

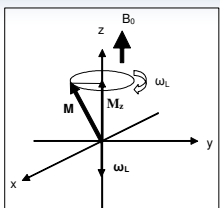
# MRI

FYS-KJM 4740

# Chap 1

Bloch equations and main principles

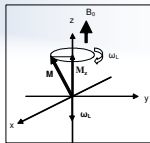
### The Bloch equation



$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M} \times \mathbf{B})$$

The magnetic moment  $\mathbf{M}$  rotates around the static  $\mathbf{B}$ -field at the Larmor frequency

### Larmor equation




$\omega_0 \text{ (rad/s)} = \gamma \text{ (rad/s/Tesla)} \times B_0 \text{ (Tesla)}$

$\gamma_{\text{hydrogen}} = 2.68 \times 10^8 \text{ rad/s/Tesla}$

/2π

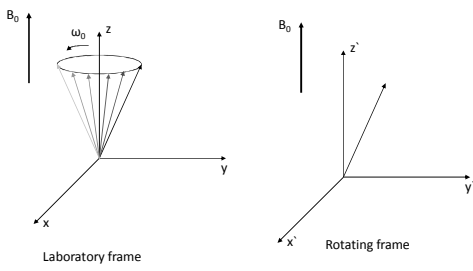
$f_L \text{ (MHz)} = \gamma \text{ (MHz/Tesla)} \times B_0 \text{ (Tesla)}$

$\gamma_{\text{hydrogen}} = 42.58 \text{ MHz/Tesla}$



Joseph Larmor

### Rotating frame of reference

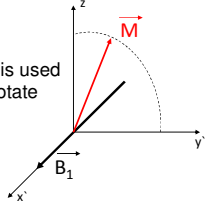


Laboratory frame      Rotating frame

### Flipping away the Magnetization from its equilibrium

In MRI, short RF pulses are used to change the direction of the magnetization  $\mathbf{M}$

To get  $\mathbf{M}$  to rotate around x or y axis, A linearly polarized magnetic field  $\mathbf{B}_1$  is used during short time (pulse) to get  $\mathbf{M}$  to rotate around  $\mathbf{B}_1$  axis



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### Nuclear Induction

E. Ruck  
Stanford University, California  
(Received July 20, 1964)

The magnetic resonance of nuclei in external magnetic fields is a well-known phenomenon. The observation of nuclear induction in a constant magnetic field is a new phenomenon. It is shown that a radiofrequency field at right angles to the constant field causes a forced precession of the nuclear magnetic moment about the constant field. This precession is observed as a nuclear induction signal. It is shown that the nuclear induction signal is proportional to the square of the constant field. The nuclear induction signal is observed in a constant magnetic field. The nuclear induction signal is observed in a constant magnetic field. The nuclear induction signal is observed in a constant magnetic field.

**1. INTRODUCTION**

The method of magnetic resonance<sup>1</sup> has been successfully applied to measure the magnetic moment of the neutron<sup>2</sup> and of various nuclei.<sup>3</sup> The principal feature of this method is the observation of transitions, caused by resonance, in the absorption of a radiofrequency field with the Larmor precession of the magnetic moment in a constant magnetic field. In the application of this method to the detection of nuclear induction, the detection of nuclear induction in an inhomogeneous field was used as a means of detecting the occurrence of nuclear induction. This method of detection has proven to be very difficult but it was clear, at the time, that the connection between nuclear induction and magnetic resonance was not of basic character. The question arose, in particular, whether nuclear induction could not be detected by far simpler electromagnetic methods, the attempt to do this was made, and was undertaken by Corser and Brown<sup>4</sup> whose arrangement was designed to induce magnetic resonance absorption by a slight change in frequency of an electric oscillator. The experiment was based upon considerations which apply strictly to radiofrequency fields which are so small that change is not observed by its relatively small reaction upon the driving circuit, but by directly observing the induced electromotive force in a coil, due to the precession of the nuclear moments around the constant field and in a direction perpendicular to it.

The conditions upon which our work was based have several features in common with the experiments previously mentioned, but differ essentially in others. In the first place, the radiofrequency field is deliberately chosen large enough so to cause a resonance, a considerable change of orientation of the nuclear moments. In the second place, this change is not observed by its relatively small reaction upon the driving circuit, but by directly observing the induced electromotive force in a coil, due to the precession of the nuclear moments around the constant field and in a direction perpendicular to it.

<sup>1</sup> E. Ruck, Phys. Rev. **21**, 432 (1953).  
<sup>2</sup> E. Ruck and P. Ruck, Phys. Rev. **97**, 111 (1955).  
<sup>3</sup> E. Ruck, S. Wilson, P. Ruck, and J. R. Zacharias, Phys. Rev. **101**, 101 (1956).

### Resonance Absorption by Nuclear Magnetic Moments in a Solid

E. M. Purcell, R. C. Torrey, and H. V. Dreyfus  
Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts  
(Received December 15, 1946)

The well-known magnetic resonance method for the detection of nuclear magnetic moments in solids is based upon the observation of the absorption of a radiofrequency field by the nuclei. The absorption is observed as a change in the amplitude of the field. The absorption is observed as a change in the amplitude of the field. The absorption is observed as a change in the amplitude of the field.

The distribution of nuclear magnetic moments in solids is a function of the orientation of the nuclei. The distribution of nuclear magnetic moments in solids is a function of the orientation of the nuclei. The distribution of nuclear magnetic moments in solids is a function of the orientation of the nuclei.

The distribution of nuclear magnetic moments in solids is a function of the orientation of the nuclei. The distribution of nuclear magnetic moments in solids is a function of the orientation of the nuclei. The distribution of nuclear magnetic moments in solids is a function of the orientation of the nuclei.

