Coronal loops

The corona is build up by coronal loops. The energy equation that describes these loops can be written as :

$$E_H + \mathbf{g}\rho \cdot \mathbf{u} = \nabla \cdot \mathbf{F_C} - E_R + \nabla \cdot \left(\left[\frac{1}{2}\rho u^2 + U \right] \mathbf{u} + \rho \mathbf{u} \right) \tag{1}$$

Where E_H is the local heating rate, F_C is the conductive heat flux, E_R is the radiative energy loss and U is the potential energy. The conductive heat flux is called the Spitzer conductivity and is given by:

$$\mathbf{F_C} = -\kappa T^{5/2} \left. \frac{dT}{d\mathbf{r}} \right|_{\hat{\mathbf{B}}} \tag{2}$$

and $\kappa = 7.8\ 10^{-12} \mathrm{JK}^{-7/2} \mathrm{s}^{-1} \mathrm{m}^{-1}$. It is here written as if it was only along the magnetic field - that is not strictly true, but the component along the magnetic field is about 10 orders of magnitude greater than perpendicular to the magnetic field so it is almost always ignored. We can assume that the conductive heat flux is parallel to the magnetic field and that the plasma only moves along the magnetic field, which means that this equation actually describes a one dimensional system. We can therefore rewrite the equation:

$$E_H + \mathbf{g}\rho \cdot \mathbf{u} = \nabla F_C - E_R + \nabla \left(\left[\frac{1}{2}\rho u^2 + U \right] u + \rho u \right)$$

If we look at the whole loop, then we can simplify equation **??** into:

$$\int_V (E_H + \mathbf{g}\rho \cdot \mathbf{u})\, \mathrm{d}\mathbf{r}^3 = -\int_V E_R \mathrm{d}\mathbf{r}^3 + L_{footpoints} + L_{sides} \tag{3}$$

where $L_{footpoints}$ is the energy lost through the footpoints (both advected heat and conducted heat) of the loop, and L_{sides} is the energy loss through the sides of the loop. Since we know no simple physical process that can make L_{sides} very large, it is often set to zero. For a loop that has the a cross section that is circular and constant we can simplify this further, using the same argument as above to get:

$$A \int_0^{2L} (E_H + \mathbf{g}\rho \cdot \mathbf{u})\, \mathrm{ds} = -A \int_0^{2L} E_R \mathrm{ds} + L_{footpoints} \tag{4}$$

Assuming here that there is no internal structure within the cross section of the loop.

Example 1:

As an example we will look at a loop which is a half-circle and along its whole length has a circular cross section with diameter of 100 km, and the loop top being at a height of 30 Mm. Since the loop is observed to shine as much at the top as at the footpoints, we know that the temperature and density must be almost constant along the whole loop, so we assume that the loop has a temperature of 2 million K and that the density is $\rho = 1.67\,10^{-12}\,\mathrm{kg\,m^{-3}}$ and that the radiative energy loss is given by $E_R = 3.3\,10^{21}\,\frac{\mathrm{J\,m^3\,kg^{-2}\,K^{1/2}}}{\mathrm{s}}\rho^2 T^{-1/2}$. Assume furthermore that foot point ($s = 0$ and $s = 2L$ is at the base of the corona and that the temperature in the transition zone decreases from the coronal temperature of 2 million K to 20000 K over a distance of 1000 km and assume that the velocity at the footpoints is zero. Calculate the amount of heat that needs to be added to the coronal loop every second in order for it to remain at 1 million K. If the heating was turned of, what would the cooling rate be (i.e. dT/dt)?

