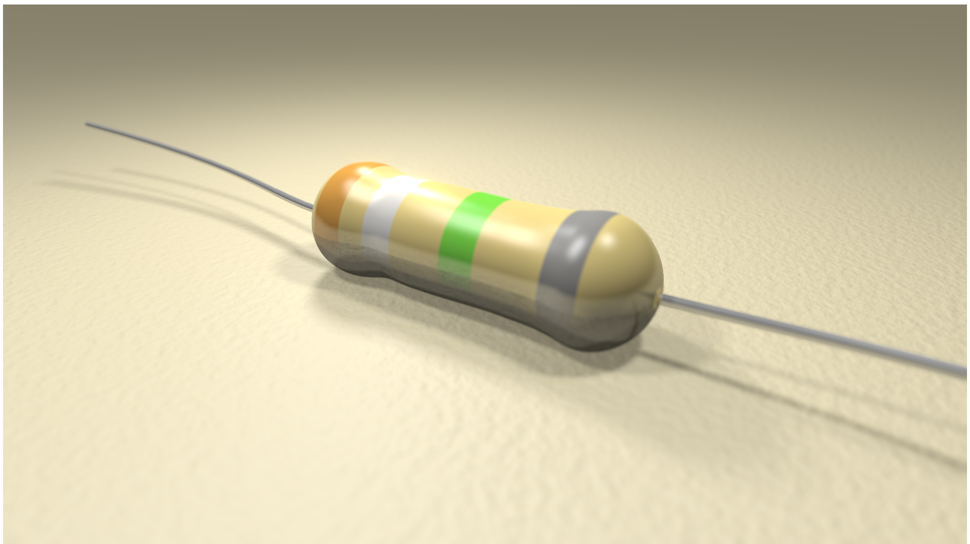
Week 5 - Conductors, capacitors and circuits



Each metal has a certain power, which is different from metal to metal, of setting the electric fluid in motion.

Count Alessandro Giuseppe Antonio Anastasio Volta

Exercise 5.1: Conductor properties

(i) $\mathbf{E} = 0$ inside a conductor.
(ii) $\rho = 0$ inside a conductor.
(iii) Any net charge resides on the surface. $\sigma \geq 0$
(iv) A conductor is an equipotential.
(v) $\mathbf{E}$ is perpendicular to the surface just outside the conductor.

If you look around, you probably have conductors all around you. They are invaluable in our technological society. In this exercise we'll study their *electrostatic* properties by studying hollow spherical conductor.

a) Consider the neutral spherical conductor in figure 1. We place a charge $-q$ inside a spherical cavity of radius a. Find the surface charge σ_a and σ_b.

Answer: $\sigma_a = \frac{q}{4\pi a^2}$ $\sigma_b = -\frac{q}{4\pi b^2}$

b) What is the electric field inside the cavity, as well as inside and outside the conductor?

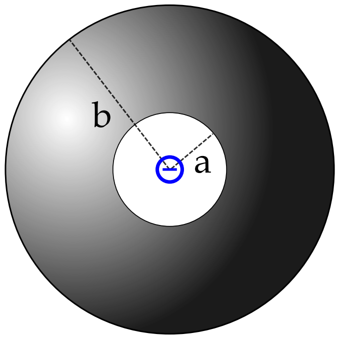


Figure 1: A hollow conductor with $-q$ inside.

Answer:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$$

inside the cavity and outside the conductor. Inside the conductor $\mathbf{E} = 0$.

c) What is the force on the charge inside?

Answer: Zero.

d) If we remove the charge inside and expose the conductor to some huge electric fields from the outside. Do you know whether or not it would be safe to sit inside the cavity? Are there any fields inside? Explain.

Exercise 5.2: Coaxial cylinders

Consider the coaxial conducting cylinders in figure 2 of radii a and b and length L.

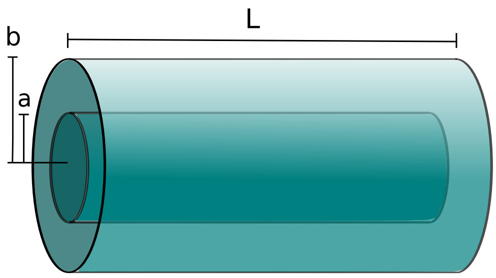


Figure 2: Capacitor consisting of two coaxial cylinders.

a) Given that the inner cylinder has charge $-Q$ and the outer has Q. Ignore the fringing fields and use Gauss law to find the electric field between the cylinders.