### Baseline, no phase coherence in transverse (X,Y) plane

Zero net  $M_x$  and  $M_y$  components

### Additional B1-field induces phase coherence in transverse (X,Y) plane

Net  $M_{x,y}$  component > 0

### Rotation

The magnetic field due to the RF pulse,  $B_1$ , is generated by two circularly polarized fields with opposing direction of rotation with angular frequency  $\pm \Omega$

$$B_1 = B_{1+} + B_{1-} = B_1 \begin{bmatrix} \cos(-\Omega t) \\ \sin(-\Omega t) \\ 0 \end{bmatrix} + B_1 \begin{bmatrix} \cos(\Omega t) \\ \sin(\Omega t) \\ 0 \end{bmatrix} = 2B_1 \begin{bmatrix} \cos(\Omega t) \\ 0 \\ 0 \end{bmatrix}$$

The RF-coil generates a magnetic field  $B_1$  along the x-axis

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### Bloch equation

The 'rotating frame' ( $x', y', z'$ -coordinates)

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}} \quad \mathbf{B}_{\text{eff}} = \mathbf{B}_0 + \mathbf{B}_1 + \Omega/\gamma \quad \Omega = \begin{bmatrix} 0 \\ 0 \\ -\Omega \end{bmatrix}$$

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### RF-eksitasjon med Larmor frekvens (rotating frame)

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}}$$

$$\Omega = \gamma B_0$$

$$\mathbf{B}_{\text{eff}} = \mathbf{B}_1$$

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### Using Matrix formalism

$$\frac{d\mathbf{M}}{dt} = \gamma \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B_{1x} \\ 0 & -B_{1x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

$$\downarrow$$

$$M_y = A \sin(\gamma B_{1x} t) + B \cos(\gamma B_{1x} t)$$

$$\omega_1 = -\gamma B_1$$

Using the boundary conditions  $M_y = M_y(0)$  and  $M_z = M_z(0)$  at  $t=0$ , we get

$$\mathbf{M}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega t) & \sin(\omega t) \\ 0 & -\sin(\omega t) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{pmatrix} = \mathbf{R} \cdot \mathbf{M}_0$$

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### RF pulse

$$\omega_1 = -\gamma B_1$$

RF pulse duration is proportional to the wanted flip angle,  $\alpha$ .

$$t_{B_1} = \alpha / \gamma B_1$$

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### Relaxation

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = -\frac{M_y}{T_2}$$

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \gamma \Delta B_0$$

RF-pulse

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### Relaxation

$$\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1}$$

RF-pulse

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### Relaxation

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{eff} - \mathbf{R}(\mathbf{M} - \mathbf{M}_0)$$

$$\mathbf{R} = \begin{bmatrix} 1/T_2 & 0 & 0 \\ 0 & 1/T_2 & 0 \\ 0 & 0 & 1/T_1 \end{bmatrix} \quad \mathbf{M}_0 = \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

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Condition: relaxation during RF excitation is neglected

$$\frac{d\mathbf{M}}{dt} = -\mathbf{R}(\mathbf{M} - \mathbf{M}_0)$$

$$M_z(t) = M_0 [1 - \exp(-t/T_1)] + M_z(0) \exp(-t/T_1)$$

$$M_{xy}(t) = M_{xy}(0) \exp(-t/T_2)$$

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### Summary with excitation and relaxation

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{eff} - \mathbf{R}(\mathbf{M} - \mathbf{M}_0)$$

$$\frac{d\mathbf{M}}{dt} = \begin{bmatrix} -1/T_2 & 0 & 0 \\ 0 & -1/T_2 & \gamma B_{1x} \\ 0 & -\gamma B_{1x} & -1/T_1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

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## Chap. 2

Slice-Selective RF excitation  
Image formation

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### Slice Selective RF pulse

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### Gradients → coding in space

Use of field gradient pulses in the 3 directions

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### Magnetic Field Gradients

$\delta B_{\theta,z}(z) = G_z z$   
 $\delta B_{\theta,z}(y) = G_y y$   
 $\delta B_{\theta,z}(x) = G_x x$

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### Principles of Slice Selection

RF frequency  
Band width  
Field gradient G  
Slice thickness

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### Magnetfelt gradient

Gradient styrke (mT/m)  
Stigningstid (ms)  
Slew rate (mT/m/ms) =  $\frac{\text{Gradient styrke}}{\text{Stigningstid}}$

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### RF pulse generation

FAVOURABLE SITUATION

B1 applied for a certain duration (ms)    B1 pulse frequency profile (HZ)

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### Effect of Magnetic Field Gradient

Example: use of z field gradient

$\omega = \gamma \times (B_0 + g z)$   
 Spatial information included in the precession frequency  
 Applied field gradient (g)  
 precession frequency  $\omega$ :  
 $\omega_0 = \gamma \times B_0$

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### Slice selective RF excitation

$$\mathbf{B}_{\text{eff}} = \mathbf{B}_0 + \mathbf{G} \cdot \mathbf{r} + \mathbf{B}_1 + \frac{\Omega}{\gamma} = \begin{bmatrix} B_1 \\ 0 \\ B_0 + G_z z_1 - \frac{\Omega}{\gamma} \end{bmatrix}$$

If we set:  $\Omega = \gamma(B_0 + G_z z_1)$ , then the effective field at  $z = z_1$  becomes:

$$\mathbf{B}_{\text{eff}} = \begin{bmatrix} B_1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{Slice-selective excitation @ } z = z_1$$

What would happen if  $B_0 + G_z z_1 - \Omega/\gamma \gg B_1$  ??

