

UNIVERSITY OF OSLO

AST 4210 Radiation I

Second mid-term take-home exam
(to be followed by oral examinations!)

Deadline: Monday 17/11, 2003 at 16.15

Problem 1

The expression for the transition rates for radiative transitions (absorption, stimulated or spontaneous emission) between two atomic states a and b all contain the matrix element $\langle b | \hat{\mathbf{e}} \cdot \mathbf{r} | a \rangle$. In this problem will study the resonance line of the HI spectrum. We neglect all effects of fine structure and external magnetic or electric fields.

- What is the wavelength of this line?
- Describe the atomic states involved in the formation of this line (electron configurations and spectral terms).
- Give the detailed expression for the wave functions $|a\rangle$ and $|b\rangle$ and the matrix element $\langle b | \hat{\mathbf{e}} \cdot \mathbf{r} | a \rangle$ for the resonance line transition corresponding to $\Delta m_\ell = 0$.
- Perform the integrals involved.
- Can you give an order of magnitude estimate for the natural lifetime of the excited state involved?

Problem 2

Assuming $L - S$ coupling, list all possible spectral levels $^{2S+1}L_J$ for the electron configuration nd^2 in the order of increasing energy. Which state represent the ground state? Which principles are your result based on?

Problem 3

In figure 5.7 in the compendium the $v = 0$ to $v = 1$ vibrational-rotational absorption spectrum for HBr is shown. The non-constant line spacing indicates that the harmonic oscillator, rigid rotor approximation is not strictly valid, but that the rotational constant B_v will depend on the vibrational quantum number v .

- a) With this assumption argue that the frequencies of the lines of the P- and R-branches in the spectrum can be expressed as

$$h\nu_P(N) = h\nu_0 + B_1(N-1)N - B_0N(N+1) \quad (1)$$

$$h\nu_R(N) = h\nu_0 + B_1(N+1)(N+2) - B_0N(N+1). \quad (2)$$

- b) Identify the P- and R-branches in the spectrum. Also identify the “ N ”-value (according to (1) and (2)) for each of the lines in the spectrum.
- c) Show that

$$\begin{aligned} h\nu_R(N) - h\nu_P(N) &= 2B_1(2N+1) \\ h\nu_R(N) - h\nu_P(N+2) &= 2B_0(2N+3). \end{aligned}$$

- d) Determine the values for B_0 , B_1 , the equilibrium distance between the two nuclei R_e , and the “spring stiffness” k of the molecule.

In electron-vibrational-rotational transitions, that is transitions where also the electron configuration changes, even larger differences in the values for the rotational constants for the two states must be expected.

- e) Discuss the following claim: “Electronic-vibrational-rotational spectra of diatomic molecules may display *band-head structures*, that is, a concentration of individual lines - either upward or downward - toward a limiting frequency”.

[*Hint*: Discuss the limiting behavior of (1) and (2) for large N for the two cases $B_1 < B_0$ and $B_1 > B_0$.]