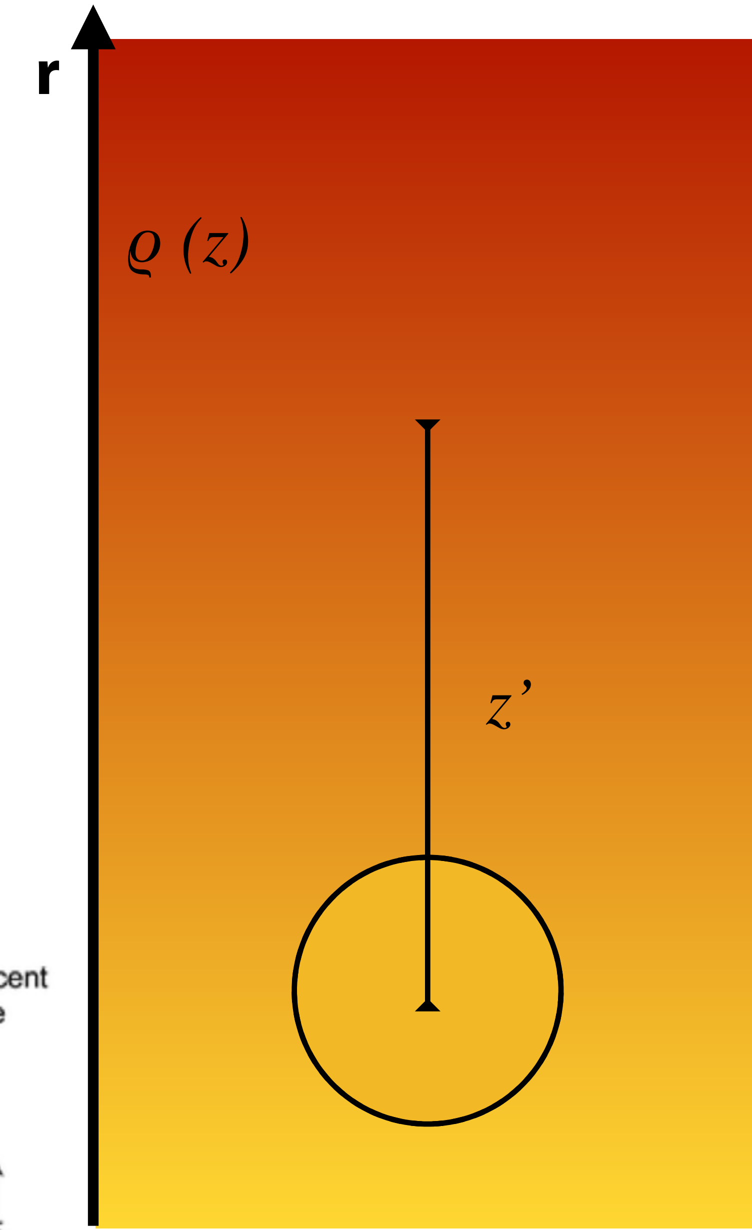
Oscillation modes

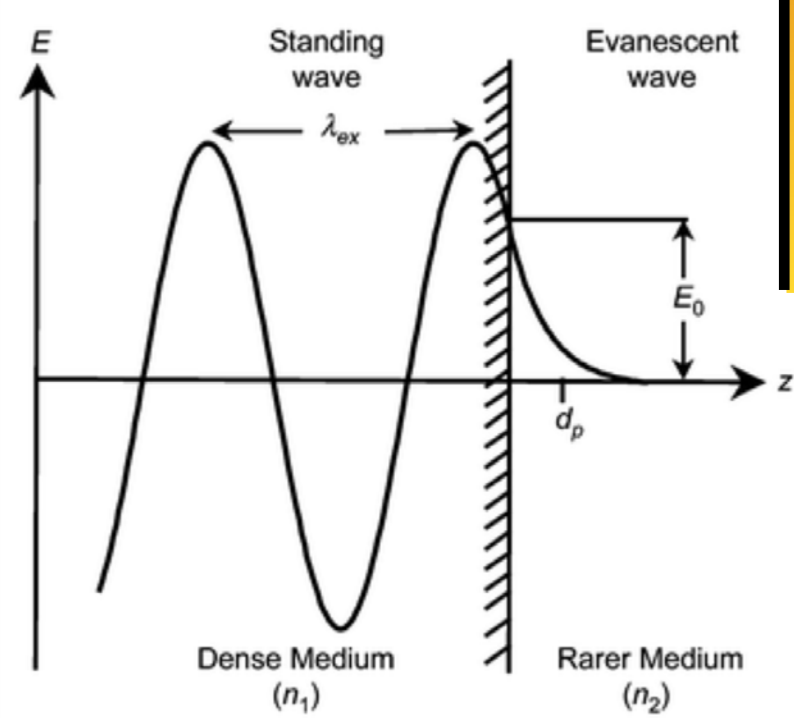


- Stability of a vertically displaced gas element against convection evaluated in terms of the **Brunt–Väisälä frequency** N
- Oscillation with gravity as restoring force occurs when $N^2 > 0$

➡ **internal gravity modes (g-modes)**

- This type of oscillation requires a medium stable against convection and is thus expected
  - In the radiative interior of the Sun
  - In the solar atmosphere.

- **Evanescent in the convection zone** (= amplitude drops exponentially there)
- Amplitudes of g-modes from the interior radiative zone reduced to very small values at the surface)



Helioseismology

Types of solar eigenmodes:

p-modes (sound waves)	**g-modes** (buoyancy modes)
restoring force = pressure • Excited by **turbulence** associated with convection, mainly by the more vigorous motions at the **surface** (granulation)	restoring force = gravity • Randomn vertical displacements (**buoyancy**) in a convectively stable medium
• Propagate in the interior but evanescent in the solar atmosphere • So far only p-modes have been detected on the Sun with certainty!	• Propagate in the radiative interior and in the atmosphere but evanescent in the convection zone • No definite observational proof for g-modes on the Sun yet.
• p-modes propagate throughout the solar interior at sound speed c_s ➡ Spend most time where c_s is lowest ➡ Spend most time at the surface (as c_s is lowest there, remember $c_s \sim T^{1/2}$) ➡ Detectable at the surface	• Current upper limit on solar interior g-modes lies below 1 cm/s. • Would probe the centre of the Sun!