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### All effects together: excitation, precession and relaxation

$\Omega = \omega_L ; B_{eff} = B_{1x}$

$$\frac{d\mathbf{M}}{dt} = \begin{pmatrix} -1/T_2 & \gamma \mathbf{G} \cdot \mathbf{r} & 0 \\ -\gamma \mathbf{G} \cdot \mathbf{r} & -1/T_2 & \gamma B_{1x} \\ 0 & -\gamma B_{1x} & -1/T_1 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ M_0/T_1 \end{pmatrix}$$

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### All effects together: excitation, precession and relaxation

Transverse ( $M_{xy}$ ) relaxation

Precession around z-axis

Excitation around x-axis

Longitudinal ( $M_z$ ) relaxation

$$\frac{d\mathbf{M}}{dt} = \begin{pmatrix} -1/T_2 & \gamma \mathbf{G} \cdot \mathbf{r} & 0 \\ -\gamma \mathbf{G} \cdot \mathbf{r} & -1/T_2 & \gamma B_{1x} \\ 0 & -\gamma B_{1x} & -1/T_1 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ M_0/T_1 \end{pmatrix}$$

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### Transverse magnetization, $M_{xy}$ , Relaxation and precession

$M_T = M_x + jM_y$   $B_{1x} = 0$

derive!

$$M_T = M_T(0) \exp(-j\mathbf{r} \cdot \int \mathbf{G}(t) dt) \exp\left(-\frac{t}{T_2}\right)$$

Longitudinal magnetization,  $M_z$  derive!

$$M_z(t) = M_0 \left[ 1 - \exp\left(-\frac{t}{T_1}\right) \right] + M_z(0) \exp\left(-\frac{t}{T_1}\right)$$

As we have seen before

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### Transverse Magnetization, $M_{xy}$ , Excitation and Precession

$M_T = M_x + jM_y$   $T_2 = \infty$

Condition:  $M_z = M_0$  (How can this be achieved?)

$$\frac{dM_T}{dt} = -j\gamma(\mathbf{G} \cdot \mathbf{r})M_T + j\gamma B_1 M_0$$

General Solution:

$$M_T = A(t) \exp\left(-j\mathbf{r} \cdot \int_{t_i}^t \mathbf{G}(t') dt'\right)$$

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### Transversal magnetisering, $M_{xy}$ , Eksitasjon og presesjon

$T_2 = \infty$

Dersom vi sier at RF puls starter ved  $-T/2$  og varer i  $T$  sek:

$$M_T(T/2, \mathbf{r}) = j\gamma M_0 \int_{-T/2}^{T/2} B_1(t) \exp\left(-j\mathbf{r} \cdot \int_t^{T/2} \mathbf{G}(t') dt'\right) dt$$

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### For a constant gradient along the z-axis: $G(t) = G_z$

$$M_T(T/2, z) = j\gamma M_0 \exp(-j\gamma G_z T/2) \int_{-T/2}^{T/2} B_1(t) \exp(j\gamma G_z t) dt$$

Phase dispersion

Fourier transform of the  $B_1$  "envelope" (green shape)

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For a constant gradient along the z-axis:  $G(t) = G_z$

$$M_T(T/2, z) = j\gamma M_0 \exp(-j\gamma G_z T/2) \int_{-T/2}^{T/2} B_1(t) \exp(j\gamma G_z t) dt$$

Slice profile = Fourier transform of  $B_1(t)$   
The phase of  $M_T(z)$  in the x-y plane is a function of z

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Elimination of the phase dispersion in x-y plane use of an extra gradient of opposite polarity and half the length:  $-G_z$

This gives

$$M_T(T, z) = jM_0 \int_{-k_T}^{k_T} \frac{B_1(k)}{G_z} \exp(jkz) dk$$

$k = \gamma G_z t$  and  $k_T = \gamma G_z T/2$

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We wish to have a 'block' excitation:  $M_T(z) = M_0 \sin(\alpha)$  between  $-d/2$  and  $d/2$  and  $M_T=0$  resten

Find the  $B_1(t)$  profile from the Fourier integral:  
Develop at home!

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z) dz = G_z d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$

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### RF pulse generation

B1 pulse frequency profile (Hz)  $\longleftrightarrow$   $B_1(t)$  applied for a certain duration (ms)

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### Slice selective RF excitation

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### ... in practice

Gradient can not be perfectly rectangular (Hardware limitations)  
Slew rate = Gradient strength / Time to reach the strength  
Slew rate (mT/m/ms)

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### B1(t) and truncation

B1(t) duration can not be too long  
 → The shape is "truncated" as a compromise between duration and achieved RF pulse at the cost of the "slice profile"

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### Summary

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### Slice profile & "cross-talk"

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### Slice selective 180° inversion RF pulse

