FYS-KJM 4740

MR-teori og medisinsk diagnostikk

Kap 5
Steady-state sekvenser

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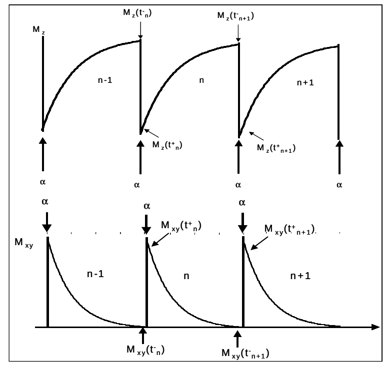
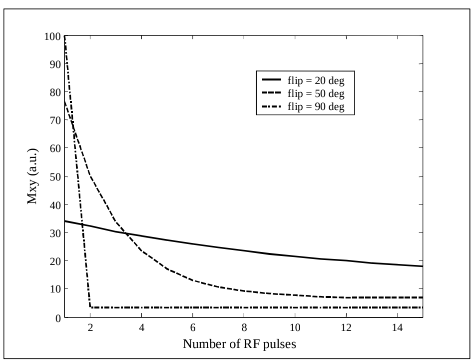
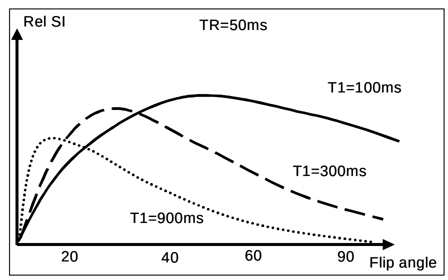


STEADY STATE: $M(t^+_{n+1})=RM(t^-_{n+1})=M(t^+_n)$



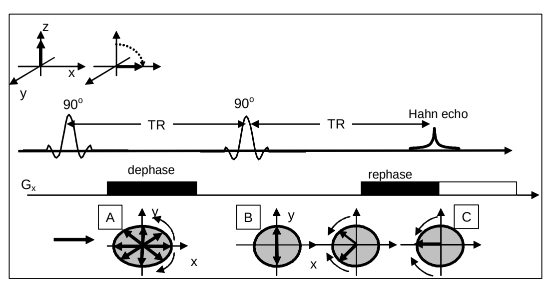
‘Spoiled’ Gradient Echo



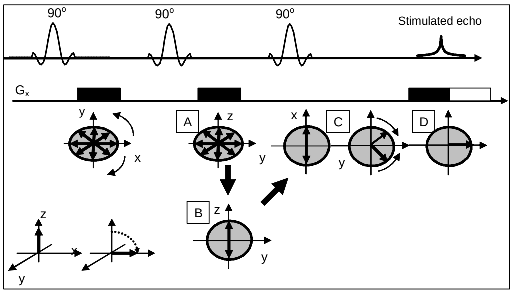
Steady state sequences

- Multiple RF-pulses generate a multitude of ‘echoes’
- The magnetization (x,y, and z) in each TR-interval will in general carry history from previous TR-intervals.
- The magnitude of the different echo signals is generally position (gradient) dependent.

