UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in FYS-MEK 1110
Day of exam: March 26, 2015
Exam hours: 10:00 a.m. – 1:00 p.m. (3 hours)
This examination paper consists of …page(s).
Appendices: Useful formulae
Permitted materials:
Øgrim og Lian: *Størrelser og enheter i fysikk og teknikk* or
Angell, Lian, Øgrim: *Fysiske størrelser og enheter: Navn og symboler*
Rottmann: *Matematisk formelsamling*
Electronic calculator of approved type.

Make sure that your copy of this examination paper is complete before answering.
Remember to explain how you solve the problems and justify your answers.

Problem 1 (6 points)

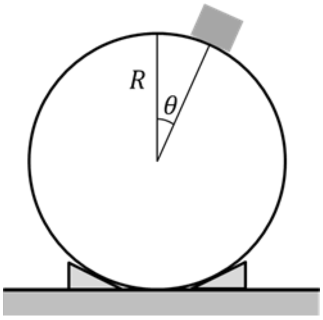
Which of the following statements are true? Explain and justify your answers.

a. A coin falls through a tube from which the air has been evacuated. As it is falling
(1) only the momentum of the coin is conserved.
(2) only the mechanical energy of the coin is conserved.
(3) both the momentum and the mechanical energy of the coin are conserved.
(4) the kinetic energy of the coin is conserved. (3 points)
Gravity is an external force acting on the coin during the fall. Momentum is therefore not conserved. Since gravity is a conservative force, the mechanical energy is conserved. Potential gravitational energy is transformed into kinetic energy as the coin falls. Statement (2) is correct.

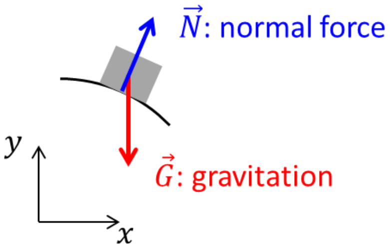
b. Two pieces of clay collide and stick together. During the collision
(1) only the momentum of the clay is conserved.
(2) only the mechanical energy of the clay is conserved.
(3) both the momentum and the mechanical energy of the clay are conserved.
(4) the kinetic energy of the clay is conserved. (3 points)
Since the two pieces of clay stick together the collision is completely inelastic. The mechanical energy is therefore not conserved. The only external force acting during the collision process is gravity. The impulse from gravity during the short duration of the collision is negligible and momentum is therefore conserved. Statement (1) is correct.

Problem 2 (12 points)

A small block is resting on the top of a large sphere. There is no friction between the block and the surface of the sphere. The block starts sliding with an infinitesimally small velocity from the top of the sphere to one side.



a. Draw a free-body diagram of the block and name the forces while it is at a finite angle θ from the top. (3 points)



b. Determine the speed of the block as a function of the angle θ.
(4 points)

The normal force is orthogonal to the direction of motion and does not do any work. Gravitation is a conservative force. We can therefore use energy conservation. If we define $U = 0$ in the center of the sphere:

$$mgR = \frac{1}{2}mv^2 + mgR\cos\theta$$
$$v = \sqrt{2gR(1-\cos\theta)}$$

c. Find the angle at which the block loses contact with the surface of the sphere. (5 points)

The net force in radial direction must provide the centripetal acceleration:

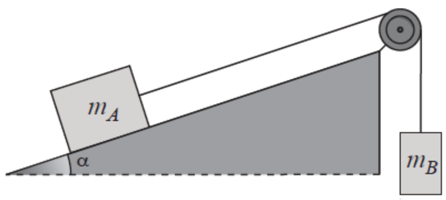
$$N - mg\cos\theta = -m\frac{v^2}{R} = -m\frac{2gR(1-\cos\theta)}{R}$$
$$N = mg\cos\theta - 2mg(1-\cos\theta) = 3mg\cos\theta - 2mg$$

The block loses contact when the normal force becomes zero:

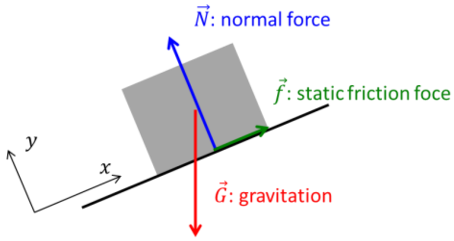
$$\theta = \cos^{-1}\left({}^{2}/_{3}\right)$$

Problem 3 (16 points)

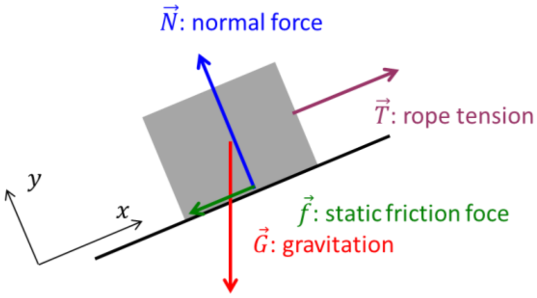
Block A of mass m_A is resting on an inclined plane which has an angle α with the horizontal. A rope connects the block to the weight B with mass m_B over a pulley as shown in the figure. Both the rope and the pulley can be considered massless, and the pulley turns without friction. The static coefficient of friction between the block A and the incline is μ_s, the dynamic coefficient of friction is μ_d. The gravitational acceleration is g.



a. Draw the free-body diagram for block A and name the forces for the case where there is no weight B attached to the other end of the rope. (3 points)



b. Draw the free-body diagram for block A and name the forces for the case where a weight B is attached. Assume that the mass m_B is just slightly smaller than the maximum mass $m_{B,\max}$ for which the system remains at rest. (3 points)



c. Find an expression for the maximum mass $m_{B,\max}$ that you can hang onto the rope without block A starting to slide up the incline. Express the maximum mass $m_{B,\max}$ in terms of the mass m_A, the angle α, and the static coefficient of friction μ_s. (5 points)

Block A remains at rest. Newton's second law for block A in x direction:

$$T - f - m_A g \sin\alpha = 0$$

Newton's second law for block A in y direction:

$$N - m_A g \cos\alpha = 0$$

Newton's second law for weight B at rest:

$$T - m_B g = 0$$

For the static friction force it is then:

$$f = T - m_A g \sin\alpha = m_B g - m_A g \sin\alpha \leq \mu_s N = \mu_s m_A g \cos\alpha$$
$$m_B \leq m_A(\sin\alpha + \mu_s \cos\alpha)$$

d. You attach a mass that is larger than the maximum mass from part c., $m_B > m_{B,\max}$, to the rope and block A starts to slide up the incline. Find the acceleration of the two blocks, expressed in terms of the two masses m_A and m_B, the angle α, the coefficient of dynamic friction μ_d, and the gravitational acceleration g. (5 points)

The two masses are moving with the same acceleration. Newton's second law for block A in x direction:

$$T - f - m_A g \sin\alpha = m_A a$$

The dynamic friction force is: $f = \mu_d N = \mu_d m_A g \cos\alpha$

Newton's second law for weight B:

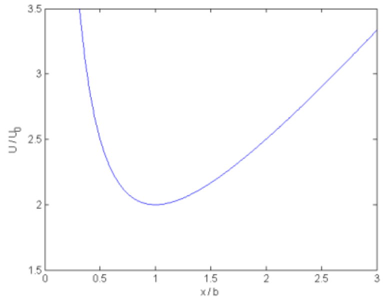
$$m_B g - T = m_B a$$

Summing the two equations gives:

$$m_B g - f - m_A g \sin\alpha = (m_A + m_B)a$$
$$a = \frac{m_B - m_A(\mu_d \cos\alpha + \sin\alpha)}{(m_A + m_B)} g$$

Problem 4 (12 points)