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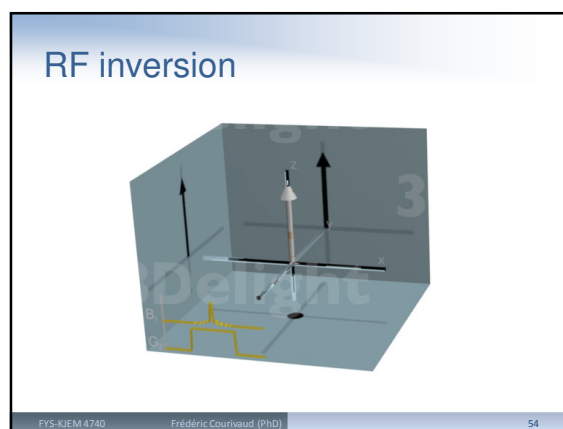
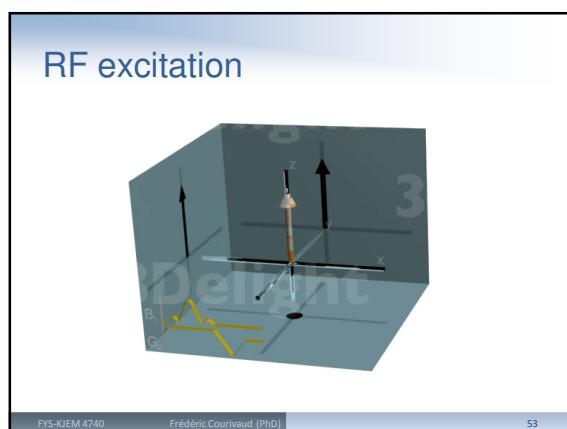
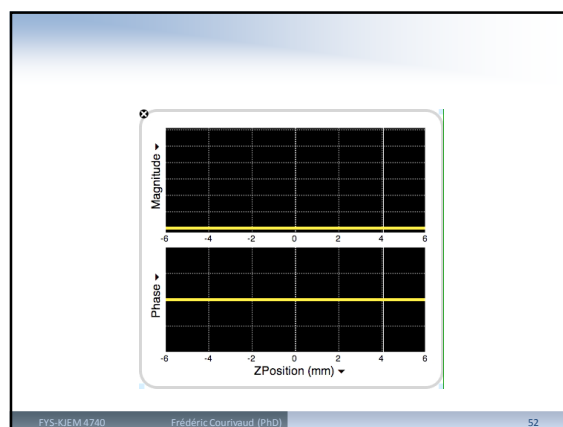
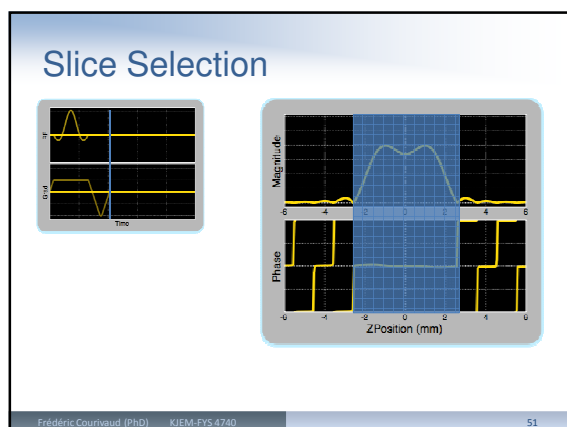
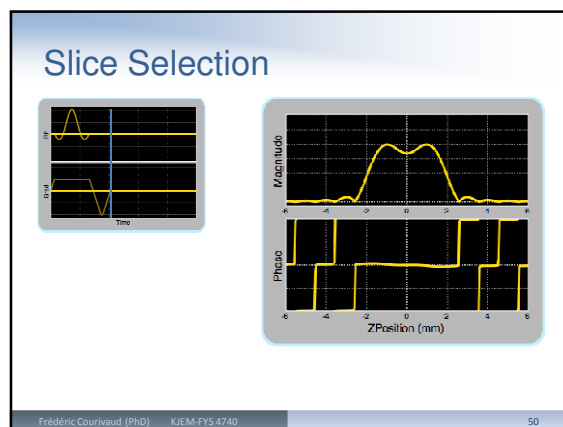
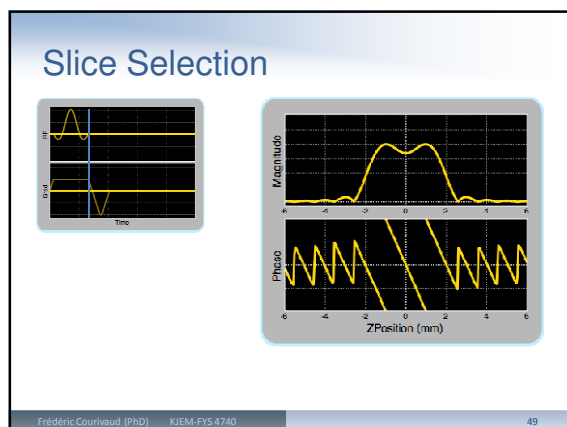
### Slice Selection

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### Slice Selection

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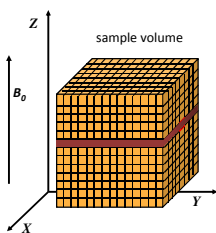
### Exercise

- We want a slice excitation with a given slice thickness of 3mm and a gradient strength of 10mT/m. We Assume the use of a sinc pulse shape for  $B_1(t)$ .
- Find the half-width duration of the  $B_1(t)$  envelope (shape)
- Now, having this  $B_1(t)$  envelope, find the intensity of  $B_1$  ( $\mu T$ ) to get a 90 degree RF pulse.

### Image formation principles

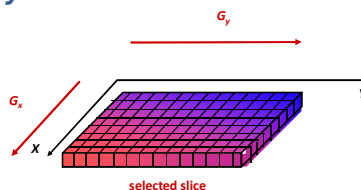
and digital sampling

### Gradients $\rightarrow$ coding in space



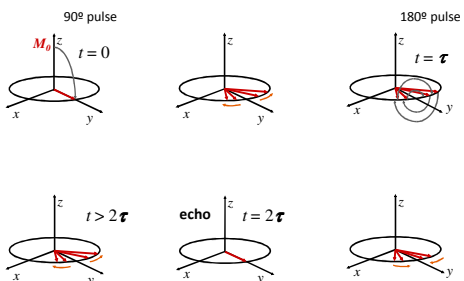
Slice selective excitation: done!

