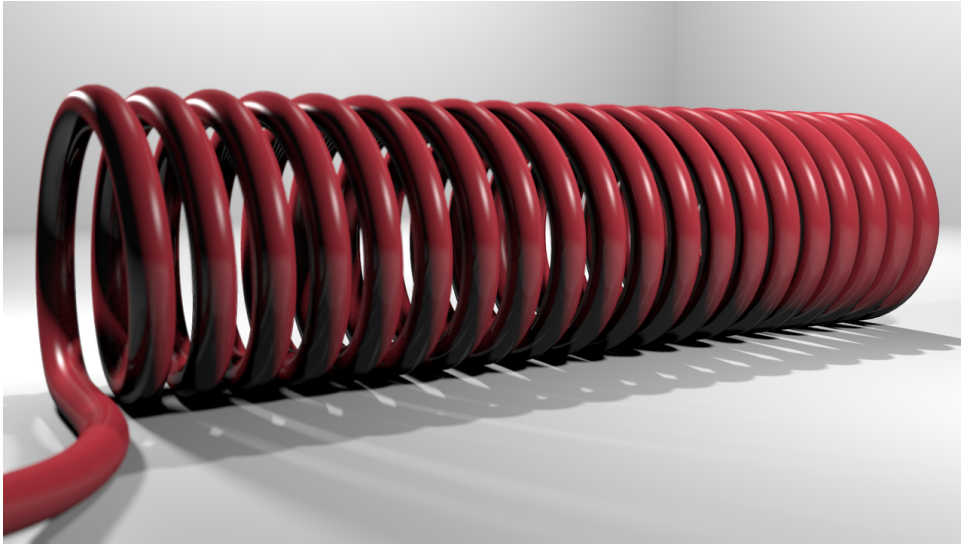


## Week 9 - Solenoids, Ampère and Maxwell's 4th equation



Ampere was the Newton of Electricity.

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James C. Maxwell

Gauss' law is to electricity what *Ampère's Law* is to magnetism. Ampère's law states that currents tend to cause circulating magnetic fields,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

We can turn it into its integral equivalent by using *Stoke's theorem* on the left side of the following equation:

$$\int_S \nabla \times \mathbf{B} \cdot d\mathbf{A} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{A}$$

Which gives

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{A} = \mu_0 I_{enclosed}$$

where the integration is taken around a *closed* loop defining the boundary of the surface  $S$ .

Like Gauss' law, Ampère's law is a fundamental statement about static electromagnetic fields that always holds, but it isn't always useful. Only in cases with great symmetry it can be used to deduce fields that would otherwise be very complicated to find directly from Biot-Savart.

### Exercise 9.1: Selected exercises from the book

Do exercise 23.19 and 23.20 from Generell Fysikk.

Answer: 23.19:

$$B(r < R) = \frac{\mu_0 I}{2\pi R^2} r$$

23.20: a)  $I_{max} = 5.34 \times 10^{-3}$  A b)  $B_{max} = 2.67 \times 10^{-9}$  T c) 8.5 V

## Exercise 9.2: Field from an infinite cylinder

A steady current  $I$  flows down a long cylindrical wire of radius  $a$ . Find the magnetic field, both inside and outside the wire if

a) The current is uniformly distributed over the outside surface of the wire.

Answer: Outside:

$$B = \frac{\mu_0 I}{2\pi r}$$

Inside:

$$B = 0$$

b) The current is distributed such that  $J = kr$ .

Answer: Outside:

$$B = \frac{\mu_0 I}{2\pi r}$$

Inside:

$$B = \frac{\mu_0 k r^2}{3}$$

## Exercise 9.3: The solenoid

In many applications we require constant electric and magnetic fields. The easiest way to generate constant electric fields is to use a parallel plate capacitor. To create a constant magnetic field we do on the other hand use a *solenoid*.

a) Consider a cylinder of radius  $a$  wound up by a wire carrying a current  $I$ . The wire is wound with  $n_1$  turns per unit length. Find the magnetic field inside the solenoid.

Answer:

$$B = \mu_0 n_1 I$$

b) Suppose we introduce yet another solenoid with  $b > a$ , coaxial with the first one and with  $n_2$  turns per unit length, but with current running in the opposite direction. Find the field everywhere.

## Exercise 9.4: Selected exercises from the book and exams

Do exercise 26.1 and 26.3 from Generell Fysikk.

Do exercise 4 from the 2009 exam. Exams from previous years are posted here:

<http://www.uio.no/studier/emner/matnat/fys/FYS1120/h11/undervisningsmateriale/eksamensoppgaver/>

*Answer: 26.1:*

a)  $3.77 \times 10^{-2} \text{ T}$

b)  $3.78 \times 10^{-2} \text{ T}$