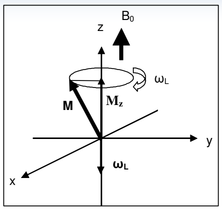
MRI

FYS-KJM 4740

Chap 1

Bloch equations and main principles

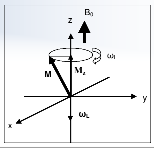
The Bloch equation



$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M} \times \mathbf{B})$$

The magnetic moment **M** rotates around the static B-field at the Larmor frequency

Larmor equation



Joseph Larmor

ω_0 (rad/s) = γ (rad/s/Tesla) x B_0 (Tesla)

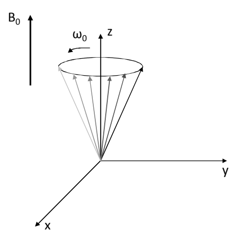
$\gamma_{hydrogen}$ = 2.68 *10^8 rad/s/Tesla

/2π

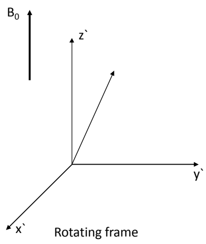
f_L (MHz) = γ (MHz/Tesla) x B_0 (Tesla)

$\gamma_{hydrogen}$ = 42.58 MHz/Tesla

Rotating frame of reference



Laboratory frame

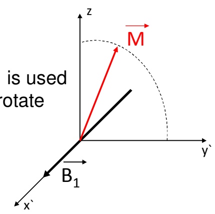


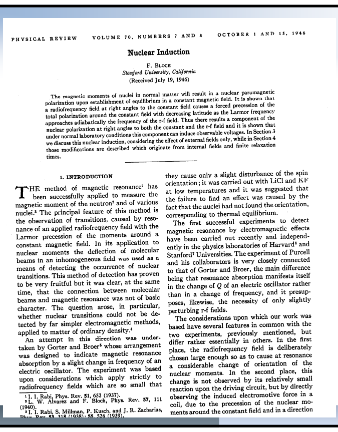
Rotating frame

Flipping away the Magnetization from its equilibrium

In MRI, short RF pulses are used to
To change the direction of the magnetization
M

To get M to rotate around x or y axis,
A linearly polarized magnetic field B1 is used
during short time (pulse) to get M to rotate
around B1 axis



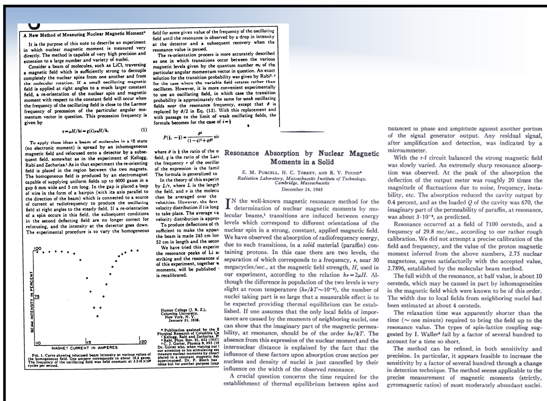

PHYSICAL REVIEW VOLUME 70, NUMBERS 7 AND 8 OCTOBER 1 AND 15, 1946

Nuclear Induction

F. Bloch

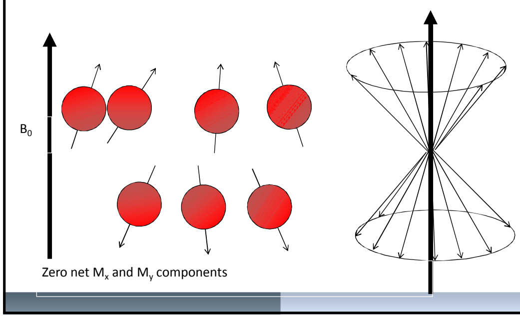
Stanford University, California

(Received July 19, 1946)

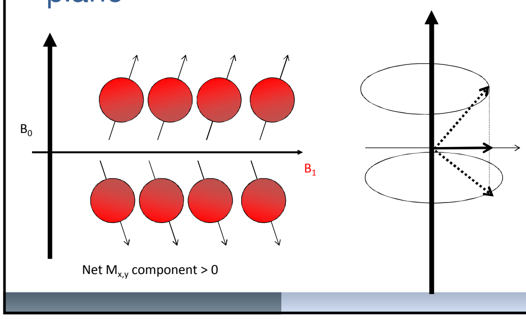

Resonance Absorption by Nuclear Magnetic Moments in a Solid

E. M. Purcell, H. C. Torrey, and R. V. Pound

Baseline, no phase coherence in transverse (X,Y) plane



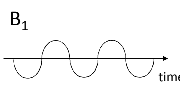
Additional B1-field induces phase coherence in transverse (X,Y) plane

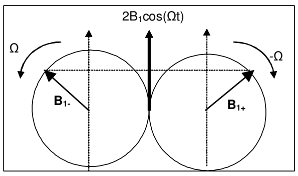


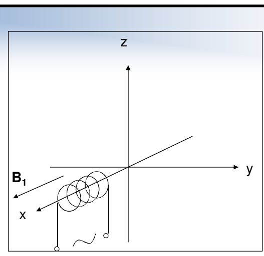
Rotation

The magnetic field due to the RF pulse, B_1, is generated by two circularly polarized fields with opposing direction of rotation with angular frequency +/- Ω

$$B_1 = B_{1+} + B_{1-} = B_1\begin{bmatrix}\cos(-\Omega t)\\ \sin(-\Omega t)\\ 0\end{bmatrix} + B_1\begin{bmatrix}\cos(\Omega t)\\ \sin(\Omega t)\\ 0\end{bmatrix} = 2B_1\begin{bmatrix}\cos(\Omega t)\\ 0\\ 0\end{bmatrix}$$

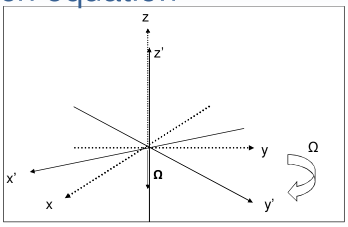






The RF-coil generates a magnetic field B1 along the x-axis

Bloch equation



The 'rotating frame' (x', y', z'-coordinates)

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}}$$

$B_{eff} = B_0 + B_1 + \Omega/\gamma$

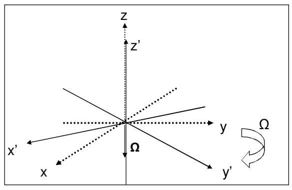
$$\Omega = \begin{bmatrix} 0 \\ 0 \\ -\Omega \end{bmatrix}$$

RF-eksitasjon med Larmor frekvens (rotating frame)

