Lecture 2010-03-16

- K-space
- Intro to k-space sampling (chap 3)
  - Frequency encoding and phase encoding
- Discrete sampling (chap 2)
  - Point Spread Function
  - K-space properties
- K-space sampling principles (chap 3)
- Basic Contrast mechanism (chap 4)

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k-space

$$\mathbf{k} = \gamma\int_0^t \mathbf{G}(\tau)d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

Limiting discussion to a slice (2D – xy plane), magnetization distribution is given by the 2-dimensional Fourier transform of the spin distribution across the slice

$$M_T(t) = \iint_{slice} \rho(\mathbf{r})\cdot \exp(-j\mathbf{k}\cdot \mathbf{r})d\mathbf{r}$$

*ρ(**r**)* is obtained from the inverse Fourier transform of $M_T(t)$ under the influence of a known gradient configuration

$$\rho(x,y) = \frac{1}{2\pi}\int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y$$

k-space = visualization of the distribution of spatial frequencies in the image.
k-space = Fourier transform of the MR image

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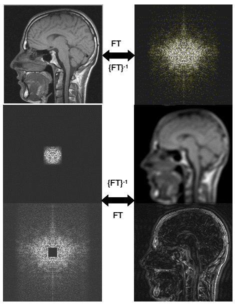
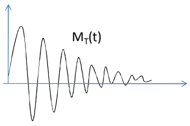
k-space illustrations

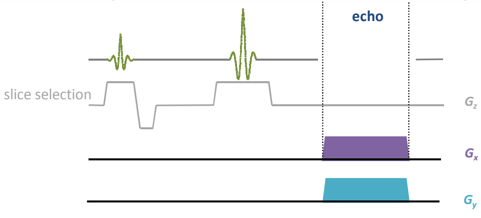




$$\mathbf{k} = \gamma\int_0^t \mathbf{G}(\tau)d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

$$\rho(x,y) = \frac{1}{2\pi}\int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y$$

Use of gradients to make an image



Combination of G_x and G_y to "rotate" the total gradient orientation → reconstruction by back projection

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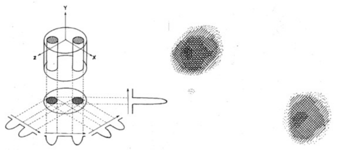
Zeugmatography



Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Lauterbur PC. Image formation by induced local interactions: examples of employing nuclear magnetic resonance. Nature 1973; 242: 190-191.

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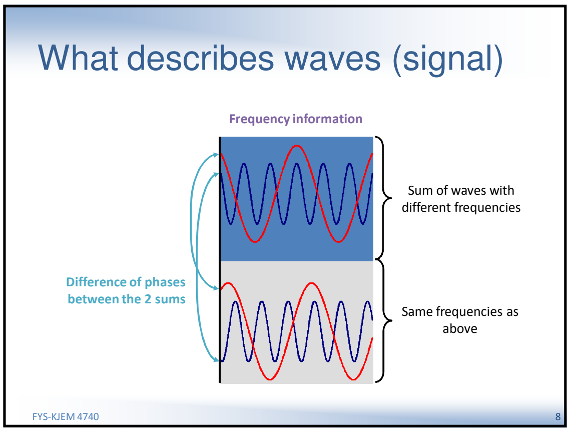
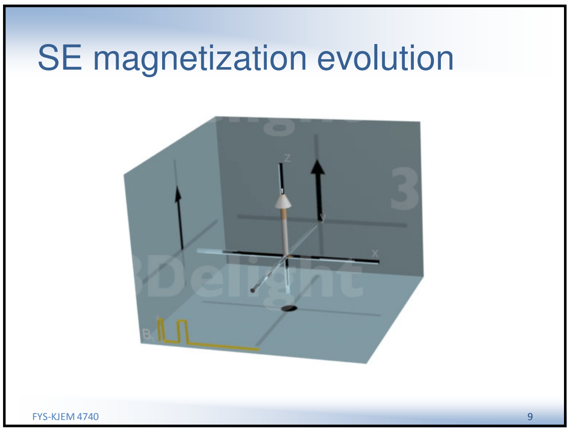
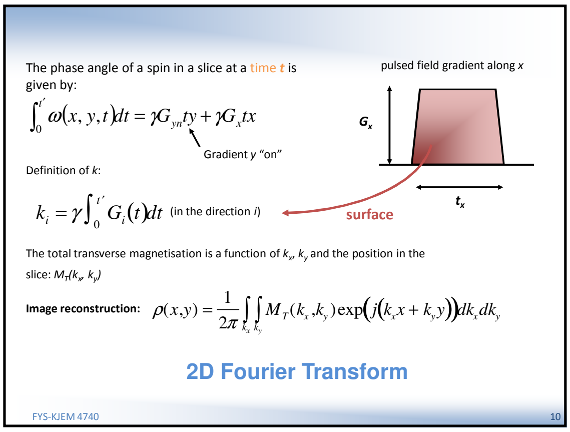
- Basic Contrast mechanisms



What describes waves (signal)



SE magnetization evolution



The phase angle of a spin in a slice at a time *t* is given by:

$$\int_0^{t'} \omega(x,y,t)dt = \gamma G_{yn} ty + \gamma G_x tx$$

Gradient *y* “on”

Definition of *k*:

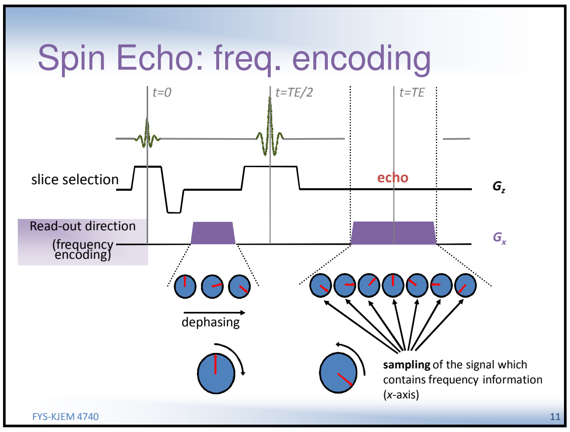
$$k_i = \gamma \int_0^{t'} G_i(t)dt$$ (in the direction *i*)

The total transverse magnetisation is a function of k_x, k_y and the position in the slice: $M_T(k_x, k_y)$

Image reconstruction:

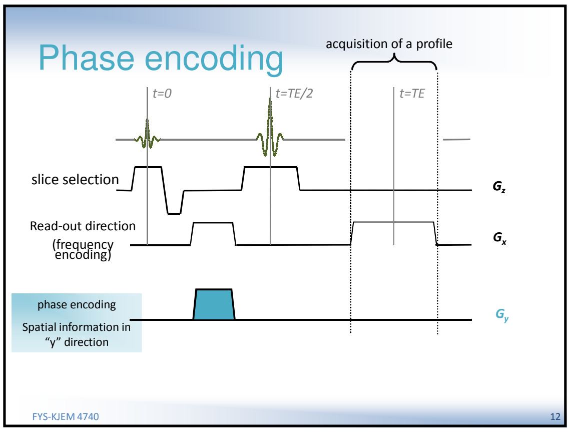
$$\rho(x,y) = \frac{1}{2\pi} \int_{k_x} \int_{k_y} M_T(k_x,k_y)\exp\left(j(k_x x + k_y y)\right)dk_x dk_y$$

2D Fourier Transform

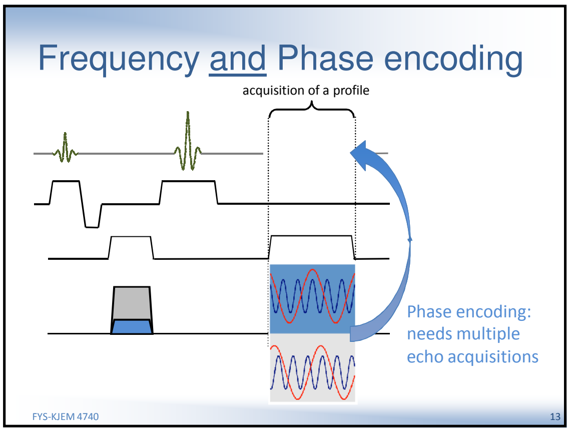


Spin Echo: freq. encoding

sampling of the signal which contains frequency information (*x*-axis)