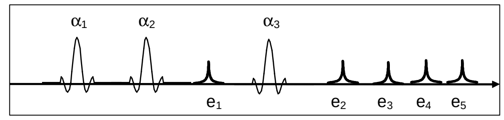
2 RF-pulses = ‘Eight-ball’ echo

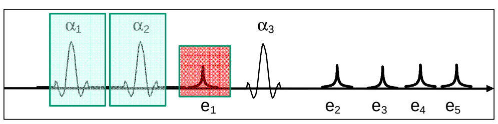


3 RF-pulses = 'Stimulated' echo

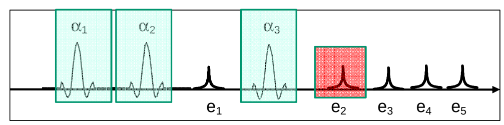


3 RF-pulses gives a total of 5 echoes:

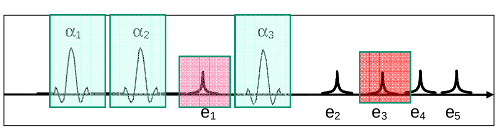




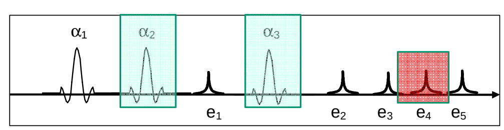
e_1 = eight-ball echo, α_1, α_2



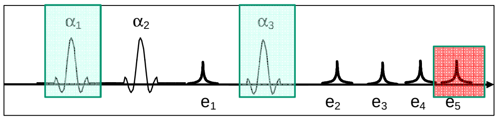
e_2 = Stimulated echo, α_1, α_2, α_3



e_3 = refocussed (by α_3) α_1



e_4 = eight-ball echo, α_2, α_3

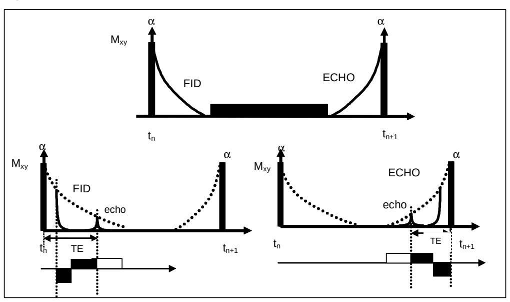


e_5 = eight-ball echo, a_1, a_3

The FID and ECHO signals

- ECHO (position dependent): total transverse magnetization prior to an RF-pulse
- FID (position dependent): total longitudinal magnetization prior to an RF-pulse

Generation of echoes either from the FID or the ECHO signal



Steady-state signal behaviour

Fra kap 2:

Excitation

$$\mathbf{R}_{\alpha,x'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

Precession

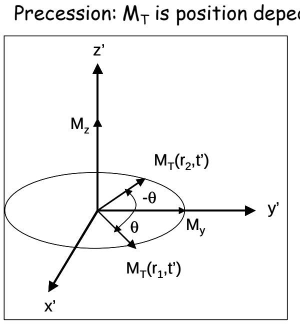
$$\mathbf{P}_{\theta,z'} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Relaxation

$$\mathbf{M}(t^-{}_{n+1}) = \begin{bmatrix} E_2 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix} \mathbf{M}(t^+{}_n) + (1-E_1)\mathbf{M_0}$$

$\mathbf{M} = [M_x, M_y, M_z]^T$, $\mathbf{M_0}=[0,0,M_0]^T$, $E_2 = \exp(-t/T2)$ and $E_1=\exp(-t/T1)$.

Precession: M_T is position depedent



$\theta(\mathbf{r},t) = -\gamma \int \mathbf{G} \cdot \mathbf{r} dt$

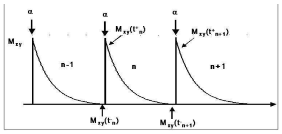
Steady-state signal behaviour

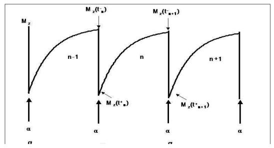
Relaxation and Precession occurs simultaneously and can be combined:

$$\mathbf{M}(t^-{}_{n+1}) = \begin{bmatrix} E_2\cos(\theta) & E_2\sin(\theta) & 0 \\ -E_2\sin(\theta) & E_2\cos(\theta) & 0 \\ 0 & 0 & E_1 \end{bmatrix} \mathbf{M}(t^+{}_n) + (1-E_1)\mathbf{M_0}$$

$$\mathbf{M}(t^-{}_{n+1}) = \mathbf{Q}(E1, E2, \theta)\mathbf{M}(t^+{}_n) + (1-E_1)\mathbf{M_0}$$

Steady-state





$$\mathbf{M}(t^-{}_{n+1}) = \mathbf{Q}(E1, E2, \theta)\mathbf{M}(t^+{}_n) + (1 - E_1)\mathbf{M_0}$$

$$\mathbf{M}(t^+{}_{n+1}) = \mathbf{R_\alpha} M(t^-{}_{n+1}) = \mathbf{M}(t^+{}_n)$$

$$(\mathbf{U} - \mathbf{QR}_\alpha)\mathbf{M}(t^+{}_n) = (1 - E_1)\mathbf{R_\alpha M_0}$$