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}}$$

$\Omega = \gamma B_0$

$B_{eff} = B_1$

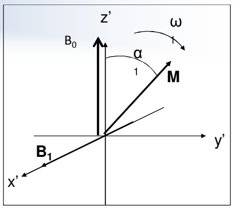


Using Matrix formalism

$$\frac{d\mathbf{M}}{dt} = \gamma \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B_{1x} \\ 0 & -B_{1x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

⇓

$$M_{y'} = A\sin(\gamma B_{1x} t) + B\cos(\gamma B_{1x} t)$$



$$\omega_1 = -\gamma B_1$$

Using the boundary conditions $M_{y'}=M_{y'}(0)$ and $M_z=M_z(0)$ at t=0, we get

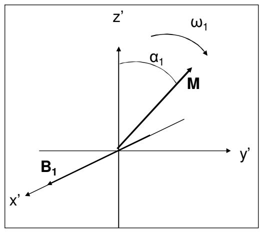
$$\mathbf{M}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1 t) & \sin(\omega_1 t) \\ 0 & -\sin(\omega_1 t) & \cos(\omega_1 t) \end{pmatrix} \begin{pmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{pmatrix} = \mathbf{R}\cdot\mathbf{M}_0$$

RF pulse

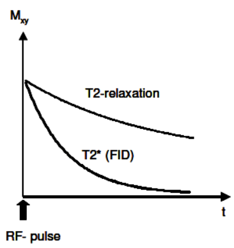
$$\omega_1 = -\gamma B_1$$

RF pulse duration is proportional to the wanted flip angle, α.

$$t_{B_1} = \alpha / \gamma B_1$$



Relaxation

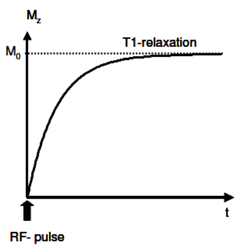


$$\frac{dM_x}{dt} = -\frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = -\frac{M_y}{T_2}$$

$$\frac{1}{T2^*} = \frac{1}{T2} + \gamma \Delta B_0$$

Relaxation



$$\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1}$$

Relaxation

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{eff} - \mathbf{R}(\mathbf{M} - \mathbf{M}_0)$$

$$\mathbf{R} = \begin{bmatrix} \frac{1}{T_2} & 0 & 0 \\ 0 & \frac{1}{T_2} & 0 \\ 0 & 0 & \frac{1}{T_1} \end{bmatrix} \quad \mathbf{M}_0 = \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

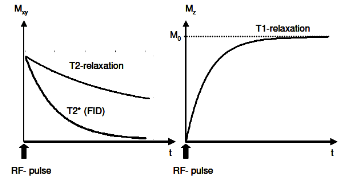
FYS-KJEM 4740 Frédéric Courivaud (PhD) 19

Condition: relaxation during RF excitation is neglected

$$\frac{d\mathbf{M}}{dt} = -\mathbf{R}(\mathbf{M} - \mathbf{M}_0)$$

$$M_z(t) = M_0[1 - \exp(-t/T_1)] + M_z(0)\exp(-t/T_1)$$

$$M_{xy}(t) = M_{xy}(0)\exp(-t/T_2)$$



FYS-KJEM 4740 Frédéric Courivaud (PhD) 20

Summary with excitation and relaxation

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{eff} - \mathbf{R}(\mathbf{M} - \mathbf{M}_0)$$

$$\frac{d\mathbf{M}}{dt} = \begin{bmatrix} -1/T_2 & 0 & 0 \\ 0 & -1/T_2 & \gamma B_{1x} \\ 0 & -\gamma B_{1x} & -1/T_1 \end{bmatrix} \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

FYS-KJEM 4740 Frédéric Courivaud (PhD) 21

Chap. 2

Slice-Selective RF excitation

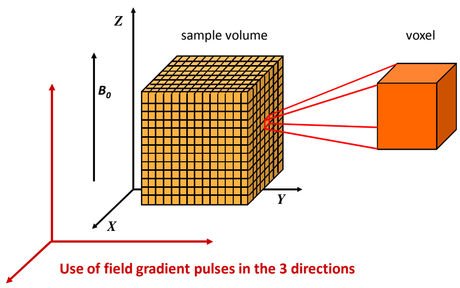
Image formation

FYS-KJEM 4740 Frédéric Courivaud (PhD) 22

Slice Selective RF pulse

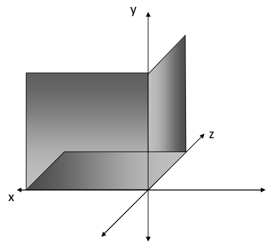
FYS-KJEM 4740 Frédéric Courivaud (PhD) 23

Gradients → coding in space



Frédéric Courivaud (PhD) KJEM/FYS 4740 24

Magnetic Field Gradients

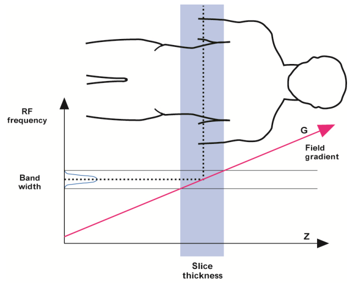


$\delta B_{G,z}(z) = G_z z$

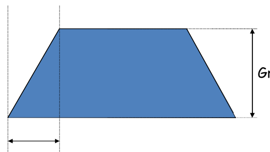
$\delta B_{G,z}(y) = G_y y$

$\delta B_{G,z}(x) = G_x x$

Principles of Slice Selection



Magnetfelt gradient



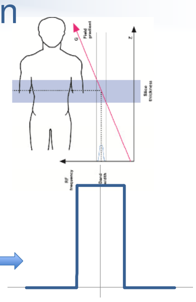
Gradient styrke (mT/m)

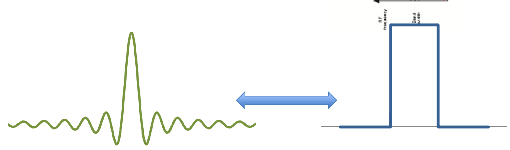
Stigningstid (ms)

$$\text{Slew rate (mT/m/ms)} = \frac{\text{Gradient styrke}}{\text{Stigningstid}}$$

RF pulse generation

FAVOURABLE SITUATION

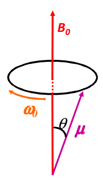




B1 applied for a certain duration (ms)

B1 pulse frequency profile (HZ)

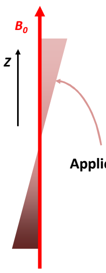
Effect of Magnetic Field Gradient



precession frequency ω:

$$\omega_0 = \gamma \times \mathbf{B}_0$$

Example: use of *z* field gradient



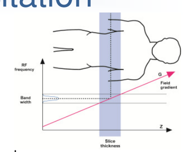
$$\omega = \gamma \times (B_0 + gz)$$

Spatial information included in the precession frequency

Applied field gradient (*g*)

Slice selective RF excitation

$$\mathbf{B}_{eff} = \mathbf{B}_0 + \mathbf{G}\cdot\mathbf{r} + \mathbf{B}_1 + \frac{\Omega}{\gamma} = \begin{bmatrix} B_1 \\ 0 \\ B_0 + G_z z_1 - \frac{\Omega}{\gamma} \end{bmatrix}$$



If we set: $\Omega = \gamma(B_0 + G_z z_1)$, then the effective field at $z = z_1$ becomes:

$$\mathbf{B}_{eff} = \begin{bmatrix} B_1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \text{Slice-selective excitation @ } z = z_1$$

What would happen if $B_0 + G_z z_1 - \Omega/\gamma >> B_1$??