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations

- Simplifying assumptions:
    - ***Cowling approximation***
      (neglects any perturbations to the gravitational potential)
    - Radial changes of stratification small over the scales of the considered oscillations
    - Linear perturbation in radial direction, adiabatic
    - Usage of spherical harmonics for any non-radial
      (i.e. horizontal) component (here primarily set by degree l)

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations

$$\frac{1}{r^2}\frac{d}{dr}(r^2\xi_r) - \frac{\xi_r g}{c^2} + \frac{1}{\rho_0}\left(\frac{1}{c^2} - \frac{l(l+1)}{r^2\omega^2}\right)P_1 = 0$$

$$\frac{1}{\rho_0}\frac{dP_1}{dr} + \frac{g}{\rho_0 c^2}P_1 - (\omega^2 - N^2)\xi_r = 0 \; .$$

Unperturbed background

ϱ_0: Density

P_0: Pressure

c: sound speed

Perturbation:

P_1: Perturbed pressure

ξ_r: radial displacement

Linear adiabatic perturbation

- Analytical solutions of these equations for an isothermal medium:

$$\xi_r \sim \rho_0^{-1/2} \exp(ik_r r)$$

$$P_1 \sim \rho_0^{1/2} \exp(ik_r r) \; .$$

- Solutions are oscillations but can also be evanescent (complex k_r)

Helioseismology

Oscillation equations

- Equation of continuity and momentum equation, describing stratification — now perturbed

➡ Equations describing radial structure of adiabatic oscillations at frequency ω

➡ **Dispersion relation**:

$$k_r^2 = \frac{\omega^2 - \omega_{\mathrm{A}}^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

Unperturbed background

c_s: sound speed

Perturbation:

ξ_r: radial displacement

ω: angular frequency

k_r: radial wavenumber

N

Brunt–Väisälä frequency

$$N^2 = g\left(\frac{1}{\Gamma_1 P}\frac{dp}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)$$

S_l

Lamb frequency

$$S_l^2 = \frac{l(l+1)}{r^2} c^2$$

ω_A

Acoustic cutoff frequency

$$\omega_{\mathrm{A}} = c/2H$$

- density scale height H: locally approx. constant $H \equiv -\rho_0/(d\rho_0/dr) = \left(\frac{g}{c^2} + \frac{N^2}{g}\right)^{-1}$

A dispersion relation describes how the frequency and wavelength of a wave are related in a particular medium or system.

Helioseismology

Oscillation equations — k_h-ω-plane

- **Dispersion relation**:

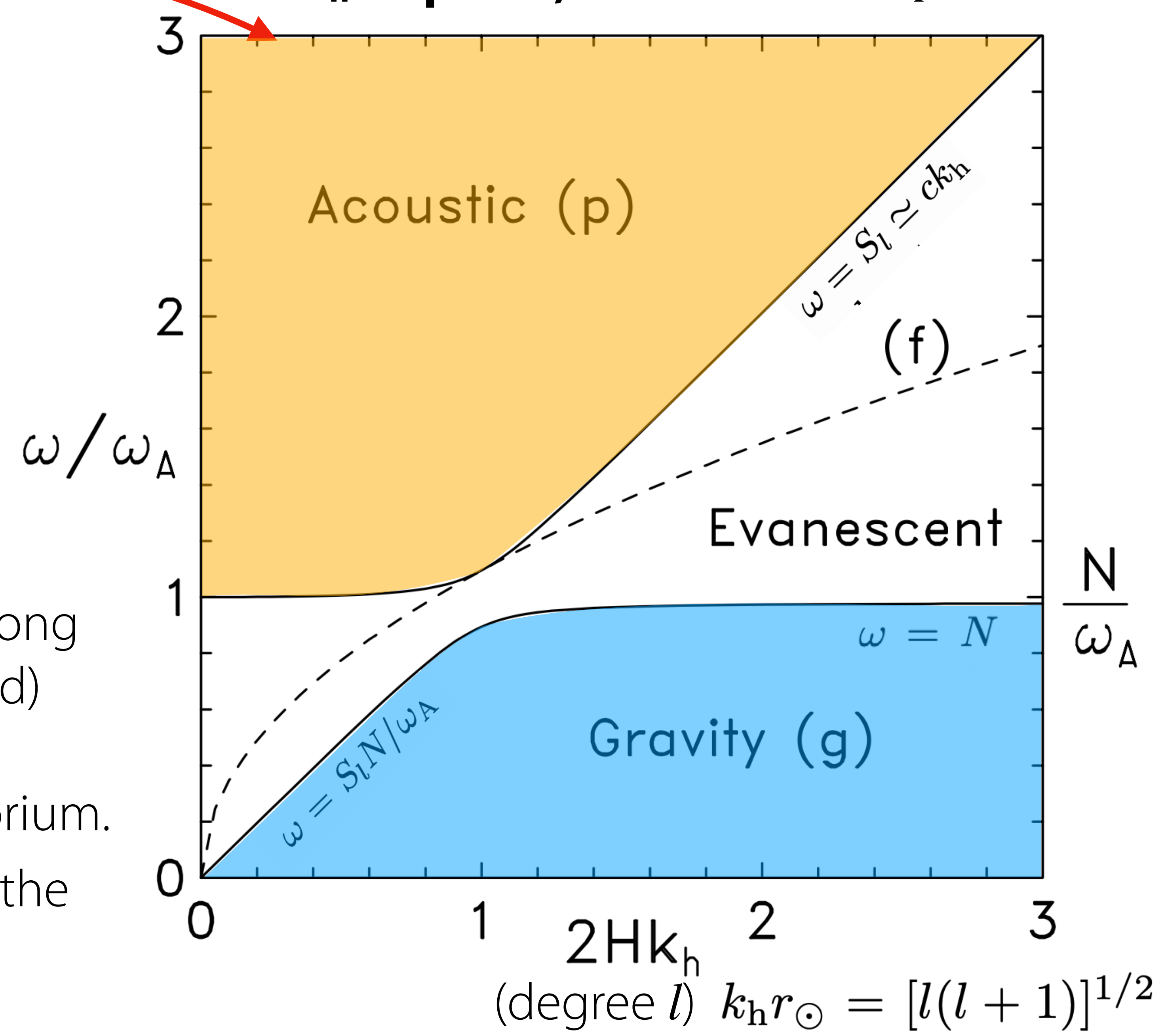
$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

- In the two regimes of acoustic waves and gravity waves:
  $k_r^2 > 0$
- Between: regime of evanescent waves (exponential damping)
  $k_r^2 < 0$
  - Evanescent waves occur if period so long that the whole (exponentially stratified) medium has time to adapt to the perturbation, achieving a new equilibrium.

