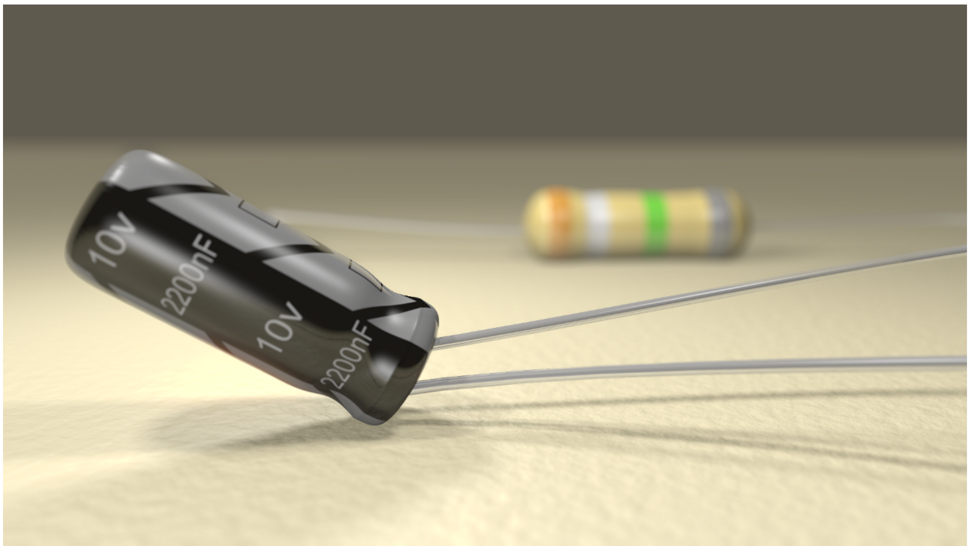
Week 7 - Current, Ohm's law and RC circuits



The equation governing the flow of currents is *Ohm's law*

$$\mathbf{J} = \sigma \mathbf{E}$$

where σ is called the *conductivity* and $\mathbf{J}$ is the volume current density. The traditional current trough a surface $\mathcal{S}$ is then found by

$$I = \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}$$

Ohm's law isn't really a law, but rather a rule of thumb. Materials who obey this equation is said to be *Ohmic materials.*

Exercise 7.1: Current

a) Consider two cylinders of radii a and b where $b > a$ and where $J = kr^2$ for $a \leq r \leq b$ and is directed along the wire. What is the total current in the wire?

Exercise 7.2: Ohm's law

a) Derive the more familiar form of Ohm's law, $V = RI$, by considering a cylinder of cross sectional area A and length l and show that $R = \frac{l}{\sigma A}$. Does this expression for R make sense?

b) A 3000-km long cable consists of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheet. Calculate the resistance of the cable. Use $3 \times 10^{-6}\,\Omega \cdot \text{cm}$ for the resistivity of copper.

Exercise 7.3: RC circuit

Consider an RC circuit, where a battery with emf ε, a resistor with resistance R, a switch and a capacitor with capacitance C are connected in series. The switch is turned on at the time $t = 0$.

a) What is the current immediately after the switched is turned on, i.e. for $t = 0$?

b) What is the current when the switch has been turned on for a long while, i.e. when $t \to \infty$?

After leaving the circuit on for a long time the capacitor will be charged with a charge $q = Q_0$. We turn the switch off, and replace the battery with a wire. Shortly afterwards we turn the switch on again. For simplicity, we say that $t = 0$ when the switch is turned on.

c) What is the current immediately after the switched is turned on, i.e. for $t = 0$?

d) What is the current when the switch has been turned on for a long while, i.e. when $t \to \infty$?

e) What difference would it make if the capacitor and resistor were wired in parallel?

Exercise 7.4:

Consider another RC circuit connected in series. This time with a given resistance $R = 100\,\mathrm{M\Omega}$ and capacitance $C = 1.0\,\mathrm{nF}$. The battery has an emf $\varepsilon = 6\,\mathrm{V}$. The capacitor has zero charge when the switch is turned on.

a) What is time constant τ for this RC circuit?

b) What is the charge on the capacitor after $t = 100\,\mathrm{s}$?

c) At this time, what is the current through the circuit?

Exercise 7.5: Spherical current

The expession for R you found in in exercise 7.2.a depended on the length of the cable, the cross sectional area and the conductivity of the material. In other words: geometric properties of the configuration aswell as intrinsic properties of the material itself. It turns out that this is a direct consequence of the more fundamental $\mathbf{J} = \sigma\mathbf{E}$, but the expressions for R will vary from configuration to configuration because the *geometry* might be different.

Consider for example two concentric metal spherical shells of radius a and b which are seperated by a weakly conducting material of conductivity σ.

a) Assume that at time $t = 0$ there is a charge Q is on the inner shell and $-Q$ on the outer shell. Find the current density as a function of position between the spherical shells.

b) What is the total current $I(t = 0)$ flowing from the inner to the outer shell?

c) What is the resistance between the shells? *Hint: Find the potential difference first.*

Hint: Go back to the expression for R in the case with the cylinder. What made the resistance go down there?

d) Find the charge on the inner shell at a time t. How does this compare to the discharging capacitor in the above exercise?

Exercise 7.6: Lorentz' force

The Lorentz' force is given as

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Consider a particle with charge $q = -e$ and mass $m = m_e$ moving with a velocity of $\mathbf{v} = 10\hat{\mathbf{i}}\,\mathrm{m/s}$ into a magnetic rectangular magnetic field $\mathbf{B} = 0.02\hat{\mathbf{k}}\,\mathrm{T}$ perpendicular on the particle's velocity. See figure 1.

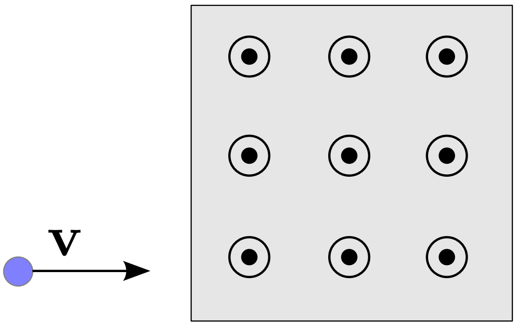


Figure 1: A charged particle entering a magnetic field.

a) What is the force on the particle once it enters?

b) Does the magnitude of the force change as the particle moves throughout the field?

c) What is the work done on the particle?

d) What is the bending radius of the particle's motion throughout the magnetic field?