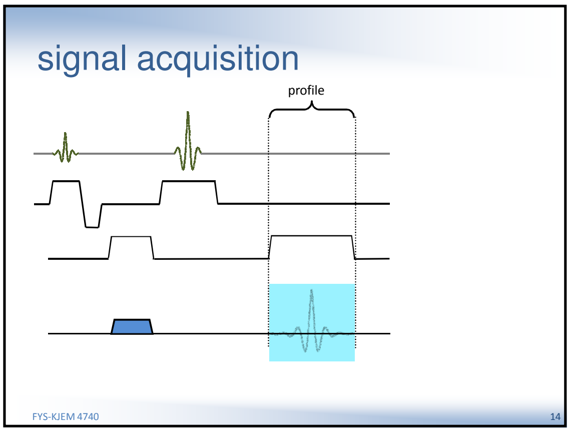


Phase encoding

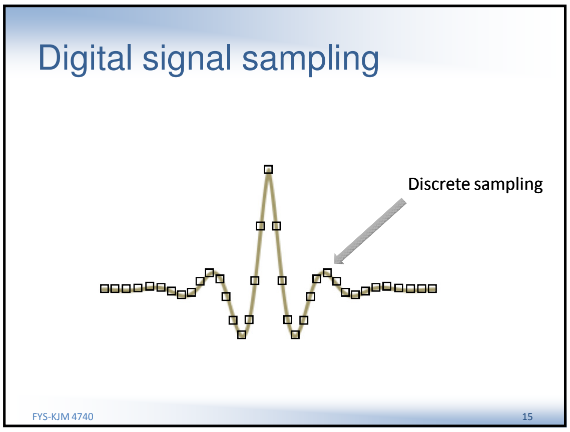


Frequency and Phase encoding

Phase encoding: needs multiple echo acquisitions



signal acquisition



Digital signal sampling

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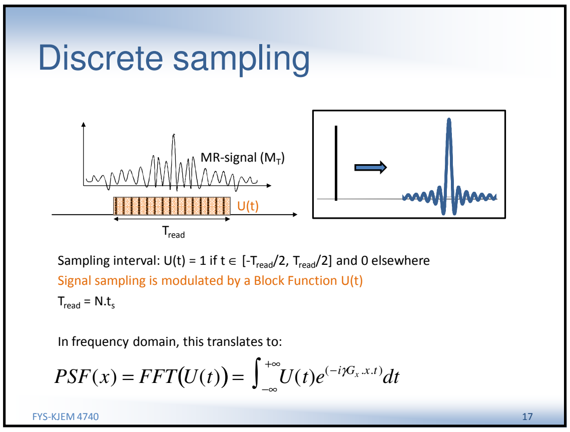


Discrete sampling

Sampling interval: U(t) = 1 if $t \in [-T_{read}/2, T_{read}/2]$ and 0 elsewhere

Signal sampling is modulated by a Block Function U(t)

$T_{read} = N.t_s$

In frequency domain, this translates to:

$$PSF(x) = FFT(U(t)) = \int_{-\infty}^{+\infty} U(t) e^{(-i\gamma G_x.x.t)} dt$$

$$PSF(x) = \int_{-T_{read}/2}^{+T_{read}/2} A.e^{(-i\gamma G_x.x.t)} dt$$

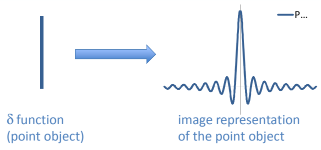
$$PSF(x) = \frac{-A}{i\gamma G_x x} \left[e^{(-i\gamma G_x.x.t)}\right]_{-T_{read}/2}^{T_{read}/2}$$

$$PSF(x) = \frac{A\sin(\gamma G_x.x.T_{read}/2)}{\gamma G_x x}$$

$$\boxed{PSF(x) = T_{read} \frac{\sin\left(\frac{\gamma G_x x T_{read}}{2}\right)}{\left(\frac{\gamma G_x x T_{read}}{2}\right)}}$$

periodic function (See eq. 2-24)

PSF(x)



We can calculate the Full Width at Half Height (**FWHH**)

$$\Delta x = \frac{\pi}{\gamma \cdot G_x \cdot T_{read}}$$

Field of View (FOV)

$$\rho(x,y) = \frac{1}{2\pi}\int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y \Rightarrow \rho(x,y) = \frac{1}{2\pi}\int_{k_x} M_T(k_x)\exp\big(j(k_x x)\big)dk_x$$

$k_{x,\max} = \gamma G_x \cdot T_{read}/2$ (smallest wavelength)

wavelength $\lambda = 2\pi / k_x$

$T_{read} = N.t_s$

$$\lambda_{\max} = \frac{2\pi}{k_{x,\min}} = \frac{2\pi}{\gamma G_x t_s} = FOV_x$$

$$G_x = \frac{2\pi}{\gamma \cdot FOV_x \cdot t_s} = \frac{2\pi}{\gamma}\frac{f_s}{FOV_x}$$

G_x is automatically calculated by the scanner
FOV_x is entered by the user (see Eq. 2-28)

FOV_y

Same definition can be done for the phase encoding direction

$$FOV_y = \frac{2\pi}{k_{y,\min}} = \frac{2\pi}{\gamma\big(G_{y,n} - G_{y,n-1}\big)T_y}$$

π phase difference at the edge of the FOV_y

Consider a square matrix: $N^2 = N_x.N_y$

$$G_{y,n} - G_{y,n-1} = \Delta G_y$$

We need to have $\Delta G_y \cdot \frac{N}{2} = G_{y,\max}$

$$G_{y,\max} = \frac{\pi N_y}{\gamma T_y FOV_y}$$ (See Eq. 2-33)

K-space properties

Resolution (x):

$$\delta x = \frac{2\pi}{\gamma G_x N_x t_s}$$

Field of view (x):

$$\lambda_{x,\max} = \frac{2\pi}{k_{x,\min}} = \frac{2\pi}{\gamma G_x t_s} = FoV_x$$

Field of view (y):

$$\lambda_{y,\max} = FoV_y = \frac{\pi N_y}{\gamma G_{y\_\max} T_y}$$

Maximum frequency in read-out (x) direction

$$\pm\omega_{\max} = \pm\gamma G_x FoV_x / 2$$

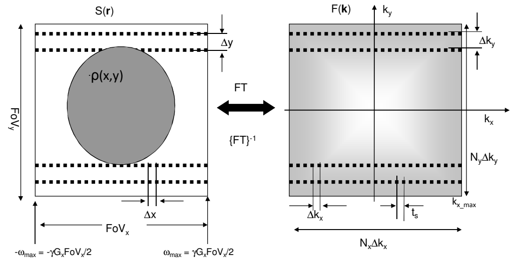
Min sampling rate (x):

$$1/t_s \geq \gamma G_x FoV_x / 2\pi$$

‘Sampling rate ‘ (y):

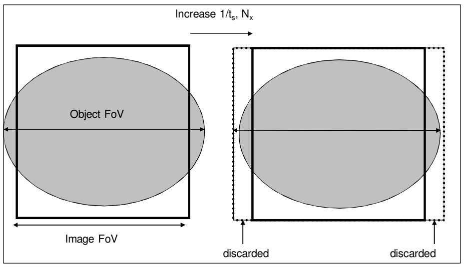
$$N_y = \gamma G_{y\_\max} T_y FoV_y$$

K-space vs image space

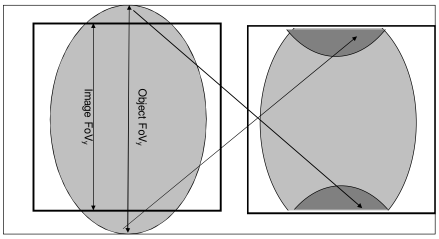


$$\rho(x,y) = \frac{1}{2\pi}\int_{k_x}\int_{k_y} M_T(k_x,k_y)\exp\big(j(k_x x + k_y y)\big)dk_x dk_y \qquad \mathbf{k} = \gamma\int_0^t \mathbf{G}(\tau)d\tau = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

