

Lecture 2011-03-15

- Multislice acquisition (MS, 3D)
- GRE: Simple Steady-State model
- RF pulse train (multiple echoes)
 - Eight-Ball Echo
 - Stimulated Echo
 - Multiples echoes
- GRE FID and ECHO components
- Mathematical Model of GRE
- GRE contrast properties and techniques

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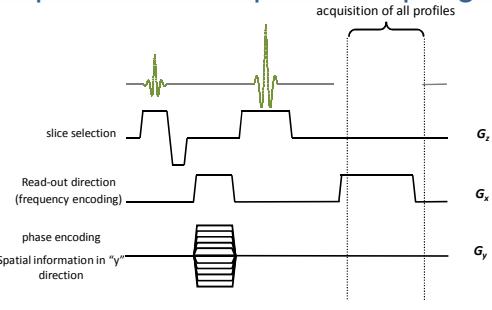
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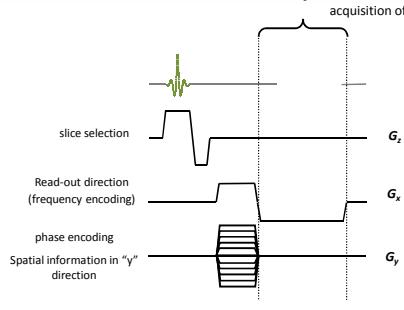
Spin Echo – k-space sampling



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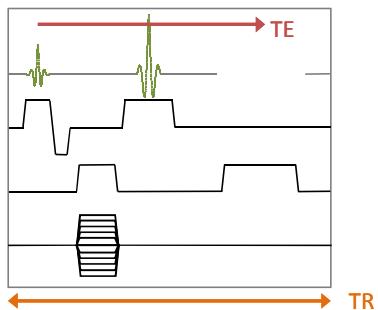
Gradient Echo – k-space sampling



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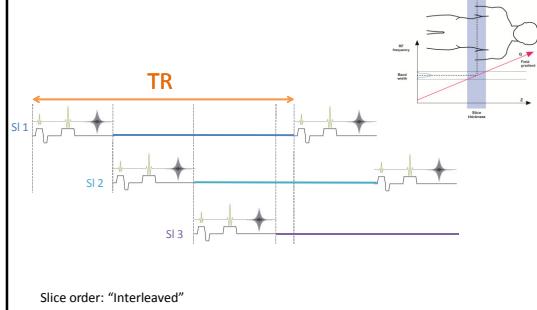
TE and TR



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Multislice acquisition

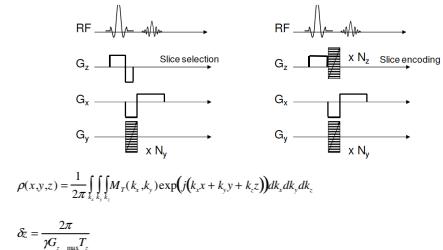


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3D acquisition

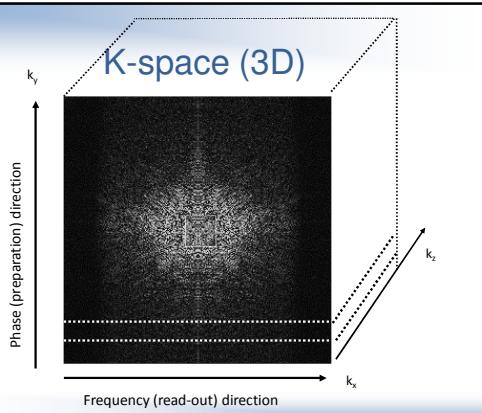
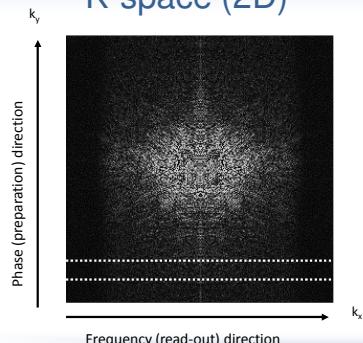
Same principle of phase encoding is applied to the slice direction (refocusing lobe of the slice selection)



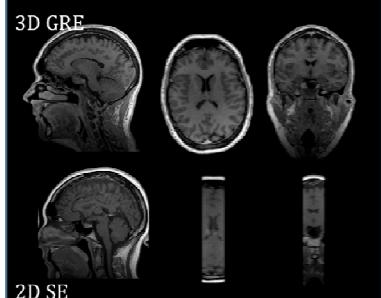
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K-space (2D)



2D versus 3D acquisition



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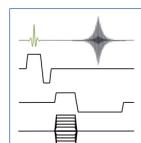
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GRE properties

- Requires less time than SE to form an Echo (i.e. TE shorter for 1 k-space profile acquisition)
- RF pulse angle $\alpha \leq 90^\circ$
- $SI = f(T1, T2, T2^*)$



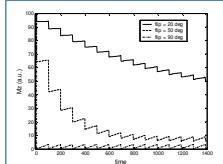
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GRE: Simple steady-state model

TR short, $T2^* \ll TR$, pulse angle α

Repeated RF pulses \rightarrow dynamic equilibrium called "Steady State"

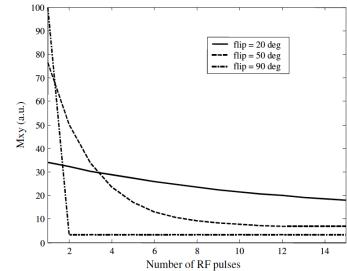


Steady-state reached at the 2nd pulse if $\alpha = 90^\circ$.