### Linear combination of gradients $x$ and $y$

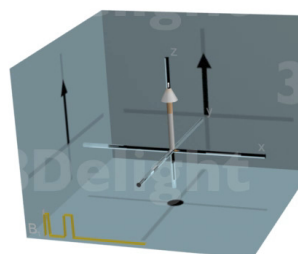


The precession of the spins depends on the field gradients  $G_x, x$  and  $G_y, y$   
the precession depends on the position

### Reminder: spin-echo experiment



### SE magnetization evolution



### Use of gradients to make an image

slice selection

echo

$G_z$

$G_x$

$G_y$

Combination of  $G_x$  and  $G_y$  to "rotate" the total gradient orientation → reconstruction by back projection

### 1st 2D MR image

1973 - Paul Lauterbur produces first two-dimensional MR image using magnetic field gradients and back projection.

### Zeugmatography

Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Lauterbur PC. Image formation by induced local interactions: examples of employing nuclear magnetic resonance. *Nature* 1973; 242: 190-191.

### Nature Editor letter to Paul C. Lauterbur

"With regret I am returning your manuscript which we feel is **not sufficiently wide significance** for inclusion in *Nature*. This action should not in any way be regarded as an adverse criticism of your work, nor even an indication of editorial policies on studies in this field. A choice must inevitably be made from the many contributions received. It is not even possible to accommodate all those manuscripts which are recommended for publication by referees."

**Paul C. Lauterbur answered:**  
 "Several of my colleagues have suggested that the style of the manuscript was too dry and spare, and that the more exuberant prose style of the grant application would have been more appropriate. If you should agree, after reconsideration, that the substance meets your standards,... I would be willing to incorporate some of the material below in the revised manuscript..."

**Nature answered short and positive:**  
 "would it be possible to modify the manuscript so as to make the applications more clear?"

### From last Chap.: definition of k variable

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z) dz = G_z \cdot d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$

This equation can be simplified by introducing the following notation

$$k = \gamma G_z t \quad \text{in general, this can be written} \quad \mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$B_1(t) = G_z \cdot d \cdot \frac{\sin(k \cdot d/2)}{k \cdot d/2}$$

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### k notation

Use of k notation is VERY IMPORTANT in MRI, we will see why...

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

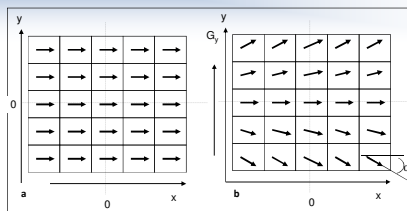
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### Phase effect given by a gradient pulse (e.g. y direction)

$$\alpha(\mathbf{r}, t) = -\gamma \int_0^t G_y(t) \cdot \mathbf{r} \cdot d\tau$$

From Bloch's equation, the Transverse magnetization is then given by (Assume negligible relaxation here):

$$M_{xy} = M_T(\mathbf{r}, t) = M_T(\mathbf{r}, 0) \cdot \exp\left(-j\gamma \int_0^t G_y(t) \mathbf{r} d\tau\right)$$



The phase angle of the transverse magnetization vector before (a) and after (b) the application of a magnetic field gradient in the y-direction.

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad M_T(t) = \iint_{\text{slice}} \rho(\mathbf{r}) \cdot \exp(-j\mathbf{k} \cdot \mathbf{r}) d\mathbf{r}$$

### MR signal = integral of transverse magnetization over entire volume

$$M_T(t) \propto S(t) \propto \iiint \rho(\mathbf{r}) \exp\left(-j\gamma \int_0^t G_y(t) \mathbf{r} d\tau\right) d\mathbf{r}$$

↓ NB: Fourier transform

$$F(k) = \iiint f(\mathbf{r}) \exp(-j\mathbf{k} \cdot \mathbf{r}) d\mathbf{r} \quad \text{Fourier transform from } k \text{ space to geometric "space" of a continuous function } f(\mathbf{r})$$

$$S(t) \propto \iiint \rho(\mathbf{r}) \exp\left(-j\gamma \int_0^t G(t) \mathbf{r} d\tau\right) d\mathbf{r}$$

### k-space

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

Limiting discussion to a slice (2D - xy plane), magnetization distribution is given by the 2-dimensional Fourier transform of the spin distribution across the slice

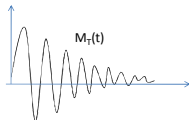
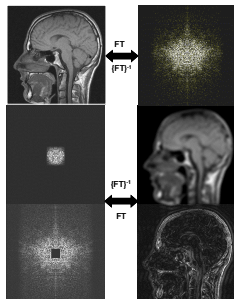
$$M_T(t) = \iint_{\text{slice}} \rho(\mathbf{r}) \cdot \exp(-j\mathbf{k} \cdot \mathbf{r}) d\mathbf{r}$$

$\rho(\mathbf{r})$  is obtained from the inverse Fourier transform of  $M_T(t)$  under the influence of a known gradient configuration

$$\rho(x, y) = \frac{1}{2\pi} \int_{k_x} \int_{k_y} M_T(k_x, k_y) \exp(j(k_x x + k_y y)) dk_x dk_y$$

k-space = visualization of the distribution of spatial frequencies in the image.  
k-space = Fourier transform of the MR image

### k-space illustrations



$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$\rho(x, y) = \frac{1}{2\pi} \int_{k_x} \int_{k_y} M_T(k_x, k_y) \exp(j(k_x x + k_y y)) dk_x dk_y$$