Back-folding / x direction

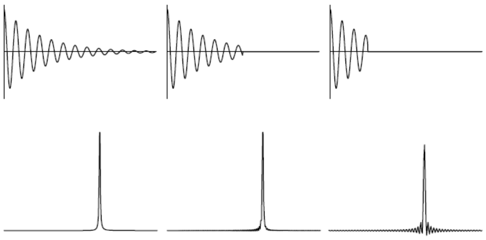


Back-folding / y direction



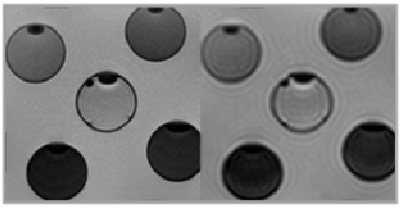
Fold-Over artefact

FFT 1D: Truncation Artefact

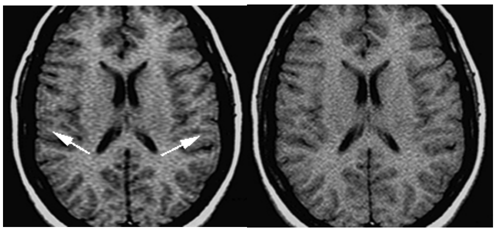


FFT 2D: Truncation artefact

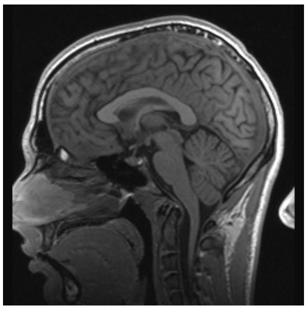
Ringing- (or truncation) artifacts in regions with high spatial frequencies (edges) in a phantom. The artifacts are more evident in the right image due to a lower matrix (N=112, vs N=256 in the left image).



Truncation artifact



Fold-Over artefact



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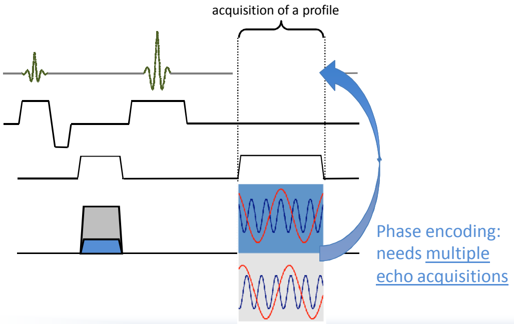
Frequency and Phase encoding



Pulse sequence introduction

Field gradients → Spatial coding

- frequency encoding (gradient x "on" during signal acquisition)
- Phase encoding (gradient y "on" before signal acquisition, repeated at different amplitudes)

Spatially encoded Echoes

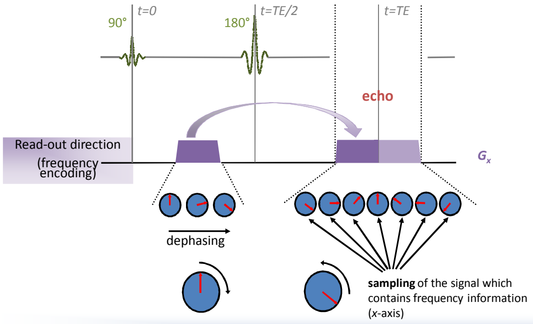
- Spin-Echo: SE

  Use of gradient in a spin-echo experiment to induce spatially dependent "dephasing" and "re-phasing"

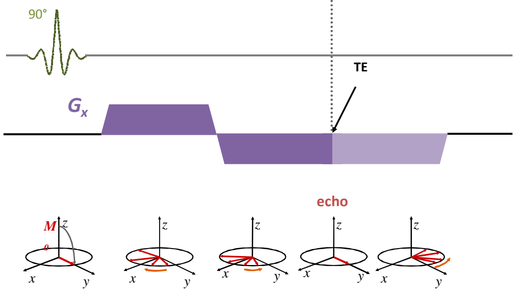
- Gradient Echo: GRE

  Gradient "induced" echo

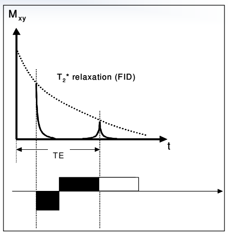
Spin Echo: freq. encoding



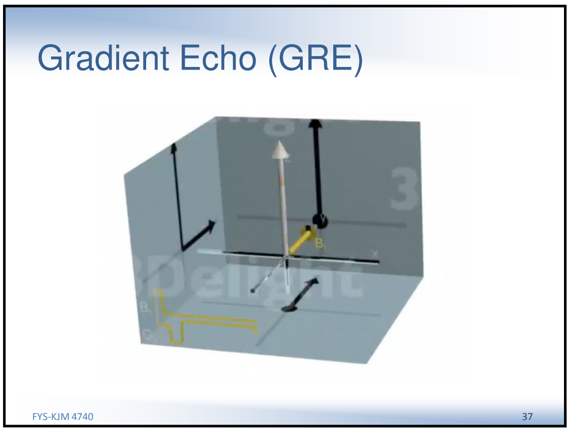
Gradient Echo



Gradient Echo

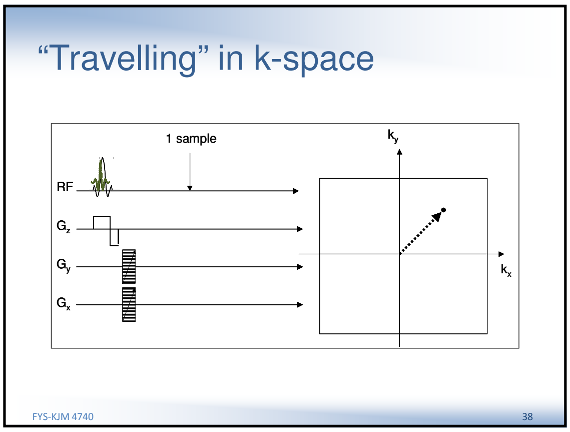


$$k_x = \frac{\gamma}{2\pi}\left[\int_0^{T_y} G_{x\_rew}\,dt + \int_{T_y}^{T_y+T_{read}/2} G_{x\_rew}\,dt\right] = 0 \Rightarrow G_{x\_rew}T_y + G_{x\_r}\frac{T_{read}}{2} = 0$$



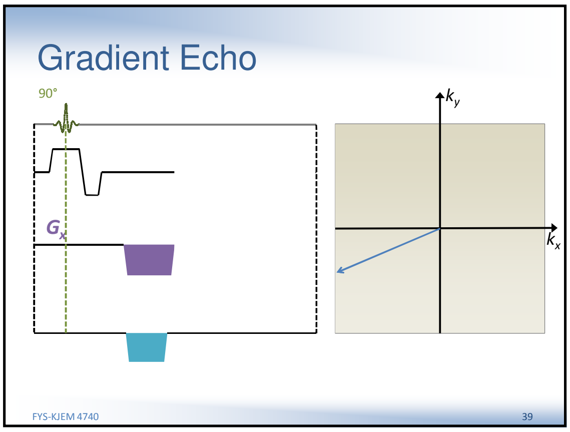
Gradient Echo (GRE)