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GRE steady-state

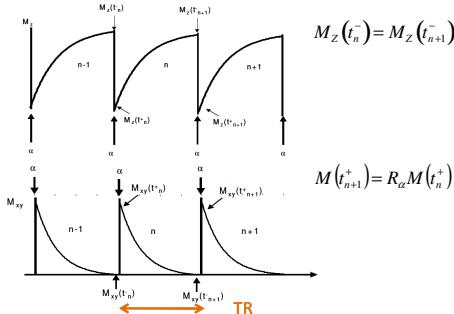


Note that the highest signal is obtained for $\alpha = 20^\circ$

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GRE: Simple steady-state model



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Simple mathematical model

'Steady state' condition:

$$M_y(t_{n+1}^-) = M_y(t_n^-)$$

$$M_z(t_{n+1}^-) = M_z(t_n^-) E_i + M_0(1 - E_i) \quad (See \ Eq. \ 2-8 \ in \ compendium)$$

Looking at the rotation by the α RF pulse along x' axis, we get:

$$\begin{bmatrix} M_y(t_{n+1}^-) \\ M_z(t_{n+1}^-) \end{bmatrix} = R_\alpha \begin{bmatrix} M_y(t_n^-) \\ M_z(t_n^-) \end{bmatrix} \quad R_\alpha = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

We assume that $M_y(t_n^-) = 0$ which is true if $T2^* \ll TR$

$$\begin{bmatrix} M_y(t_{n+1}^-) \\ M_z(t_{n+1}^-) \end{bmatrix} = \begin{bmatrix} \sin(\alpha) \\ \cos(\alpha) \end{bmatrix} M_z(t_n^-)$$

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Simple mathematical model (cont.)

We have now the following equations

$$\begin{bmatrix} M_y(t_{n+1}^-) \\ M_z(t_{n+1}^-) \end{bmatrix} = \begin{bmatrix} \sin \alpha \\ \cos \alpha \end{bmatrix} M_z(t_n^-) \quad \text{and} \quad M_z(t_{n+1}^-) = M_z(t_n^-) E_i + M_0(1 - E_i)$$

At equilibrium (steady-state): $M_z(t_n^-) = M_z(t_{n+1}^-) = \cos \alpha \cdot M_z(t_{n+1}^-)$

and writing $M_z(t_{n+1}^-) = M_z$

$$\text{Which then gives } M_z = \frac{M_0(1 - E_i)}{(1 - \cos \alpha E_i)}$$

$$\text{Applying an RF pulse of angle } \alpha \quad M_y(t_{n+1}^-) = \frac{\sin \alpha M_0(1 - E_i)}{(1 - \cos \alpha E_i)} \quad \text{Eq. 4-8 (Compendium)}$$

Moreover, the net magnetization of GRE readout is modulated by $\exp(-TE/T2^*)$

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Simple mathematical model (cont.)

Finding the maximum of this function: $\frac{dM_y(t_{n+1}^-)}{d\alpha} = 0$

$$\frac{dM_y(t_{n+1}^-)}{d\alpha} = \frac{M_0((1 - E_i)(\cos \alpha - E_i))}{(1 - \cos \alpha E_i)^2}$$

$$\cos \alpha_e = E_i = \exp(TR/T1) \quad \alpha_e \text{ is the Ernst angle}$$

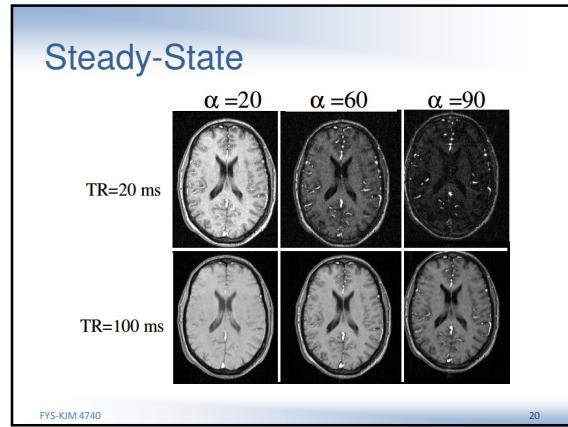
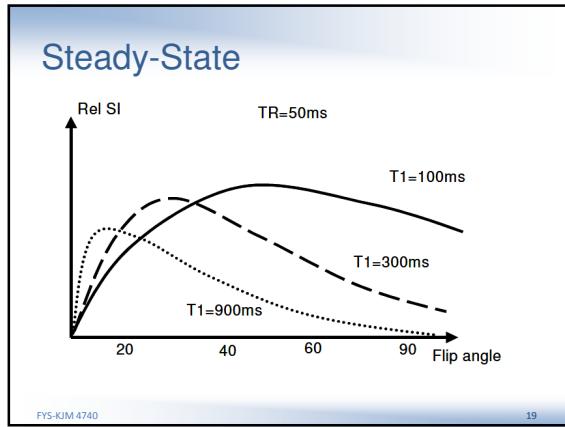
contrast weighting in a spoiled GRE sequence

- T1-weighted
 - T2*-weighted
- (weighting depends on the TR/T1 at given α)

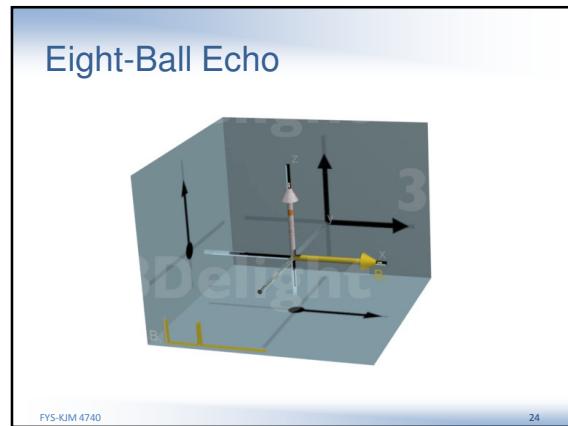
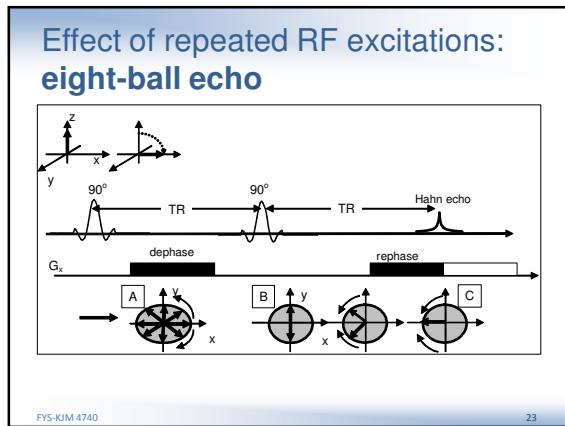
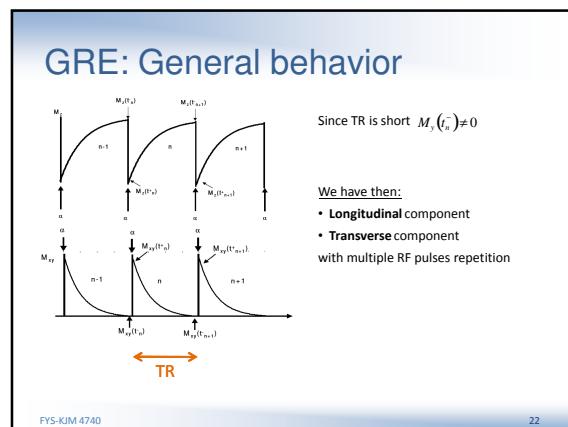
Short TR (TR/T1 $\ll 1$)	Large α	T1 weighting
Long TR (TR/T1 ≈ 1)	Low α	T2* weighting