U = unit matrix

Steady-state

$$(\mathbf{U} - \mathbf{QR}_\alpha)\mathbf{M}(t^+{}_n) = (1 - E_1)\mathbf{R_\alpha M_0}$$

$$\mathbf{M}(t^+{}_n) = (\mathbf{U} - \mathbf{QR}_\alpha)^{-1}(1 - E_1)\mathbf{R_\alpha M_0}$$

$M_T = M_{x'} + jM_{y'}$

$$M_T(x, y, t^+{}_n) = M_0(x, y)\frac{(1 - E_1)\sin(\alpha)(1 - E_2\exp(-j\theta))}{C\cos(\theta) + D} = M_0(x, y)F^+(\alpha, \theta)$$

$$C = E_2(E_1 - 1)(1 + \cos(\alpha)) \quad D = (1 - E_1\cos(\alpha)) - (E_1 - \cos(\alpha))E_2{}^2$$

Steady-state

Transverse magn. immediately after n[th] RF pulse:

$$M_T(x, y, t^+{}_n) = M_0(x, y)\frac{(1 - E_1)\sin(\alpha)(1 - E_2\exp(-j\theta))}{C\cos(\theta) + D} = M_0(x, y)F^+(\alpha, \theta)$$

$E_2=0$

$$M_T(x, y, t^+{}_n) = M_0(x, y)\frac{\sin(\alpha)(1 - E_1)}{1 - \cos(\alpha)E_1}$$

- The equation reduces to the 'spoiled' GRE expression in when E_2=exp(-t/T2)=0 (TR >> T2 or active spoiling). In this case, the factor D in the denominator depends on T1-weighting (through E_1) only.

Steady-state

$$M_T(x, y, t^+{}_n) = M_0(x, y)\frac{(1 - E_1)\sin(\alpha)\boxed{1 - E_2\exp(-j\theta)}}{C\cos(\theta) + D} = M_0(x, y)F^+(\alpha, \theta)$$

- The term 1-E_2exp(-jθ) in the numerator implies a varying degree of T2-weighting, dependent on θ, and hence position. This means that, if E_2 >0 the image will contain 'bands' of varying degree of T1- and T2-weighting depending on the value of exp(-jθ(**r**)) at a given position .

Steady-state: magnetization evolution between RF-excitations

Transverse magn. during n[th] TR-interval:

$$M_T(x, y, t) = M_T(x, y, t^+{}_n)\boxed{\exp(-t/T2)}\boxed{\exp(j\varphi(r, t))}$$

relaxation

Gradient effects

$$\varphi(r, t) = \gamma\int_{t_n}^{t} G(r, t)dt$$

Steady-state: from magnetization to FID signal

Transverse magn. during n[th] TR-interval:

$$M_T(x, y, t) = M_T(x, y, t^+{}_n)\exp(-t/T2)\exp(j\varphi(r, t))$$

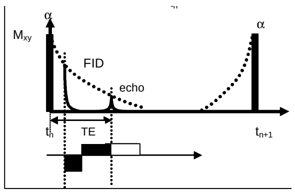
$\Phi(r,t)=G_x tx + G_{y,n}T_y y$ (see chapt 2)

FID signal:

$$S(t') = \exp(-t/T2^*)\int_x\int_y M_0(x, y)F^+(\alpha, \theta)\exp(j\gamma(G_x t' x + G_{y,n}T_y y))dxdy$$

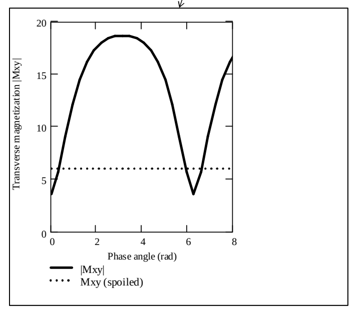
The FID signal

$$M_T(x,y,t^+_n) = M_0(x,y)\frac{(1-E_1)\sin(\alpha)(1-E_2\exp(-j\theta))}{C\cos(\theta)+D} = M_0(x,y)F^+(\alpha,\theta)$$



$$S(t') = \exp(-t/T2^*)\int_x\int_y M_0(x,y)F^+(\alpha,\theta)\exp(j\gamma(G_x t' x + G_{y,n}T_y y))dxdy$$

$$S(t') = \exp(-t/T2^*)\int_x\int_y M_0(x,y)F^+(\alpha,\theta)\exp(j\gamma(G_x t' x + G_{y,n}T_y y))dxdy$$



