№ 14.1.

Given	SI	
$f = 100\ MHz$	$10^8\ Hz$	
$E_0 = 0{,}05\ V/m$		
λ - ?		
I - ?		
P_{rad} - ?		
B_0, H_0 - ?		

a) Wavelength of the signal is

$$\lambda = \frac{c}{f},$$

where $c \approx 3 \cdot 10^8\ m/s$ is speed of light in vacuum.

Thus,

$$\lambda = \frac{3 \cdot 10^8\ m/s}{10^8\ Hz} = 3\ m$$

b) For electromagnetic wave in vacuum we have

$$E_0 = c B_0 \quad \Rightarrow \quad B_0 = \frac{E_0}{c}$$

that gives

$$B_0 = \frac{5 \cdot 10^{-2}\ V/m}{3 \cdot 10^8\ m/s} \approx 1.667 \cdot 10^{-10}\ T$$

H- and B- fields are related by:

$$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H},$$

where $\mu_0 = 4\pi \cdot 10^{-7}\ \frac{Wb}{A \cdot m}$ is permeability of vacuum, μ_r is relative permeability ($\mu_r \approx 1$ for air).

Therefore:

$$H_0 = \frac{B_0}{\mu_0} = \frac{E_0}{\mu_0 c},$$

or

$$H_0 = \frac{1.667 \cdot 10^{-10}\ T}{4\pi \cdot 10^{-7}\ Wb/A \cdot m} \approx 1.326 \cdot 10^{-4}\ A/m$$

c) Poynting vector in vacuum is given by

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} \cdot \vec{E} \times \vec{B}$$

$\vec{E} \perp \vec{B}$ for electro-magnetic waves.

Intensity of radiation is

$$I = \langle S \rangle \quad \Rightarrow \quad I = \frac{1}{\mu_0} \langle \vec{E} \times \vec{B} \rangle = \frac{1}{\mu_0} \langle E \cdot B \rangle$$

that gives for sinusoidal waves:

$$\langle E \rangle = \frac{E_0}{\sqrt{2}} \ ; \quad \langle B \rangle = \frac{1}{T} \int_0^T B_0 \cos \omega t \, dt = \frac{B_0}{\sqrt{2}}$$

Or finally we have ;

$$\langle E\,B\rangle = \langle E\rangle\langle B\rangle = \frac{E_0 B_0}{\sqrt{2}\sqrt{2}} = \frac{1}{2}E_0 B_0 .$$

or

$$I = \frac{1}{2\mu_0} E_0 B_0 = \frac{E_0 H_0}{2} \qquad I = \frac{1}{2}\varepsilon_0 c\, E_0^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0^2, \text{ where } c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Numerical value of the intensity is

$$I = \frac{1}{2\cdot 4\pi\cdot 10^{-7}\,\frac{Wb}{A\cdot m}} \cdot 0.05\,V/m \cdot 1.667\cdot 10^{-10}\,T \approx 3.32\cdot 10^{-6}\,\frac{W}{m^2}$$

Radiatio pressure when wave totally absorbed is

$$P_{rad} = \frac{\langle S\rangle}{c} = \frac{I}{c} \Rightarrow P_{rad} = \frac{1}{2}\varepsilon_0 E_0^2$$

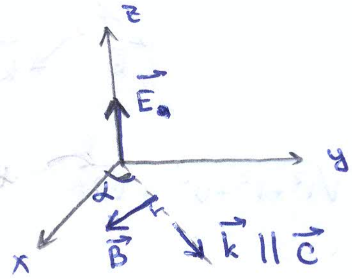
so

$$P_{rad} = \frac{3.32\cdot 10^{-6}\,W/m^2}{3\cdot 10^8\,m/s} \approx 1.105\cdot 10^{-14}\,Pa$$

$\alpha = 45^\circ$,
$\lambda = 5\cdot10^{-7}\,m$
$\langle S\rangle = 2\,W/m^2$

ω - ?
k - ?
E_0 - ?
$\vec{E}(\vec{x},t)$ - ?
$\vec{B}(\vec{x},t)$ - ?

The sketch of the problem



a) Frequency is related to wavelengs by

$$f = \frac{c}{\lambda}, \qquad c = 3\cdot10^8\,m/s$$

that gives for angular frequency

$$\omega = 2\pi f = 2\pi\frac{c}{\lambda}$$

or

$$\omega = 2\pi\cdot\frac{3\cdot10^8\,m/s}{5\cdot10^{-7}\,m} = 3.77\cdot10^{15}\,rad/s \; ; \quad f = 6\cdot10^{14}\,Hz = 6\cdot10^5\,GHz = 600\,THz$$

The wavelength $\lambda = 500\,nm$ corresponds to green light of visible spectrum.