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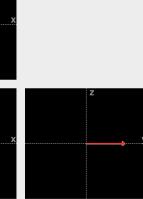
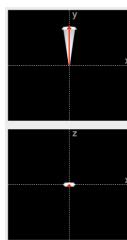
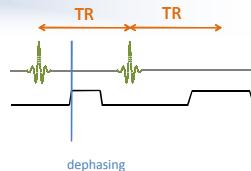
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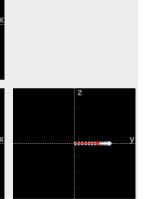
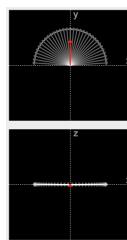
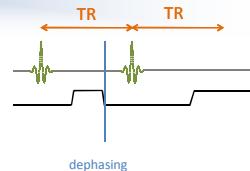
Eight-Ball Echo



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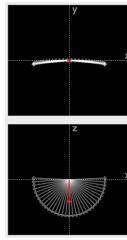
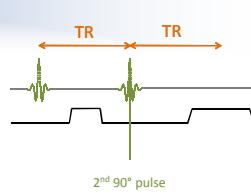
Eight-Ball Echo



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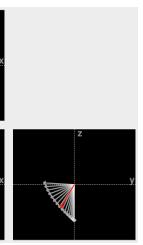
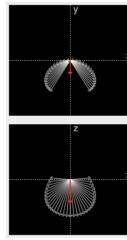
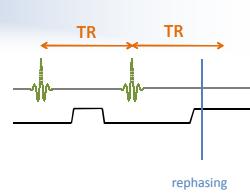
Eight-Ball Echo



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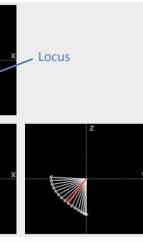
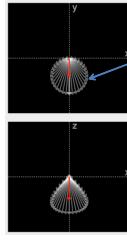
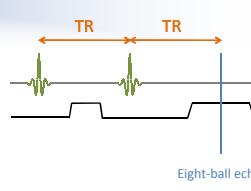
Eight-Ball Echo



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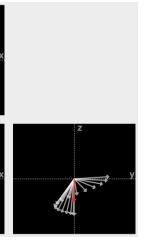
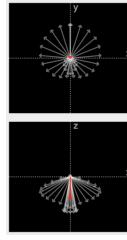
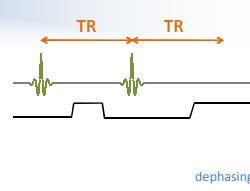
Eight-Ball Echo



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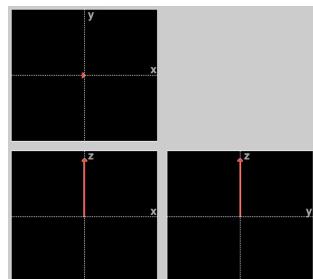
Eight-Ball Echo



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Eight-Ball Echo



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Stimulated Echo

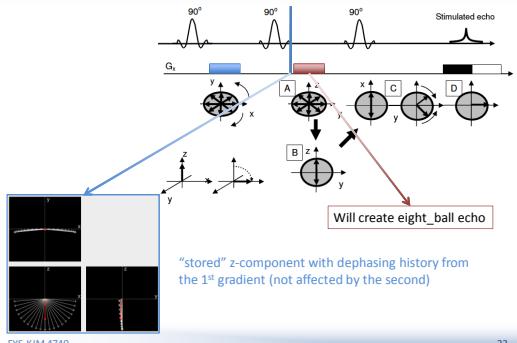
- two 90° pulses \rightarrow eight-ball echo
- The 2nd 90° pulse flipped x' component of the transverse magnetization along z-axis (this z-magnetization carries magnetization history from before 2nd RF pulse, i.e. gradient dephasing as function of position)*
- 3rd 90° pulse then flips back this longitudinal component into the transverse plane. The 3rd G_x gradient will refocus these components previously stored along z-axis

* The longitudinal component is not affected by the second gradient after RF pulse

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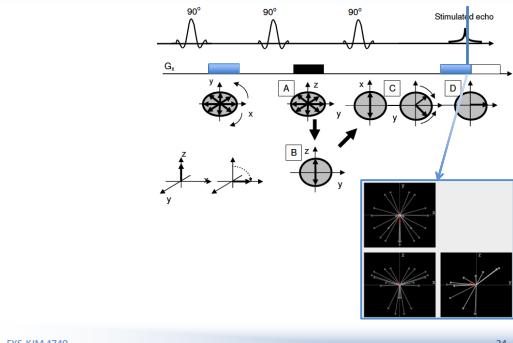
Stimulated Echo



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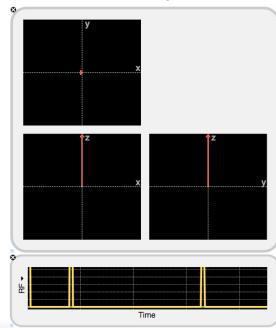
Stimulated Echo



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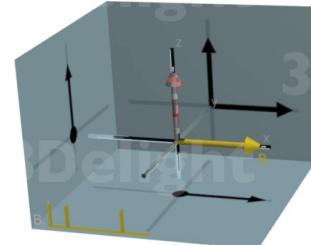
Stimulated Echo (simulation)



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Stimulated Echo (simulation)



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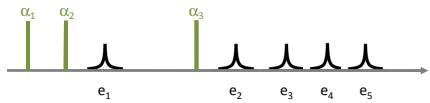
Multiple Echoes

- From Steady-State simplified model / Ernst angle $\alpha < 90^\circ$
- Both transverse and longitudinal components are present at any time
- RF pulse train generates multiple echoes**