FYS-KJM 4740 37



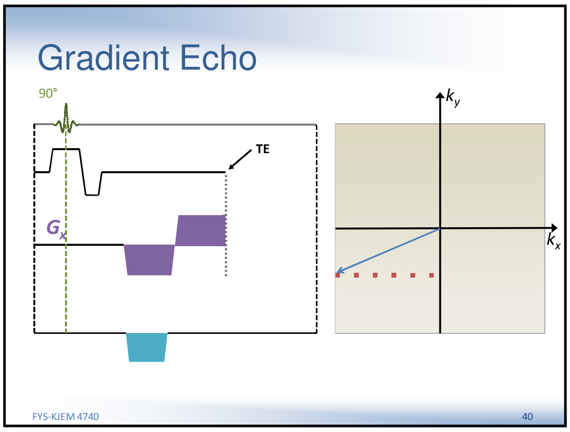
“Travelling” in k-space

FYS-KJM 4740 38



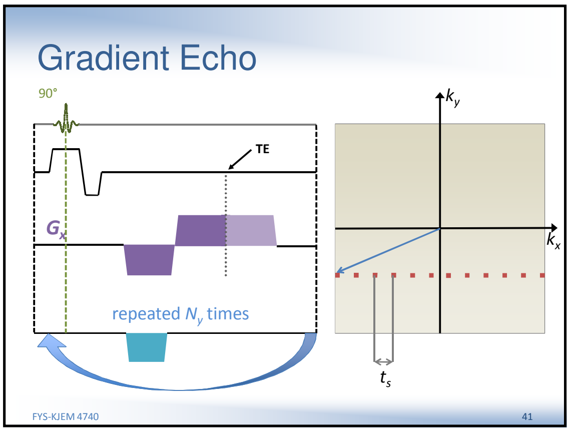
Gradient Echo

FYS-KJEM 4740 39



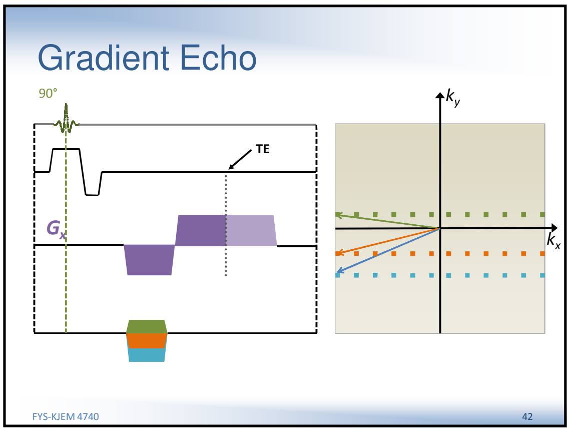
Gradient Echo

FYS-KJEM 4740 40



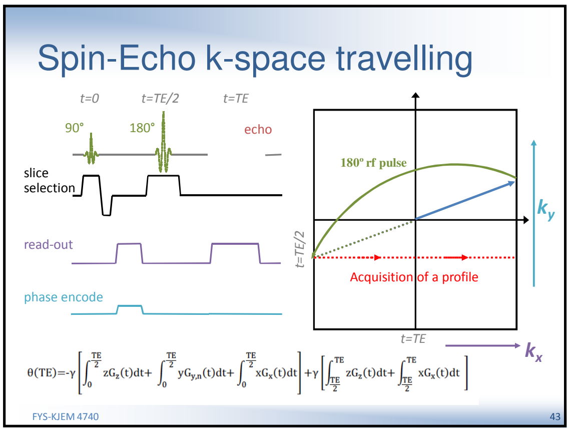
Gradient Echo

FYS-KJEM 4740 41



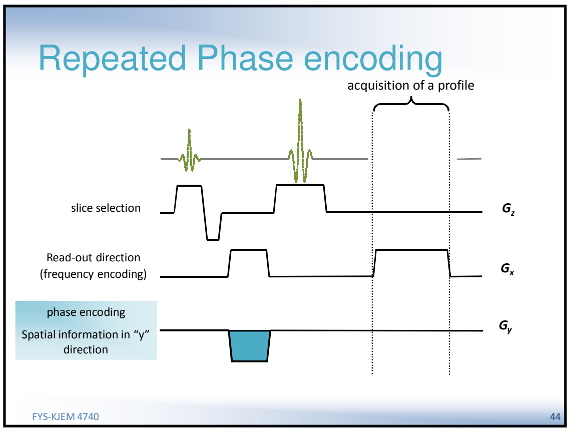
Gradient Echo

FYS-KJEM 4740 42

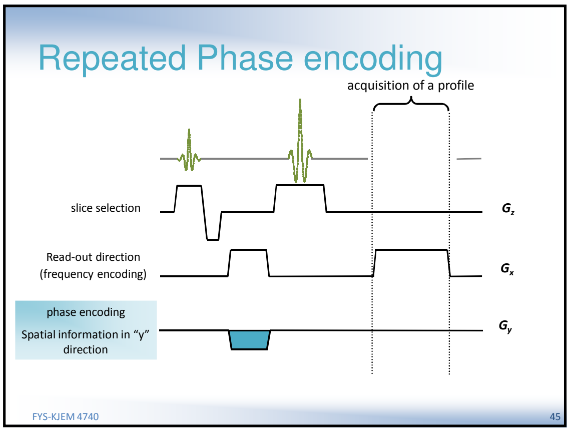


Spin-Echo k-space travelling

$$\theta(TE)=-\gamma\left[\int_0^{\frac{TE}{2}} zG_z(t)dt+\int_0^{\frac{TE}{2}} yG_{y,n}(t)dt+\int_0^{\frac{TE}{2}} xG_x(t)dt\right]+\gamma\left[\int_{\frac{TE}{2}}^{TE} zG_z(t)dt+\int_{\frac{TE}{2}}^{TE} xG_x(t)dt\right]$$



Repeated Phase encoding



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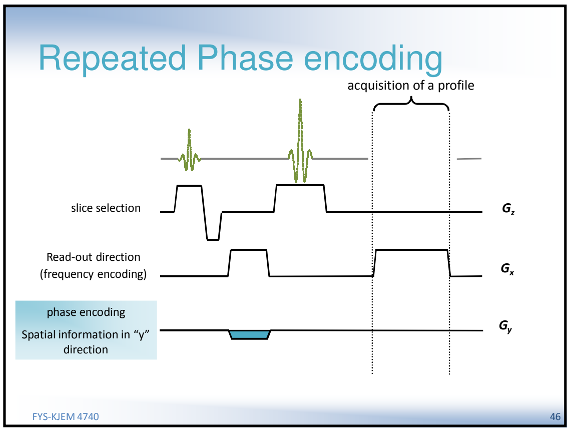
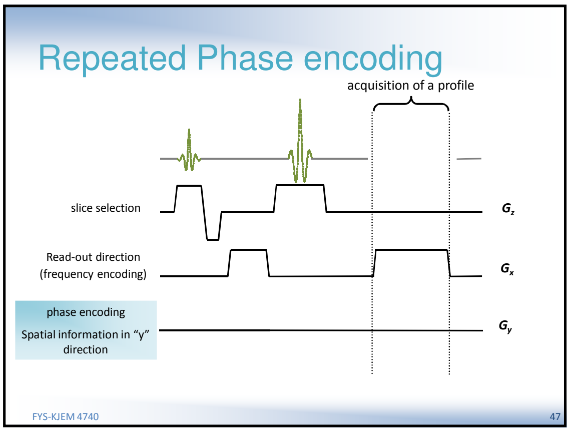
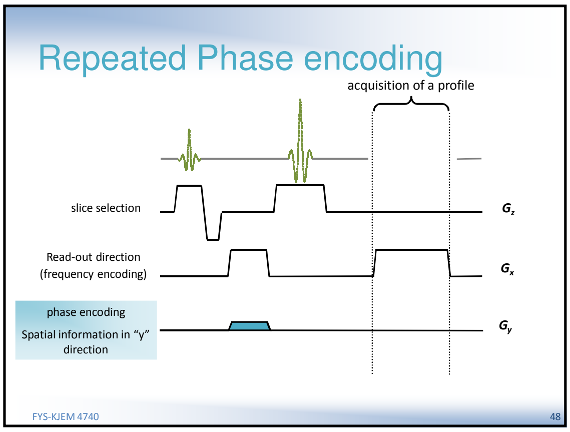