The force acting on a particle with mass m is characterized by the potential $U(x) = U_0\left(\frac{b}{x} + \frac{x}{b}\right)$, where U_0 and b are positive constants and the position x can only take positive values.



a. Determine the force acting on the particle in the position x. (3 points)

$$F = -\frac{dU}{dx} = -U_0\left(-\frac{b}{x^2} + \frac{1}{b}\right) = U_0\left(\frac{b}{x^2} - \frac{1}{b}\right)$$

b. Describe the motion of the particle. How can you characterize the position $x = b$? (3 points)

Since the force has a potential it is conservative and the mechanical energy is conserved. The particle will oscillate around the stable equilibrium point $x = b$. If the particle is located in the position $x = b$ without kinetic energy it will remain there. The position $x = b$ is therefore a stable equilibrium point. If it starts from a position $x_0 < b$ it is accelerated in positive x direction. It will reach its maximum energy in $x = b$, then slow down and finally turn around at the position $x = \frac{1}{x_0}$. The motion is then reversed and the particle moves back and forth between the same positions.

c. The particle is located at the position $x_0 = \frac{1}{2}b$ and released without initial velocity. Find the velocity of the particle at the position $x = b$. (3 points)

Since the force is conservative we can use energy conservation:

$$U(x_0) + K(x_0) = U(x) + K(x)$$

$$\frac{5}{2}U_0 + 0 = 2U_0 + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{U_0}{m}}$$

d. How far does the particle move? (3 points)

We use again energy conservation to find the position where all energy is in the form of potential energy with the same value as in x_0.

$$U_0\left(\frac{b}{x} + \frac{x}{b}\right) = \frac{5}{2}U_0$$

$$x^2 - \frac{5}{2}bx + b^2 = 0$$

$$x = \frac{5}{4}b \pm \sqrt{\frac{25}{16}b^2 - b^2} = \frac{5}{4}b \pm \frac{3}{4}b$$

The particle moves back and forth between the points $x = \frac{1}{2}b$ and $x = 2b$.

e. In three dimensions the potential can be written as $U(\vec{r}) = U_0\left(\frac{b}{r} + \frac{r}{b}\right)$, where $r = |\vec{r}|$. Determine the force acting on the particle in the position $\vec{r}$. (3 points)

$$\vec{F} = -\vec{\nabla}U$$

In spherical coordinates:

$$\vec{F} = -\frac{\partial U}{\partial r}\hat{u}_r = U_0\left(\frac{b}{r^2} - \frac{1}{b}\right)\hat{u}_r$$

Problem 5 (18 points)

A plane that is moving horizontally with constant velocity $\vec{v} = v_0\hat{\imath}$ drops a parcel from a height h above the ground. At first you can ignore air resistance.

a. Find the velocity vector of the parcel when it hits the ground, expressed in terms of the speed v_0, the height h and the gravitational acceleration g. (5 points)
   The only force acting on the parcel is gravity. The initial conditions are: $\vec{v}_0 = v_0\hat{\imath}$ and $\vec{r}_0 = h\hat{\jmath}$
   We can write the equations of motion:

$$\vec{a} = -g\hat{\jmath}$$
$$\vec{v}(t) = \vec{v}_0 - gt\hat{\jmath}$$
$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t - \frac{1}{2}gt^2\hat{\jmath}$$

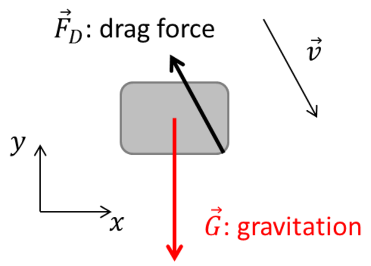
   In components: $x(t) = v_0 t$ and $y(t) = h - \frac{1}{2}gt^2$