### Reminder: Fourier... and frequencies



### Low and high frequencies

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### Introduction to k-space image sampling

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### What describes waves (signal)

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### Spin Echo: freq. encoding

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### Phase encoding

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The phase angle of a spin in a slice at a time  $t$  is given by:

$$\int_0^t \omega(x, y, t) dt = \gamma G_y t y + \gamma G_x t x$$

Definition of  $k$ :

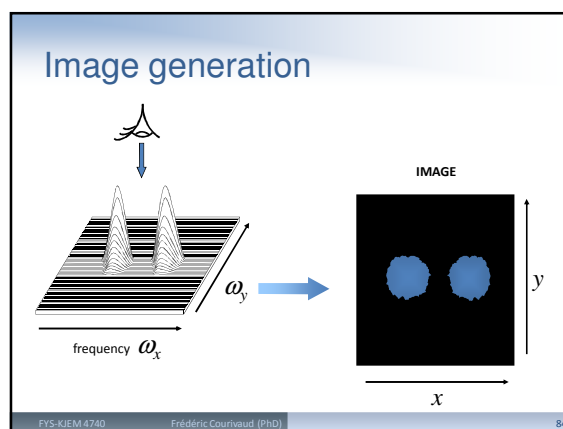
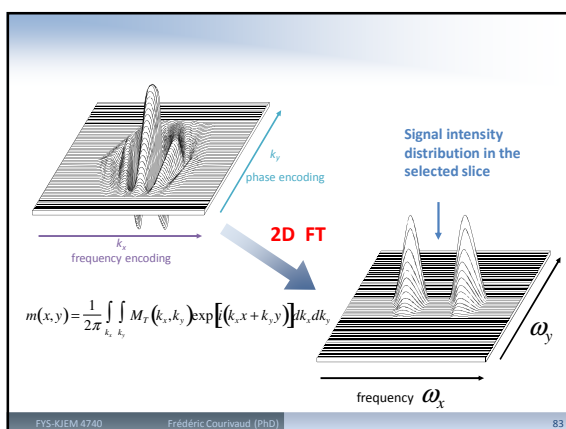
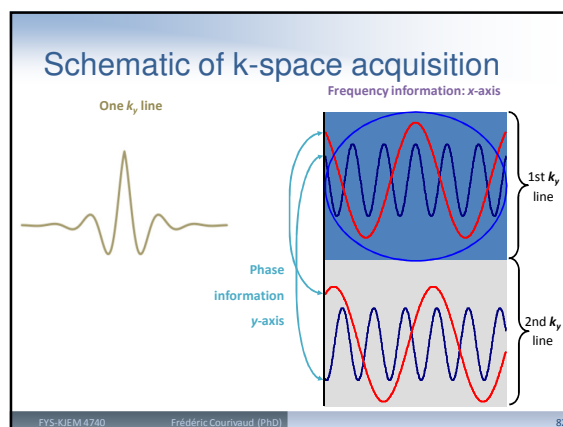
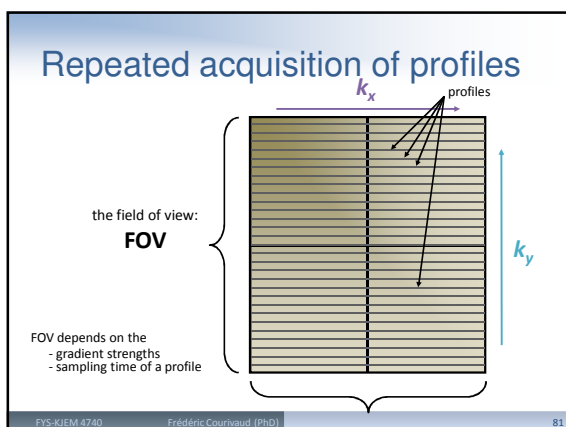
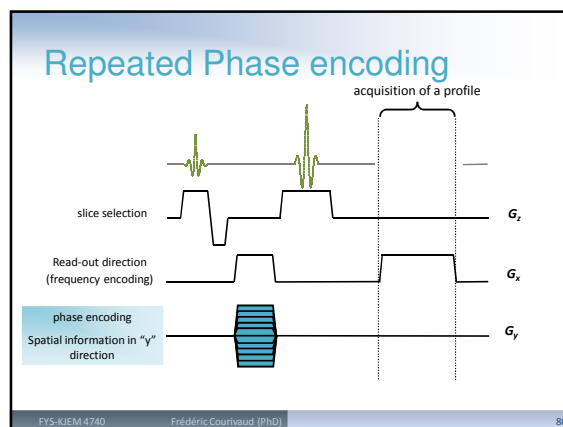
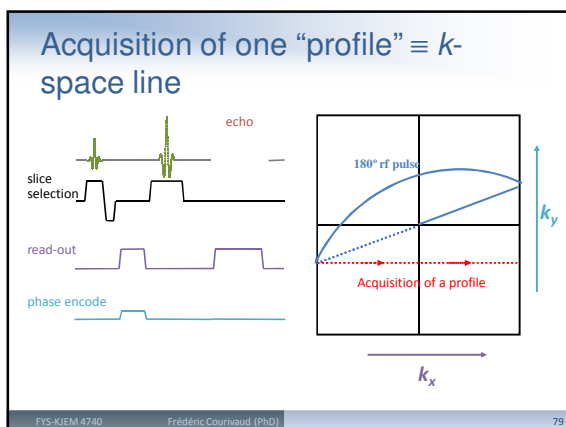
$$k_i = \gamma \int_0^{t'} G_i(t) dt \text{ (in the direction } i \text{)}$$

The total transverse magnetisation is a function of  $k_x, k_y$  and the position in the slice:  $M_T(k_x, k_y)$