  ➡ Wave does not propagate, but rather the medium as a whole oscillates

- ω_A : Acoustic cutoff frequency
- N : Brunt–Väisälä frequency
- S_l : Lamb frequency

k_h-ω-plane, solid curves: $k_r^2 = 0$



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Oscillation equations — k_h-ω-plane

- **Dispersion relation**:

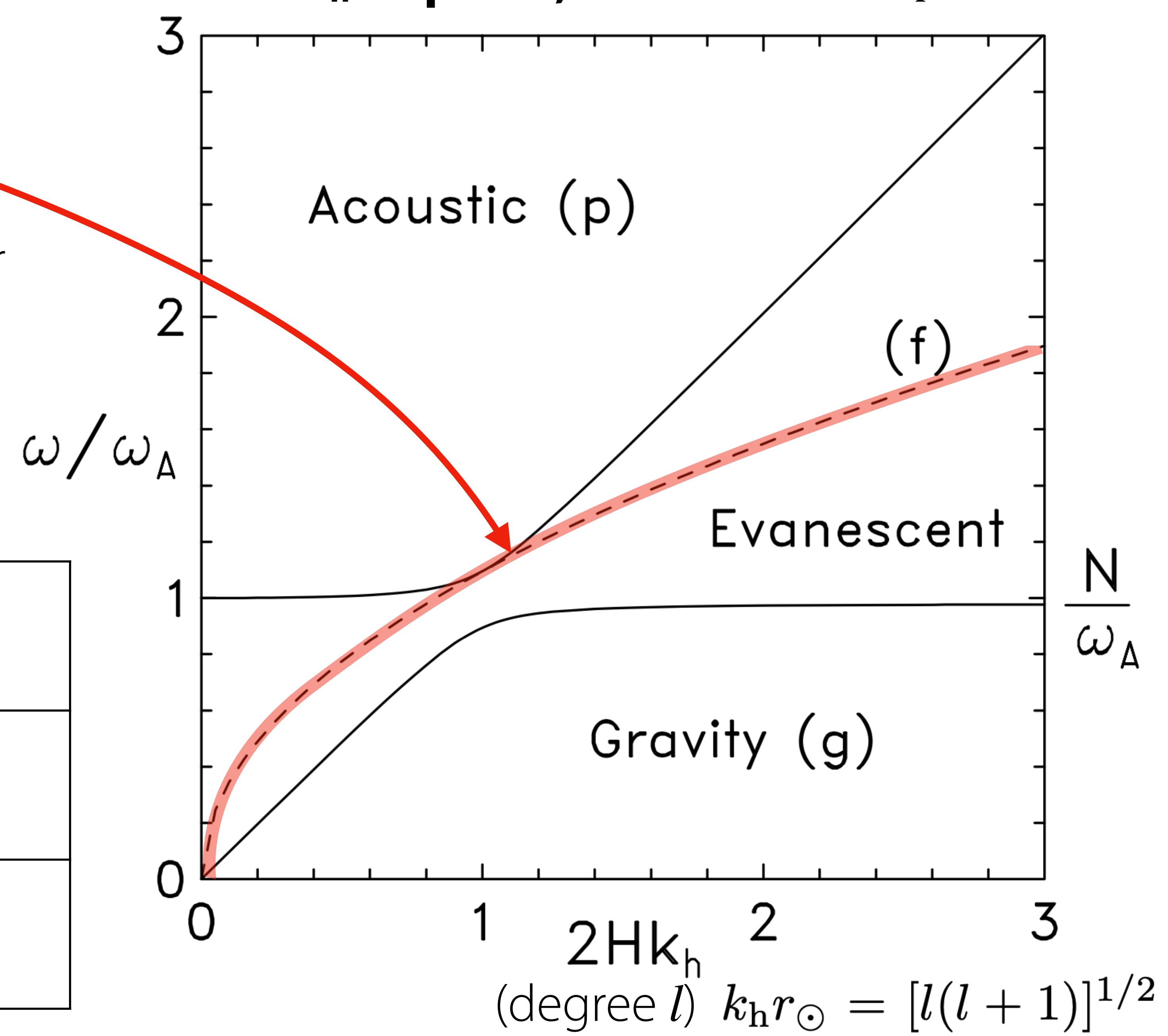
$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

- **Fundamental mode (f-mode):** essentially without compression, resembles a surface wave on deep water

$$\omega = \sqrt{g k_h}$$

- ω_A : Acoustic cutoff frequency
- N : Brunt–Väisälä frequency
- S_l : Lamb frequency

k_h-ω-plane, solid curves: $k_r^2 = 0$



Acoustic waves	➡ p-modes
Buoyancy waves	➡ g-modes
Surface gravity waves	➡ f-mode

Helioseismology

Oscillation equations — k_h-ω-plane

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- **Fundamental mode (f-mode):** essentially without compression, resembles a surface wave on deep water

