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NOTE: There might be errors in the solution. If you find something which doens't look right, please let me know.

Partial solutions to problems: Part 3B

Exercise 3B.1

The easiest way is to solve this problem using center of mass coordinates. Write the velocities in terms of $\vec{v_1}^{\mathrm{CM}}$ and $\vec{v_2}^{\mathrm{CM}}$ and use expressions from the lectures on celestial mechanics to write these in terms of the relative velocity $\vec{v}$ which is the velocity of one object as observed from the other (remember that this is the frame from which Kepler's law is valid). Finally write the velocity in terms of the period P.

Exercise 3B.2

1. $32' = 2\pi/60/60/360 = 0.0093$ radians. The radius (for *small radial* values of θ) is given as $r = \theta d = 0.8Mpc$.

2. We assume all galaxies are like the milky way, and that there are 200 billion stars in each galaxy. Assuming each star to have a mass of $2 \cdot 10^{30} kg$, we end up with an assumed mass of the 100 galaxies:

$$M_{total} = 100\text{galaxies} \cdot 2 \cdot 10^{11}\text{stars} \cdot 2 \cdot 10^{30}\text{kg} = 4 \cdot 10^{43} kg$$

   which is the *luminous* mass of the cluster.

3. Result given in the question.

4. We will use the following Python code:

```
from scitools.all import *


#Function that reads galaxydata from file
def read_data():
    ...
    ...
    return ang_x, ang_y, dist, obs_lambda


#Function that calculates the radial velocity of the cluster of galaxies,
```

```
#and writes the result to screen
def rad_vel_cluster():
   v_gal = ...              #Radial velocity of galaxies - use Doppler's formula
   v_clus = sum(..)/len(..) #Compute velocity of the cluster (peculiar velocity)
   v_rel =                  #Compute the relative velocity of the galaxies (velo
                            #relative to center of the cluster - center of mass
   print '...'
   return v_gal, v_clus, v_rel


#Function that plots (and stores) the cluster as seen from a telescope
#as an eps-file
def plot_cluster():
   plot(ang_x, ang_y, 'o')         #Plot galaxy as a ring
   ...


#Function that calculates the mass of a galaxy in the cluster
def mass_galaxy():
   #Find the position of the galaxy in the xyz-coord.frame
   dist_galaxy = ...
   rad_x = ...              #Convert from arcmin to radians
   rad_y = ...
   x = ...                  #Trigonometry
   y = ...
   z = ...

   #Calculate the mass of the galaxy by eq. (7)
   # --> m = m_1/(G*m_2)
   m_1 = ...                #Calculate SUM(v_i^2) (Relative velocity)
   step1 = range(len(dist_galaxy))
   m_2 = 0.0
   for i in step1:
      step2 = seq(i+1, len(dist_galaxy)-1)
      for j in step2:
          m_2 += ...        #Calculte 1/r_ij, use sqrt()-function
   mass = ...               #Calculate mass by eq. (7)
   print '...'



#____
#MAIN
```

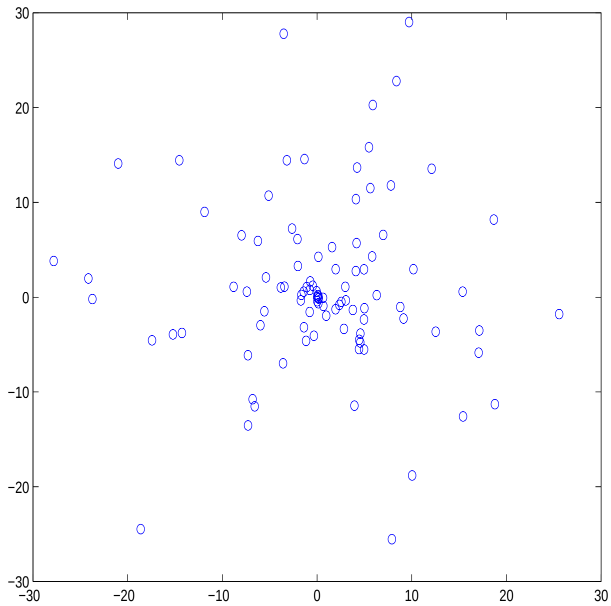

Figure 1: The galaxies in the cluster. The unit on the axes is in arcminutes.

```
#Constants
...

#Variables
...

#Calculations/Call functions
ang_x, ang_y, dist, obs_lambda = read_data()
v_gal, v_clus, v_rel = rad_vel_cluster()
plot_cluster()
mass_galaxy()
```

(a) $v_{\text{pec}} = 1.24 \cdot 10^6$ m/s.

(b) The plot is shown in figure 1

(c) $m \approx 8.7 \cdot 10^{41} kg$, about 2.17 times the estimated luminous mass. This suggests that more than 50% of the galaxy consists of something unknown and non-luminous.

(d)

$$\langle sin^2 i \rangle = \frac{\int_0^{\pi/2} \sin^2 i \, di}{\int_0^{\pi/2} di} = \frac{1}{2}$$

(e) Using the result from the previos question, we see that the results from 2.4.c should be **doubled**: The "more correct" mass is $m \approx$

$1.7 \cdot 10^{42} kg$, about 4.4 times the estimated galaxy mass found in 2.2. This means that only about 20% of the total mass of a galaxy is represented as luminous matter, the rest consists of something else and more sinister, namely "dark matter".

Exercise 3B.3

1. For small values of θ, the diameter is given as $D = d \cdot \theta = 200pc \cdot 3.5' \approx 0.2pc$. The radius is then $d = D/2 = 0.1pc$.

2. The volume of a sphere is given as $V = \frac{4}{3}\pi r^3$, so assuming a uniformly distributed mass density ρ we obtain

   $$M = \rho \cdot V = 3 \cdot 10^{-17} kg/m^3 \cdot \frac{4}{3}\pi(0.1pc)^3 \approx 3.62 \cdot 10^{30} kg$$

   which approximately is 1.8 solar masses.

3. The mass of hydrogen is $m_h = 1.71 \cdot 10^{-27}$kg, while the mean molecular weight is assumed to be $\mu = 1$ (that is, there are only hydrogen atoms in the cloud). The Jean mass is defined as

   $$M_J = \Big(\frac{5kT}{G\mu m_H}\Big)^{3/2} \Big(\frac{3}{4\pi\rho}\Big)^{1/2}$$

   and describes the mass threshold for whether a molecular cloud will collapse to a more compact object ($M > M_J$) or not ($M < M_J$). Inserting the values (where T=$10K$), we obtain a Jeans mass of $M_J \approx 4.2 \cdot 10^{31}$kg, or 21 solar masses. This is more than the result obtained in 13.2.2, so this cloud will not collapse (alone) and form a protostar.

4. Recall that the condition for a cloud to collapse is that $2K < |U|$, where U is the potential energy and K kinetic energy. If a supernova in the vicinity contributes to compressing the gas, the gravitational attraction becomes stronger. This is because the mass density increases while the radius of the cloud decreases, thus U grows. But why would K on average not grow? Increasing the mass density should decrease the jeans mass ($M_J \propto \frac{1}{\sqrt{\rho}}$). It is therefore plausible that a supernova could contribute to the creation of protostars.

5. See the last answer.

6. The spiral shaped pressure wave will compress the gas at the tops of the wave and thus increase the probability for star birth in these areas.