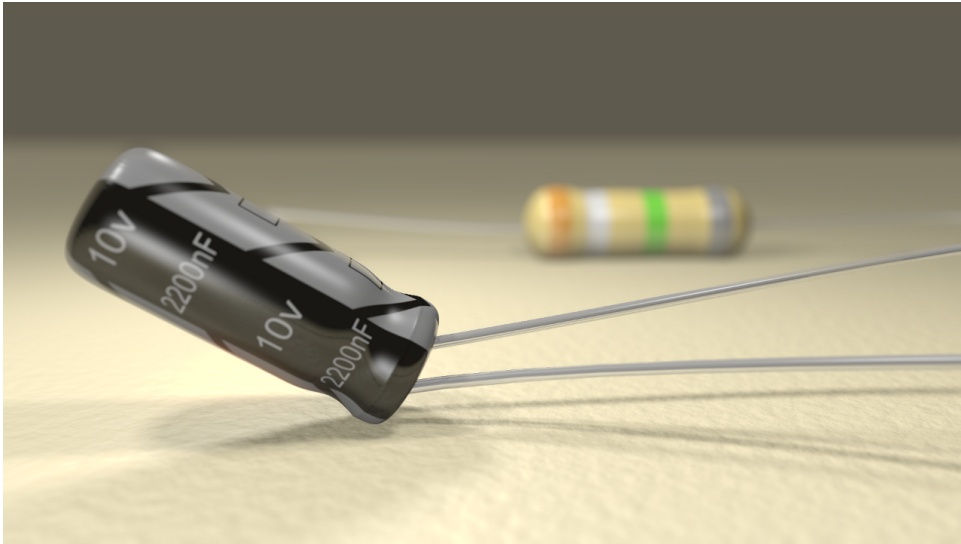


## Week 7 - Current, Ohm's law and RC circuits



The equation governing the flow of currents is *Ohm's law*

$$\mathbf{J} = \sigma \mathbf{E}$$

where  $\sigma$  is called the *conductivity* and  $\mathbf{J}$  is the volume current density. The traditional current through a surface  $\mathcal{S}$  is then found by

$$I = \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}$$

Ohm's law isn't really a law, but rather a rule of thumb. Materials who obey this equation is said to be *Ohmic materials*.

### Exercise 7.1: Current

- a) Consider two cylinders of radii  $a$  and  $b$  where  $b > a$  and where  $J = kr^2$  for  $a \leq r \leq b$  and is directed along the wire. What is the total current in the wire?

### Exercise 7.2: Ohm's law

- a) Derive the more familiar form of Ohm's law,  $V = RI$ , by considering a cylinder of cross sectional area  $A$  and length  $l$  and show that  $R = \frac{l}{\sigma A}$ . Does this expression for  $R$  make sense?
- b) A 3000-km long cable consists of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheet. Calculate the resistance of the cable. Use  $3 \times 10^{-6} \Omega \cdot \text{cm}$  for the resistivity of copper.

### Exercise 7.3: RC circuit

Consider an  $RC$  circuit, where a battery with emf  $\varepsilon$ , a resistor with resistance  $R$ , a switch and a capacitor with capacitance  $C$  are connected in series. The switch is turned on at the time  $t = 0$ .

- a) What is the current immediately after the switch is turned on, i.e. for  $t = 0$ ?

*Answer:*

$$I = \varepsilon/R$$

- b) What is the current when the switch has been turned on for a long while, i.e. when  $t \rightarrow \infty$ ?

*Answer:*

$$I = 0$$

After leaving the circuit on for a long time the capacitor will be charged with a charge  $q = Q_0$ . We turn the switch off, and replace the battery with a wire. Shortly afterwards we turn the switch on again. For simplicity, we say that  $t = 0$  when the switch is turned on.

- c) What is the current immediately after the switch is turned on, i.e. for  $t = 0$ ?

