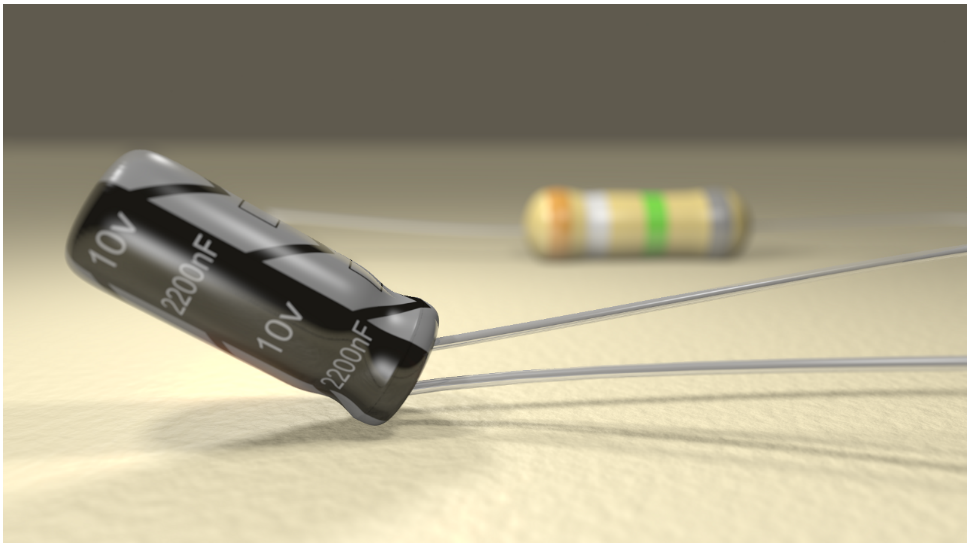
Week 7 - Current, Ohm's law and RC circuits



The equation governing the flow of currents is *Ohm's law*

$$\mathbf{J} = \sigma \mathbf{E}$$

where σ is called the *conductivity* and $\mathbf{J}$ is the volume current density. The traditional current trough a surface $\mathcal{S}$ is then found by

$$I = \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}$$

Ohm's law isn't really a law, but rather a rule of thumb. Materials who obey this equation is said to be *Ohmic materials*.

Exercise 7.1: Current

a) Consider two cylinders of radii a and b where $b > a$ and where $J = kr^2$ for $a \leq r \leq b$ and is directed along the wire. What is the total current in the wire?

Exercise 7.2: Ohm's law

a) Derive the more familiar form of Ohm's law, $V = RI$, by considering a cylinder of cross sectional area A and length l and show that $R = \frac{l}{\sigma A}$. Does this expression for R make sense?

b) A 3000-km long cable consists of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheet. Calculate the resistance of the cable. Use $3 \times 10^{-6}\,\Omega \cdot \mathrm{cm}$ for the resistivity of copper.

Exercise 7.3: RC circuit

Consider an RC circuit, where a battery with emf ε, a resistor with resistance R, a switch and a capacitor with capacitance C are connected in series. The switch is turned on at the time $t = 0$.

a) What is the current immediately after the switched is turned on, i.e. for $t = 0$?

Answer:

$$I = \varepsilon / R$$

b) What is the current when the switch has been turned on for a long while, i.e. when $t \to \infty$?

Answer:

$$I = 0$$

After leaving the circuit on for a long time the capacitor will be charged with a charge $q = Q_0$. We turn the switch off, and replace the battery with a wire. Shortly afterwards we turn the switch on again. For simplicity, we say that $t = 0$ when the switch is turned on.

c) What is the current immediately after the switched is turned on, i.e. for $t = 0$?

Answer:

$$I = \frac{Q_0}{CR} = \varepsilon / R$$

d) What is the current when the switch has been turned on for a long while, i.e. when $t \to \infty$?

Answer:

$$I = 0$$

e) What difference would it make if the capacitor and resistor were wired in parallel?

Answer: For the discharging circuit, the parallel connection would short circuit the current coming from the capacitor. For the charging circuit, the circuit would at $t = 0$ almost be a short circuit, while it at $t \to \infty$ would be as if the capacitor was a infinite resistance. The voltage drop would be ε across both the capacitor and resistor.

Exercise 7.4:

Consider another RC circuit connected in series. This time with a given resistance $R = 100\,\mathrm{M\Omega}$ and capacitance $C = 1.0\,\mathrm{nF}$. The battery has an emf $\varepsilon = 6\,\mathrm{V}$. The capacitor has zero charge when the switch is turned on.

a) What is time constant τ for this RC circuit?

b) What is the charge on the capacitor after $t = 100\,\mathrm{s}$?

c) At this time, what is the current through the circuit?

Exercise 7.5: Spherical current

The expession for R you found in in exercise 7.2.a depended on the length of the cable, the cross sectional area and the conductivity of the material. In other words: geometric properties of the configuration aswell as intrinsic properties of the material itself. It turns out that this is a direct consequence of the more fundamental $\mathbf{J} = \sigma \mathbf{E}$, but the expressions for R will vary from configuration to configuration because the *geometry* might be different.

Consider for example two concentric metal spherical shells of radius a and b which are seperated by a weakly conducting material of conductivity σ.

a) Assume that at time $t = 0$ there is a charge Q is on the inner shell and $-Q$ on the outer shell. Find the current density as a function of position between the spherical shells.

b) What is the total current $I(t = 0)$ flowing from the inner to the outer shell?

c) What is the resistance between the shells? *Hint: Find the potential difference first.*

Hint: Go back to the expression for R in the case with the cylinder. What made the resistance go down there?

d) Find the charge on the inner shell at a time t. How does this compare to the discharging capacitor in the above exercise?

Exercise 7.6: Lorentz' force

The Lorentz' force is given as

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Consider a particle with charge $q = -e$ and mass $m = m_e$ moving with a velocity of $\mathbf{v} = 10\hat{\mathbf{i}}\,\mathrm{m/s}$ into a magnetic rectangular magnetic field $\mathbf{B} = 0.02\hat{\mathbf{k}}\,\mathrm{T}$ perpendicular on the particle's velocity. See figure 1.

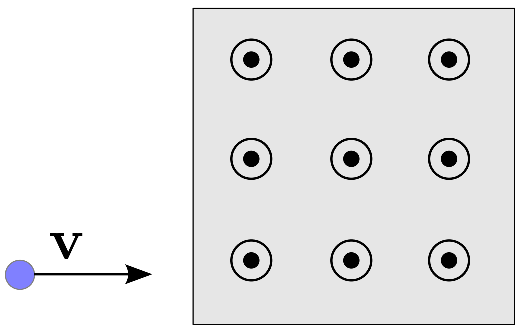


Figure 1: A charged particle entering a magnetic field.

a) What is the force on the particle once it enters?

Solution:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Answer: $\mathbf{F} = -3.204 \times 10^{-21}\hat{\mathbf{j}}\,\mathrm{N}$

b) Does the magnitude of the force change as the particle moves throughout the field?

Answer: No. The direction of the velocity changes, but the force remains the same.

c) What is the work done on the particle?

Answer: Zero. The force is always perpendicular to the velocity and thus does no work.

d) What is the bending radius of the particle's motion throughout the magnetic field?

Solution:

$$R = \frac{mv}{|q|B}$$

Answer:

$$R = 2.843 \times 10^{-9}\,\mathrm{m}$$