The ECHO signal

M_T just before the n+1th RF-pulse

$$M_T(x,y,t^-_{n+1}) = M_T(x,y,t^+_n)\exp(-TR/T2)\exp(j\theta(x,y))$$

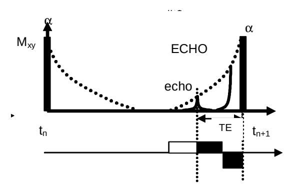
$E_2=\exp(-TR/T2)$

$$M_T(x,y,t^-_{n+1}) = M_0(x,y)\frac{\sin(\alpha)(1-E_1)(\exp(j\theta)-E_2)E_2}{C\cos(\theta)+D}$$

$$S(t') = \exp(TE/T2)\int_x\int_y M_0(x,y)F^+(\alpha,\theta)\exp(j\gamma(G_x t' x + G_{y,n}T_y y))dxdy$$

The ECHO signal

$$M_T(x,y,t^-_{n+1}) = M_0(x,y)\frac{\sin(\alpha)(1-E_1)(\exp(j\theta)-E_2)E_2}{C\cos(\theta)+D}$$



$$S(t') = \exp(TE/T2)\int_x\int_y M_0(x,y)F^+(\alpha,\theta)\exp(j\gamma(G_x t' x + G_{y,n}T_y y))dxdy$$

FID vs ECHO

FID:

$$M_T(x,y,t^+_n) = M_0(x,y)\frac{(1-E_1)\sin(\alpha)(1-E_2\exp(-j\theta))}{C\cos(\theta)+D}$$

ECHO:

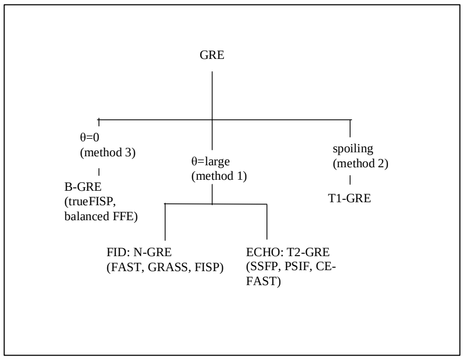
$$M_T(x,y,t^-_{n+1}) = M_0(x,y)\frac{\sin(\alpha)(1-E_1)(\exp(j\theta)-E_2)E_2}{C\cos(\theta)+D}$$

For both signals, T2-contribution to magnetization is function of position (through exp(jθ))

Methods to eliminate phase effect

1. Make θ large (large net gradient surface) relative to the voxel size so that the bands of varying T1- and T2-weighting is averaged out over each voxel.
2. Apply RF- or gradient spoiling (E2=0); removing all transverse coherence prior to each RF-pulse.
3. Make θ = 0 by balancing all gradients at the time of the signal readout

Steady-state sequences



Steady-state sequences in practice:

T1-GRE (fid, spoiling; E2=0)

$$M_T(x,y,t^+_n) = M_0(x,y)\frac{\sin(\alpha)(1-E_1)}{1-\cos(\alpha)E_1}$$

N-GRE (FID, large θ)

$$\langle M^+_T(x,y)\rangle = M_0(x,y)\frac{(1-E_1)\sin(\alpha)}{C}\left(\frac{C+DE_2}{\sqrt{D^2-C^2}}-E_2\right)$$

T2-GRE (ECHO, large θ)

$$\langle M^-_T(x,y)\rangle = M_0(x,y)\frac{(1-E_1)E_2\sin(\alpha)}{C}\left(1-\frac{D+CE_2}{\sqrt{D^2-C^2}}\right)$$

Steady-state sequences in practice:

B-GRE (balanced FFE, θ=0)

$$M_T = M_0\frac{\sin(\alpha)(1-E_1)\sqrt{E_2}}{1-(E_1-E_2)\cos(\alpha)-E_1E_2}$$