Answer:

$$\mathbf{E} = -\frac{Q}{2\pi\varepsilon_0 L r}\hat{\mathbf{r}}$$

b) Show that the capacitance is given by

$$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

c) Find the energy stored in the electric field between the cylinders given that $a = 1\,\text{mm}$, $b = 2\,\text{mm}$ and $Q = 0.2\,\text{nC}$.

Answer:

$$U = \frac{1}{2}\frac{Q^2}{C}$$

d) Suppose now that we place this capacitor partially in water with $\kappa = 80$ and that the total charge on the capacitor was unaffected. See figure 3. Find an expression for V in the two regions.

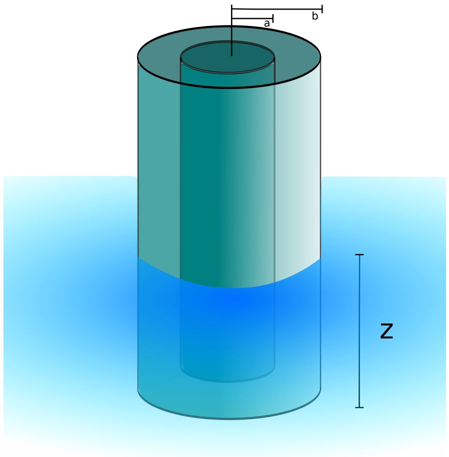


Figure 3: Cylinder capacitor used as a water height sensor.

Hint: The polarized water will change the surface charge on the part of the capacitor immersed

Answer:

$$V_{air} = \frac{\sigma_{air} a}{\varepsilon_0}\ln\frac{b}{a}$$

$$V_{water} = \frac{\sigma_{water} a}{\kappa\varepsilon_0}\ln\frac{b}{a}$$

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e) Now we know that a conductor is an equipotential. Can you use this to find a relation between the surface charge in the two regions? Use this to find the capacitance $C = C(z)$.

Answer:

$$\sigma_{water} = \kappa\sigma_{air}$$

$$C = \frac{Q}{V} = 2\pi\varepsilon_0\frac{(z(\kappa - 1) + L)}{\ln\frac{b}{a}}.$$

Exercise 5.3: Electric circuits

Consider the circuit shown in figure 4 where $R_1 = 10\,\Omega$, $R_2 = 4\,\Omega$ and $R_3 = 8\,\Omega$. The battery has a voltage of $\varepsilon = 3\,\mathrm{V}$.

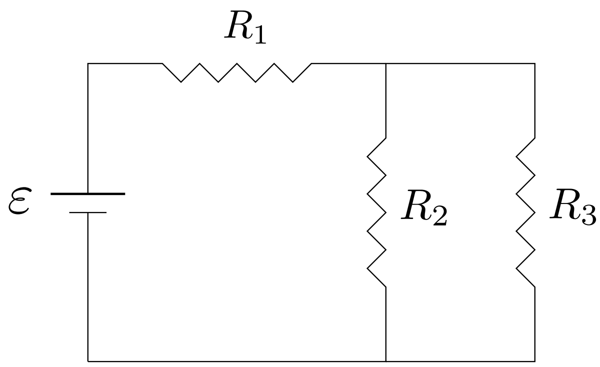

Figure 4: One resistor in series and two in parallel.

a) What is the current through the battery?

Answer:

$$I_{\mathrm{tot}} = I_1 = 0.237\,\mathrm{A}$$

b) What is the total resistance in the circuit?

Answer:

$$R_{\mathrm{tot}} = 12.7\,\Omega$$

c) What are the currents through R_2 and R_3?

Answer:

$$I_{R_2} = 0.15$$
$$I_{R_3} = 0.075$$

d) What is the power generated of the battery?

Answer:

$$P = 3\,\mathrm{V} \cdot 0.24\,\mathrm{A} = 0.7\,\mathrm{W}$$

e) What is the total power consumed in the resistors?

Answer:

$$P = (0.24\,\mathrm{A})^2 \cdot 12.7\,\Omega = 0.7\,\mathrm{W}$$