FYS-KJM 4740

MR-teori og medisinsk diagnostikk

Kap 2 (forts)
Bildedannelse & K-space

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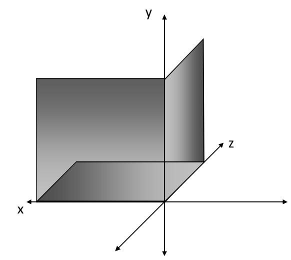
Magnetfelt-gradient



$\delta B_{G,z}(z) = G_z z$

$\delta B_{G,z}(y) = G_y y$

$\delta B_{G,z}(x) = G_x x$

Fase-effekt av gradient (f.eks i y-retn):

$$\alpha(\mathbf{r},t) = -\gamma \int_0^t G_y(t) \cdot \mathbf{r} \cdot d\tau$$

Transversal magnetisering er da gitt ved (fra Bloch's likn – ser bort fra T2-relaks):

$$M_{xy} = M_T(\mathbf{r},t) = M_T(\mathbf{r},0) \cdot \exp\left(-j\gamma \int_0^t G_y(t)\mathbf{r}d\tau\right)$$

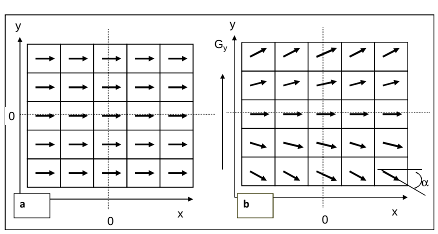
MR-signal =integral av transv. magnetisering over posisjon:

$$M_T(t) \propto S(t) \propto \iiint \rho(\mathbf{r}) \exp\left(-j\gamma \int_0^t G_y(t)\mathbf{r}d\tau\right) d\mathbf{r}$$

NB: Fourier transform

$$F(k) = \iiint_R f(\mathbf{r}) \exp(-jk\mathbf{r}) d\mathbf{r}$$

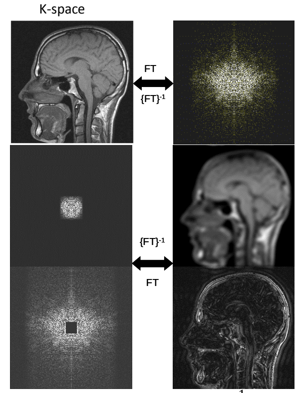
$$S(t) \propto \iiint_R \rho(\mathbf{r}) \exp\left(-j\gamma \int_0^t G(t)\mathbf{r}d\tau\right) d\mathbf{r}$$

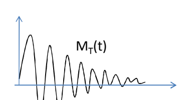


The phase angle of the transverse magnetization vector before (a) and after (b) the application of a magnetic field gradient in the y-direction.

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \qquad M_T(t) = \iint_{slice} \rho(\mathbf{r}) \cdot \exp(-j\mathbf{k} \cdot \mathbf{r}) d\mathbf{r}$$

K-space

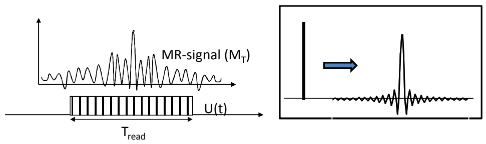




$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$\rho(x,y) = \frac{1}{2\pi} \int_{k_x} \int_{k_y} M_T(k_x,k_y) \exp\left(j(k_x x + k_y y)\right) dk_x dk_y$$

Discrete sampling

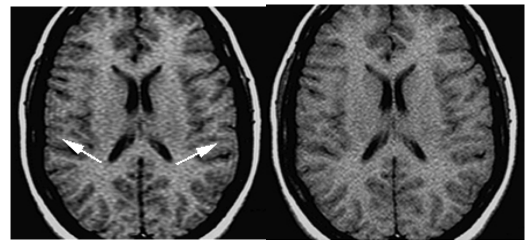


Sampling intervall: $-T_{read}/2$ - $T_{read}/2$

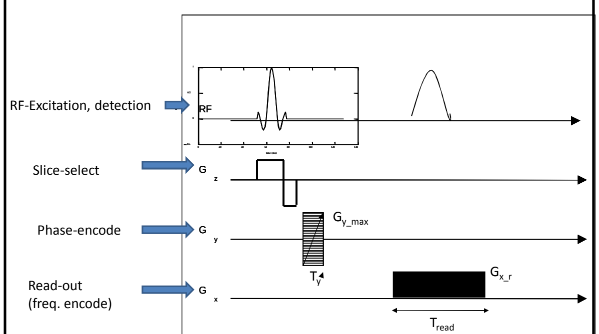
$$PSF(x) = FT(U(t)) = \int_{-T_r/2}^{T_r/2} \exp(-j\gamma G_x x t)\, d\tau = T_r \frac{\sin\left(\frac{\gamma G_x x T_r}{2}\right)}{\frac{\gamma G_x x T_r}{2}}$$

NB: feil i kompendiet (eq. 2.24)!

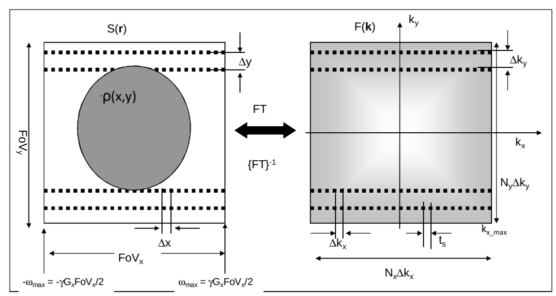
Truncation artifact



Pulse sequence



K-space vs image space



$$\rho(x,y) = \frac{1}{2\pi} \int_{k_x}\int_{k_y} M_T(k_x,k_y) \exp\left(j\left(k_x x + k_y y\right)\right) dk_x\, dk_y$$

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)\, d\tau$$

K-space egenskaper

Resolution (x):

$$\delta x = \frac{2\pi}{\gamma G_x N_x t_s}$$

Field of view (x):

$$\lambda_{x,\max} = \frac{2\pi}{k_{x,\min}} = \frac{2\pi}{\gamma G_x t_s} = FoV_x$$

Field of view (y):

$$\lambda_{y,\max} = FoV_y = \frac{\pi N_y}{\gamma G_{y\_\max} T_y}$$

Maximum frequency in read-out (x) direction

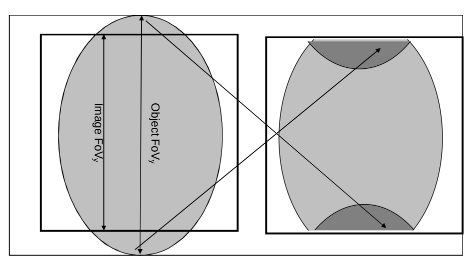
$$\pm \omega_{\max} = \pm \gamma G_x FoV_x / 2$$

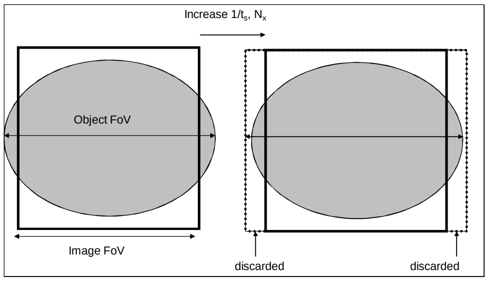
Min sampling rate (x):

$$1/t_s \geq \gamma G_x FoV_x / 2\pi$$

'Sampling rate ' (y):

$$N_y = \gamma G_{y\_\max} T_y FoV_y$$




Increase 1/t_s, N_x
Object FoV
Image FoV
discarded
discarded