*Answer:*

$$I = \frac{Q_0}{CR} = \varepsilon/R$$

- d) What is the current when the switch has been turned on for a long while, i.e. when  $t \rightarrow \infty$ ?

*Answer:*

$$I = 0$$

- e) What difference would it make if the capacitor and resistor were wired in parallel?

*Answer:* For the discharging circuit, the parallel connection would short circuit the current coming from the capacitor. For the charging circuit, the circuit would at  $t = 0$  almost be a short circuit, while it at  $t \rightarrow \infty$  would be as if the capacitor was a infinite resistance. The voltage drop would be  $\varepsilon$  across both the capacitor and resistor.

### Exercise 7.4:

Consider another  $RC$  circuit connected in series. This time with a given resistance  $R = 100 \text{ M}\Omega$  and capacitance  $C = 1.0 \text{ nF}$ . The battery has an emf  $\varepsilon = 6 \text{ V}$ . The capacitor has zero charge when the switch is turned on.

- a) What is time constant  $\tau$  for this  $RC$  circuit?

- b) What is the charge on the capacitor after  $t = 100$  s?
- c) At this time, what is the current through the circuit?

### Exercise 7.5: Spherical current

The expression for  $R$  you found in exercise 7.2.a depended on the length of the cable, the cross sectional area and the conductivity of the material. In other words: geometric properties of the configuration as well as intrinsic properties of the material itself. It turns out that this is a direct consequence of the more fundamental  $\mathbf{J} = \sigma\mathbf{E}$ , but the expressions for  $R$  will vary from configuration to configuration because the *geometry* might be different.

Consider for example two concentric metal spherical shells of radius  $a$  and  $b$  which are separated by a weakly conducting material of conductivity  $\sigma$ .

- a) Assume that at time  $t = 0$  there is a charge  $Q$  on the inner shell and  $-Q$  on the outer shell. Find the current density as a function of position between the spherical shells.
- b) What is the total current  $I(t = 0)$  flowing from the inner to the outer shell?
- c) What is the resistance between the shells? *Hint: Find the potential difference first.*  
*Hint: Go back to the expression for  $R$  in the case with the cylinder. What made the resistance go down there?*
- d) Find the charge on the inner shell at a time  $t$ . How does this compare to the discharging capacitor in the above exercise?

### Exercise 7.6: Lorentz' force

The Lorentz' force is given as

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Consider a particle with charge  $q = -e$  and mass  $m = m_e$  moving with a velocity of  $\mathbf{v} = 10\hat{\mathbf{i}}$  m/s into a magnetic rectangular magnetic field  $\mathbf{B} = 0.02\hat{\mathbf{k}}$  T perpendicular on the particle's velocity. See figure 1.

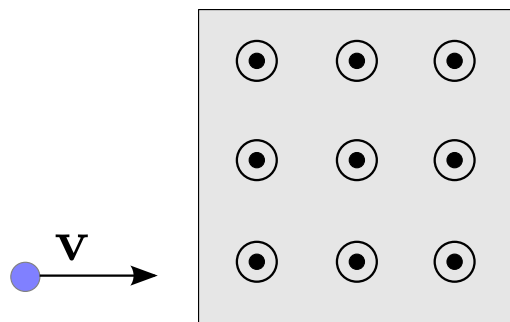


Figure 1: A charged particle entering a magnetic field.

- a) What is the force on the particle once it enters?

*Answer:*  $\mathbf{F} = -3.204 \times 10^{-21}\hat{\mathbf{j}}$  N

b) Does the magnitude of the force change as the particle moves throughout the field?

*Answer:* No. The direction of the velocity changes, but the force remains the same.

c) What is the work done on the particle?

*Answer:* Zero. The force is always perpendicular to the velocity and thus does no work.

d) What is the bending radius of the particle's motion throughout the magnetic field?

*Answer:*

$$R = 2.843 \times 10^{-9} \text{ m}$$