Definition of wave number is

$$k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{2\pi}{5\cdot10^{-7}\,m} \approx 1.257\cdot10^{7}\,m^{-1}.$$

Amplitude of the wave can be defined from the intensity:

$$I = \langle S\rangle = \frac{1}{2}\varepsilon_0 c E_0^2, \quad \text{where } \varepsilon_0 = 8.854\cdot10^{-12}\,\frac{C^2}{m^2\cdot N}$$

that gives

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} \quad \text{or} \quad E_0 = \sqrt{\frac{2\cdot2\,W/m^2}{8.854\cdot10^{-12}\cdot3\cdot10^8\,m/s}} \approx 38.8\,V/m.$$

b) Mathematical expression of the plane wave is

$$\vec{E} = \vec{E}_0\sin(\vec{k}_0\vec{r} - \omega t)$$

In our cas $\vec{E}_0 \parallel 0z \Rightarrow \vec{E}_0 = E_0\cdot\vec{k}$

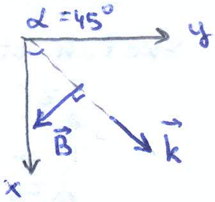
$$\vec{k}\vec{r} = k_x\cdot x + k_y\cdot y = \left[\begin{matrix} k_x = \cos\alpha\cdot k \\ k_y = \sin\alpha\cdot k\end{matrix}\right] = \frac{1}{\sqrt{2}}k_0 x + \frac{1}{\sqrt{2}}k_0 y = \overset{\uparrow}{\sqrt{2}}k_0(x+y)$$

So

$$\vec{E} = \vec{k}\cdot E_0\sin\left(\frac{k_0}{\sqrt{2}}(x+y) - \omega t\right)$$

For magnetic field we have

$$\vec{B} \perp \vec{E} \quad ; \quad \vec{B} \perp \vec{k}_0$$



$\vec{B} = B_0 \vec{n}$, where $\vec{n}$ is unit vector:

$$\vec{n} = \frac{\vec{N}}{|\vec{N}|} \quad ; \quad \vec{N} = (1,-1,0) \Rightarrow |\vec{N}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$$

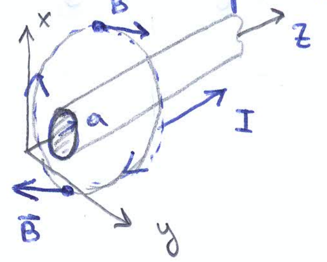
$$\Rightarrow \vec{n} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right) = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j} = \frac{1}{\sqrt{2}}(\vec{i}-\vec{j}).$$

Therefore we obtain

$$\vec{B} = \frac{B_0}{\sqrt{2}}(\vec{i}-\vec{j}) \cdot \sin\left(\frac{k_0}{\sqrt{2}}(x+y) - \omega t\right)$$

№ 14. 3

a, σ, I, l	The sketch of the problem
$\vec{E}$ - ?	
$\vec{H}$ - ?	
$\vec{S}$ - ?	



a) We can use Ohm's law :

$$\vec{J} = \sigma \vec{E}$$, where current density is

$J = \frac{I}{A}$, where $A = \pi a^2$ - for circular cross-section.

Therefore :

$$\vec{E} = \frac{\vec{I}}{\sigma A} = \frac{\vec{I}}{\pi a^2 \sigma}$$, where $\vec{I} = I \cdot \vec{k}$, $\vec{k} = (0, 0, 1)$.

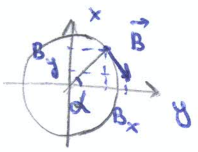
so

$$\vec{E} = \frac{I}{\pi a^2 \sigma} \vec{k}$$

b) Magnetic field due to long straight conductor is

$$B = \frac{\mu_0}{2\pi r} I$$ (can be found by Ampere's law).

Its direction is defined by right-hand rule , so



$$\begin{cases} B_x = B \cdot \sin\alpha \\ B_y = -B \cdot \cos\alpha \end{cases}$$, where $\cos\alpha = \frac{x}{r}$; $\sin\alpha = \frac{y}{r}$

$r = \sqrt{x^2 + y^2}$.

Therefore

$$\begin{cases} B_x = \frac{B}{r} y \\ B_y = -\frac{B}{r} x \end{cases} \Rightarrow \vec{B} = \frac{\mu_0}{2\pi r^2} I (y\vec{i} - \vec{j} x) = \frac{\mu_0}{2\pi r^2} I (y \cdot \vec{i} - x\vec{j})$$

c) Poynting vector is

$$\vec{S} = \vec{E} \times \vec{H}$$, where $\vec{H} = \frac{\vec{B}}{\mu_0}$ for vacuum.

so $$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{I}{2\pi r^2} (y \cdot \vec{i} - x \cdot \vec{j})$$

when $r = a$ (on the wire surface) :

$$\vec{H} = \frac{I}{2\pi a^2} (y \cdot \vec{i} - x \cdot \vec{j})$$, $x^2 + y^2 = a^2$