   The parcel hits the ground when $y = 0 \Rightarrow t = \sqrt{\frac{2h}{g}}$

   The velocity at this time is: $\vec{v} = v_0\hat{\imath} - \sqrt{2gh}\hat{\jmath}$

From now on we take air resistance into account. The drag force on the parcel can be described as $\vec{F}_D = -D\vec{v}|\vec{v}|$, where D is a positive constant. The plane drops the parcel from large altitude and the parcel reaches terminal velocity.

b. Draw a free-body diagram for the parcel and name the forces. (3 points)



c. Determine the velocity with which the parcel hits the ground. (4 points)
   The parcel reaches terminal velocity when the horizontal component of the drag force has stopped the horizontal motion and the vertical component of the drag force compensates gravity:

   $Dv^2 = mg \Rightarrow v = \sqrt{\frac{mg}{D}}$

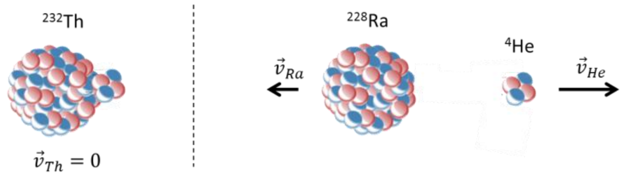
d. Write a program to determine the velocity and position of the parcel as a function of time. It is sufficient to write the integration loop only. (6 points)

```
clf;
m = 1.0;
g = 9.81;
h = 1000.0;
v0 = 180.0;
D = 0.006;
time = 30.0;
dt = 0.01;
n = round(time/dt);
t = zeros(n,1);
```

```
r = zeros(n,2);
v = zeros(n,2);
t(1) = 0.0;
r(1,:) = [0.0 h];
v(1,:) = [v0 0.0];
i = 1;
while (r(i,2) > 0.0)
    F = -m*g*[0 1]-D*v(i,:)*norm(v(i,:));
    a = F/m;
    v(i+1,:) = v(i,:)+a*dt;
    r(i+1,:) = r(i,:)+v(i+1,:)*dt;
    t(i+1) = t(i)+dt;
    i = i+1;
end
v(i,:)
figure(1);
plot(r(1:i,1),r(1:i,2));
xlabel('x (m)');
ylabel('y (m)');
figure(2);
subplot(2,1,1);
plot(t(1:i),v(1:i,1));
xlabel('t (s)');
ylabel('vx (m/s)');
subplot(2,1,2);
plot(t(1:i),v(1:i,2));
xlabel('t (s)');
ylabel('vy (m/s)');
```

Problem 6 (6 points)

A ^{232}Th (thorium) nucleus at rest undergoes alpha decay, i.e. it breaks up into a ^{228}Ra (radium) nucleus and a ^{4}He (helium) nucleus. The total energy Q is released in the decay process and appears as kinetic energy of the radium and helium fragments, so that $Q = K_{Ra} + K_{He}$. The mass of the radium and helium fragments are $m_{Ra} = 228$ u and $m_{He} = 4$ u, respectively, where u is the atomic mass unit. What fraction of the total energy Q goes into the kinetic energy of the ^{4}He fragment?



The impulse from other forces can be neglected during the short decay process and we can use conservation of momentum: $m_{Ra}v_{Ra} + m_{He}v_{He} = 0 \;\Rightarrow\; v_{He} = -\frac{m_{Ra}}{m_{He}}v_{Ra}$

$$K_{He} = \frac{1}{2}m_{He}v_{He}^2 = \frac{1}{2}m_{He}\left(\frac{m_{Ra}}{m_{He}}v_{Ra}\right)^2 = \frac{m_{Ra}}{m_{He}}\frac{1}{2}m_{Ra}v_{Ra}^2 = \frac{228}{4}K_{Ra} = 57K_{Ra}$$

$$K_{He} + K_{Ra} = 58K_{Ra} = Q$$

$K_{Ra} = \frac{1}{58}Q$ and $K_{He} = \frac{57}{58}Q$