UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam AST 4210 — Radiation I
Date of exam: December 9, 2004
Time of exam: 1430-1730
This exercise set contains 3 pages
Attachments: None
Allowed sources of help: Any non-communicating source
Check that the exercise set is complete before you start solving the problems.

Exercise I

With the help of a spectroscope the Na I resonace line (Na I D line) is studied under 4 different situations:

i) from a flame sprinkled with salt (NaCl) against a dark background
ii) from a flame sprinkled with salt (NaCl) against an intense continuous light source
iii) from the Solar disk
iv) from the Chromosphere (just outside the Solar disk) during a Solar eclipse.

Under two of these situations an emission line is observed, for the other two an absorption line results. We want to investigate the reasons for the different results. Give your answers in **precise** and **brief** form.

a) State the radiative transport equation, explain the different parameters and variables involved and give their physical dimensions.

Radiation is emerging from a homogeneous thick (semi-infinite) slab in thermal equilibrium (TE) at temperature $\mathcal{T}_R$ (see figure Ia). We consider the frequency range (ν_0, ν_1) for which the extinction coefficent α_ν is of the form illustrated in figure Ib.

b) Which physical processes can be responsible for the particular form of α_ν?

c) Find the spesific intensity I_ν^R that will be observed as a function of frequency ν and angle θ, $\mu = \cos\theta$.

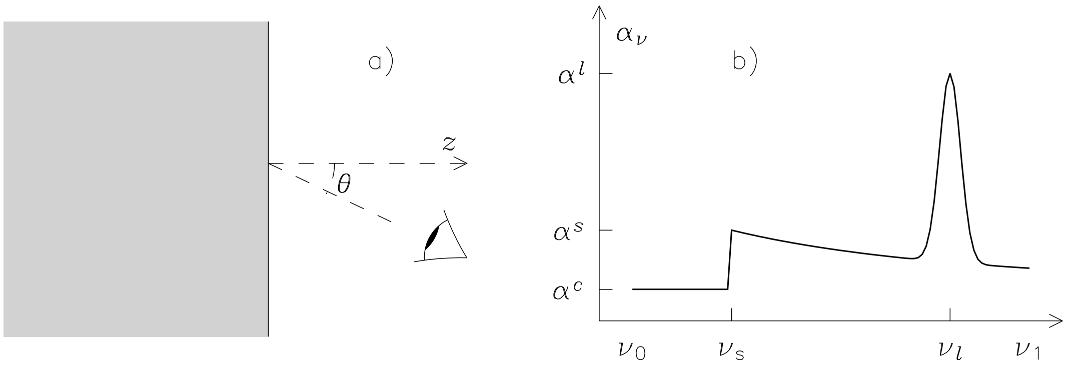

Figure 1: Semi-infinite slab and extinction coefficient

We next consider the radiation emerging from a homogeneous thin slab of thickness L. For simplicity we assume the slab to be held in TE, now at temperature $\mathcal{T}_L$. Let the extinction coefficient α_ν for the thin slab again be of the form given figure Ib. The slab is irradiated from behind by a spesific intensity I_ν^R.

d) If $\alpha_\nu^\ell L > 1$, $\alpha_\nu^s L < 1$ and $\alpha_\nu^c L \ll 1$, describe qualitatively the intensity I_ν^L that will be observed this time as a function of frequency ν and direction μ for the case that 1) $\mathcal{T}_L > \mathcal{T}_R$
and 2) $\mathcal{T}_L < \mathcal{T}_R$.
In particular, give the explicit value of I_ν^L for $\nu = \nu_l$.

e) On the basis of the results above explain what type of line you expect to observe under the 4 situations described in the introduction.

Exercise II

An external electric field $\boldsymbol{E}$ will modify the energy levels of atoms. This effect is called Stark effect. For the excited energy states of H I linear Stark effect is the dominating one, thus the $n = 2$ energy level is split into $2n - 1 = 3$ equidistant levels with energy separtion

$$\Delta W = 3 a_B e E, \tag{1}$$

where a_B is the Bohr radius.

The line widths in stellar spectra, for instance the lines in the hydrogen Balmer series, are determined by several factors: thermal and rotational Doppler effects, natural line width due to finite lifetime of excited states, collisional broadening, ..., but also Stark effect. We shall consider the Stark effect due to the electric microfields from "relatively slow-moving" H^+ ions in a stellar atmosphere consisting mainly of partially ionized hydrogen gas.

a) The distance between a given H I atom and its nearest H^+ ion varies with time. Argue that a typical maximum distance is $r_{\max} \approx n_e^{-1/3}$ where n_e is the electron density.

A typical minimum distance will be a certain fraction f of this maximum value, $r_{\text{min}} \approx f r_{\text{max}}$. [A statistical analysis will be needed to estimate f. For the following discussion we assume that f is in the range 0.1-0.01.]

b) What is the corresponding typical electric field E seen by the atom?

c) Based on (1) give an estimate for a typical line width $(\Delta\lambda_S/\lambda)$ due to the Stark effect for the Balmer series lines. [For a minimum estimate the energy splitting of the $n > 2$ levels can be neglected.]

For a certain line number k in the Balmer series this line width will be comparable to the wavelength distance between subsequent lines.

d) Express the distance $(\Delta\lambda_k/\lambda)$ between subsequent lines in the Balmer series as a function of line number k.

e) Estimate the line number k where the distance between subsequent lines equals the (minimum) Stark effect line width.

f) Check that thermal Doppler effect $(\Delta\lambda_D/\lambda)$ is not completely masking the Stark effect line broadening. Give a brief argument for your choice of typical temperature T. What is your conclusion?

It is claimed that: "The line number of the highest frequency Balmer line that is still separately discernable allows for a rough first estimate of the electron density in the stellar atmosphere".

g) Do you find this claim reasonable? If so, give an expression for n_e as a function of this line number k.