Poynting vector on the surface of the wire is

$$\vec{S} = \vec{E} \times \vec{H} = \frac{I}{\pi a^2 \cdot \sigma} \cdot \frac{I}{2\pi a^2} \cdot \vec{k} \times (y\vec{i} - x \cdot \vec{j}),$$

we know that

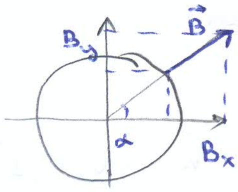
$$[\vec{k} \times \vec{i}\,] = \vec{j} \;;\quad [\vec{k} \times \vec{j}\,] = -\vec{i}$$

$$\Rightarrow \vec{k} \times (y\vec{i} - x\vec{j}) = y \cdot \vec{j} + x \cdot \vec{i}$$

So

$$\vec{S} = \frac{I^2}{(\pi a^2)^2 \cdot 2\sigma} (x \cdot \vec{i} + y \cdot \vec{j}) = \frac{I^2}{\pi a^2 \cdot 2\pi a \cdot \sigma} \left(\frac{x}{a}\vec{i} + \frac{y}{a}\vec{j}\right)$$

That describes the following vector: (radial vector)



$$\begin{cases} B_x = B \cdot \cos\alpha = B\,\dfrac{x}{r} \\ B_y = B \cdot \sin\alpha = B \cdot \dfrac{y}{r} \end{cases}$$

So in cylindric coordinate system:

$$\vec{S} = S \cdot \vec{e_r}, \quad \text{where} \quad S = \frac{I^2}{\pi a^2 \cdot 2\pi a \sigma}$$

It means that wire emites power through its surface for l-length:

$$P_{emit.} = S \cdot A_0, \quad \text{wher} \quad A_0 = 2\pi a \cdot l$$

$$\Rightarrow \quad P_{emit} = \frac{I^2 \cdot l}{\pi a^2 \sigma}$$

On the other hand resistance of wire with length l is

$$R = \frac{1}{\sigma} \frac{l}{A}, \quad \text{where} \quad A = \pi a^2 \; - \text{cross-sectional area.}$$

$$\Rightarrow \quad R = \frac{l}{\pi a^2 \sigma}$$

So Joule heat of the resistor is

$$P = I^2 R \quad \rightarrow \quad P = \frac{I^2 l}{\pi a^2 \sigma}$$

It means that emited energy by the electro-magnetic field is equivalent (and converts to) Joule heat.

№ 14.4

Given	
$P = 60\ W$	
$r = 3\ m$	
$P_0 = 10^5\ Pa$	
I - ?	
P_{rad} - ?	
E_0 - ?	
B_0 - ?	

We assume that lamp emites radiation uniformly.

a) The intensity of radiation is

$$I = \frac{P}{A}, \text{ where } A \text{ is area.}$$

In case of uniform radiation we can choose

$$A = 4\pi r^2 \quad \text{- area of sphere.}$$

so

$$I = \frac{P}{4\pi r^2}$$

that gives numerical value:

$$I = \frac{60\ W}{4\pi \cdot (3m)^2} \approx 0.531\ W/m^2$$

b) Radiatio pressure on totally "black" surface is

$$P_{rad} = I/c, \text{ where } c \text{ is speed of light}$$

so

$$P_{rad} = \frac{0.531\ W/m^2}{3 \cdot 10^8\ m/s} = 1.768 \cdot 10^{-9}\ Pa$$

We can see that $P_{rad} << P_0$:

$$\frac{P_{rad}}{P_0} \approx 1.8 \cdot 10^{-14} \quad \text{is negligible quantity.}$$

c) On the other hand the intensity can be expressed in terms of electric field amplitude:

$$I = \frac{1}{2}\varepsilon_0 c E_0^2 \quad \Rightarrow \quad E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

or numerical value is

$$E_0 = \sqrt{\frac{2 \cdot 0.531\ W/m^2}{8.854 \cdot 10^{-12} \frac{C^2}{N \cdot m^2} \cdot 3 \cdot 10^8\ m/s}} \approx 20\ V/m$$

Magnetic field can be found from the expression

$$E_0 = cB_0$$

so

$$B_0 = \frac{E_0}{c} \quad \text{or} \quad B_0 = \frac{20\ V/m}{3 \cdot 10^8\ m/s} = 6.67 \cdot 10^{-8}\ T$$

Note: We can check the calculation by relation

$$I = \langle S \rangle = \frac{1}{\mu_0} \langle \vec{E}_0 \rangle \langle \vec{B} \rangle = \frac{1}{2\mu_0} E_0 B_0$$

so

$$I = \frac{1}{2 \cdot 4\pi \cdot 10^{-7} \frac{H}{m}} \cdot 20 \, V/m \cdot 6.67 \cdot 10^{-8} \, T \approx 0.53 \, W/m^2 \qquad \text{Q.E.D.}$$