All effects together: excitation, precession and relaxation

$\Omega=\omega_L$;$B_{eff}=B_{1x}$

$$\frac{d\mathbf{M}}{dt}=\begin{pmatrix}-1/T_2 & \gamma\mathbf{G}\cdot\mathbf{r} & 0\\ -\gamma\mathbf{G}\cdot\mathbf{r} & -1/T_2 & \gamma B_{1x}\\ 0 & -\gamma B_{1x} & -1/T_1\end{pmatrix}\begin{pmatrix}M_{x'}\\ M_{y'}\\ M_{z'}\end{pmatrix}+\begin{pmatrix}0\\ 0\\ M_0/T_1\end{pmatrix}$$

All effects together: excitation, precession and relaxation

Transverse (M_{xy}) relaxation

Precession around z-axis

$$\frac{d\mathbf{M}}{dt}=\begin{pmatrix}-1/T_2 & \gamma\mathbf{G}\cdot\mathbf{r} & 0\\ -\gamma\mathbf{G}\cdot\mathbf{r} & -1/T_2 & \gamma B_{1x}\\ 0 & -\gamma B_{1x} & -1/T_1\end{pmatrix}\begin{pmatrix}M_{x'}\\ M_{y'}\\ M_{z'}\end{pmatrix}+\begin{pmatrix}0\\ 0\\ M_0/T_1\end{pmatrix}$$

Excitation around x-axis

Longitudinal (M_z) relaxation

Transverse magnetization, $M_{xy,}$ Relaxation and precession

$M_T = M_x + jM_y$ $B_{1x}=0$

derive!

$$M_T = M_T(0)\exp\left(-j\gamma\mathbf{r}\cdot\int\mathbf{G}(t)dt\right)\exp\left(-\frac{t}{T_2}\right)$$

Longitudinal magnetization, M_z

derive!

$$M_z(t)=M_0\left[1-\exp\left(-\frac{t}{T_1}\right)\right]+M_z(0)\exp\left(-\frac{t}{T_1}\right)$$

As we have seen before

Transverse Magnetization, $M_{xy,}$ Excitation and Precession

$M_T = M_x + jM_y$ $T_2=\infty$

Condition: $M_z \approx M_0$ (How can this be achieved?)

$$\frac{dM_T}{dt}=-j\gamma(\mathbf{G}\cdot\mathbf{r})M_T + j\gamma B_1 M_0$$

General Solution:

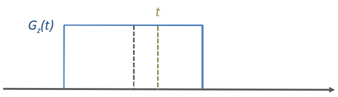
$$M_T = A(t)\exp\left(-j\gamma\mathbf{r}\cdot\int_{t_1}^{t}\mathbf{G}(t')dt'\right)$$

Transversal magnetisering, $M_{xy,}$ Eksitasjon og presesjon

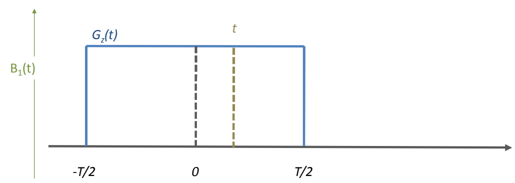
$T_2=\infty$

Dersom vi sier at RF puls starter ved –T/2 og varer i T sek:

$$M_T(T/2,\mathbf{r})=j\gamma M_0\int_{-T/2}^{T/2}B_1(t)\exp\left(-j\gamma\mathbf{r}\cdot\int_{t}^{T/2}\mathbf{G}(t')dt'\right)dt$$



For a constant gradient along the z-axis: $G(t) = G_z$



$$M_T(T/2,z)=j\gamma M_0\exp(-j\gamma zG_zT/2)\int_{-T/2}^{T/2}B_1(t)\exp(j\gamma zG_zt)dt$$

Phase dispertion

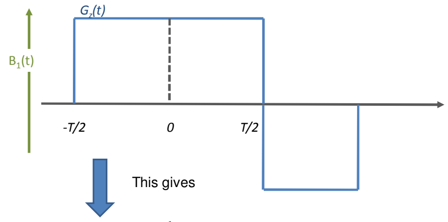
Fourier transform of the B1 ”envelope” (green shape)

For a constant gradient along the z-axis: *G(t) = G_z*

$$M_T(T/2,z) = j\gamma M_0 \exp(-j\gamma z G_z T/2)\int_{-T/2}^{T/2} B_1(t)\exp(j\gamma z G_z t)dt$$

Slice profile = Fourier transform of $B_1(t)$
The phase of $M_T(z)$ in the x-y- plane is a function of z

Elimination of the phase dispertion in x-y plane use of an extra gradient of opposite polarity and half the length: *-G_z*



This gives

$$M_T(T,z) = jM_0\int_{-k_T}^{k_T}\frac{B_1(k)}{G_z}\exp(jkz)dk$$

k=γG_zt and *k_T=γG_zT/2.*

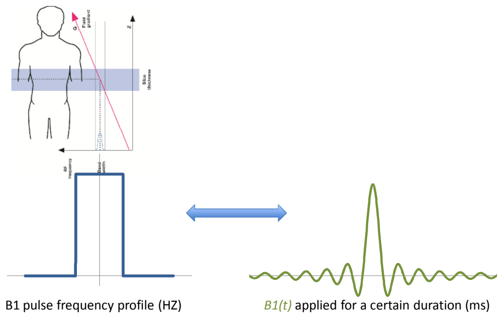
We wish to have a 'block' excitation: $M_T(z) = M_0\sin(\alpha)$ between –d/2 og d/2 og M_T=0 resten

Find the $B_1(t)$ profile from the Fourier integral:

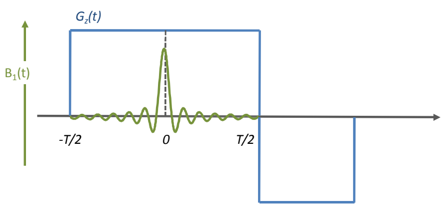
Develop at home!

$$B_1(t) = G_z\int_{-d/2}^{d/2}\exp(j\gamma G_z t\cdot z)dz = G_z d\cdot\frac{\sin(\gamma G_z t\cdot d/2)}{\gamma G_z t\cdot d/2}$$

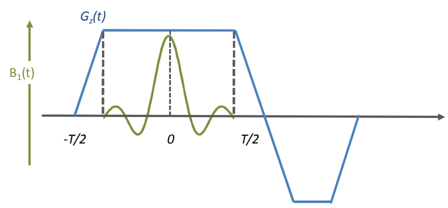
RF pulse generation



Slice selective RF excitation



… in practice

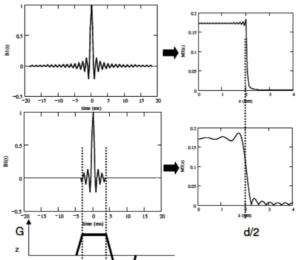


Gradient can not be perfectly rectangular (Hardware limitations
Slew rate = Gradient strength / Time to reach the strength
Slew rate (mT/m/ms)

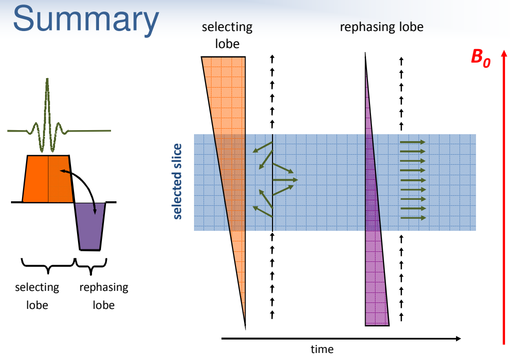
B1(t) and truncation

B1(t) duration can not be too long

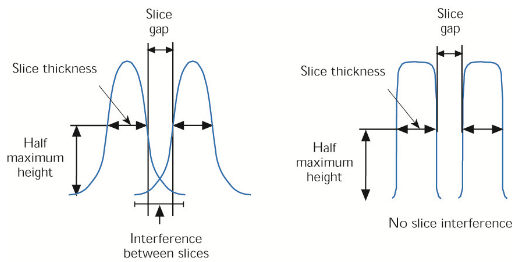
→ The shape is "truncated" as a compromise between duration and achieved RF pulse at the cost of the "slice profile"



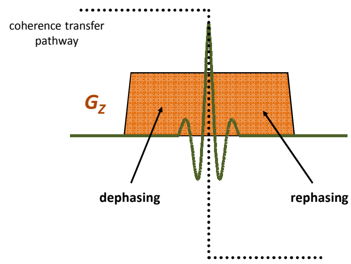
Summary



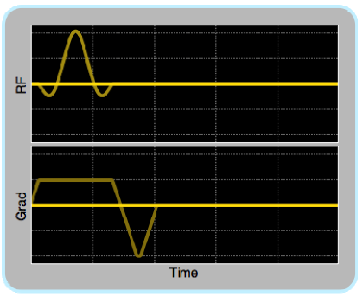
Slice profile & "cross-talk"



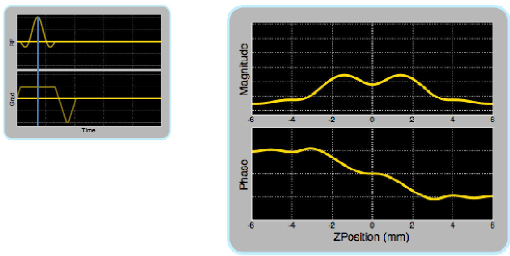
Slice selective 180° inversion RF pulse



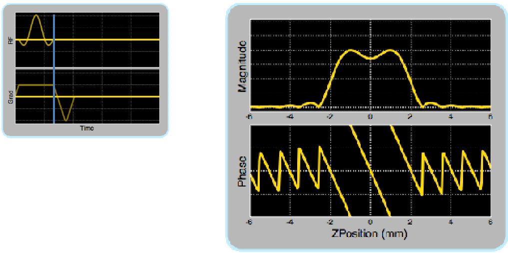
Slice Selection



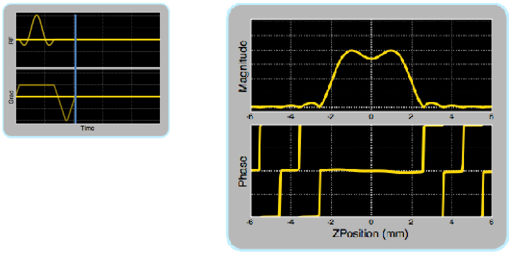
Slice Selection



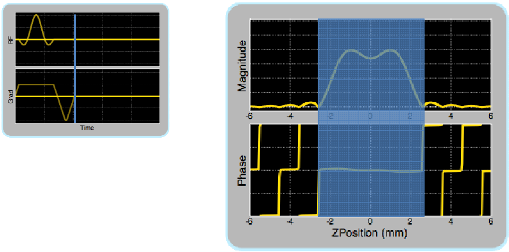
Slice Selection

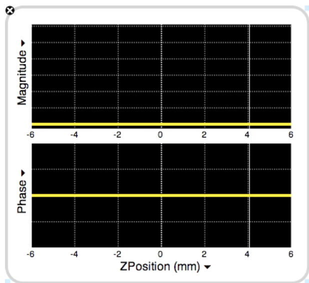


Slice Selection

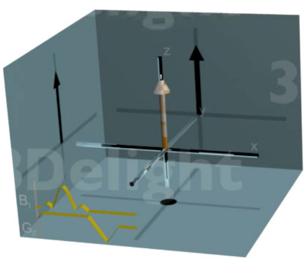


Slice Selection

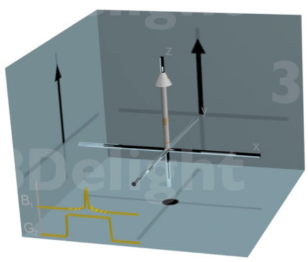




RF excitation



RF inversion



Exercise

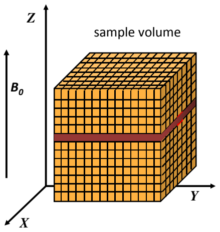
- We want a slice excitation with a given slice thickness of 3mm and a gradient strength of 10mT/m. We Assume the use of a sinc pulse shape for B1(t).

