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NOTE: There might be errors in the solution. If you find something which doens't look right, please let me know

Partial solutions to problems: part 2A

Exercise 2A.1

1. If cylinder B gets a smaller radius, it could pass through the inside of cylinder A.
2. From the reference of cylinder B, then cylinder A will have its radius shortened. Hence cylinder A will pass through the inside of cylinder B. This contradicts the results in the previous question.
3. $y \neq y'$ and $z \neq z'$ would give causal contradictions, as discussed in the previous question.

Exercise 2A.2

1. In the Earth-frame, a muon travelling $15km$ at $0.999c$ spends $15km/0.999c \approx 5.005 \cdot 10^{-5}$s
2. Ignoring relativistic effects: the muons on average live for $2 \cdot 10^{-6}$ seconds each travelling at $0.999c$ and can therefore travel $2 \cdot 10^{-6} \cdot 0.999c \approx 600m$, which is less than $15km$. We should therefore *not* expect to detect muons at the surface of Earth, when ignoring relativistic effects.
3. We will calculate the distance to the Earth and the time it takes to reach the Earth in both the Earth frame and the muon frame.
   (a) In the Earth frame we have that $\Delta x = 15km = 5 \cdot 10^{-5}$s and $\Delta t = 5.005 \cdot 10^{-5}$s.
   (b) $\Delta x' = 0$
   (c) $\Delta t' = \sqrt{\Delta t^2 - \Delta x^2} = 2.2\mu s.$

   The muon decays in $2\mu s$ so some neutrinos which live a little longer than the average will reach the surface of the Earth (remember that $2\mu s$ is the mean life time).
4. In the reference frame of the relativistic particle, $\Delta s^2 = \Delta t^2 - 0^2 = \Delta t^2$. In the reference frame of the galaxy, $\Delta s'^2 = \Delta t'^2 - \Delta x'^2$. Equating the line elements and solving for Δt, we find

$$\Delta t = \sqrt{\Delta t'^2 - \Delta x'^2} = \sqrt{\left(\frac{100000ly}{0.999999999999}\right)^2 - (100000ly)^2} \approx 50days$$

Whether this gives hope to future space travels or not: Transversing the galaxy in 50 days seems like a good thing, but there are a few negative sides:

1. The universe will have aged considerably during these 50 days (calculate for yourself)
2. The energies needed to accelerate / decelerate to 0.999999999999c are tremendous ($E \sim E/(1-v^2) \to \infty$ as $v \to 1$).
3. The acceleration process would take much more than 50 days. A person can only handle a couple of G before turning liquid - accelerating to almost c + decelerating + assuming constant 5G would take over 140 days alone.

Exercise 2A.3

1. In the reference frame of the clock, the time it takes for the light to travel between the two mirrors is L_0 (remember $c = 1$).

2. We use this representation: $(coordinate, time)$.
   In the stationary frame:
   $A : (0, 0)$
   $B : (L_0, L_0)$
   $C : (L_0 \cdot v, L_0)$

   The line element in the stationary frame is given as $\Delta s^2_{AB} = |(B - A)|^2 = (L_0 - 0)^2 - (L_0 - 0)^2 = 0$. This reflects that the line element for light is always zero.

   In the train frame:
   $A : (0, 0)$
   $B : (x'_B, t'_B)$
   $C : (0, t'_C)$

   The line element in the train frame is given as $\Delta s'^2_{AB} = |(B - A)|^2 = t'^2_B - x'^2_B$.

3. Setting the line elements equal ($\Delta s^2_{AB} = \Delta s'^2_{AB}$) shows that $t'_B = x'_B$. Since the speed of light is the same in all frames, light follows $\Delta x = \Delta t$ along all points of the tracetory.