$$\omega = \sqrt{g k_h}$$

- In diagnostic diagrams: f-mode = lowest ridge

- ω_A : Acoustic cutoff frequency
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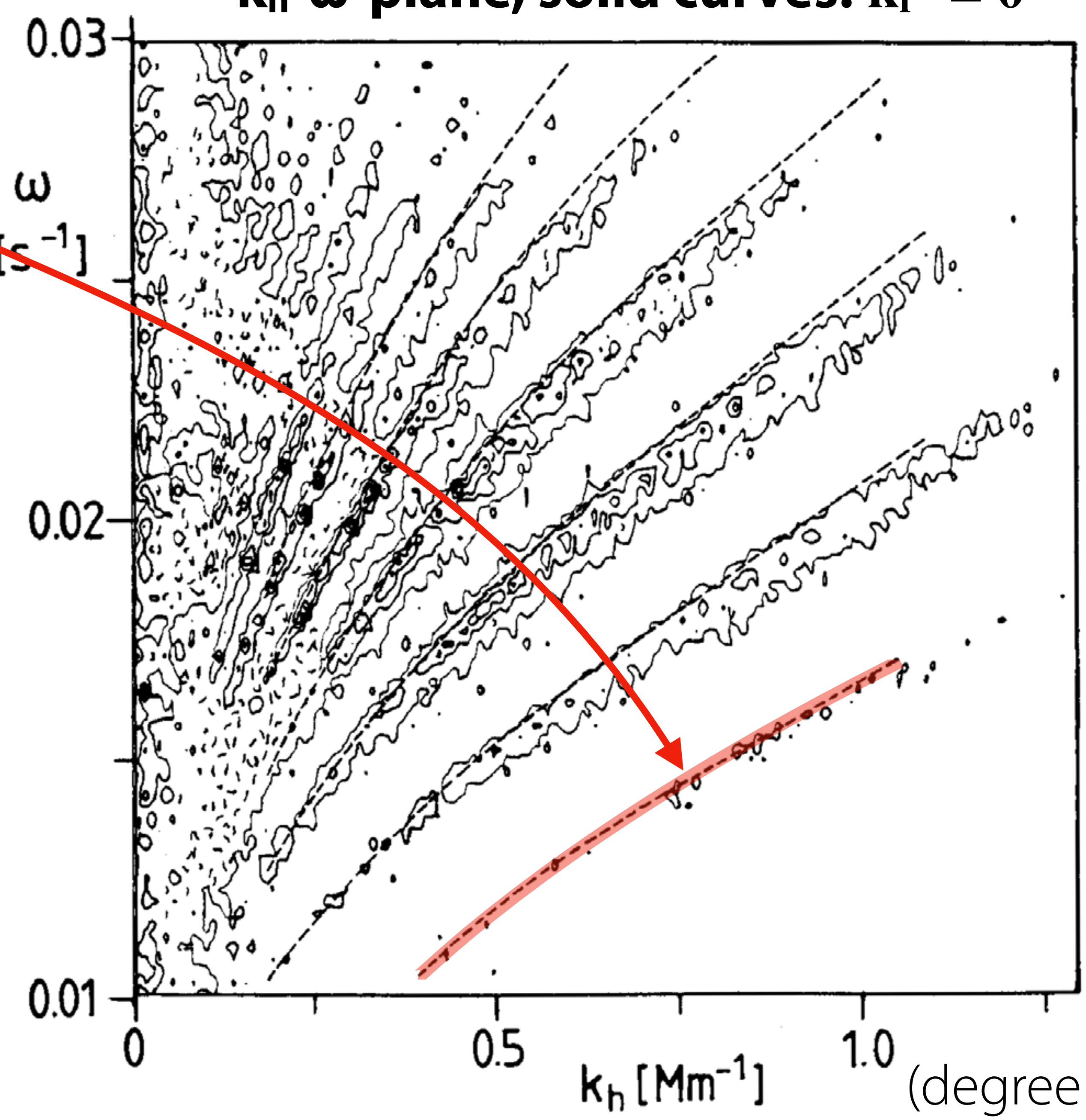
- S_l : Lamb frequency

k_h-ω-plane, solid curves: $k_r^2 = 0$



Deubner et al. (1979)

Helioseismology

Critical frequencies in the Sun

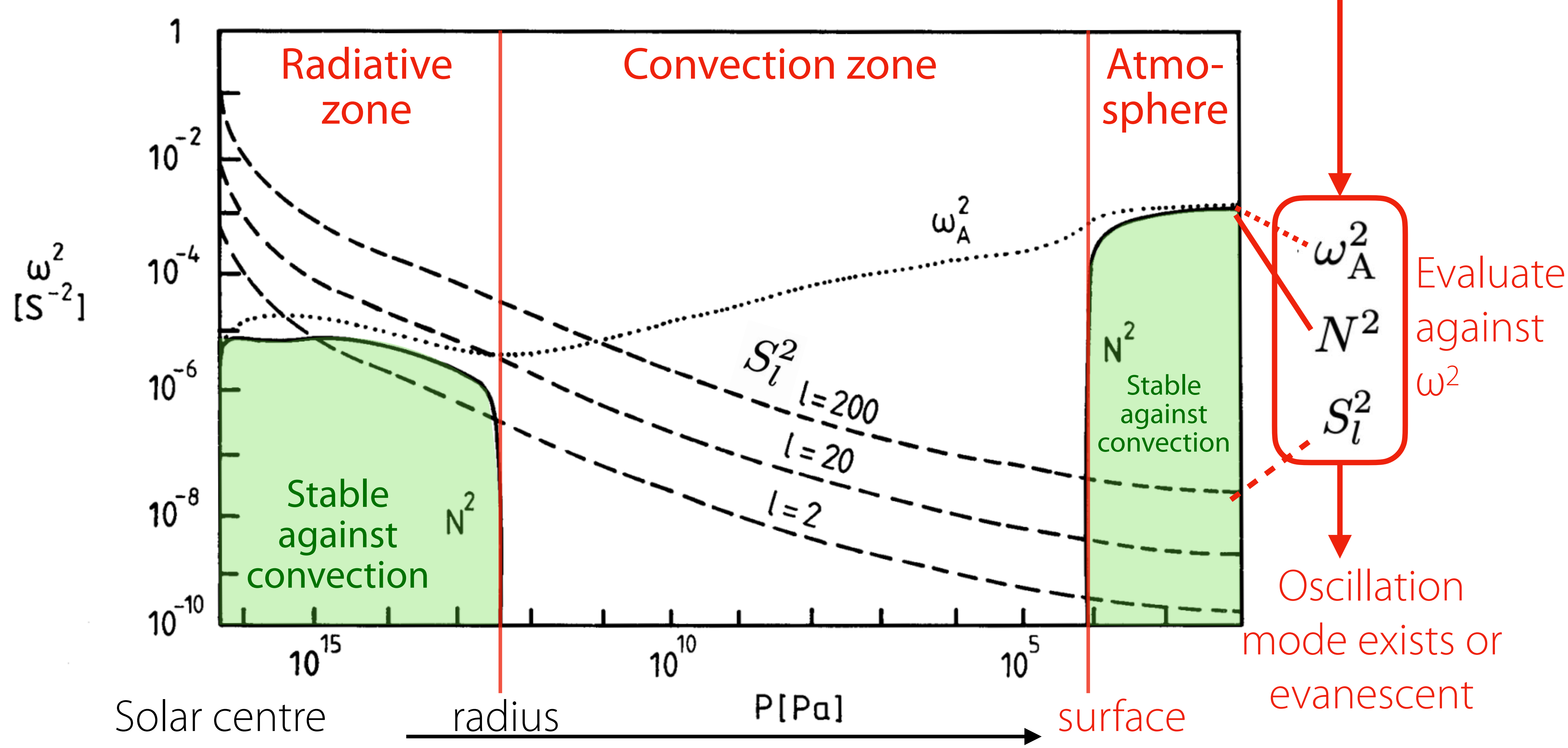
- Mode "trapped" in a layer with oscillatory wave behaviour for this type of mode if the layer is in-between two evanescent layers

➡ Spectrum of oscillation frequencies is discrete.