- Find the half-width duration of the B1(t) envelope (shape)
- Now, having this B1(t) envelope, find the intensity of B1 (μT) to get a 90 degree RF pulse.

Image formation principles

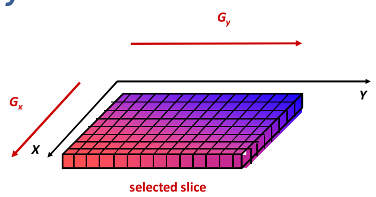
and digital sampling

Gradients → coding in space



Slice selective excitation: done!

Linear combination of gradients *x* and *y*

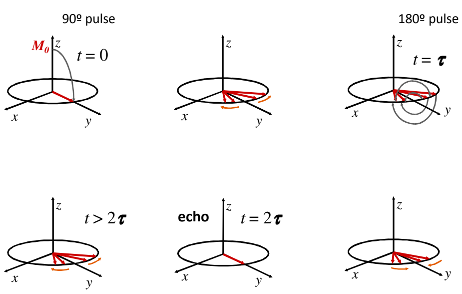


The precession of the spins depends on the field gradients

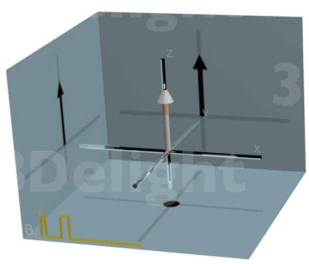
$G_x x$ and $G_y y$

the precession depends on the position

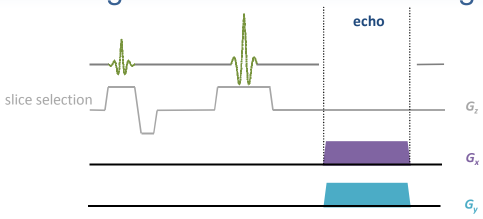
Reminder: spin-echo experiment



SE magnetization evolution

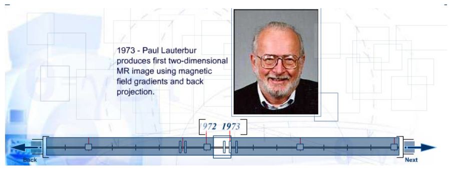


Use of gradients to make an image

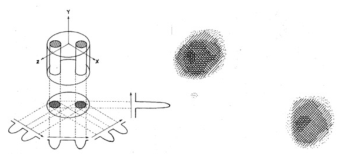


Combination of G_x and G_y to "rotate" the total gradient orientation → reconstruction by back projection

1st 2D MR image



Zeugmatography



Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Lauterbur PC. Image formation by induced local interactions: examples of employing nuclear magnetic resonance. Nature 1973; 242: 190-191.

Nature Editor letter to Paul C. Lauterbur

"With regret I am returning your manuscript which we feel is **not sufficiently wide significance** for inclusion in *Nature*. This action should not in any way be regarded as an adverse criticism of your work, nor even an indication of editorial policies on studies in this field. A choice must inevitably be made from the many contributions received; It is not even possible to accommodate all those manuscripts which are recommended for publication by referees."

Paul C. Lauterbur answered:

"Several of my colleagues have suggested that the style of the manuscript was too dry and spare, and that the more exuberant prose style of the grant application would have been more appropriate. If you should agree, after reconsideration, that the substance meets your standards,… I would be willing to incorporate some of the material below in the revised manuscript…"

Nature answered short and positive:

"would it be possible to modify the manuscript so as to make the applications more clear?"

From last Chap.: definition of k variable

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z)dz = G_z d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$

This equation can be simplified by introducing the following notation

$k=\gamma G_z t$ in general, this can be written $\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{vmatrix} k_x \\ k_y \\ k_z \end{vmatrix}$

$$B_1(t) = G_z d \cdot \frac{\sin(k \cdot d/2)}{k \cdot d/2}$$

k notation

Use of k notation is VERY IMPORTANT in MRI, we will see why…

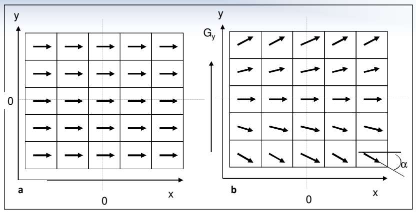
$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{vmatrix} k_x \\ k_y \\ k_z \end{vmatrix}$$

Phase effect given by a gradient pulse (e.g. y direction)

$$\alpha(\mathbf{r},t) = -\gamma \int_0^t G_y(t) \cdot \mathbf{r} \cdot d\tau$$

From Bloch's equation, the Transverse magnetization is then given by (Assume negligible relaxation here):

$$M_{xy} = M_T(\mathbf{r},t) = M_T(\mathbf{r},0) \cdot \exp\left(-j\gamma \int_0^t G_y(t)\mathbf{r}d\tau \right)$$



The phase angle of the transverse magnetization vector before (a) and after (b) the application of a magnetic field gradient in the y-direction.

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \qquad M_T(t) = \iint_{slice} \rho(\mathbf{r}) \cdot \exp(-j\mathbf{k} \cdot \mathbf{r})d\mathbf{r}$$

MR signal = integral of transverse magnetization over entire volume

$$M_T(t) \propto S(t) \propto \iiint \rho(\mathbf{r})\exp\left(-j\gamma \int_0^t G_y(t)\mathbf{r}d\tau\right)d\mathbf{r}$$

NB: Fourier transform

$$F(k) = \iiint_R f(\mathbf{r})\exp(-jk\mathbf{r})d\mathbf{r}$$

Fourier transform from *k* space to geometric "space" of a continuous function *f(r)*

$$S(t) \propto \iiint_R \rho(\mathbf{r})\exp\left(-j\gamma \int_0^t G(t)\mathbf{r}d\tau\right)d\mathbf{r}$$

k-space

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

Limiting discussion to a slice (2D – xy plane), magnetization distribution is given by the 2-dimensional Fourier transform of the spin distribution across the slice

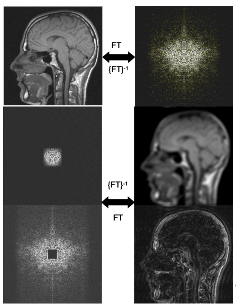
$$M_T(t) = \iint_{slice} \rho(\mathbf{r}) \cdot \exp(-j\mathbf{k} \cdot \mathbf{r})d\mathbf{r}$$