4. In the stationary frame

$$\Delta s^2_{AC} = |(C - A)|^2 = L_0^2 - (vL_0)^2$$

In the train frame

$$\Delta s'^2_{AC} = |(C - A)|^2 = t'^2_C$$

Invariance of the line element gives

$$t'^2_C = L_0^2 - (vL_0)^2$$

such that

$$t'_C = L_0\sqrt{1 - v^2} = L_0/\gamma$$

5.

$$\Delta s^2_{BC} = |(C - B)|^2 = (L_0 - L_0)^2 - (L_0 v - L_0)^2 = -(L_0 v - L_0)^2$$

$$\Delta(s'_{BC})^2 = |(C - B)|^2 = (t'_c - t'_B)^2 - (0 - x'_B)^2 = (L_0/\gamma - t'_B)^2 - t'^2_B$$

Using the invariance of the line element:

$$(L_0/\gamma - t'_B)^2 - (t'_B)^2 = (L_0^2/\gamma^2 - 2L_0/\gamma t'_B) = -L_0^2(1 - v)^2$$

Using that $1/\gamma^2 = (1 - v^2)$ and solving for t'_B:

$$t'_B = \frac{\gamma L_0}{2}\left((1 - v^2) + (1 - v)^2\right) = L_0\gamma(1 - v)$$

6. In the stationary frame, $D = (0, 2L_0)$ while in the train frame, $D = (-v \cdot t'_D, t'_D)$. In the stationary frame,

$$\Delta s^2_{AD} = |(D - A)|^2 = 4L_0^2$$

while in the train frame

$$\Delta(s'_{AD})^2 = |(D - A)|^2 = t^2_D - v^2 t'^2_D$$

Equating these line elements gives:

$$4L_0^2 = |(D - A)|^2 = t^2_D - v^2 t'^2_D$$

Solving for t'_D gives

$$t'_D = \frac{2L_0}{\sqrt{1 - v^2}} = 2L_0\gamma$$

7. In the frame of the train: $\Delta t'_{AB} = t'_B - t'_A = t'_B = L_0\gamma(1 - v)$ while $\Delta t'_{BD} = t'_D - t'_B = 2L_0\gamma - L_0\gamma(1 - v) = L_0\gamma(1 + v)$. In the stationary reference frame, $\Delta t_{AB} = L_0$ while $\Delta t_{BD} = 2L_0 - L_0 = L_0$. As the speed of light is equal in all reference frames, the time experienced is different in the two coordinate systems.

Exercise 2A.4

1. We convert to the physical distance, using $d = r\theta$. We find

$$d = 2.6 \cdot 10^9 ly \cdot (2. \cdot 10^{-3}) \cdot 2\pi/60/60/360 \approx 2.7425.2ly'$$

Dividing by 3 years:

$$v = 25.2ly/3 \approx 8.4c$$

2. The observed time signal Δt_{obs} we observed equals the real time interval Δt minus the length the jet transversed in the radial coordinate. This length is given as $s = v * \Delta t$, and projecting the jet onto the radial coordinate gives $s = \Delta t v \cos\theta$, such that

$$\Delta t_{obs} = \Delta t - \Delta t v \cos\theta$$

3. We use that

$$v_{obs} = \frac{s_{obs}}{\Delta t_{obs}}$$

From the figure in the exercise description, we note that the transverse length is given as $\Delta t v \sin\theta$ such that

$$v_{obs} = \frac{\Delta t v \sin\theta}{\Delta t - \Delta t v \cos\theta} = \frac{v \sin\theta}{1 - v \cos\theta}$$

4. We use that $\sin\theta = \cos\theta = 1/\sqrt{2}$. Insert and ask what values of observed velocity will be greater than the speed of light:

$$\frac{v/\sqrt{2}}{1 - v/\sqrt{2}} > 1$$

$$\frac{v}{\sqrt{2} - v} > \sqrt{2}$$

Solve for v:

$$v > \frac{c\sqrt{2}}{2} \approx 0.7$$

Exercise 2A.5

1. The coordinates are defined as (position, time). Event A happens at $(0, 0)$ in both the laboratory and moving frame, while event B happens at (x, t) in the laboratory frame and $(0, t')$ in the moving frame. Then $\Delta t_{AB} = t$ and $\Delta t'_{AB} = t'$. We then use that $\Delta t = \gamma \Delta t'$, or $t = \gamma t'$. This resembles the first equation (8) from the problem text:

$$t = f(v)x' + g(v)t' = \gamma t'.$$

Since $x' = 0$ we have that $g(v) = \gamma$.