$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2}$$

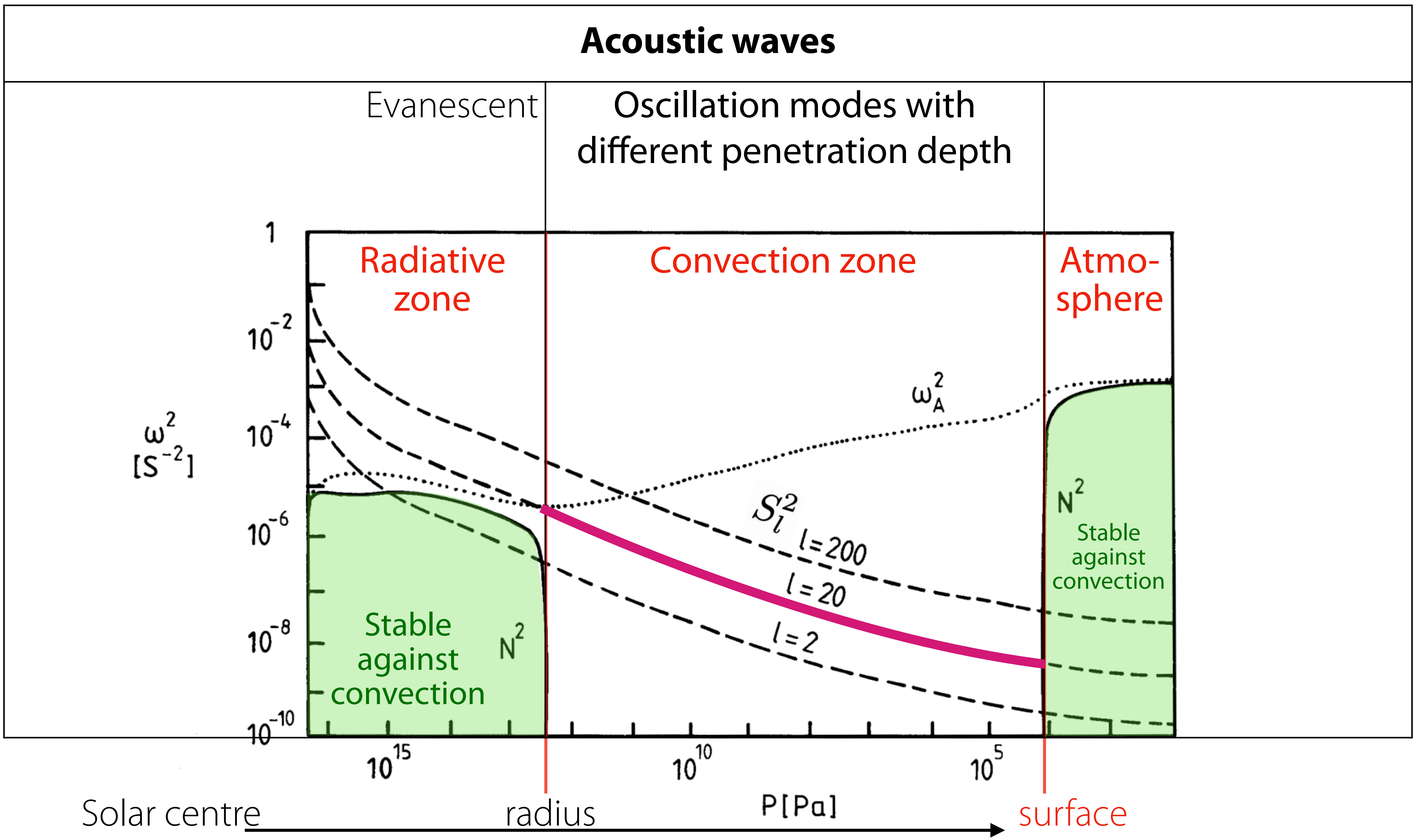
$k_r^2 > 0$: Oscillation mode exists

$k_r^2 < 0$: Evanescent



Helioseismology

Critical frequencies in the Sun

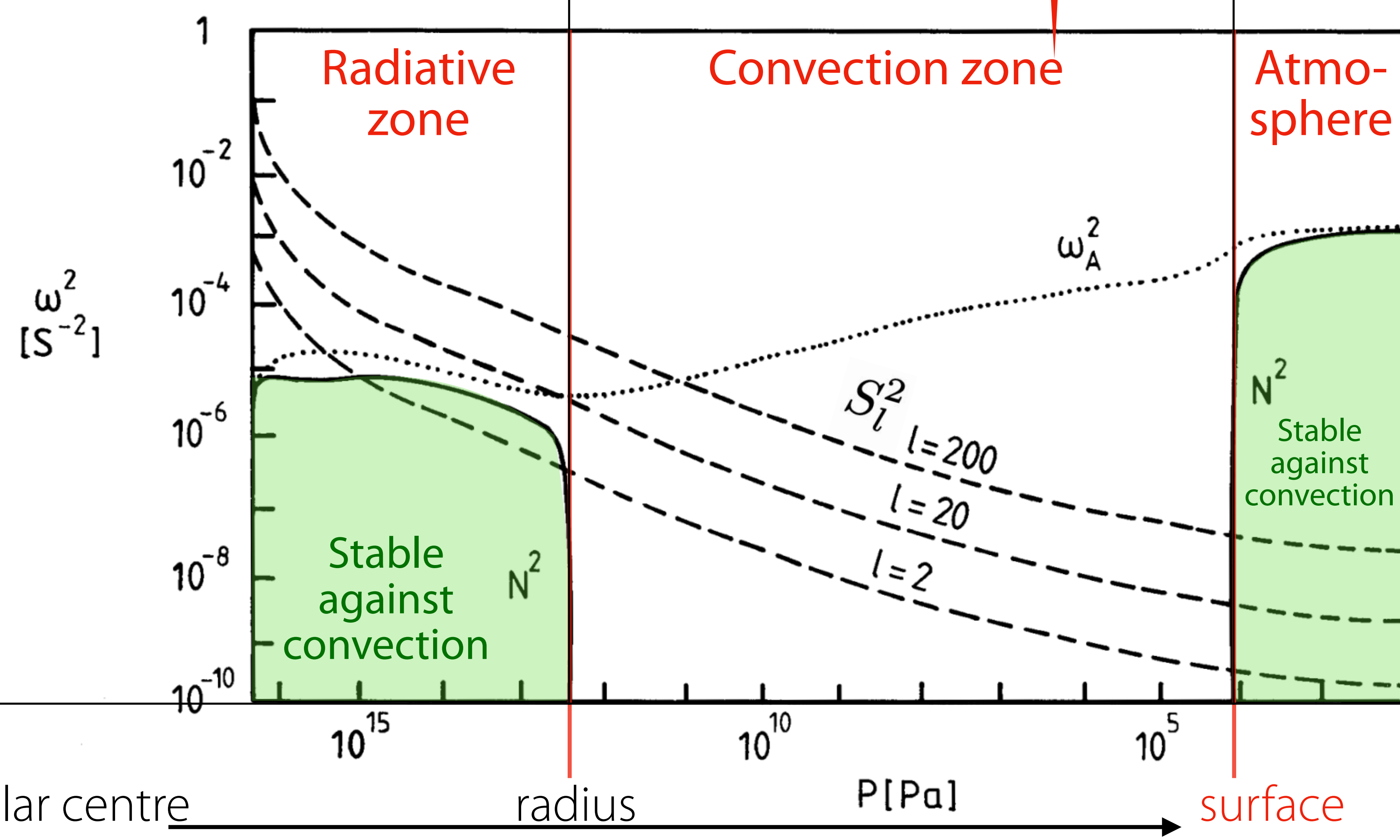


Helioseismology

Critical frequencies in the Sun

Note: The exact mode spectrum is set by the interior structure and thus