Since there is no difference between the two halfs of the loop, then we can just treat half of the loop and multiply by two and as there if no velocities at the foot points we get:

$$2A\int_0^L E_H \mathrm{ds} = -2A\int_0^L (E_R + \mathbf{g}\rho\cdot\mathbf{u})\,ds + \frac{d}{ds}\left[2A\kappa T^{5/2}\left.\frac{dT}{ds}\right|_{s=0}\right] \quad (5)$$

This is of course only the radiative energy loss and the conductive energy loss at the boundary. Since the work of the gravitational force through the boundaries is zero ($\mathbf{u}(0) = \mathbf{u}(2L) = 0$) That means that the work by the gravitaional force is zero.

The energy loss at the footpoints are only through conduction, since there is no velocity at the footpoints. The energy flux at the footpoints can be calculated since we know that the temperature gradient can be calculated to be:

$$\frac{dT}{ds} = \frac{T_{loop} - T_{chromosphere}}{1000km} = 1.98\,\mathrm{K\,m^{-1}} \quad (6)$$

Since there is no conduction into the loop, the total loss is just the energy conducted out, so we do not have to do the second derivative. From that we can now calculate $L_{footpoints}$:

$$\begin{aligned} L_{footpoints} &= 2A\kappa T^{5/2}\left.\frac{dT}{ds}\right|_{s=0} & (7)\\ &= -2A\,7.8\;10^{-12}\mathrm{JK^{-7/2}s^{-1}m^{-1}}\;5.7\;10^{15}\,\mathrm{K^{5/2}}1.98\,\mathrm{K\,m^{-1}} & (8)\\ &= -2A\,8.7\;10^4\,\mathrm{J\,m^{-2}\,s^{-1}} & (9) \end{aligned}$$

The radiative loss is given by:

$$\begin{aligned} E_R &= 3.3\,10^{21}\frac{\mathrm{J\,m^3\,K^{1/2}}}{\mathrm{s}}\rho^2 T^{-1/2} & (10)\\ &= 3.3\,10^{21}\frac{\mathrm{J\,m^3\,K^{1/2}}}{\mathrm{s}}2.79\,10^{-24}\,\mathrm{kg^2\,m^{-6}}\;7.1\;10^{-4}\,\mathrm{K^{-1/2}} & (11)\\ &= 6.5\,10^{-6}\,\mathrm{J\,s^{-1}\,m^{-3}} & (12) \end{aligned}$$

and so the total loss of energy is given by

$$\int_V E_R \mathrm{d}\mathbf{r}^3 = 2A\int_0^L E_R ds = 2A\frac{\pi}{2}hE_R = 2A\,3.1\;10^2\,\mathrm{J\,s^{-1}\,m^{-2}} \quad (13)$$

We can see from these two results that a coronal loop with a temperature of 2 million K, cools mainly by conducting heat down it's footpoints, and not by radiation. If there was no heating ($E_H = 0$) then the total cooling rate of the loop is (it is here pr unit cross section):

$$\frac{dE_{loop}}{dt} = -\int_0^L E_R ds + L_{footpoints} = 8.7\ 10^4\,\mathrm{J\,s^{-1}\,m^{-2}} \tag{14}$$

If we assume that the plasma is an ideal gas then we know that:

$$E = \frac{1}{\gamma - 1}\frac{\rho}{m_p \mu} kT \tag{15}$$

where k is boltzmanns constant, m_p is the proton mass, $\mu = 0.8$ is the molecular mass and $\gamma = {}^5\!/_3$ is the adiabatic index. Since we now have the energy loss pr unit time for the whole loop, we have to convert that to pr unit volume and from above we can convert that to a temperature change:

$$\frac{dT}{dt} = \frac{1}{L}(\gamma - 1)\frac{\mu m_p}{\rho k}\frac{dE_{loop}}{dt} \tag{16}$$

$$= \frac{1}{30\ 10^6\,\mathrm{m}} 3.4\ 10^{12}\,\mathrm{K\,s^{-1}} \tag{17}$$

$$= 1.1\ 10^5\,\mathrm{K\,s^{-2}} \tag{18}$$

That means the loop will cool from 2 million to 1 million in just 10s if this cooling rate was constant. Since it is not, it takes somewhat longer.