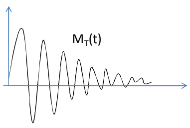
$\rho(\mathbf{r})$ is obtained from the inverse Fourier transform of $M_T(t)$ under the influence of a known gradient configuration

$$\rho(x,y) = \frac{1}{2\pi} \int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y$$

k-space = visualization of the distribution of spatial frequencies in the image.
k-space = Fourier transform of the MR image

k-space illustrations

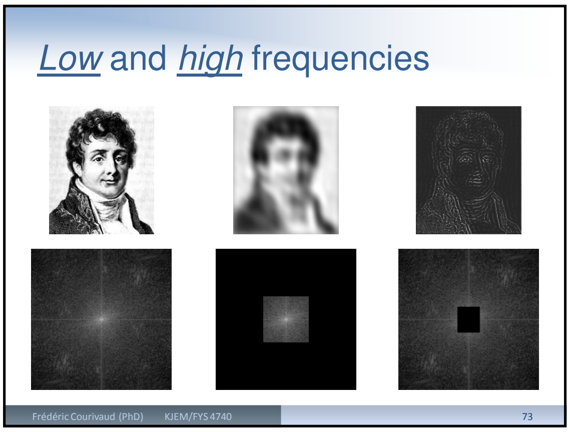




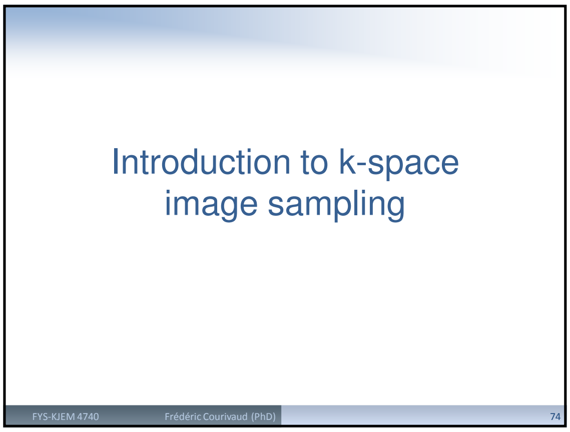
$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$\rho(x,y) = \frac{1}{2\pi} \int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y$$

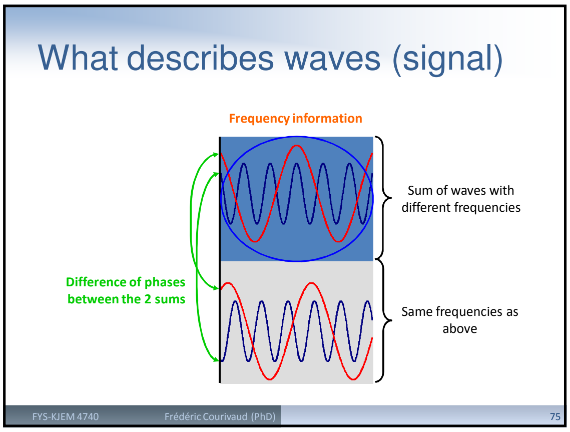
Reminder: Fourier… and frequencies



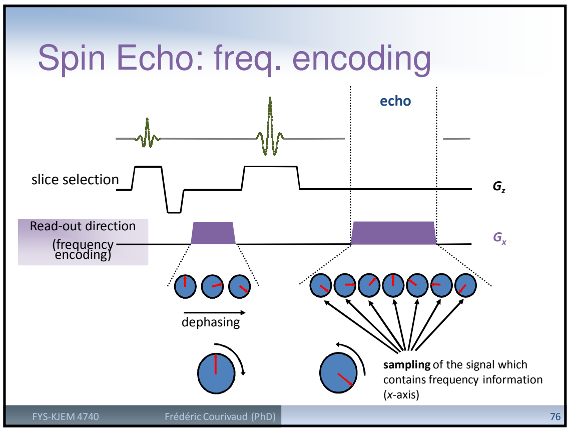
Low and high frequencies
Frédéric Courivaud (PhD) KJEM/FYS 4740 73



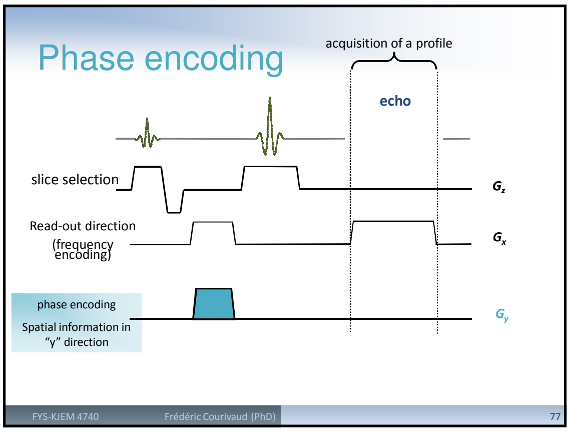
Introduction to k-space image sampling
FYS-KJEM 4740 Frédéric Courivaud (PhD) 74



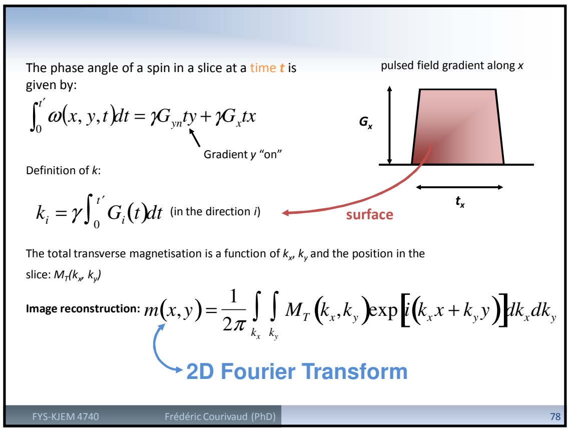
What describes waves (signal)
Frequency information
Sum of waves with different frequencies
Difference of phases between the 2 sums
Same frequencies as above
FYS-KJEM 4740 Frédéric Courivaud (PhD) 75



Spin Echo: freq. encoding
echo
slice selection
G_z
Read-out direction (frequency encoding)
G_x
dephasing
sampling of the signal which contains frequency information (x-axis)
FYS-KJEM 4740 Frédéric Courivaud (PhD) 76

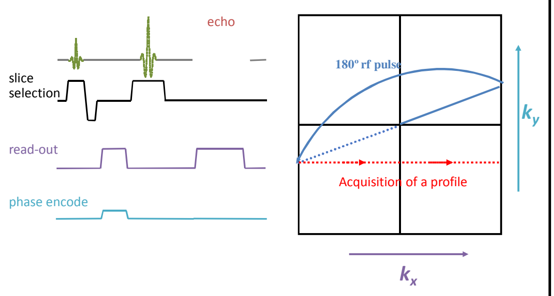


Phase encoding
acquisition of a profile
echo
slice selection
G_z
Read-out direction (frequency encoding)
G_x
phase encoding
Spatial information in "y" direction
G_y
FYS-KJEM 4740 Frédéric Courivaud (PhD) 77