expected to differ as function of **spectral type** etc.

Internal gravity waves		
trapped, form a discrete frequency spectrum of g modes	Evanescent	Should be able to occur but no direct observational proof (yet)



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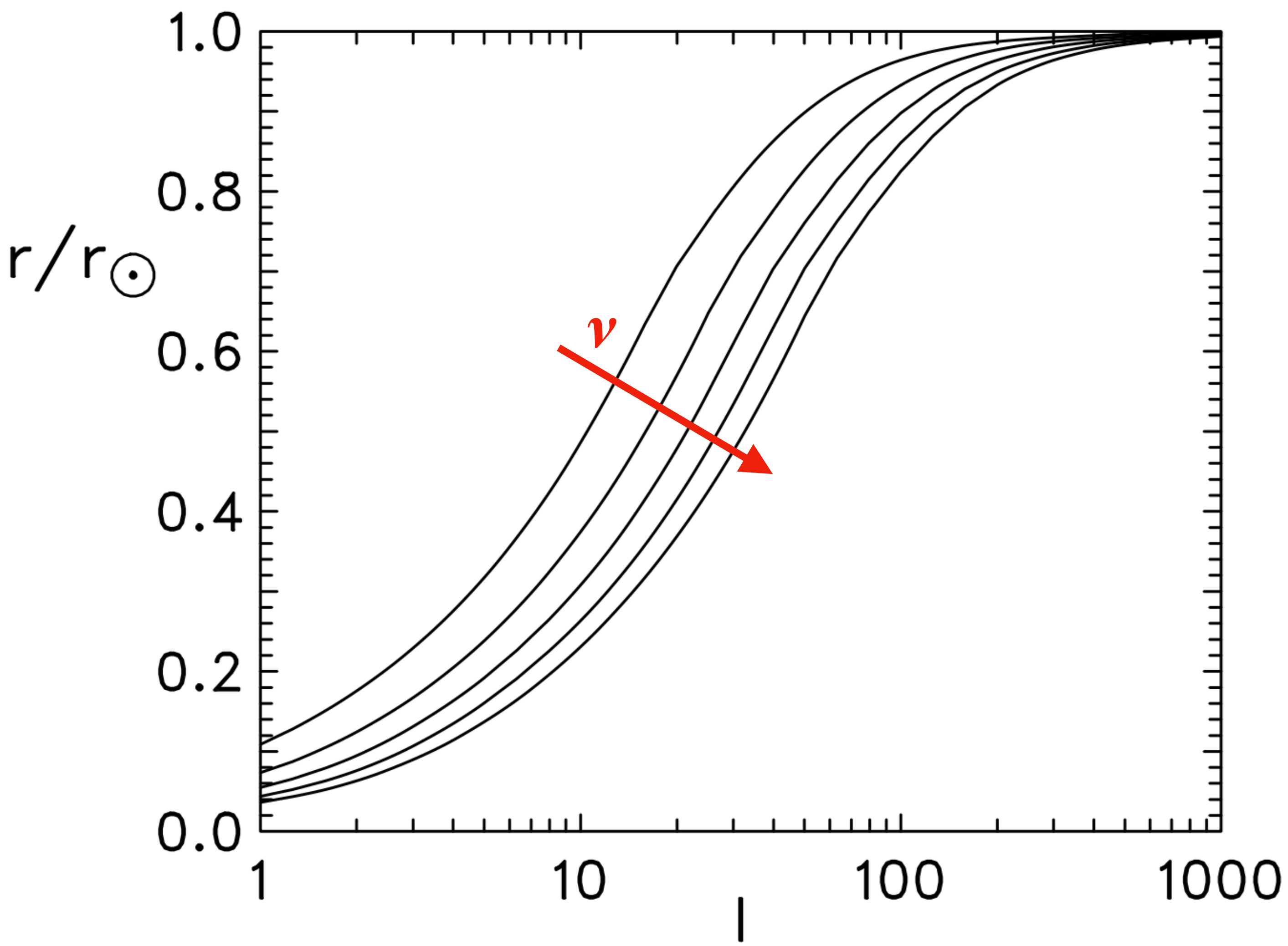
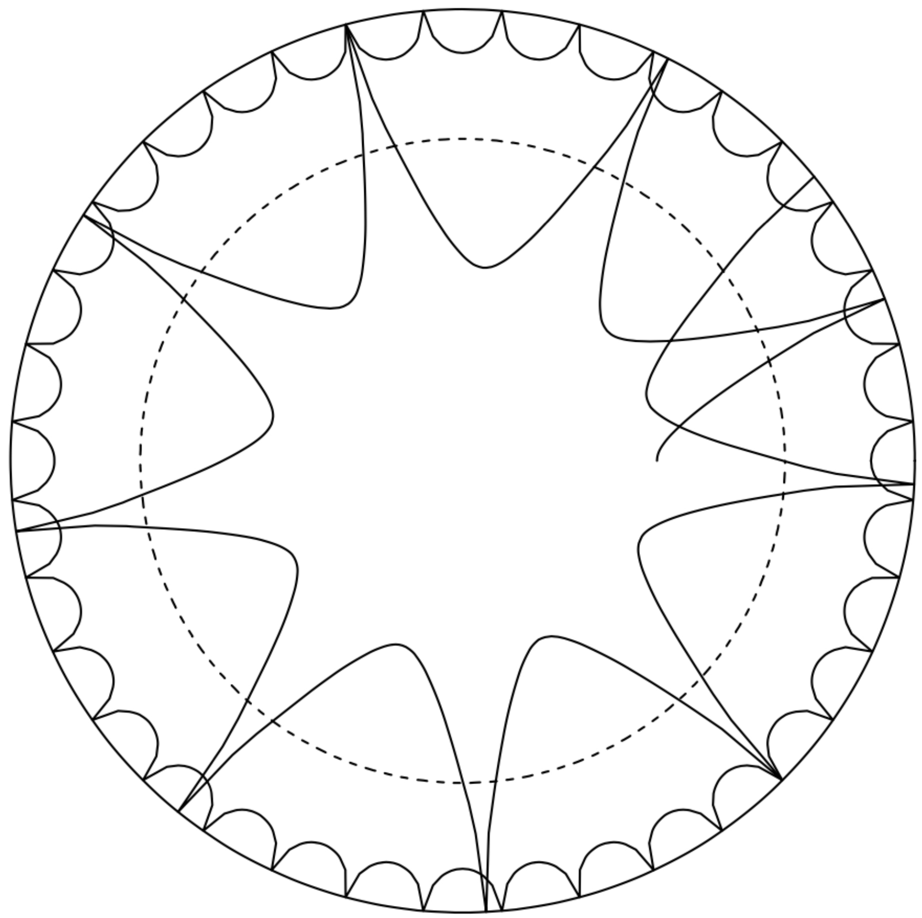
Reflection of p-modes