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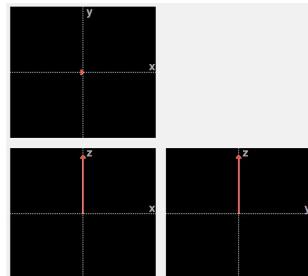
3 RF-pulses gives a total of 5 echoes



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3 plans projection of 3 RF pulses effect



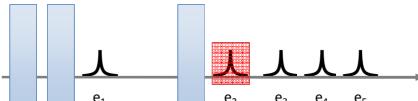
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e₁: eight-ball echo of α_1 and α_2

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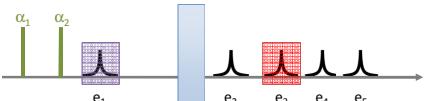
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e₂: Stimulated echo of α_1, α_2 and α_3

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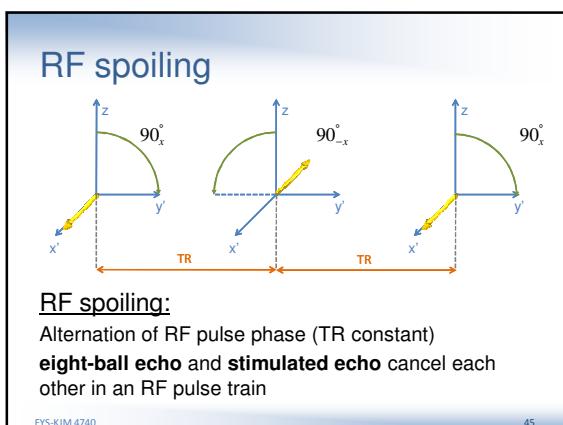
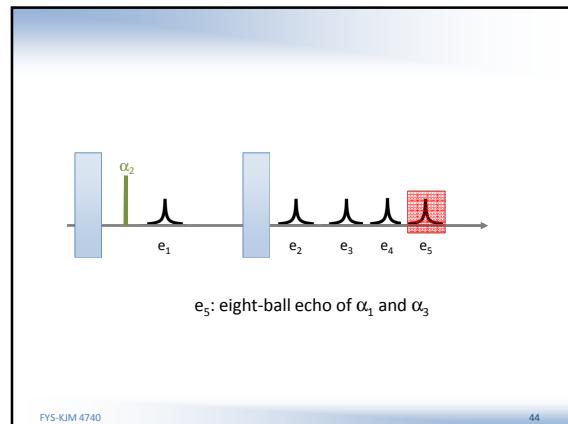
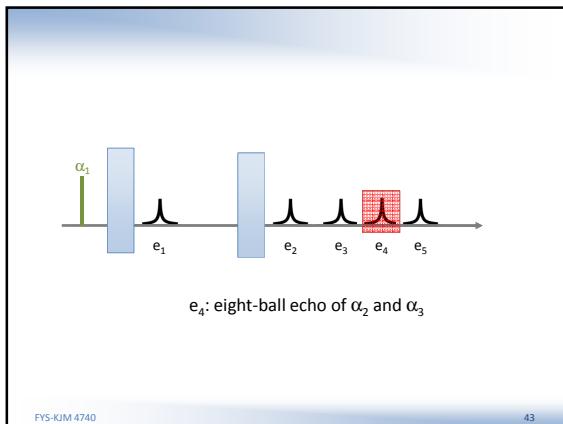
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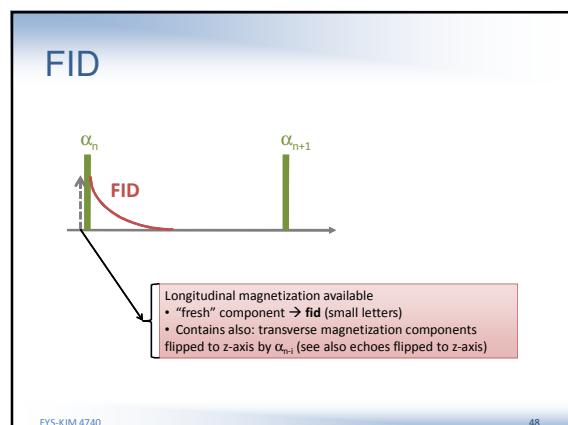
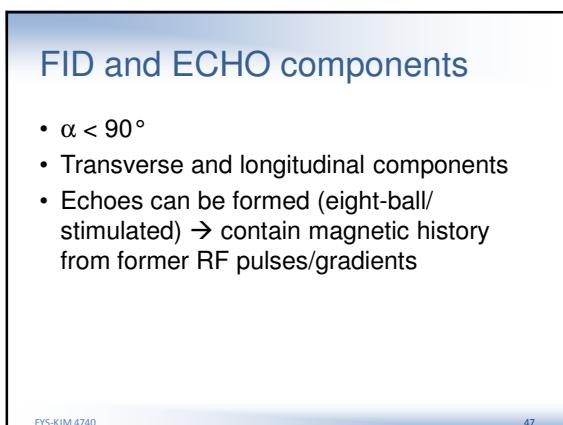
e₃: refocused e₁ by α_3

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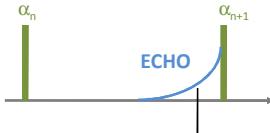
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ECHO

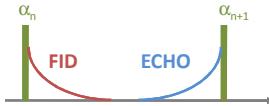


- Available transverse component preceding an RF pulse
- Refocused echoes (*position dependent history/gradients*)
 - These echoes are partially rotated out of the transverse plane ($\alpha < 90^\circ$) and a part remains in the transverse plane after the RF Pulse α_{n+1} .

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FID & ECHO



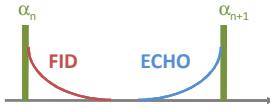
FID is mainly T1 weighted (based on available longitudinal magnetization before the RF pulse)

ECHO is mainly T2 weighted (persistence of the transverse components that form the echoes (e.g. eight-ball etc.)

