Solar Wind & the Heliosphere

The heliosphere is the region between the corona and the interstellar medium. The heliosphere is filled with a wind driven by the sun, which sends out a wind consisting mainly of protons and eletrons. The particles travel at a speed of several hundred kilometers per second until they hit the heliopause which is the interface between the solar wind and the interstellar medium. The heliopause was traversed by the pioneer satellites just a few years ago at a distance from the sun of roughly 90 AU.

The first and still used model of the solar wind was created by Eugene Parker in the 1950'ies. Until then it was assumed that the heliosphere was static, and with no net outflow. The model by Parker is rather simple, and follows from only physical characteristics of the outer and inner boundary of the heliosphere. For purely radial outflow the equation of momentum conservation looks like:

$$\rho u \frac{du}{dr} = -\frac{dp}{dr} - \rho \frac{GM_\odot}{r^2} = 0 \tag{1}$$

and the continuity equation looks like (in cylindrical coordinates):

$$\frac{1}{r^2}\frac{dr^2 \rho u}{dr} = 0 \tag{2}$$

We also need an equation of state, and in order for the calculation to be as simple as possible we choose an equation of state corresponding to an isothermal plasma:

$$p = nkT = \frac{\rho kT}{\mu m_p} = \frac{2\rho kT}{m_p} \tag{3}$$

where T is constant, and μ is the molecular weight here set to be $\mu = {}^1/_2$ because we assume that the solar wind consists only of fully ionized hydrogen and so there is as many protons as electrons.
Integrating eq. 2 we get:

$$r^2 \rho u = I \tag{4}$$

where I is a constant of integration, and the above equation just states that for a given solid angle the mass flux is independent of radius and equal to I.We can now use eqs. 1,3,4 to get:

$$\begin{aligned}
\rho u \frac{du}{dr} &= -\frac{dp}{dr} - \rho \frac{GM_\odot}{r^2} && (5)\\
u \frac{du}{dr} &= -\frac{1}{\rho}\frac{d\left[\frac{2\rho kT}{m_p}\right]}{dr} - \frac{GM_\odot}{r^2} && (6)\\
u \frac{du}{dr} &= -\frac{2kT}{m_p}\frac{r^2 u}{I} I \frac{d}{dr}\left(\frac{1}{r^2 u}\right) - \frac{GM_\odot}{r^2} && (7)\\
u \frac{du}{dr} &= \frac{2kT}{m_p}\left[\frac{1}{u}\frac{du}{dr} + \frac{2}{r}\right] - \frac{GM_\odot}{r^2} && (8)\\
\frac{1}{u}\frac{du}{dr}\left(u^2 - \frac{2kT}{m_p}\right) &= \frac{4kT}{m_p r} - \frac{GM_\odot}{r^2} && (9)
\end{aligned}$$

We now assume that the temperature in the heliosphere is lower than the temperature set by:

$$T < T_c \equiv \frac{GM_{\odot}\mu m_p}{4ka} \tag{10}$$

where a is the radius of the base of the corona. For typical coronal parameters $T_c \approx 5.8\,10^6$K which is reasonable, since the corona in general is a few million K. Looking at eq. 9 we can see that the right hand side is negative for $a < r < r_c$, where

$$\frac{r_c}{a} = \frac{T_c}{T} \tag{11}$$

and positive for $r < r_c < \infty$. At $r = r_c$ the right hand side of eq. 9 is zero, implying that the left hand side is also zero. The left hand side can be zero in two ways : Either

$$u^2(r_c) = u_c^2 \equiv \frac{2kT}{m_p} = \frac{\rho kT}{\rho\mu m_p} = \frac{p}{\rho} = c_s^2 \tag{12}$$

or

$$\frac{du(r_c)}{dr} = 0 \tag{13}$$

Note that u_c corresponds to the coronal sound speed. It can easily be seen that if eq. 12 is satisfied then ${}^{du}/_{dr}$ has the same sign for all r and that $u(r)$ is then monotonically increasing or decreasing. On the other hand if eq. 13 is satisfied then $(u - u_c)$ has the same sign for all r and u has an extremum at $r = r_c$. The flow is either supersonic for all r or subsonic for all r. This then leads to four different types of solutions:

- $u(r)$ is subsonic throughout the domain $a < r < \infty$. $u(r)$ increases with r, attains maximum value around $r = r_c$ and then decreases with r
- a unique solution for which $u(r)$ increases monotonically with r and $u(r_c) = u_c$
- a unique solution for which $u(r)$ decreases monotonically with r and $u(r_c) = u_c$
- $u(r)$ is supersonic throughout the domain $a < r < \infty$. $u(r)$ decreases with r, attains a minimum around r_c and then increases with r

Each of these four types of solution fits a certain set of boundary condition at the corona and the inter stellar medium. Type 3 and 4 can already be ruled out as a plausible solution since it demands that the velocity is supersonic at the base of the corona. Type 1 and 2 are both plausible when looking at the boundary near the solar corona, but look quite differently at the outer boundary. Eq. 9 can be rewritten to give

$$\frac{du^2}{dr}\left(1 - \frac{u_c^2}{u^2}\right) = \frac{4u_c^4}{r}\left(1 - \frac{r_c}{r}\right) \tag{14}$$

with the help of eqs. 10 and 11. Integrating the above equation we get

$$\left(\frac{u}{u_c}\right)^2 - \ln\left(\frac{u}{u_c}\right)^2 = 4\ln r + 4\frac{r_c}{r} + C \tag{15}$$

where C is a constant of integration. Looking at the type 1 solution, then in the limit $r \to \infty$ one can see that ${}^{u}/_{u_c}$ is less than unity and monotonically decreasing with as $r \to \infty$. For large r then eq. 15 can be reduced to

$$\ln\frac{u}{u_c} \simeq -2\ln r \tag{16}$$

so that

$$u \propto \frac{1}{r^2} \tag{17}$$

From eq. 4 we can now see that this solution means that the density approaches a finite, non-zero value ρ_∞ when $r \to \infty$, and the pressure will then be

$$p_\infty = \frac{2\rho_\infty kT}{m_p} \tag{18}$$

which is a pressure much larger than the interstellar pressure, so it is not physical, since if it was, the solar wind would be carving out a bigger and bigger bubble in the interstellar matter, which is not observed. So we are left with type 2 of the possible solutions. If we look at the limit $r \to \infty$ then it gives

$$\left(\frac{u}{u_c}\right)^2 \simeq 4\ln r \tag{19}$$

so that

$$u \simeq 2u_c\,(\ln r)^{1/2} \tag{20}$$

From here it follows that the velocity is supersonic and increasing with r and following the same arguments as for type 1 we see that $\rho \to 0$ as $r \to \infty$ and so will the pressure. the type 2 solution is therefore physical and it predicts that the solar corona expands radially outwards at modest subsonic speed close to the sun, and then accelerates to supersonic velocities when it comes further away from the sun.

Parker's model of the solar wind was made, before satellites had measured there was even a solar wind at all, and has later been confirmed by many measurements.