Stix

We have now a theoretical framework for oscillation modes in the Sun and their connection to the physical properties of the solar interior (incl. the stratification), which is otherwise not directly observable!

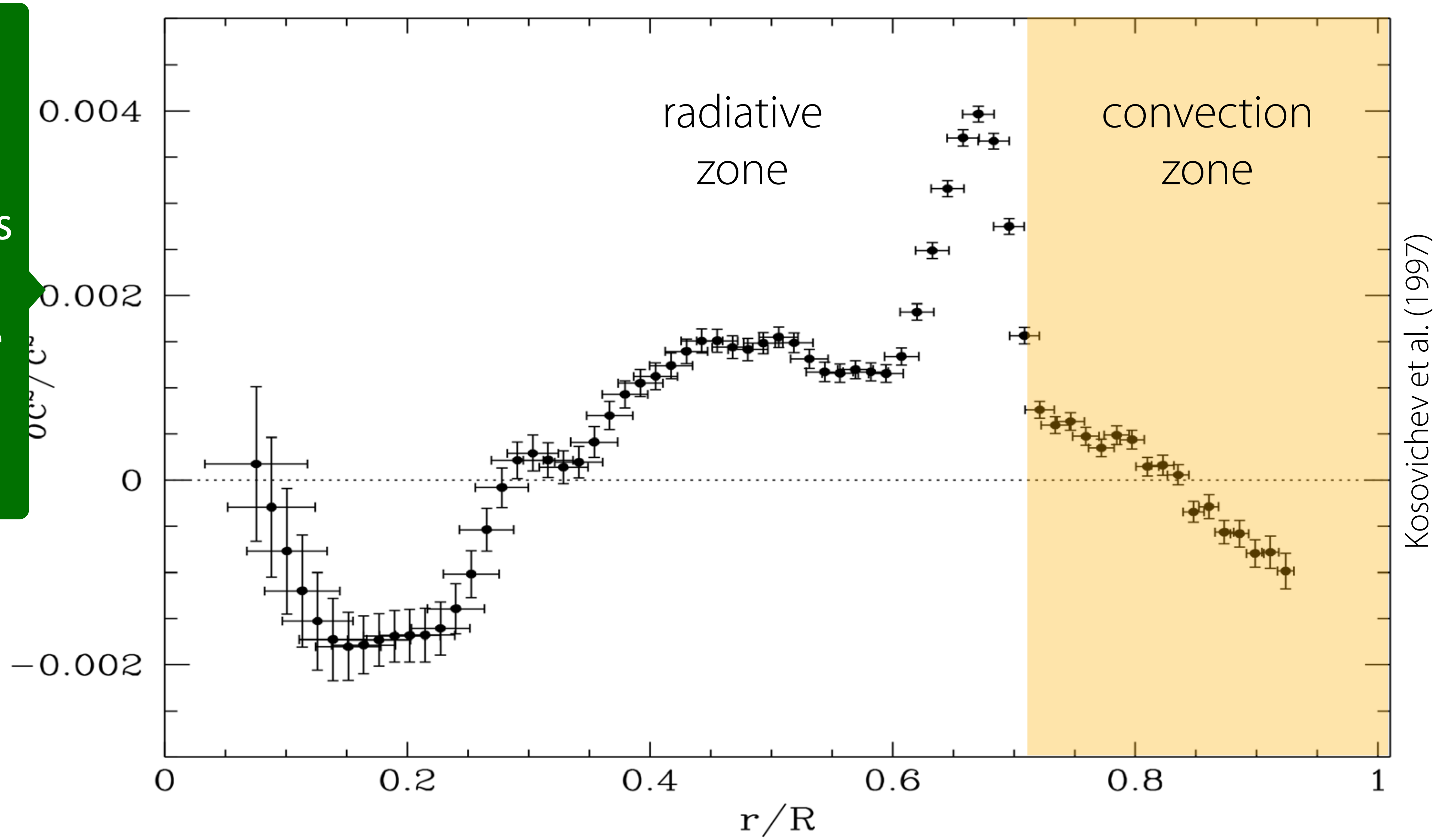
➡ Use this knowledge to invert real observations!