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FID & ECHO



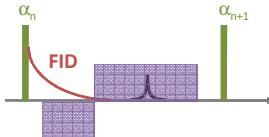
Now considering the readout gradients in this figure:

Both **FID** and **ECHO** can give a gradient echo (signal that will be encoded into k-space to form the GRE image)

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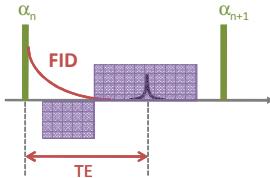
FID & ECHO



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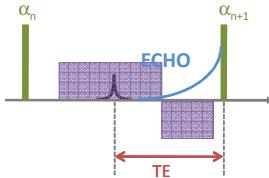
FID & ECHO



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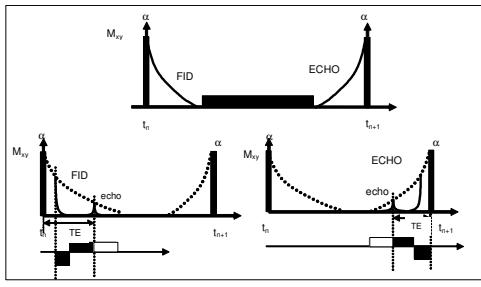
FID & ECHO



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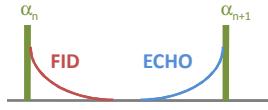
Both FID and ECHO signals are used in Imaging



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FID & ECHO



FID and ECHO properties:

FID carries partially the spatial info contains in the ECHO component which is flipped to the x-y plane by the α_n pulse

It can be shown that ECHO can add parallel to the FID or anti-parallel depending on the previously acquired phase angle (i.e. position) → Periodic "band" structure in the image with different T1/T2 weighting

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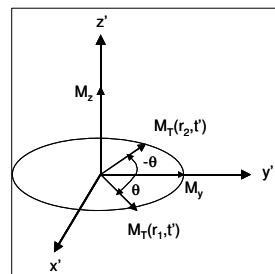
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Precession: M_T is position dependent (in-plane gradient)



$$\theta(\mathbf{r}) = \gamma \int \mathbf{r} \cdot \mathbf{G}(t) dt$$

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Steady-state signal behavior

From Chap 2:

Excitation

$$\mathbf{R}_{\alpha,x'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

Precession

$$\mathbf{P}_{0,x'} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Relaxation

$$\mathbf{M}(t^{-}_{n+1}) = \begin{bmatrix} E_2 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix} \mathbf{M}(t^{+}_n) + (1-E_i)\mathbf{M}_0$$

$$\mathbf{M} = [M_x, M_y, M_z]^T, \mathbf{M}_0 = [0, 0, M_0]^T, E_2 = \exp(-t/T2) \text{ and } E_1 = \exp(-t/T1).$$

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Steady-state signal behavior

Relaxation and Precession occurs simultaneously and can be combined:

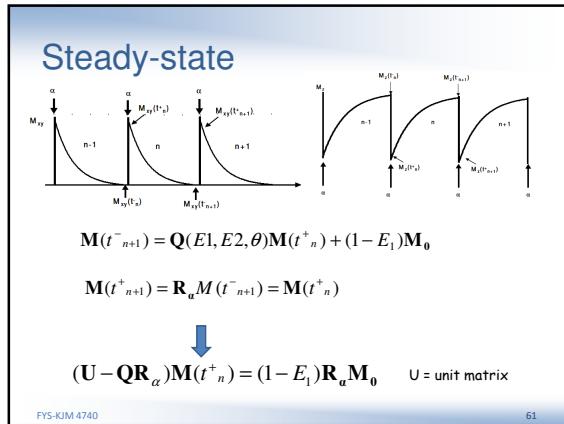
$$\mathbf{M}(t^{-}_{n+1}) = \begin{bmatrix} E_2 \cos(\theta) & E_2 \sin(\theta) & 0 \\ -E_2 \sin(\theta) & E_2 \cos(\theta) & 0 \\ 0 & 0 & E_1 \end{bmatrix} \mathbf{M}(t^{+}_n) + (1-E_i)\mathbf{M}_0$$



$$\mathbf{M}(t^{-}_{n+1}) = \mathbf{Q}(E1, E2, \theta)\mathbf{M}(t^{+}_n) + (1-E_i)\mathbf{M}_0$$

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Steady-state

$$(\mathbf{U} - \mathbf{QR}_\alpha)\mathbf{M}(t_n^+) = (1 - E_1)\mathbf{R}_a\mathbf{M}_0$$

↓

$$\mathbf{M}(t_n^+) = (\mathbf{U} - \mathbf{QR}_\alpha)^{-1}(1 - E_1)\mathbf{R}_a\mathbf{M}_0$$

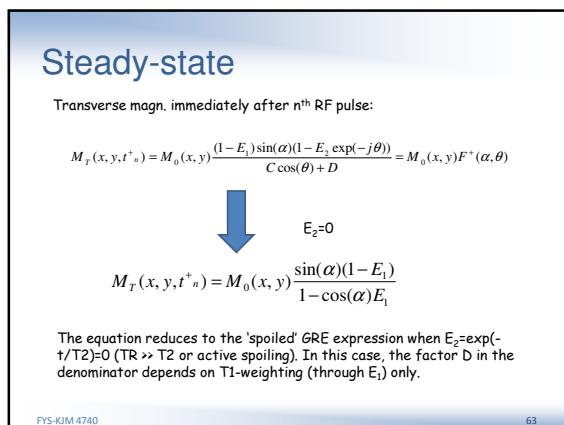
↓

$$\mathbf{M}_T = \mathbf{M}_x + j\mathbf{M}_y$$

$$M_T(x, y, t_n^+) = M_0(x, y) \frac{(1 - E_1) \sin(\alpha)(1 - E_2 \exp(-j\theta))}{C \cos(\theta) + D} = M_0(x, y) F^+(\alpha, \theta)$$

$C = E_2(E_1 - 1)(1 + \cos(\alpha))$ $D = (1 - E_1 \cos(\alpha)) - (E_1 - \cos(\alpha))E_2^2$

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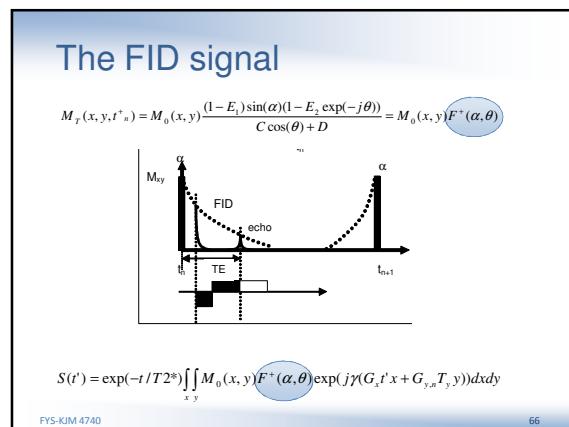
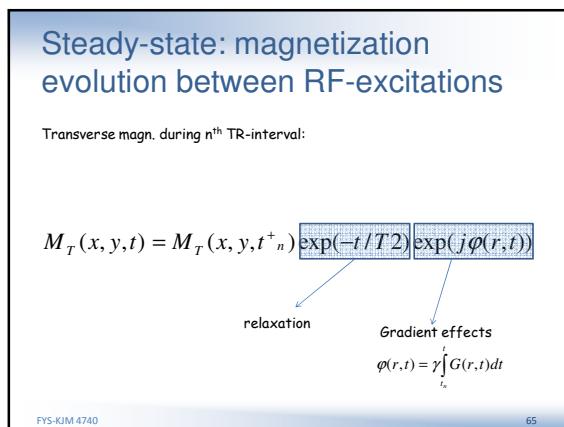