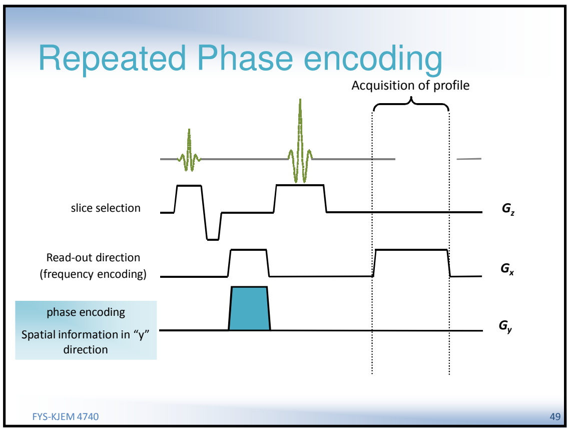
Repeated Phase encoding



Repeated Phase encoding



Repeated Phase encoding



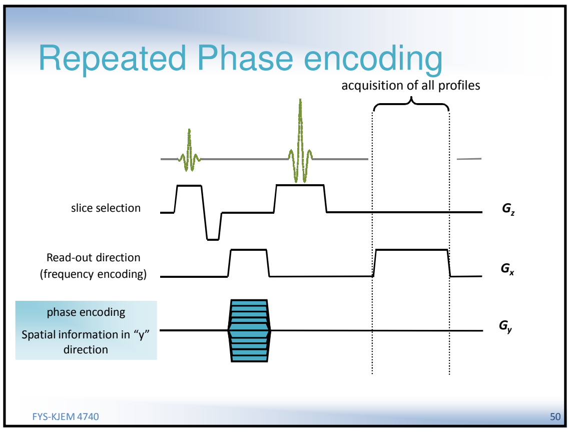
Repeated Phase encoding

Acquisition of profile

slice selection

Read-out direction
(frequency encoding)

phase encoding
Spatial information in “y” direction



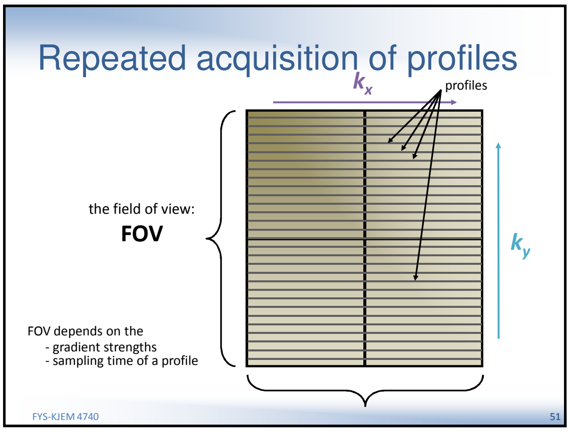
Repeated Phase encoding

acquisition of all profiles

slice selection

Read-out direction
(frequency encoding)

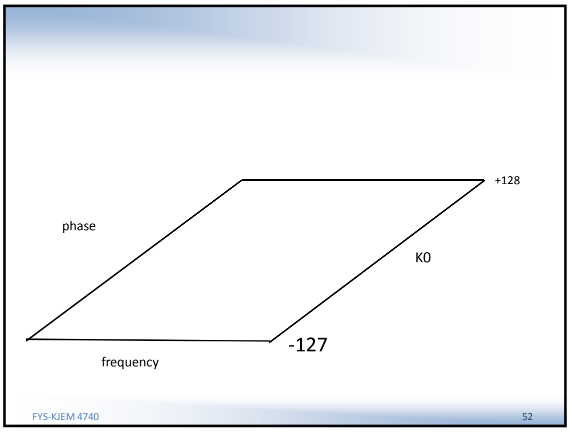
phase encoding
Spatial information in “y” direction

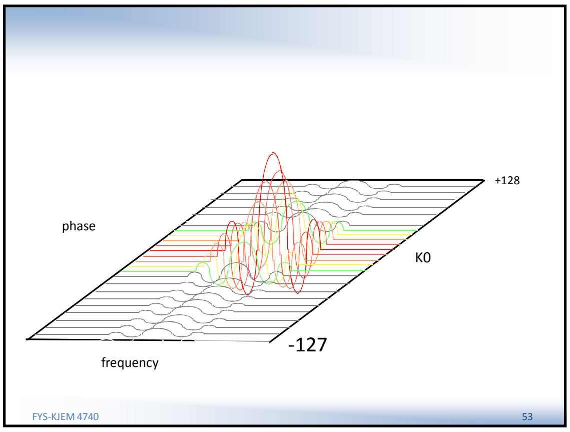


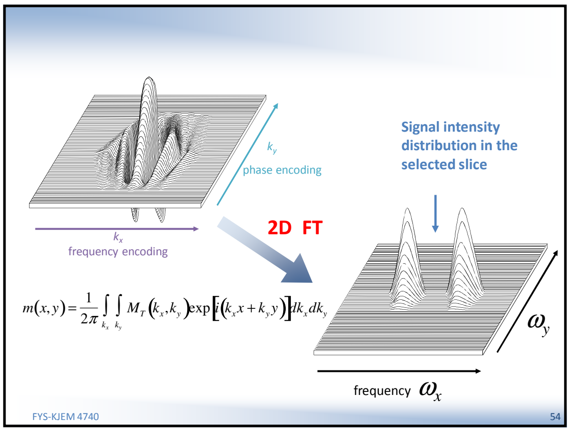
Repeated acquisition of profiles

the field of view:
FOV

FOV depends on the
- gradient strengths
- sampling time of a profile







Signal intensity distribution in the selected slice

$$m(x,y)=\frac{1}{2\pi}\int_{k_x}\int_{k_y}M_T(k_x,k_y)\exp\left[i(k_x x+k_y y)\right]dk_x dk_y$$

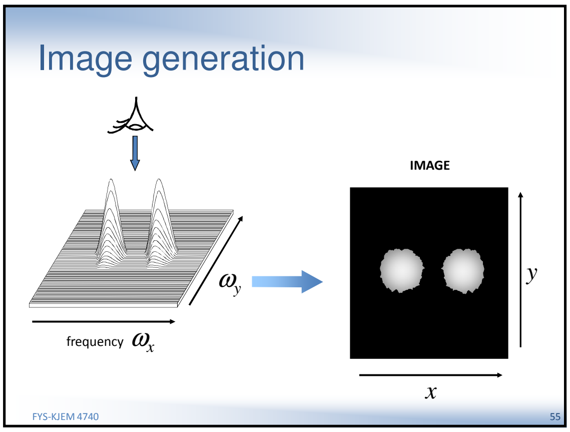
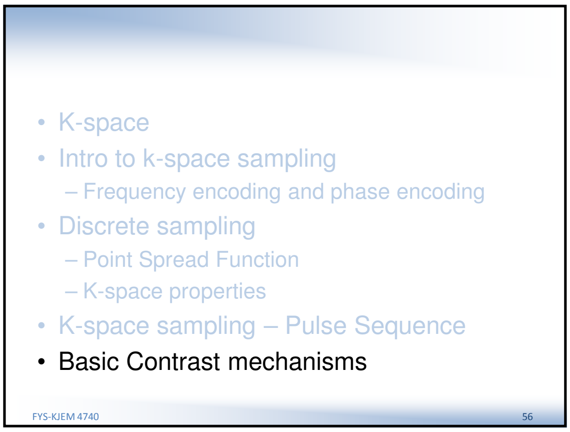