Helioseismology

Probing the solar interior — inversion results

- The measured mode spectrum now allows for deriving the stratification of the solar interior
- Derived sound speed **differs from the standard model** of the Sun!

Only small errors but improvements to the solar model can be made!

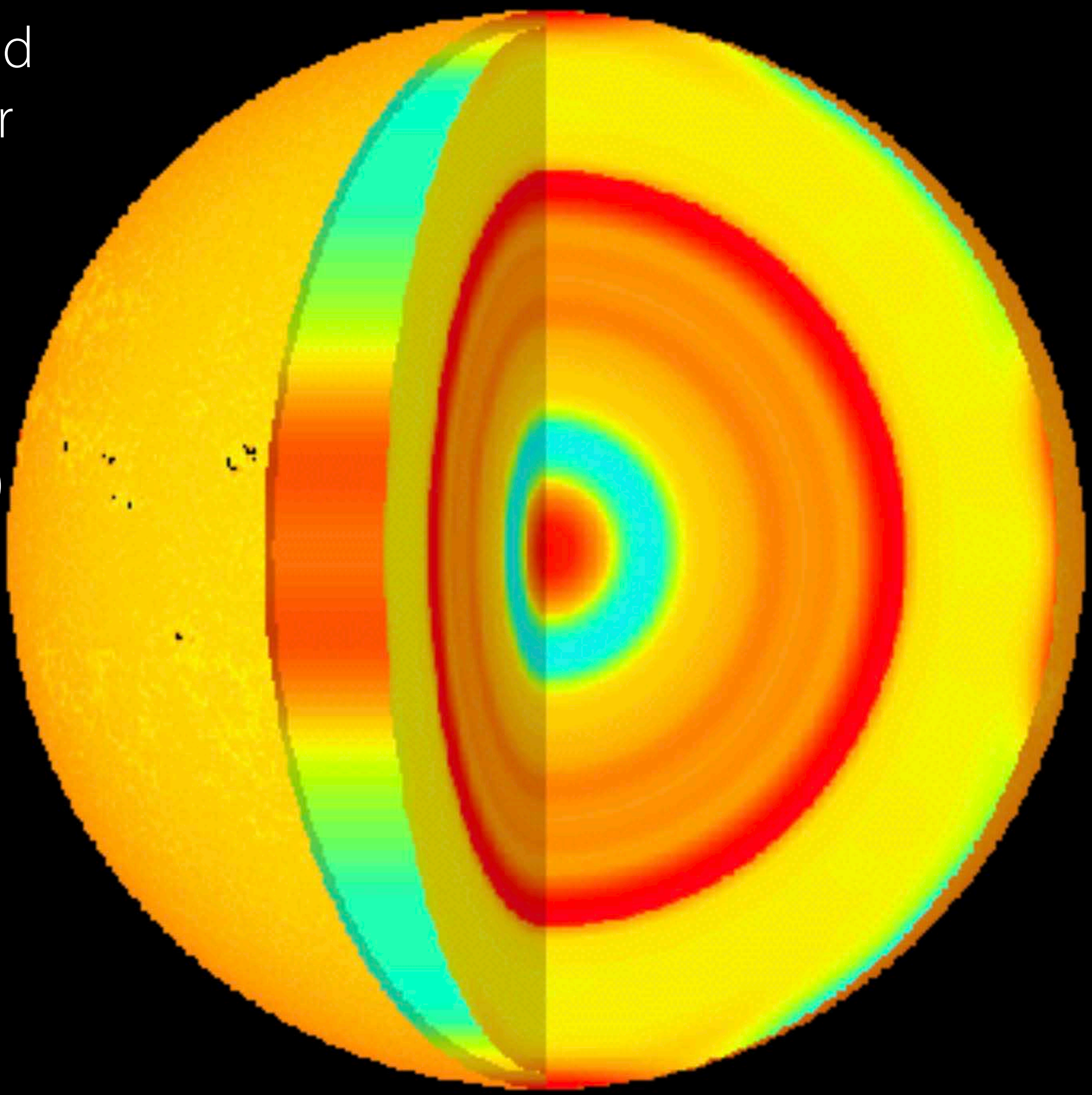


Relative difference between the squared sound speed as inferred from 2 months of MDI data and the standard solar model of Christensen-Dalsgaard et al. (1996)

Helioseismology

Probing the solar interior — inversion results

- Derived sound speed differs from the standard model of the Sun!
- Radial and latitudinal variations of the sound speed in the Sun relative to a standard solar model.
- Temperature does differ correspondingly
- Red = positive variations (`hotter' regions)
- Blue = negative variations (`cooler' regions)



Kosovichev

Helioseismology

Implications

- So far, despite (relatively small deviations): **good agreement** between sound speed predicted by models of the solar interior and helioseismological measurements

- New determination of chemical abundances — updated (suggested) values
  - Among them, C, N, O: significant **opacity sources in the solar interior**

  ➡ Changed abundances result in (slightly) different density and temperature stratification

  ➡ **Disagreement** with standard solar interior models,
  most notably just below convection zone including predicted depth of convection zone

- Different explanations debated
  - Possible: New solar abundances are not precise enough yet as a lot of effects to be taken into account
    - Active field of research, advanced use of 3D spectral line synthesis
      (based on 3D magnetohydrodynamical models)
    - Relatively new and challenging

- Please also note that the standard solar model assumes perfect spherical symmetry and no rotation!

See spectral line data files provided for the assignments.