Steady-state

$$M_T(x, y, t_n^+) = M_0(x, y) \frac{(1 - E_1) \sin(\alpha)(1 - E_2 \exp(-j\theta))}{C \cos(\theta) + D} = M_0(x, y) F^+(\alpha, \theta)$$

The term $1 - E_2 \exp(-j\theta)$ in the numerator implies a varying degree of T2-weighting, dependent on θ , and hence position. This means that, if $E_2 > 0$ the image will contain 'bands' of varying degree of T1- and T2-weighting depending on the value of $\exp(-j\theta(r))$ at a given position .

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Steady-state: from magnetization to FID signal

Transverse magn. during n^{th} TR-interval:

$$M_T(x, y, t) = M_T(x, y, t_{n+1}^+) \exp(-t/T2) \exp(j\phi(r, t))$$

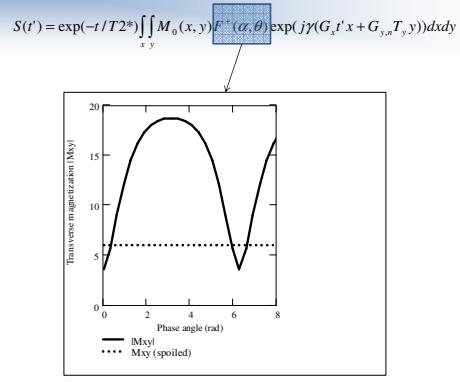
↓
 $\Phi(r, t) = G_x t x + G_{y,n} T_y y$ (see chapt 2)

FID signal:

$$S(t') = \exp(-t/T2) \int_{x,y} M_0(x, y) F^+(\alpha, \theta) \exp(j\gamma(G_x t' x + G_{y,n} T_y y)) dx dy$$

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The ECHO signal

M_T just before the $n+1^{\text{th}}$ RF-pulse

$$M_T(x, y, t_{n+1}^-) = M_T(x, y, t_{n+1}^+) \exp(-TR/T2) \exp(j\theta(x, y))$$

↓
 $E_2 = \exp(-TR/T2)$

$$M_T(x, y, t_{n+1}^-) = M_0(x, y) \frac{\sin(\alpha)(1 - E_1)(\exp(j\theta) - E_2)E_2}{C \cos(\theta) + D}$$

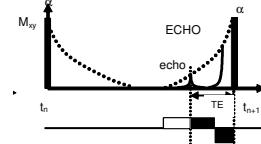
↓
 $S(t') = \exp(TE/T2) \int_{x,y} M_0(x, y) F^+(\alpha, \theta) \exp(j\gamma(G_x t' x + G_{y,n} T_y y)) dx dy$

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The ECHO signal

$$M_T(x, y, t_{n+1}^-) = M_0(x, y) \frac{\sin(\alpha)(1 - E_1)(\exp(j\theta) - E_2)E_2}{C \cos(\theta) + D}$$



$$S(t') = \exp(TE/T2) \int_{x,y} M_0(x, y) F^+(\alpha, \theta) \exp(j\gamma(G_x t' x + G_{y,n} T_y y)) dx dy$$

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FID vs ECHO

FID:

$$M_T(x, y, t_{n+1}^+) = M_0(x, y) \frac{(1 - E_1)\sin(\alpha)(1 - E_2 \exp(-j\theta))}{C \cos(\theta) + D}$$

ECHO:

$$M_T(x, y, t_{n+1}^-) = M_0(x, y) \frac{\sin(\alpha)(1 - E_1)(\exp(j\theta) - E_2)E_2}{C \cos(\theta) + D}$$

For both signals, T2-contribution to magnetization is function of position (through $\exp(j\theta)$)

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Methods to eliminate phase effect

1. Make θ large (large net gradient surface) relative to the voxel size so that the bands of varying T1- and T2-weighting is averaged out over each voxel.
2. Apply RF- or gradient spoiling ($E_2=0$): removing all transverse coherence prior to each RF-pulse.
3. Make $\theta = 0$ by balancing all gradients at the time of the signal readout

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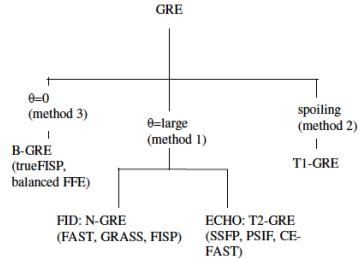
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- Multislice acquisition (MS, 3D)
- GRE: Simple Steady-State model
- RF pulse train (multiple echoes)
 - Eight-Ball Echo
 - Stimulated Echo
 - Multiples echoes
- GRE FID and ECHO components
- Mathematical Model of GRE
- GRE contrast properties and techniques

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Steady-state sequences



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Steady-state sequences in practice:

T1-GRE (fid, spoiling; E2=0)

$$M_T(x, y, t^+_{n_0}) = M_0(x, y) \frac{\sin(\alpha)(1-E_1)}{1-\cos(\alpha)E_1}$$

N-GRE (FID, large θ)

$$\langle M^+ r(x, y) \rangle = M_0(x, y) \frac{(1-E_1)\sin(\alpha)}{C} \left(\frac{C+DE_2}{\sqrt{D^2-C^2}} - E_2 \right)$$

T2-GRE (ECHO, large θ)

$$\langle M^- r(x, y) \rangle = M_0(x, y) \frac{(1-E_1)E_2 \sin(\alpha)}{C} \left(1 - \frac{D+CE_2}{\sqrt{D^2-C^2}} \right)$$

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Steady-state sequences in practice:

B-GRE (balanced FFE, θ=0)

$$M_T = M_0 \frac{\sin(\alpha)(1-E_1)\sqrt{E_2}}{1-(E_1-E_2)\cos(\alpha)-E_1E_2}$$

TR ≪ T1, T2 and α ≈ α_e:

$$M_T = M_0 \frac{\sin(\alpha)}{1+\cos(\alpha)+(1-\cos(\alpha))(T1/T2)} \approx \frac{\sin(\alpha)}{1-\cos(\alpha)} \frac{T2}{T1}$$

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Reminder: what happens with θ=0

FID:

$$M_T(x, y, t^+_{n_0}) = M_0(x, y) \frac{(1-E_1)\sin(\alpha)(1-E_2 \exp(-j\theta))}{C \cos(\theta) + D}$$

ECHO:

$$M_T(x, y, t^-_{n+1}) = M_0(x, y) \frac{\sin(\alpha)(1-E_1)(\exp(j\theta) - E_2)E_2}{C \cos(\theta) + D}$$

For both signals, T2-contribution to magnetization is function of position (through $\exp(j\theta)$)

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Steady-state sequences in practice:

B-GRE (balanced FFE, θ=0)

$$M_T = M_0 \frac{\sin(\alpha)}{1+\cos(\alpha)+(1-\cos(\alpha))(T1/T2)} \approx \frac{\sin(\alpha)}{1-\cos(\alpha)} \frac{T2}{T1}$$

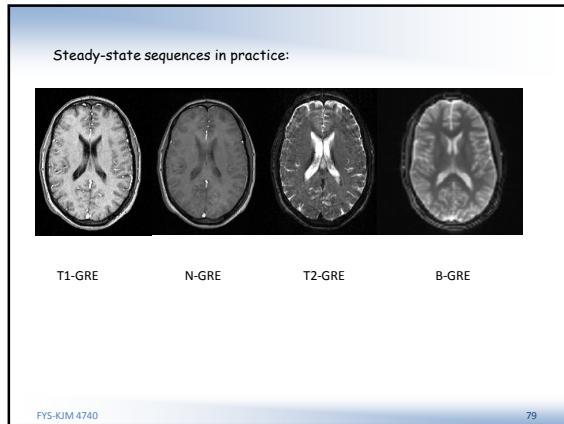
At optimal flip angle:

$$\cos(\alpha_e) = \frac{T1-T2}{T1+T2}$$

$$M_{T,peak} = M_0 \frac{1}{2} \sqrt{T2/T1} \quad \text{Very high signal-noise!}$$

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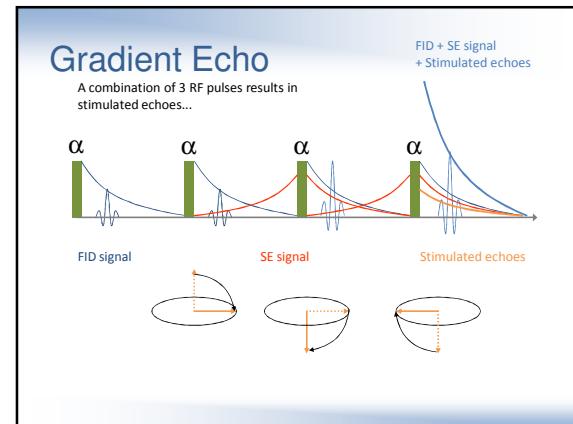
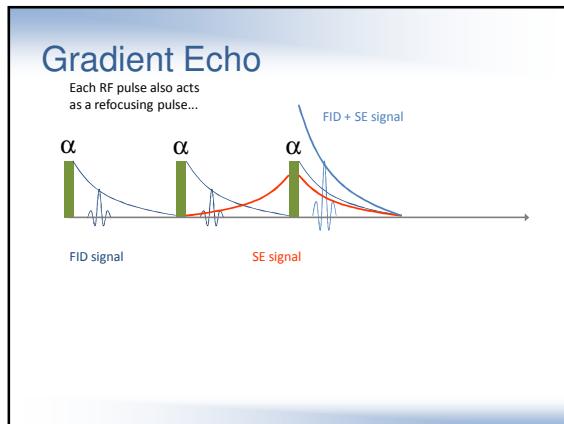
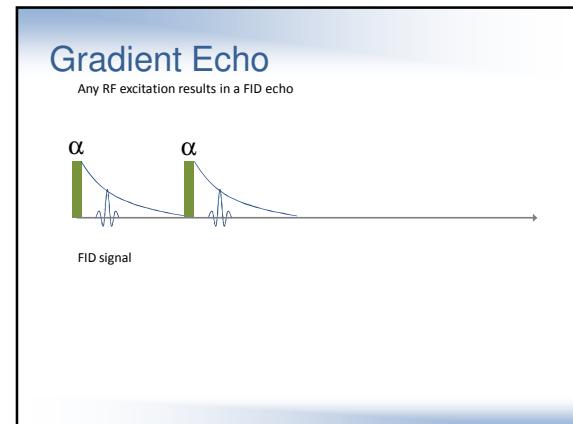
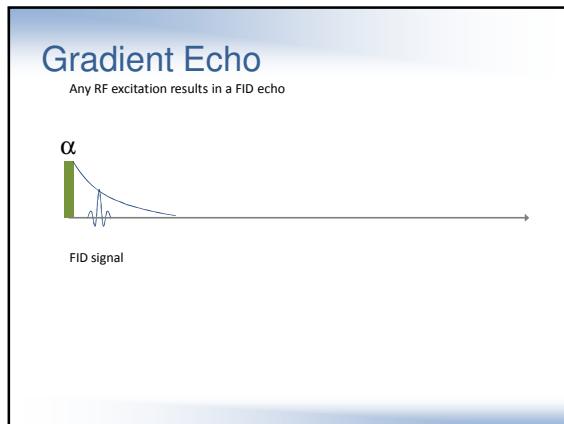


Signal in Gradient Echo

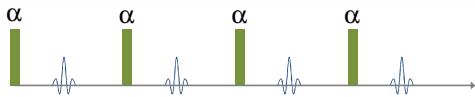
There are several ways to control the contrast in Gradient Echo techniques.

- "normal" FFE Gradient Spoiling (destroy SE's)
- T1 enhanced FFE* RF Spoiling (destroy SE's)
- T2 enhanced FFE Gradient Spoiling (destroy FID)
- Balanced FFE (SSFP) NO SPOILING
 - Very Short TR
 - TE = $\frac{1}{2}$ TR
 - High Flip Angle
 - Intrinsic Flow Compensation
 - No spoiling...