Image generation



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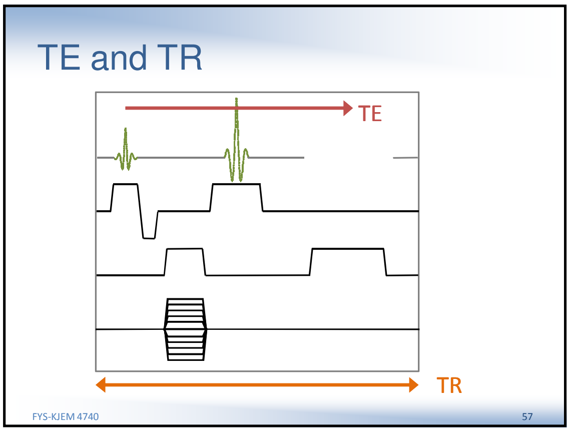
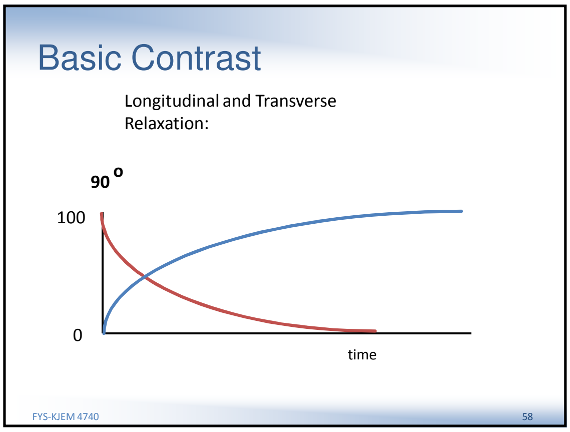
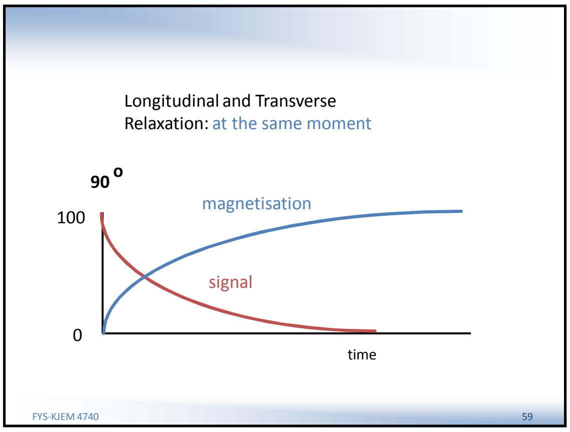


TE and TR

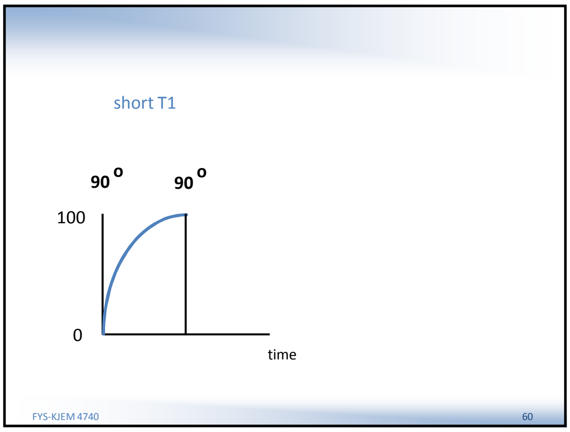


Basic Contrast

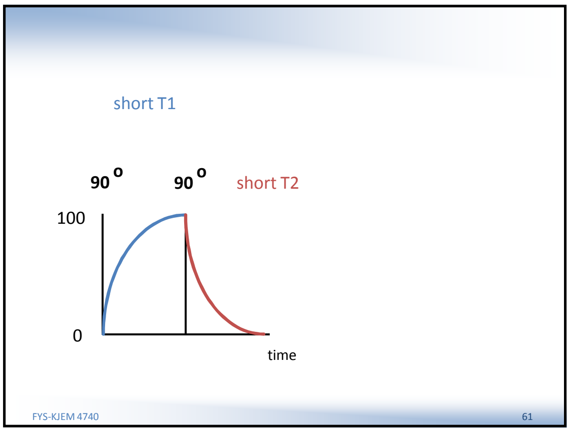
Longitudinal and Transverse
Relaxation:



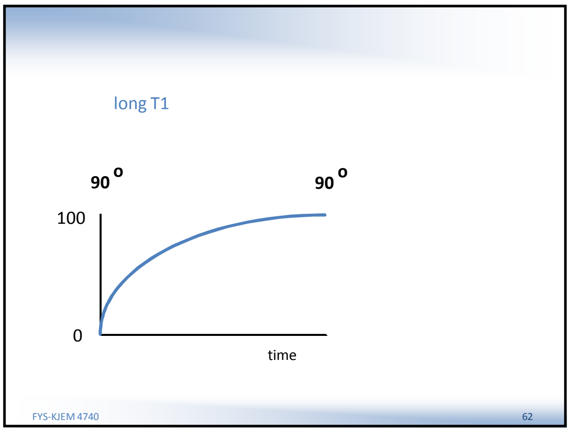
Longitudinal and Transverse
Relaxation: at the same moment