Image reconstruction:  $m(x, y) = \frac{1}{2\pi} \int_{k_x} \int_{k_y} M_T(k_x, k_y) \exp[i(k_x x + k_y y)] dk_x dk_y$

**2D Fourier Transform**

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## CONTINUE PPT FROM HERE

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### Discrete sampling

Sampling interval:  $-T_{read}/2 - T_{read}/2$

$$PSF(x) = FT(U(z)) = \int_{-T_{read}/2}^{T_{read}/2} \exp(-j\gamma G_x z) dz = T_{read} \frac{\sin(\frac{\gamma G_x x T_{read}}{2})}{\frac{\gamma G_x x T_{read}}{2}}$$

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### Schematic of k-space acquisition

Frequency information: x-axis

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### Discrete sampling and PSF

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### Pulse sequence

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### K-space vs image space

$$\rho(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M_T(k_x, k_y) \exp(j(k_x x + k_y y)) dk_x dk_y$$

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau$$

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### K-space egenskaper

Resolution (x):  
 $\delta x = \frac{2\pi}{\gamma G_x N_x t_s}$

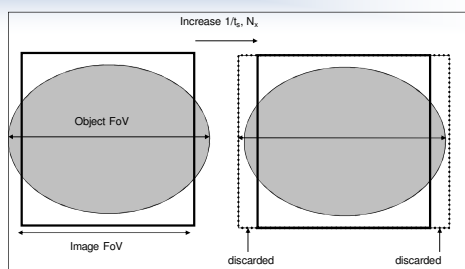
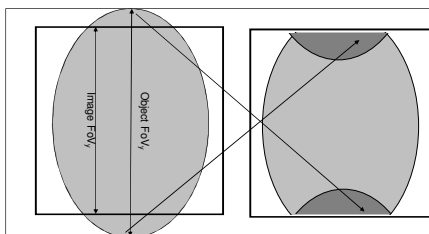
Field of view (x):  
 $\lambda_{x,max} = \frac{2\pi}{k_{x,min}} = \frac{2\pi}{\gamma G_x t_s} = FoV_x$

Field of view (y):  
 $\lambda_{y,max} = FoV_y = \frac{\pi N_y}{\gamma G_y T}$

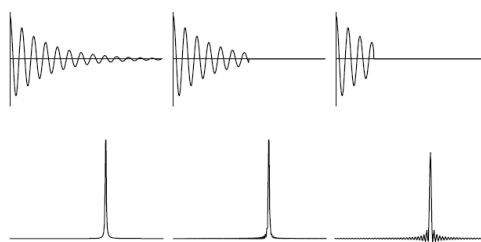
Maximum frequency in read-out (x) direction  
 $\pm \omega_{max} = \pm \gamma G_x FoV_x / 2$