2. Event B happens at position $x = vt$ in the laboratory frame, or $x = vt = v\gamma t'$. Equation (9) then gives

$$x = h(x)x' + k(v)t' = 0 + k(v)t' = v\gamma t'$$

as $x' = 0$ in the moving frame. Hence $k(v) = v\gamma$.

3. Even A is still the same as in question 1, but event B is now (L_0, t') in the moving frame and $(L, 0)$ in the laboratory frame. Equation (6) shows that $L = L_0/\gamma$ such that $x = x'/\gamma$. Then equation (11) reads

$$x' = h(-v)x - v\gamma t = h(-v)x = \gamma x$$

such that $h(v) = h(-v) = \gamma$ (we used that $t = 0$ since A and B are simultaneous in the lab frame and A happens at $t = 0$)

4. We only need to decide on $f(v)$. The space-time interval Δs^2_{AB} is then expressed as

$$\Delta s^2_{AB} = |(B - A)|^2 = t^2 - x^2 = (f(v)x' + \gamma t')^2 - (\gamma x' + v\gamma t')^2$$

while

$$\Delta s'^2_{AB} = |(B - A)|^2 = t'^2 - x'^2$$

Equating these two line elements and expanding the square:

$$t'^2 - x'^2 = f(v)^2 x'^2 + \gamma^2 t'^2 + 2f(v)x'\gamma t' - \gamma^2 x'^2 - v^2\gamma^2 t'^2 - 2\gamma^2 x' v t'$$

The simplest way to show $f(v) = \gamma v$ is by insertion:

$$t'^2 - x'^2 = \gamma^2 v^2 x'^2 + \gamma^2 t'^2 + 2vx'\gamma^2 t' - \gamma^2 x'^2 - v^2\gamma^2 t'^2 - 2\gamma^2 x' v t'$$

Extract the γ^2:

$$t'^2 - x'^2 = \gamma^2(v^2 x'^2 + t'^2 + 2vx't' - x'^2 - v^2 t'^2 - 2x'vt')$$

Notice how two of the terms cancel. So:

$$t'^2 - x'^2 = \gamma^2(v^2 x'^2 + t'^2 - x'^2 - v^2 t'^2)$$

which equals

$$t'^2 - x'^2 = \gamma^2(v^2(x'^2 - t'^2) + (t'^2 - x'^2)) = \gamma^2(t'^2 - x'^2)(1 - v^2)$$

But $\gamma^2 = 1/(1 - v^2)$, and the equation holds. Thus, $f(v) = \gamma v$

Exercise 2A.6

1. This is done as in problem 4: We use this representation: $(coordinate, time)$.
   In the stationary frame:
   $A : (0, 0)$
   $B : (L_0, L_0)$
   $D : (0, 2L_0)$

   In the train frame:
   $A : (0, 0)$
   $B : (x'_B, t'_B)$
   $D : (x'_D, t'_D)$

2. The lorentz transformations states that

   $$t'_B = f(-v)x_B + g(-v)t_B = -\gamma v L_0 + \gamma L_0 = \gamma L_0(1 - v)$$

   t'_D is then

   $$t'_D = f(-v)x_D + g(-v)t_D = 2\gamma L_0$$

   as from problem 4. For the third thick we have $x_3 = L_0$ and $t_3 = 3L_0$ giving

   $$t'_3 = -v\gamma L_0 + 3\gamma L_0 = \gamma L_0(3 - v).$$

   Finally the fourth tick happening at $x_4 = 0$ and $t_4 = 4L_0$ gives

   $$t'_4 = 4\gamma L_0,$$

   so that the time between the next to ticks is $\Delta t'_{34} = \gamma L_0(4 - (3 - v)) = \gamma L_0(1 + v)$ exactly as for the first two ticks.

Exercise 2A.7

See separate document for this exercise.