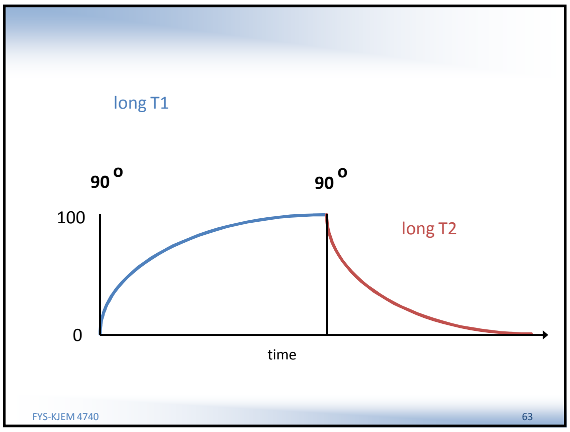
short T1



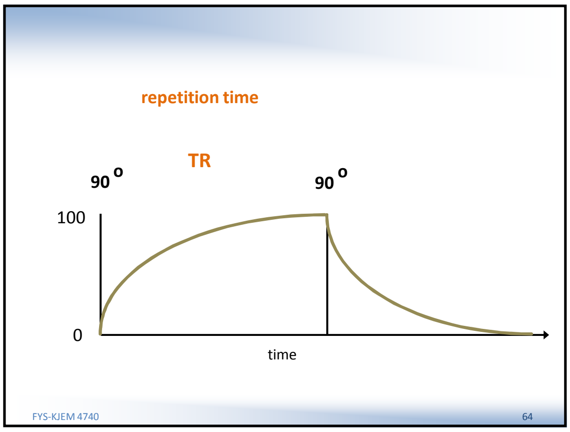
short T1



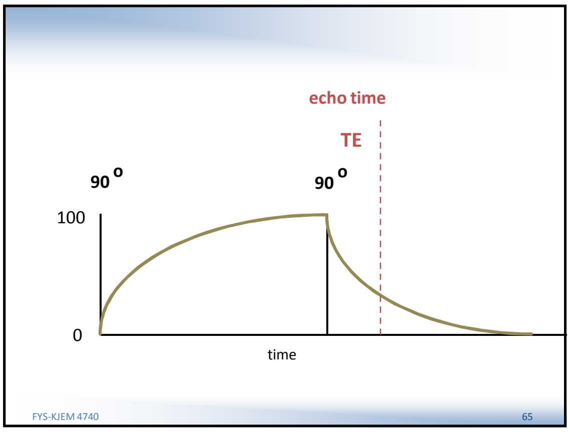
long T1



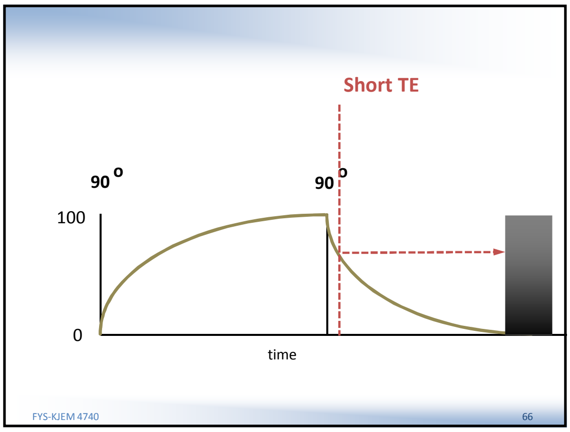
long T1



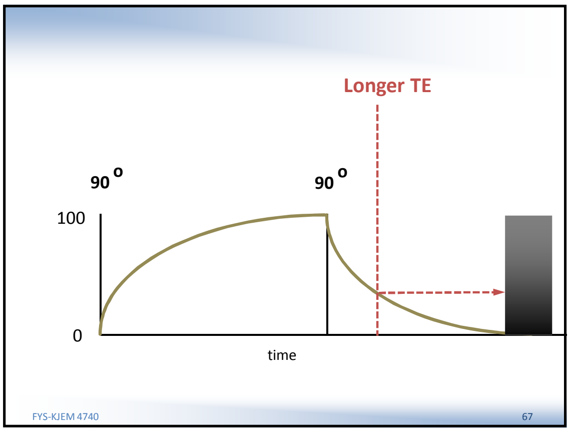
repetition time

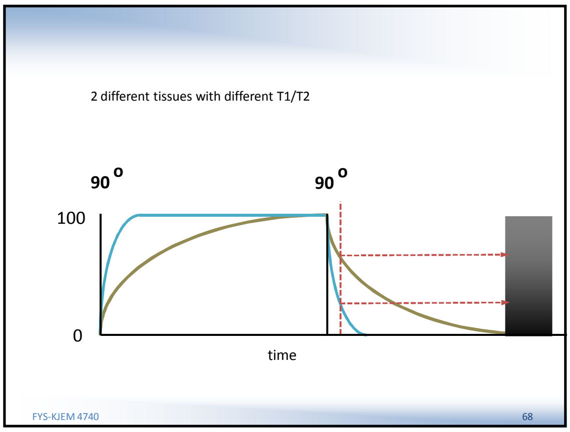


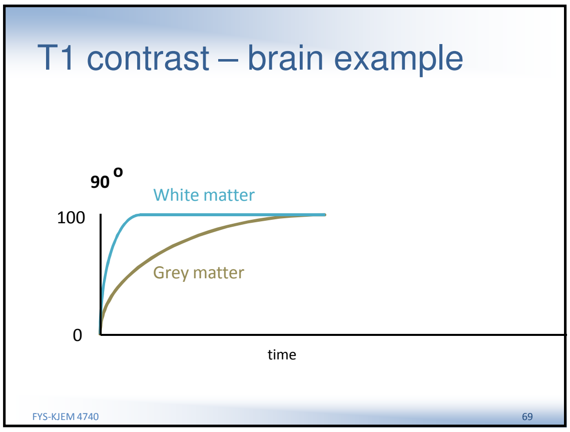
echo time

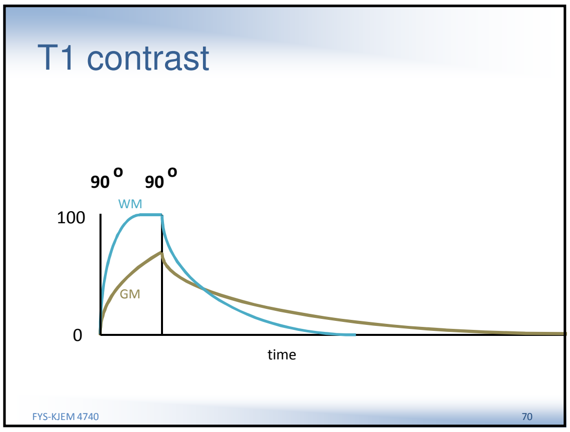


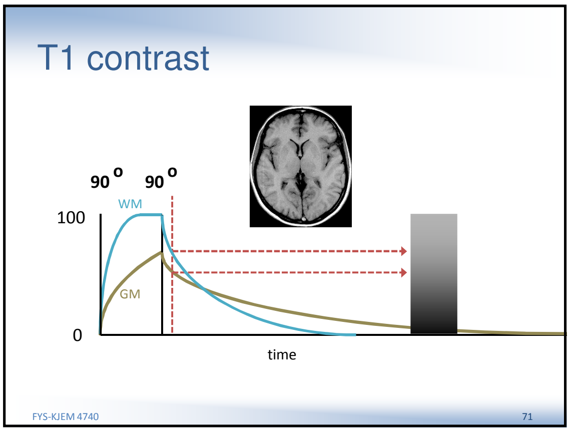
Short TE

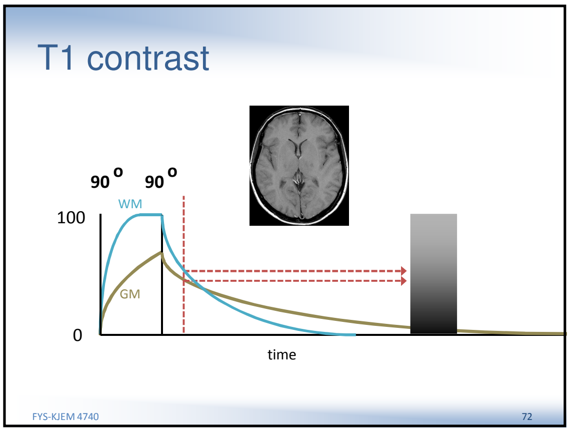



2 different tissues with different T1/T2

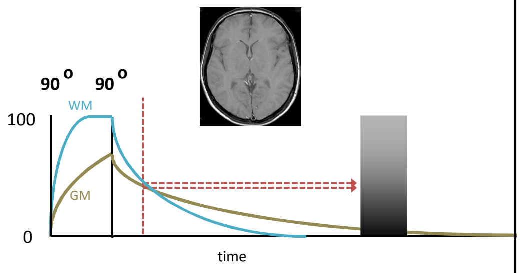

T1 contrast – brain example


T1 contrast


T1 contrast

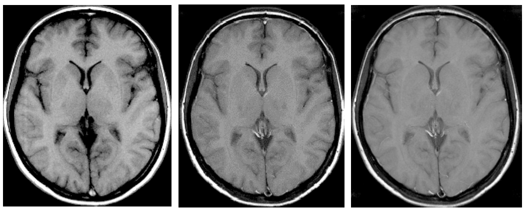

T1 contrast

T1 contrast



T1-weighted images

TR 400 ms



TE 10 ms **TE 30 ms** **TE 40 ms**

T1 weighted images:

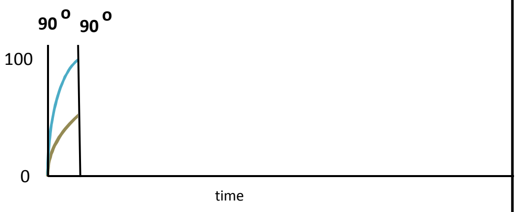
TR short (SE)	**< 600 ms (can be as low as 1.5ms)**
TE short	**< 25 ms**

		i.v.contrast
Fat	bright	
bone marrow	⇧	
white matter		
grey matter	grey	
muscle	⇩	
body fluids		
bone		
air	black	

