UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Midterm Exam AST 4210 — Radiation I
Date of exam: October 15, 2004
Time of exam: 1500-1800
This exercise set contains 2 pages
Attachments: None
Allowed sources of help: Any non-communicating source
Check that exercise set is complete before you start solving the problems

Exercise I

Let H_0 be the lowest order Hamiltonian for the one-electron atom (with an infinitely heavy nucleus) and let $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ be the total angular momentum operator.

a) Making use of the fact that H_0, $\boldsymbol{L}^2$, L_z, $\boldsymbol{S}^2$ and S_z are mutually commuting operators, show that

$$[H_0, \boldsymbol{J}^2] = 0, \quad [\boldsymbol{L}^2, \boldsymbol{J}^2] = 0 \quad \text{and} \quad [\boldsymbol{S}^2, \boldsymbol{J}^2] = 0.$$

b) Can you now argue that H_0, $\boldsymbol{L}^2$, $\boldsymbol{S}^2$, $\boldsymbol{J}^2$ and J_z are simultaneously measurable?

c) In spherical coordinates the azimuthal angular operator has the form $L_z = -\imath\hbar\frac{\partial}{\partial\phi}$. Show that L_z is a Hermitian operator. What consequences has this fact for the eigenvalues of the operator?

Exercise II

A distant star with radius R equal to the Solar radius $R_\odot$ has a rotational axis perpendicular to the line of sight. The rotational period is $P = 6$ hours. The star has a surface temperature of 6000 K. An observer has the whole star within her field of view while observing the hydrogen (^{1}H) H_α line.

a) Calculate the full width at half height of the H_α-line, $\Delta\lambda_{FWHH}$, that would result if the star was not rotating.

b) Estimate the effect of stellar rotation on the observed line width.

c) If the observer next observes the H_α line from singly ionized helium ($^4He^+$), what would the relative importance of the thermal and rotational Doppler effects then be?

d) Will your result under point b) be modified if the angle θ between the rotational axis and the line of sight is changed from 90° to 45°?

Exercise III

During an eclipse the Solar corona becomes visible due to Solar radiation scattered on (non-relativistic) free electrons in the corona. We want to estimate the effectiveness of this mechanism.

a) Show that the average power flux carried by an electromagnetic wave (in vacuum) with electric field amplitude $\boldsymbol{E}_0$ and wave vector $\boldsymbol{k}$ is given by $\overline{\boldsymbol{P}_{wave}} = \frac{1}{2}\epsilon_0 \boldsymbol{E}_0^2 c\,\hat{\boldsymbol{k}}$. Give a brief interpretion this result.

b) Express the average power per unit solid angle $\overline{\mathrm{d}P_{scat}/\mathrm{d}\Omega}$ scattered into a direction $\hat{\boldsymbol{n}}$ from an incident plane-polarized plane wave with electric field amplitude $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{\epsilon}}$ by a single free electron. Express your result in terms of the average power flux $\overline{\boldsymbol{P}_{wave}}$ of the incident plane wave.

c) A unit volume at a location in the corona, for which the direction to the observer $\hat{\boldsymbol{n}}$ is orthogonal to $\boldsymbol{k}$, contains N free electrons. If we assume that each of these scatter radiation independently of each other, what is the total scattered radiation from the given volume element per unit solid angle in direction $\hat{\boldsymbol{n}}$?

d) Let us finally assume the Solar radiation at a given frequency ω to be propagating radially outward as a plane, but randomly polarized wave. What fraction of this radiation will be scattered toward our observer per unit solid angle by the N electrons for the geometry described under point c)? What is the polarization of the scattered radiation?