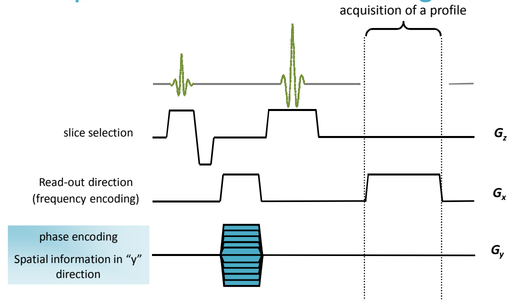


The phase angle of a spin in a slice at a time t is given by:
$\int_0^{t'} \omega(x,y,t)dt = \gamma G_{yn} t y + \gamma G_x t x$
Gradient y "on"
pulsed field gradient along x
G_x
t_x
Definition of k:
$k_i = \gamma \int_0^{t'} G_i(t)dt$ (in the direction i)
surface
The total transverse magnetisation is a function of k_x, k_y and the position in the slice: $M_T(k_x, k_y)$
Image reconstruction: $m(x,y) = \frac{1}{2\pi} \int_{k_x} \int_{k_y} M_T(k_x,k_y) \exp[i(k_x x + k_y y)] dk_x dk_y$
2D Fourier Transform
FYS-KJEM 4740 Frédéric Courivaud (PhD) 78

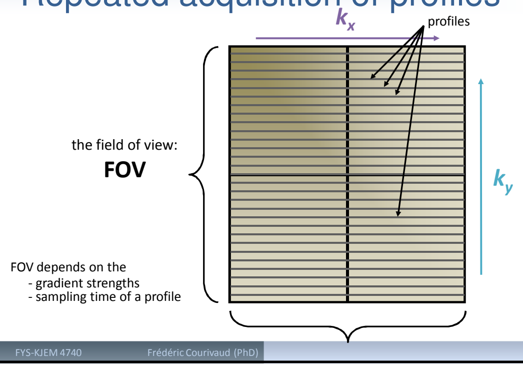
Acquisition of one "profile" ≡ *k*-space line



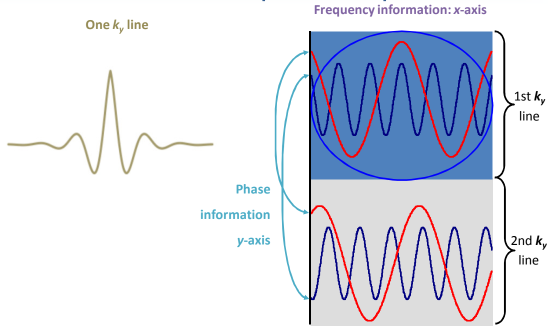
Repeated Phase encoding

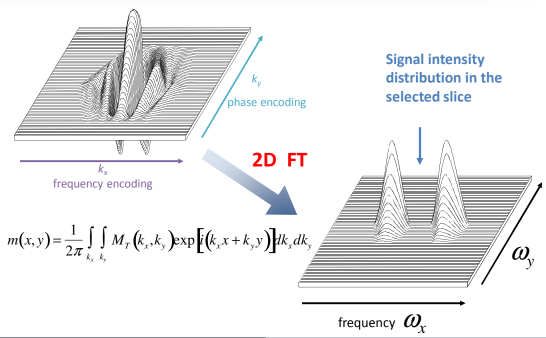


Repeated acquisition of profiles



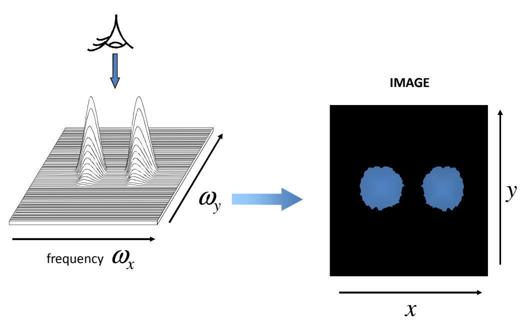
Schematic of k-space acquisition





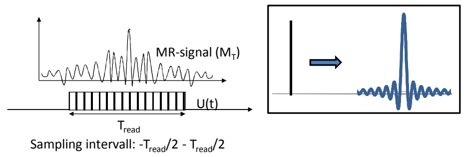
$$m(x,y)=\frac{1}{2\pi}\int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\left[i(k_x x+k_y y)\right]dk_x dk_y$$

Image generation



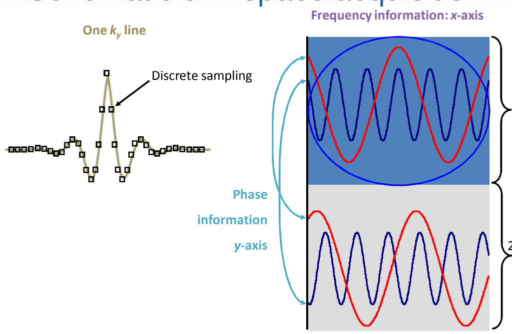
CONTINUE PPT FROM HERE

Discrete sampling

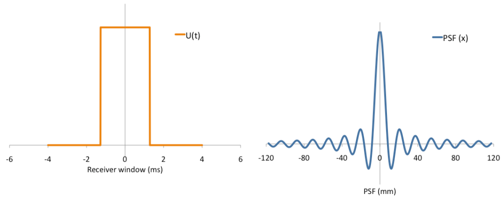


$$PSF(x) = FT(U(t)) = \int_{-T_r/2}^{T_r/2} \exp(-j\gamma G_x x t)\,dt = T_r \frac{\sin\left(\frac{\gamma G_x x T_r}{2}\right)}{\frac{\gamma G_x x T_r}{2}}$$

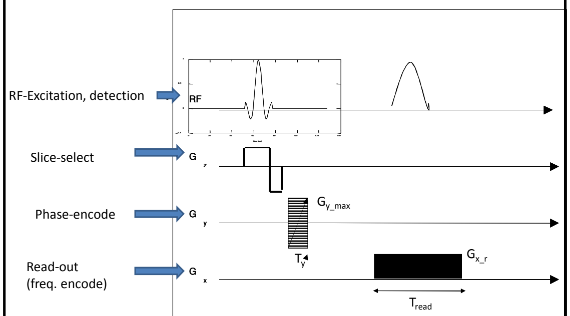
Schematic of k-space acquisition



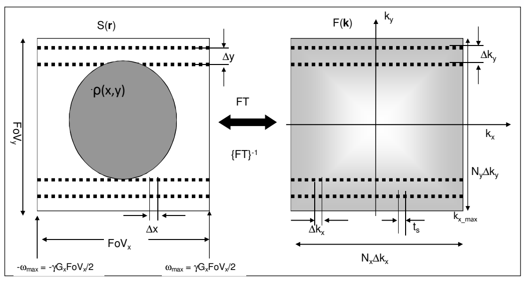
Discrete sampling and PSF



Pulse sequence



K-space vs image space



$$\rho(x,y) = \frac{1}{2\pi}\int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y \qquad \mathbf{k} = \gamma\int_0^t \mathbf{G}(\tau)d\tau$$