Min sampling rate (x):  
 $1/t_s \geq \gamma G_x FoV_x / 2\pi$

'Sampling rate' (y):  
 $N_y = \gamma G_y T FoV_y$

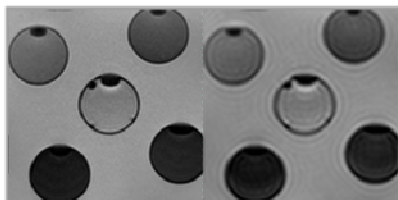


### FFT 1D: Truncation Artefact

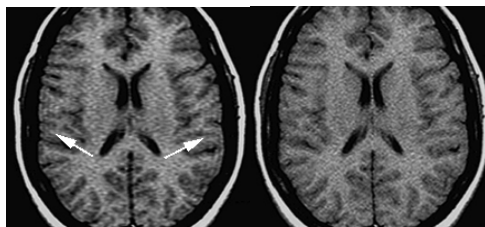


### FFT 2D: Truncation artefact

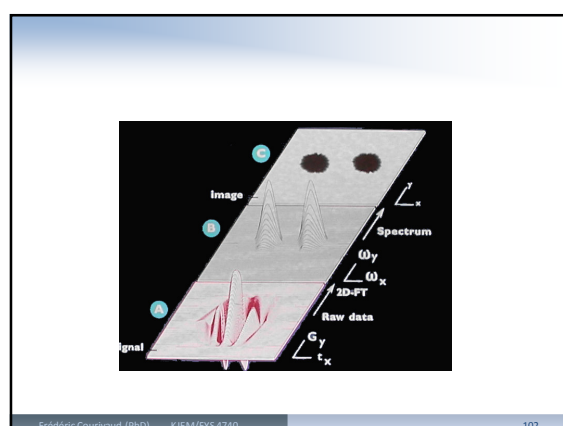
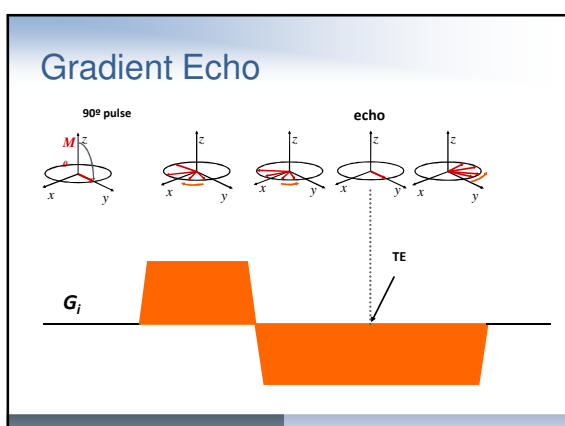
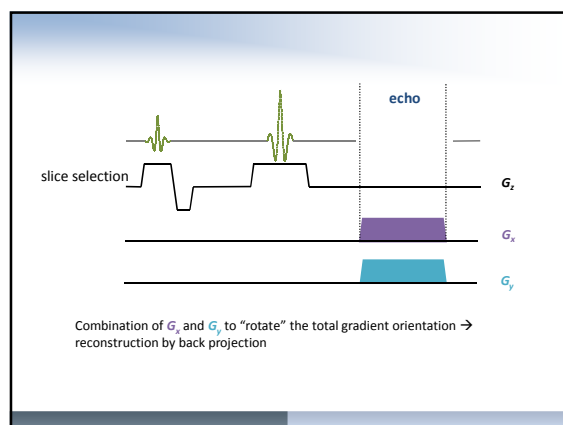
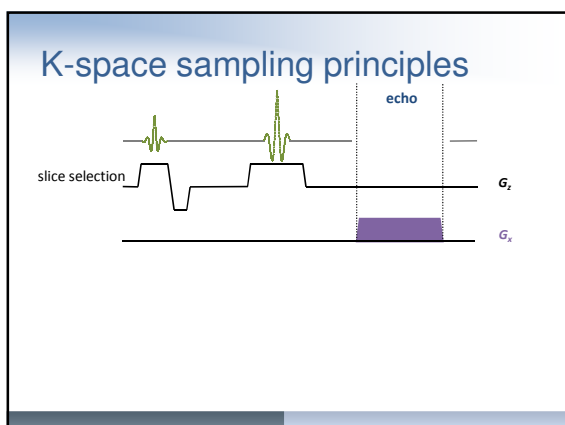
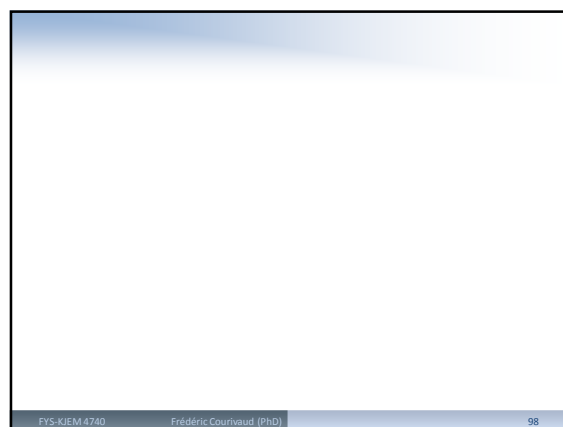
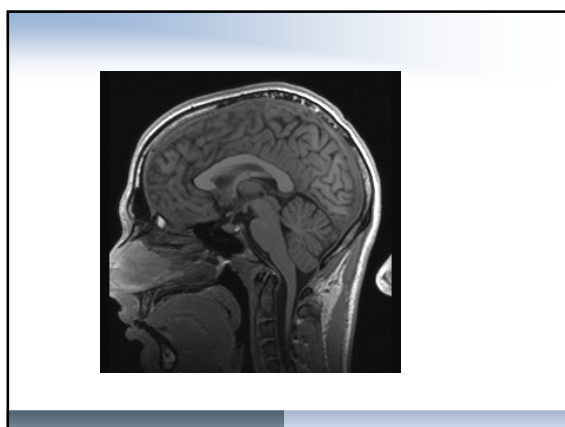
Ring- (or truncation) artifacts in regions with high spatial frequencies (edges) in a phantom. The artifacts are more evident in the right image due to a lower matrix (N=112, vs N=256 in the left image).

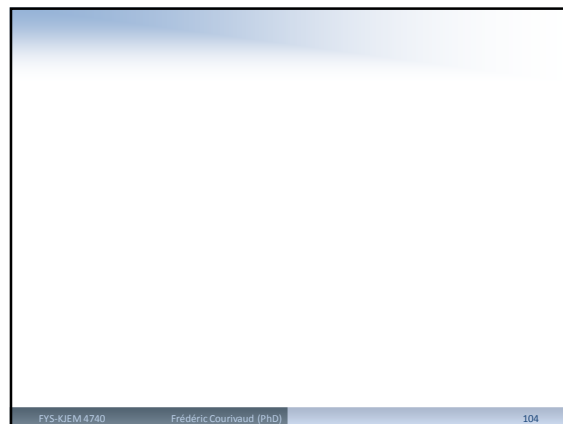
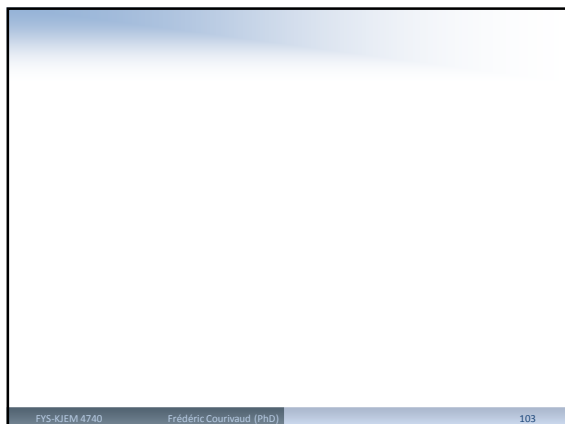


### Truncation artifact









### B1(t) for slice selective excitation

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z) dz = G_z d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$

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### Definition of k variable

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z) dz = G_z d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$

This equation can be simplified by introducing the following notation

$$k = \gamma G_z t \quad \text{in general, this can be written} \quad \mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$B_1(t) = G_z d \cdot \frac{\sin(k \cdot d/2)}{k \cdot d/2}$$

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### k notation

Use of k notation is VERY IMPORTANT in MRI, we will see why...

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau) d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

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