* RF spoiling does not lengthen the TR. Therefore it is very useful in sequences with very short TR such as 3D T1w FFE.



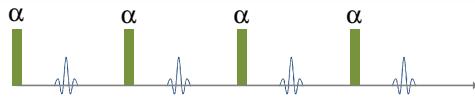
Gradient Echo



The signal in FFE techniques is always a combination of:

- FID which is more T1w / T2*w
- SE which is more T2w
- Stimulated echoes, also more T2w

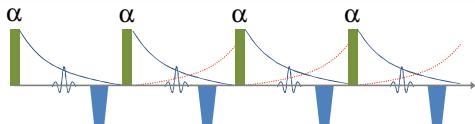
Gradient Echo



The overall contrast in FFE techniques can be controlled by controlling FID, SE and Stimulated echoes:

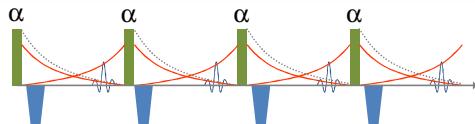
There are several ways to destroy or encourage a particular signal type.

Gradient Echo



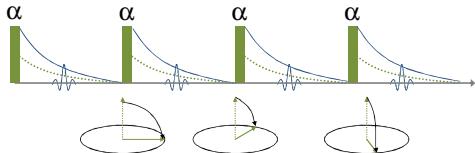
Placing "spoiler" gradients can destroy SE components to decrease T2w contrast.

Gradient Echo



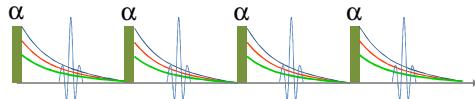
...or ignore the FID signal as done in "T2w enhancement"

Gradient Echo



RF spoiling can decrease stimulated echoes as done in "T1w enhancement"

SSFP (Balanced FFE, true FISP)

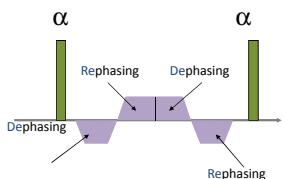


In Balanced FFE the goal is to re-use as many signals as possible:

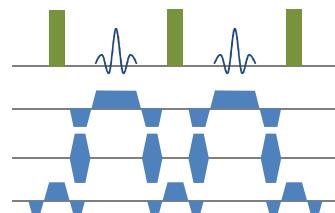
- use high flip to increase both SE and stimulated echo

SSFP (Balanced FFE, true FISP)

- In balanced FFE we MUST de-encode the signal, before the next excitation to avoid artifacts.
- ... And now we can use this signal that is left after the echo...

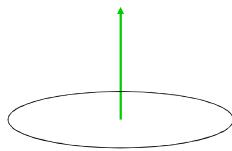


SSFP (Balanced FFE, true FISP)

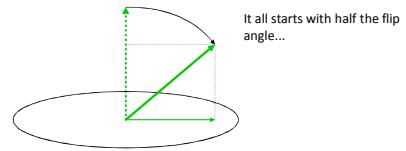


All signals are de-encoded by rephasing gradients in three directions

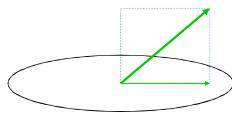
Contrast in SSFP



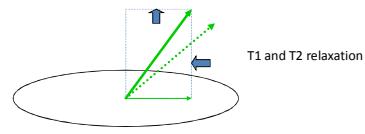
Contrast in SSFP



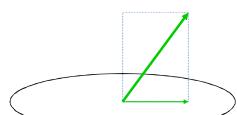
Contrast in SSFP



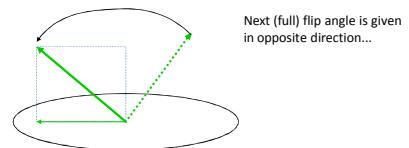
Contrast in SSFP



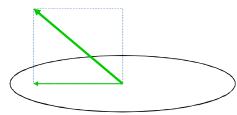
Contrast in SSFP



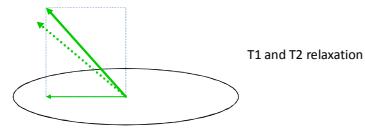
Contrast in SSFP



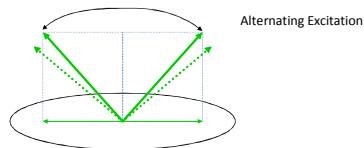
Contrast in SSFP



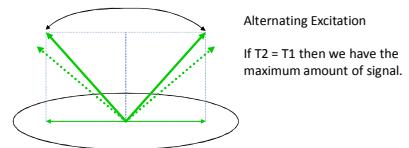
Contrast in SSFP



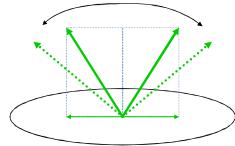
Contrast in SSFP



Contrast in SSFP



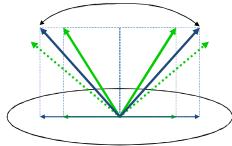
Contrast in SSFP



Alternating Excitation

If $T_2 < T_1$ then we have the less signal.

Contrast in SSFP



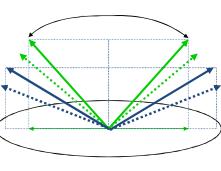
Alternating Excitation

If $T_2 < T_1$ then we have the less signal.

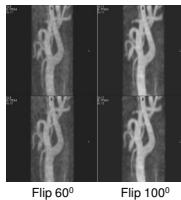
In balanced FFE tissue with high T_2/T_1 (yellow) gives the brightest signal. (Blood, Fat*, Fluid)

Optimize sequence

- Flip angle 90° to 110°



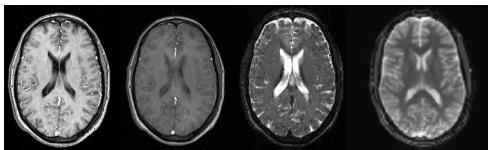
Alternating Excitation



SSFP requirements

- Low flip angle to not destroy slice profile
- Short TR: +/- 3ms
- very high B0 homogeneity
- STRONG and FAST GRADIENTS
- smallest Water Fat Shift

Steady-state sequences in practice:



T1-GRE

N-GRE

T2-GRE

B-GRE

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Next lecture: transient GRE and approach to steady-state