K-space egenskaper

Resolution (x):

$$\delta x = \frac{2\pi}{\gamma G_x N_x t_s}$$

Field of view (x):

$$\lambda_{x,\max} = \frac{2\pi}{k_{x,\min}} = \frac{2\pi}{\gamma G_x t_s} = FoV_x$$

Field of view (y):

$$\lambda_{y,\max} = FoV_y = \frac{\pi N_y}{\gamma G_{y\_\max} T_y}$$

Maximum frequency in read-out (x) direction

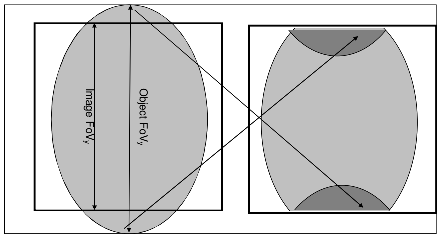
$$\pm\omega_{\max} = \pm\gamma G_x FoV_x / 2$$

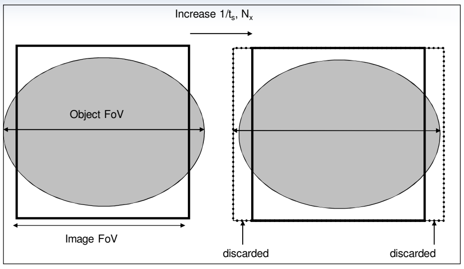
Min sampling rate (x):

$$1/t_s \geq \gamma G_x FoV_x / 2\pi$$

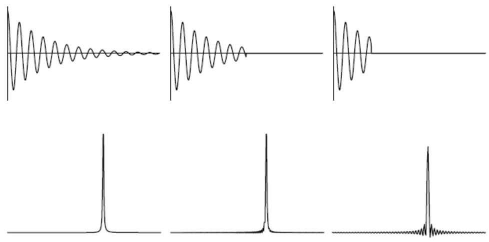
'Sampling rate ' (y):

$$N_y = \gamma G_{y\_\max} T_y FoV_y$$



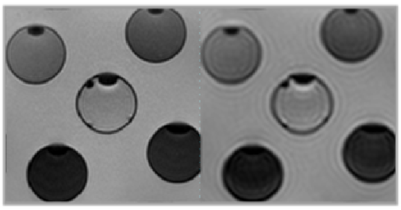


FFT 1D: Truncation Artefact

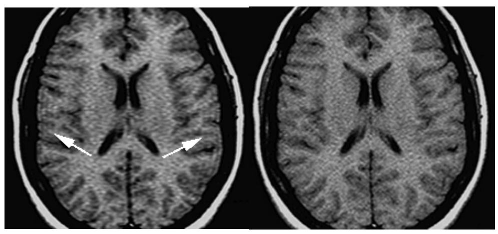


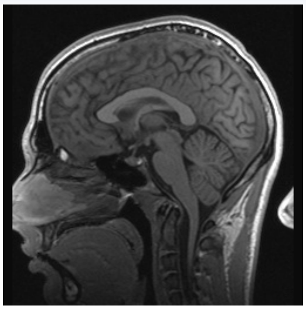
FFT 2D: Truncation artefact

Ringing- (or truncation) artifacts in regions with high spatial frequencies (edges) in a phantom. The artifacts are more evident in the right image due to a lower matrix (N=112, vs N=256 in the left image).

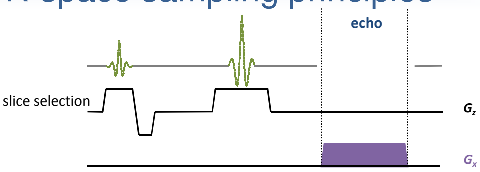


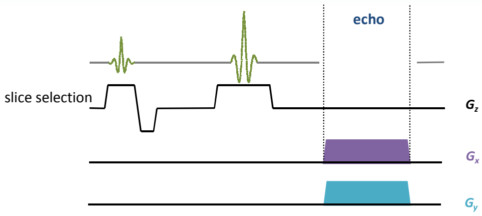
Truncation artifact





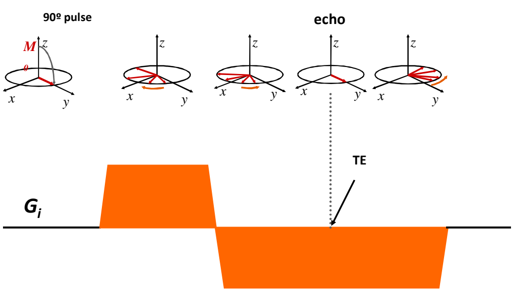
K-space sampling principles

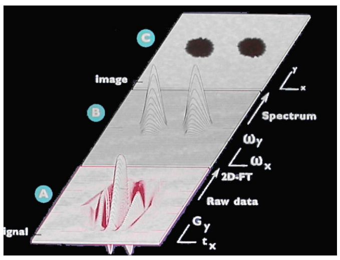




Combination of G_x and G_y to "rotate" the total gradient orientation → reconstruction by back projection

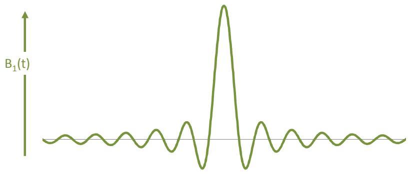
Gradient Echo





B1(t) for slice selective excitation

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z)dz = G_z d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$



Definition of k variable

$$B_1(t) = G_z \int_{-d/2}^{d/2} \exp(j\gamma G_z t \cdot z)dz = G_z d \cdot \frac{\sin(\gamma G_z t \cdot d/2)}{\gamma G_z t \cdot d/2}$$

This equation can be simplified by introducing the following notation

$k=\gamma G_z t$ in general, this can be written $\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{vmatrix} k_x \\ k_y \\ k_z \end{vmatrix}$

$$B_1(t) = G_z d \cdot \frac{\sin(k \cdot d/2)}{k \cdot d/2}$$

k notation

Use of k notation is VERY IMPORTANT in MRI, we will see why…

$$\mathbf{k} = \gamma \int_0^t \mathbf{G}(\tau)d\tau = \begin{vmatrix} k_x \\ k_y \\ k_z \end{vmatrix}$$