$TR \ll T1, T2$ and $\alpha > \alpha_e$:

$$M_T = M_0\frac{\sin(\alpha)}{1+\cos(\alpha)+(1-\cos(\alpha))(T1/T2)} \approx \frac{\sin(\alpha)}{1-\cos(\alpha)}\frac{T2}{T1}$$

Steady-state sequences in practice:

B-GRE (balanced FFE, θ=0)

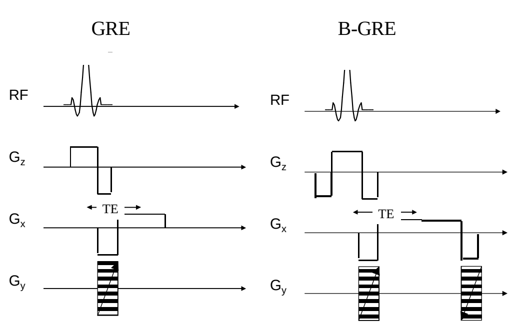
$$M_T = M_0\frac{\sin(\alpha)}{1+\cos(\alpha)+(1-\cos(\alpha))(T1/T2)} \approx \frac{\sin(\alpha)}{1-\cos(\alpha)}\frac{T2}{T1}$$

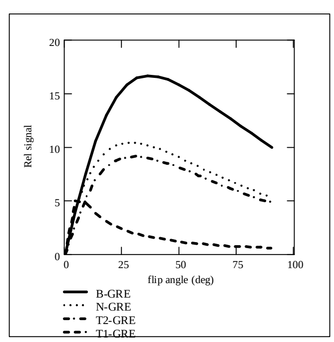
At optimal flip angle:

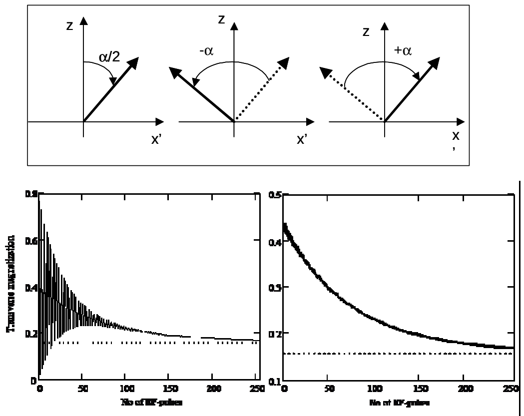
$$\cos(\alpha_e) = \frac{T1-T2}{T1+T2}$$

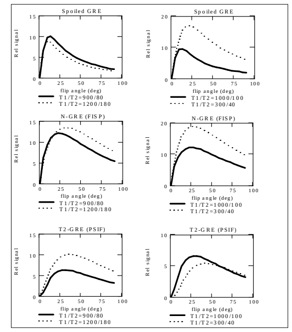
$$M_{T,peak} = M_0\frac{1}{2}\sqrt{T2/T1}$$

Very high signal-noise!

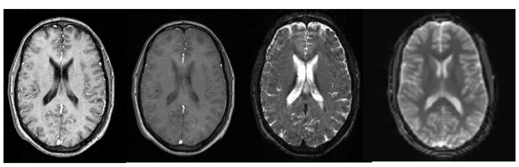




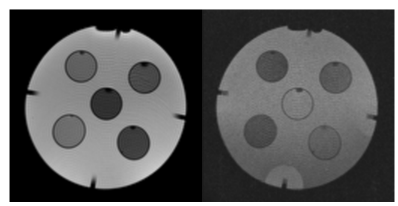




Steady-state sequences in practice:

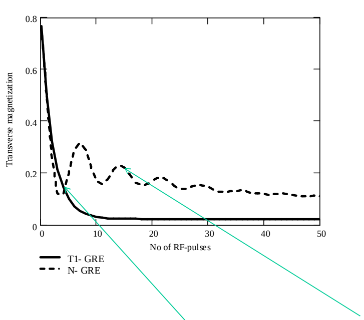


T1-GRE N-GRE T2-GRE B-GRE



B-GRE T1-GRE

Transient signal response



blurring

blurring and ghosting

Increasing flip-angle during transient state