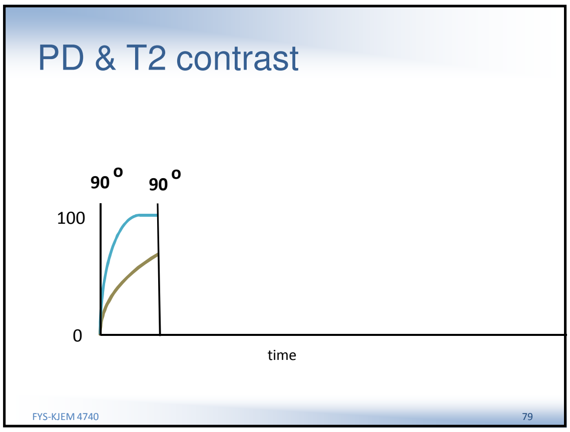
TR < 600 ms **TE < 25 ms**

T1-w knee

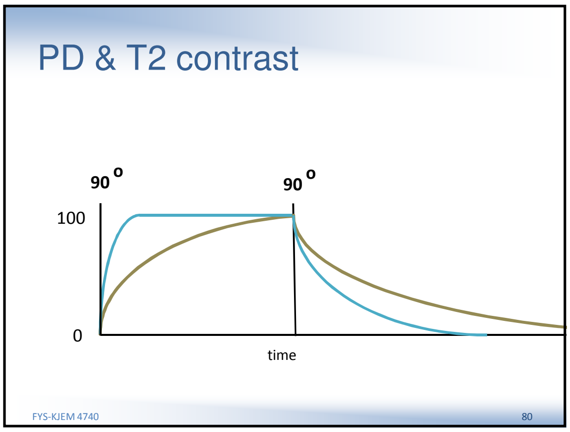
PD (ρ*, proton density) & T2 contrast



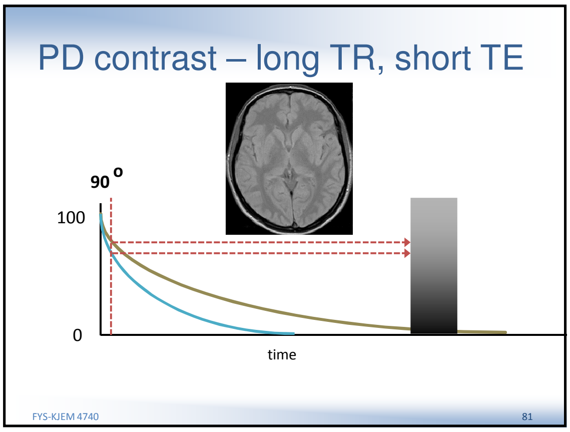
* Denoted rho contrast in the compendium



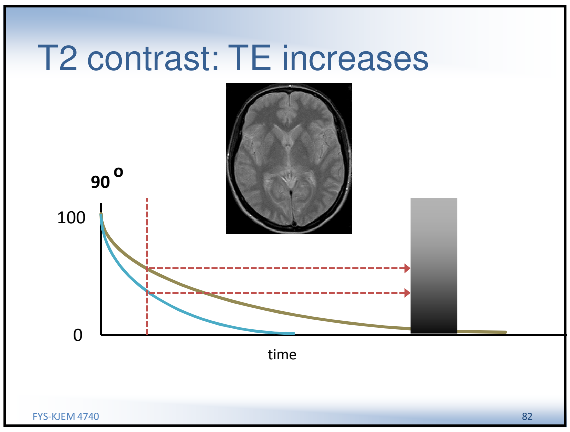
PD & T2 contrast



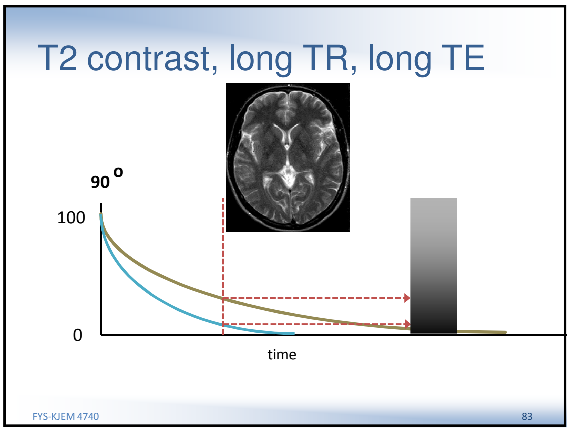
PD & T2 contrast



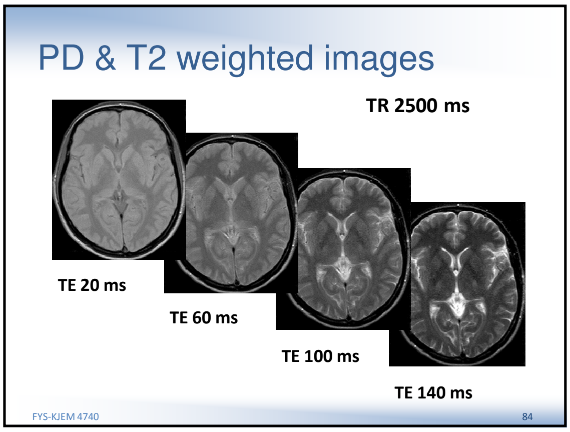
PD contrast – long TR, short TE



T2 contrast: TE increases



T2 contrast, long TR, long TE



PD & T2 weighted images

TR 2500 ms

TE 20 ms

TE 60 ms

TE 100 ms

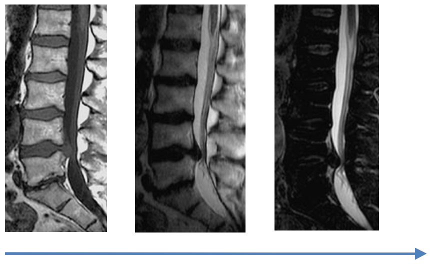
TE 140 ms

T2 weighted images:

TR long **> 1800 ms**

TE long **> 80 ms**

Spine / summary



T1 weighted

T2 weighted

Table of relaxation times

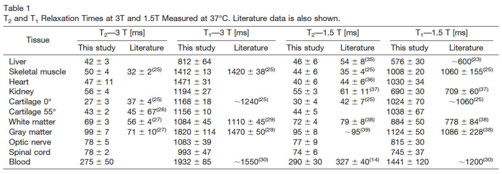


Table 1

T_2 and T_1 Relaxation Times at 3T and 1.5T Measured at 37°C. Literature data is also shown.

Tissue	T_2—3 T [ms]		T_1—3 T [ms]		T_2—1.5 T [ms]		T_1—1.5 T [ms]	
	This study	Literature	This study	Literature	This study	Literature	This study	Literature
Liver	42 ± 3		812 ± 64		46 ± 6	54 ± 8(35)	576 ± 30	∼600(23)
Skeletal muscle	50 ± 4	32 ± 2(25)	1412 ± 13	1420 ± 38(25)	44 ± 6	35 ± 4(25)	1008 ± 20	1060 ± 155(25)
Heart	47 ± 11		1471 ± 31		40 ± 6	44 ± 6(36)	1030 ± 34	
Kidney	56 ± 4		1194 ± 27		55 ± 3	61 ± 11(37)	690 ± 30	709 ± 60(37)
Cartilage 0°	27 ± 3	37 ± 4(25)	1168 ± 18	∼1240(25)	30 ± 4	42 ± 7(25)	1024 ± 70	∼1060(25)
Cartilage 55°	43 ± 2	45 ± 67(26)	1156 ± 10		44 ± 5		1038 ± 67	
White matter	69 ± 3	56 ± 4(27)	1084 ± 45	1110 ± 45(29)	72 ± 4	79 ± 8(38)	884 ± 50	778 ± 84(38)
Gray matter	99 ± 7	71 ± 10(27)	1820 ± 114	1470 ± 50(29)	95 ± 8	∼95(39)	1124 ± 50	1086 ± 228(38)
Optic nerve	78 ± 5		1083 ± 39		77 ± 9		815 ± 30	
Spinal cord	78 ± 2		993 ± 47		74 ± 6		745 ± 37	
Blood	275 ± 50		1932 ± 85	∼1550(30)	290 ± 30	327 ± 40(14)	1441 ± 120	∼1200(30)

From: Greg J. Stanisz, Magnetic Resonance in Medicine 54:507–512 (2005)